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# *Ab initio* calculations of $^5\text{H}$ resonant states

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## Abstract

By solving the 5-body Faddeev-Yakubovsky equations in configuration space with realistic nuclear Hamiltonians we have studied the resonant states of  $^5\text{H}$  isotope. Two different methods, allowing to bypass the exponentially diverging boundary conditions, have been employed providing consistent results. The existence of  $^5\text{H}$  broad  $J^\pi=1/2^+, 3/2^+, 5/2^+$  states as S-matrix poles has been confirmed and compared with the, also calculated, resonant states in  $^4\text{H}$  isotope. We have established that the positions of these resonances only mildly depend on the nuclear interaction model.

*Keywords:*  $^4\text{H}$  and  $^5\text{H}$ , Faddeev-Yakubovsky equations, Few-Nucleon problem, *ab initio* calculations

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## 1. Introduction

Neutron-proton unbalanced nuclei are a unique laboratory to study the isospin dependence of the nuclear interaction as well as the nuclear properties far beyond the stability line. Hydrogen isotopes play a special role, as they provide the extreme neutron to proton ratios and thus are actively studied both theoretically as well as experimentally.

If there seems to be a general agreement between the experimentalist on the existence of several resonances in  $^5\text{H}$  isotope [1, 2, 3, 4], the parameters of these resonances vary significantly from one experiment to another. These discrepancies might be attributed to the different reaction mechanism leading to the formation of broad resonant states and to the difficulty in identifying their quantum numbers. Indeed, in any physical observable, the contributions of broad resonances strongly interfere among themselves as well as with a non-resonant background, producing in the resonance region a response, which strongly depends on the particular reaction mechanism. Unless the reaction mechanism is well understood and one is able to isolate – analytically or numerically – both contributions, the resonance parameters are not defined in a unique way. In this case only qualitative analysis of the experimentally extracted broad resonance parameters makes sense.

According to the results of Korshennikov et al. [2] the lowest resonant state of  ${}^5\text{H}$  would be narrower than the ones of  ${}^4\text{H}$  isotope, a fact that could be naively attributed to the additional attraction provided by the  $4n$  subsystem present in  ${}^5\text{H}$  relative to  $3n$  one present in  ${}^4\text{H}$ . Or, alternatively, to the  $nn$  pairing effect over a core of  ${}^3\text{H}$ . By extending this effect to  $6n$  one could in this way obtain a natural explanation for the extremely narrow parameters of the  ${}^7\text{H}$  resonance published in [5].

The findings of ref. [2] were confirmed by Golovkov et al. [3], attributing even smaller widths to the  ${}^5\text{H}$  resonant states. They are however in conflict with a recent work [4] where the resonance width was found to be much larger and comparable with the one of the  ${}^4\text{H}$  ground state, although this difference could be partially attributed to the different reaction mechanism used in their production.

Several theoretical works on this system were performed in the past. They could be classified in two major groups:  $t$ - $n$ - $n$  models in which  ${}^3\text{H}$  nuclei is considered as a point like particle (hereafter denoted by  $t$ ) and microscopic 3-body cluster models, based on  ${}^3\text{H}+n+n$  structure. In  $t$ - $n$ - $n$  models, the Pauli principle between the valence neutrons and those contained in the  ${}^3\text{H}$  core is mimicked by a repulsive term in the effective  $t$ - $n$  interaction. In contrast, microscopic models start from a bare Nucleon-Nucleon (NN) interaction and employ properly antisymmetrized 5-nucleon wave functions, however some severe constraints on the dynamics of the  ${}^3\text{H}$  core are applied.

These studies were pioneered by Shul'gina et al. [6], where  ${}^5\text{H}$  is described by a  $t$ - $n$ - $n$  model, solved by means of hyperspherical Harmonics techniques. The  $t$ - $n$  potential was taken from the same authors [7] and the  $nn$  one from Gogny et al. [8]. Within the same approach, a theoretical study of broad states in few-body systems beyond the neutron drip line was published in [9]. No precise positions were given but studies of the sensitivity of the  ${}^5\text{H}$  spectrum to reaction mechanism predict that the "ground state" peak may be observed at 2-3 MeV as lowest position, though could be shifted to much higher energies in certain situations.

In [10] a similar 3-body model but with a refined  $t$ - $n$  interaction was considered with  $V_{nn}$  taken from [11]. The positions of  ${}^5\text{H}$  resonant states were computed by means of the Complex Scaled (CS) hyperspherical adiabatic expansion method. However, this model failed to obtain realistic  ${}^5\text{H}$  resonant states when using only two-body interactions; adjustment of an additional *ad-hoc*  $t$ - $n$ - $n$  3-body force was required to reproduce the positions of the  $J^\pi=1/2^+, 3/2^+, 5/2^+$  states. The energy distributions of the fragments after their decay were computed as well.

A  $t$ - $n$ - $n$  model was also used, in the framework of hypernuclear physics [12], to compute  ${}^5\text{H}$  resonant states. The problem was solved by combining CS with the Gaussian expansion method [12]. The  $nn$  interaction was Argonne V8' [13] and  $t-n$  one was adapted from [14]. It was found again, that this 3-body model failed to reproduce ground state resonance parameters unless some *ad-hoc* 3-body force was included and properly adjusted.

The first microscopic model for  ${}^5\text{H}$  using the Generator Coordinate Method (GCM) was used in [15]. The key ingredient of the calculation, the NN potential, was based on semi-realistic Minnesota (MN) [16] and Mertelmeier and Hofmann [17] models. Resonance parameters were identified using the analytic continuation in a coupling constant (ACCC) method [18]. In a later work [19] this last study was improved, replacing the GCM technique by a 3-body microscopic approach based on Hyperspherical Harmonics method.

A microscopic three cluster model of  ${}^5\text{H}$ , using the equivalent Resonating Group Method (RGM) approach and the MN potential, was also presented in [20].

Finally, a microscopic cluster model of  ${}^5\text{H}$  was presented in [21], based on 3-cluster J-matrix method (MJM) and Hyperspherical Harmonics basis. Resonance positions were directly computed by imposing the appropriate boundary conditions. Different components of the MN and modified Hasegawa-Nagata potentials [22] were used to build the NN interaction.

It seems firmly established that the t-n-n models of  ${}^5\text{H}$  systematically require a properly adjusted t-n-n 3-body force in order to produce  ${}^5\text{H}$  resonant states with reasonable widths. Thus, their predictions critically depends on 3-body force parameters on which there is little control. On the contrary, all microscopic models agree on the existence of a relatively broad  $J^\pi=1/2^+$  resonant state in  ${}^5\text{H}$ , which is located  $\approx 1.5$  MeV above the  ${}^3\text{H}$  threshold and has a width in the range of  $\Gamma \approx 1\text{-}3$  MeV. The presence of even broader  $J^\pi=3/2^+$  and  $J^\pi=5/2^+$  states has been also indicated. These methods however rely on 3-cluster wave functions and employ simplistic phenomenological NN interactions. As we will see in what follows, their predictions display also some dispersion even when using the same interaction.

We believe it could be of interest to determine unambiguously the S-matrix pole position of these resonant states by undertaking the *ab initio* solution of this problem with the only input of a realistic NN interaction. Indeed, our recently developed techniques, allowing to solve the 5-body Faddeev-Yakubovsky equations in configuration space, and for the first time applied to calculate n- ${}^4\text{He}$  elastic phase shifts [23, 24], provide us the opportunity to investigate the properties of  ${}^5\text{H}$  nucleus from an *ab initio* theoretical perspective. By solving the 5-nucleon problem in the complex energy plane we present here the first *ab initio* study of  ${}^5\text{H}$ , involving no approximations on the dynamics.

A new experiment has been recently performed at RIKEN [25], combining benefits of radioactive ion beam facilities and multineutron detection arrays, aiming to resolve the puzzle of observable  ${}^4\text{n}$  and  ${}^7\text{H}$  states. Understanding the evolution of H-isotope resonances, originated from S-matrix poles in the vicinity of the real energy axis, when increasing the number of neutrons was one of the main motivations of our work. Therefore our theoretical attempt is timely and hopefully will also be relevant for a better understanding and interpretation of the experimental data.

The paper is organized as follows. In Section 2 we summarize the formalism used to solve the 5-body problem and the extrapolation to the complex energies. Numerical results for  ${}^4\text{H}$  and  ${}^5\text{H}$  resonant states are given in Section 3. Section 4 contains some concluding remarks.

## 2. Formalism

Faddeev-Yakubovsky (FY) equations were formulated in [26], as an extension of the seminal work of Faddeev [27] for a 3-particle system ( $A=3$ ) to an arbitrary number of particles ( $A=N$ ). Starting from the middle 80's, they have been successfully solved both in momentum and configuration space for  $A=4$  in all possible bound and scattering channels [28, 29, 30]. The first results for  $A=5$  problem, recently obtained by one of the authors [23, 24], introduced the formalism in some detail. In this letter we will just highlight the main ideas.

In the FY approach, the intrinsic (center of mass) 5-body wave function depending on four vector variables  $\Psi(\vec{x}, \vec{y}, \vec{z}, \vec{w})$ , is decomposed into 180 components  $F_a^{(i)}$  corresponding to the five

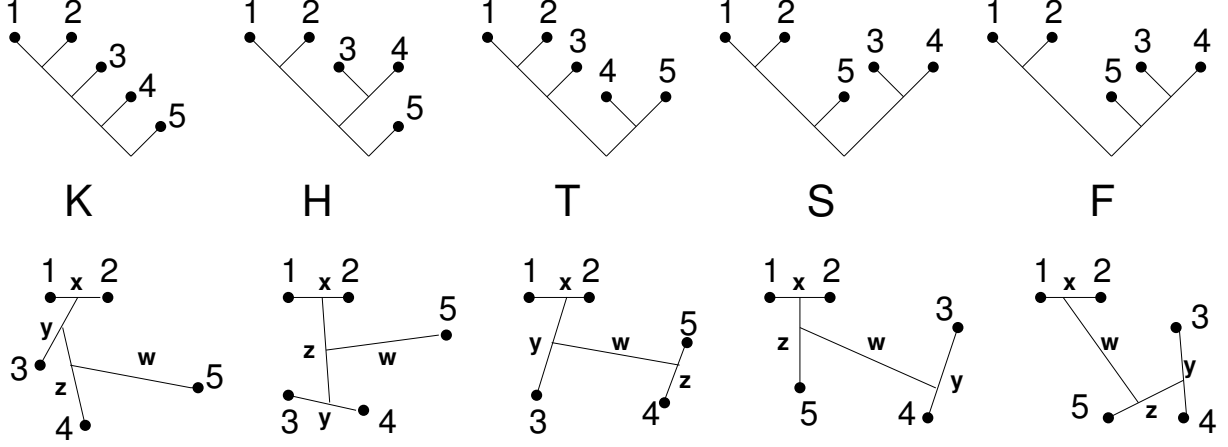


Figure 1: Different topologies for the 5-body FY amplitudes and corresponding Jacobi coordinates.

different topologies in the cluster partition of the system, which are represented in Fig. 1. Subscript  $a = K, H, S, T, F$  denotes their topological type and the superscript  $i$  the sequence of particle indexes.

These components are naturally expressed in their proper set of Jacobi coordinates  $\vec{x}_a^{(i)}, \vec{y}_a^{(i)}, \vec{z}_a^{(i)}, \vec{w}_a^{(i)}$ . These are represented in the lower panel of Fig.1 and have the generic form

$$(\vec{x}, \vec{y}, \vec{z}, \vec{w}) = \sqrt{\frac{2\mu_{st}}{m_0}} (\vec{R}_s - \vec{R}_t)$$

where  $\vec{R}_i$  denotes the center of mass positions of the two clusters (eventually single particles) connected by each coordinate and  $\mu_{st}$  their reduced mass. An arbitrary mass  $m_0$  is introduced to keep the physical units.

For a system of identical particles with mass  $m$  one chooses  $m_0 = m$ . One can also choose a preferential particle ordering  $i$ , say  $i = \{1, 2, \dots, 5\}$ , and the permutation symmetry allows to reduce the number of independent components to only five  $\{F_a\}$ , one for each topological type. The remaining components, as well as the total wave function, can be reconstructed from this subset with the help of coordinate basis transformation operators  $\hat{P}_a^{b,i}$ :

$$F_a^{(i)}(\vec{x}_b^{(i)}, \vec{y}_b^{(i)}, \vec{z}_b^{(i)}, \vec{w}_b^{(i)}) = \hat{P}_a^{b,i} F_a(\vec{x}_a, \vec{y}_a, \vec{z}_a, \vec{w}_a); \quad (1)$$

$$\Psi(\vec{x}_b^{(i)}, \vec{y}_b^{(i)}, \vec{z}_b^{(i)}, \vec{w}_b^{(i)}) = \sum_j \sum_{a=1}^5 F_a^{(j)}(\vec{x}_b^{(i)}, \vec{y}_b^{(i)}, \vec{z}_b^{(i)}, \vec{w}_b^{(i)}). \quad (2)$$

Using the completeness relation one may split any operator  $\hat{P}_a^{b,i}$  into a product of simpler transformation operators  $\hat{P}_k$ , similar to the ones used in the 3-body case [24], which affect only two vector coordinates at once and let unchanged the functional dependence of the remaining two:

$$\hat{P}_a^{b,i} = \prod_k \hat{P}_k. \quad (3)$$

As discussed in [24] a set of eleven such operators is sufficient to express any complicated transformation appearing in the solution of a 5-body problem.

The FY components  $F_a^{(i)}$  correspond to the different asymptotic channels and satisfy a coupled system of second order partial differential equations. In view of solving these equations, a partial wave expansion on each vector variables is performed

$$F_a(\vec{x}_a, \vec{y}_a, \vec{z}_a, \vec{w}_a) = \sum_{\alpha} \frac{1}{x_a y_a z_a w_a} f_{a,\alpha}(x_a, y_a, z_a, w_a) \mathcal{Y}_{\alpha}(\hat{x}, \hat{y}, \hat{z}, \hat{w})$$

where  $\mathcal{Y}_{\alpha}$  is a generalized "quadrupolar harmonic" function accounting for the angular momentum, spin and isospin couplings

$$\mathcal{Y}_{\alpha} = [[l_x, l_y]_{l_{xy}} [l_z, l_w]_{l_{zw}}]_L \{S\}_S \{T\}_{TT_z},$$

$\{S\}_S = |[[s_1, s_2]_{s_x} [s_3, s_4]_{s_y}]_{s_{xy}} s_5; SS_z\rangle$  ( $\{T\}_{TT_z} = |[[t_1, t_2]_{t_x} [t_3, t_4]_{t_y}]_{t_{xy}} t_5; TT_z\rangle$ ) holds for the spin (isospin) couplings and  $\alpha$  labels the set of quantum numbers involved in the intermediate couplings. The above expressions represents the main coupling scheme used in this work for the K, H, S and F topologies. For T topology a slightly modified coupling scheme was used, obtained by permuting particle 3 and 5 indexes for spin and isospin couplings. These coupling schemes allow to (anti)symmetrize the total wave function in a trivial way.

After projecting the angular part, one is left with a set of coupled four-dimensional integro-differential equations for the reduced radial amplitudes  $f_{a,\alpha}(x, y, z, w)$  in the form

$$[q^2 \delta_{\alpha\alpha'} + \Delta_{xyzw}^{\alpha} \delta_{\alpha\alpha'} - v_{\alpha\alpha'}(x)] f_{a\alpha'}(x, y, z, w) = v_{\alpha\alpha'}(x) \sum_{b\beta} \int d\theta d\xi d\zeta h_{a\alpha'}^{b\beta}(x, y, z, w, \theta, \xi, \zeta) f_{b\beta}(x_b, y_b, z_b, w_b) \quad (4)$$

where

$$\frac{m}{\hbar^2} \Delta_{\alpha,xyzw}^{\alpha} = \partial_x^2 + \partial_y^2 + \partial_z^2 + \partial_w^2 - \frac{l_x(l_x+1)}{x^2} - \frac{l_y(l_y+1)}{y^2} - \frac{l_z(l_z+1)}{z^2} - \frac{l_w(l_w+1)}{w^2}$$

and  $v_{\alpha\alpha'}$  is the two-body potential (multiplied by  $\frac{m}{\hbar^2}$ ).

In the right hand side of eq. (4), an integration over the angular variables  $\theta, \xi$  and  $\zeta$ , allowing to connect the Jacobi coordinate sets  $(x, y, z, w)$  and  $(x_b, y_b, z_b, w_b)$ , is performed. This essential operation is realized in the matrix form by breaking the integration in three successive steps, as briefly explained in eqs.(1-3). The radial dependence of these amplitudes is sought by expanding them in the Lagrange-Laguerre basis functions using the Lagrange-mesh method [31]. In our numerical calculation, the partial wave basis was limited to  $\max(l_x, l_y, l_z, l_w) < 4$ . The number of Lagrange-mesh points to expand the radial FY components was  $n_x = 12 - 2l_x$  for the  $x$ -coordinate and  $n_{\alpha} = 11 - 2l_{\alpha}$  for  $y, z$  and  $w$ . The errors introduced by these truncations are smaller than the uncertainties related with the methods used to calculate the positions of the resonant states.

To access the complex energy eigenvalues of a resonant state, we have used the method of Analytic Continuation in the Coupling Constant (ACCC) in the same form as in our previous works [32, 33, 34] and we have developed a variant of the Smooth Exterior Complex Scaling method (SECS) that will be detailed in what follows.

ACCC does not allow a direct determination of resonance parameters. These are obtained by artificially binding the system with an "external field" driven by a strength parameter  $\lambda$  and computing

its binding energy as a function of this parameter:  $B(\lambda)$ . In our case, we added to  ${}^5\text{H}$  Hamiltonian a fictitious 5-body hyperradial potential on the form:

$$V_5(\rho) = -\lambda \rho^p e^{-\rho^2/\rho_0^2} \quad \rho = \sqrt{x^2 + y^2 + z^2 + w^2} \quad (5)$$

Equation (4) should be accordingly modified by adding to its left hand side the term  $V_5 \delta_{\alpha\alpha'} f_{\alpha\alpha'}$ .

Several values of  $B_{5H}(\lambda)$  were calculated up to  ${}^3\text{H}+n+n$  threshold, corresponding to the critical strength  $B_{5H}(\lambda_0) \equiv B_{3H}$ , above which the  ${}^5\text{H}$  S-matrix pole moves into the resonance region. These values are used to determine a Padé-like functional dependence in the domain of real arguments

$$\sqrt{B_{5H}(\lambda) - B_{3H}} = f_{ACCC}(\sqrt{\lambda - \lambda_0})$$

which will be analytically continued in the complex plane until  $f_{ACCC}(\sqrt{-\lambda_0})$ , where the external potential  $V_5$  is fully removed ( $\lambda = 0$ ). At this point, the position of the  ${}^5\text{H}$  resonance relative to  ${}^3\text{H}$  threshold may be easily determined. As it was explained in [34, 35, 36], in the context of  $3n$  and  $4n$  systems, the use of an artificial 5-body force to bind  ${}^5\text{H}$  is motivated by the compelling need to avoid spurious branching points, related to the appearance of artificially bound  ${}^5\text{H}$  subsystems.

In our previous studies we have extensively employed the CS method to calculate resonance parameters as well as several scattering observables [37]. Nevertheless, the conventional CS fails in describing broad resonant states for they require a large rotation angle  $\theta$  to be exposed. Furthermore, some short range potentials, like e.g. chiral NN interactions, include sharp vertex form factors and start to diverge already when very moderate values of  $\theta$  are used.

To tackle this problem, the so called Exterior Complex Scaling method (ECS) was proposed [38], which applies the CS transform only in the region free from interaction, say  $r \rightarrow z(r) = r_0 + r e^{i\theta}$  with  $r_0$  sufficiently large. However, in its standard form, ECS has a formal inconsistency when applied to the  $A > 2$  problem solved via partial wave decomposition, due to the interplay between different Jacobi coordinate sets. To circumvent this problem we propose an analytic mapping  $r \rightarrow z_a(r)$  – hereafter denoted SECS – whose effect is extremely small in the interaction region and, hence, it is applied to the kinetic energy term only. If this mapping is performed by means of a sharp enough function centered outside of interaction region, SECS gives very accuracy results. In practice it can be realized by the means of a smooth step-like function  $s(r)$ :

$$r \rightarrow z_a(r) = s(r) r e^{i\theta} + r[1 - s(r)] \quad s(r) = e^{-(r_0/r)^n} \quad (6)$$

which is represented in Fig. 2 (right panel) together with its ancestors.

We illustrate this method with a two-body P-wave resonance generated by the potential:

$$V(r) = V_r \frac{e^{-\eta_r r}}{r} - V_a \frac{e^{-\eta_a r}}{r} \quad (7)$$

with  $V_r=2589.696$  MeV,  $V_a=1128.393$  MeV,  $\eta_r=3.11$  fm $^{-1}$ ,  $\eta_a=1.55$  fm $^{-1}$  and  $\hbar^2/m=41.471$  MeV fm $^2$ . In this example we took  $\theta = 40^\circ$ , but the method applies well for larger angles.

We have plotted in Figure 2 (left panel) the complex energy values of the resonance as a function of  $r_0$ , the range parameter of SECS, calculated with the mapping (6) and  $n=4$  displayed in right

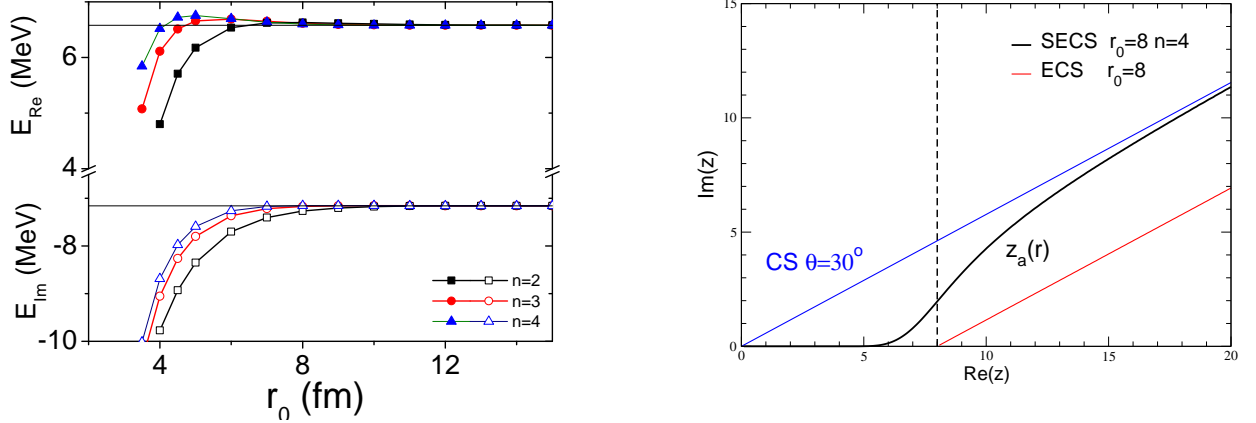


Figure 2: P-wave resonance energies in the 2-body potential eq. (7) obtained by the SECS method with  $n = 4$  as a function of  $r_0$  (left panel). If  $r_0$  is larger than the interaction range ( $r_0 \approx 8$  fm) one gets accurate resonance position. The mapping given in eq. (6) used in these calculations for the converged value is displayed in the right panel.

panel. As one may see, results are indistinguishable from the exact S-matrix pole  $E_R + iE_I = 6.578 - i7.1578$  MeV for  $r_0 \gtrsim 8$  fm. With  $r_0 = 12$  fm, one reproduces four significant digits using  $n = 3$  and  $n = 4$ , while the differences using  $n = 2$  differs only by 1 keV. Despite the fact that our approach neglects the transformation of the potential energy, it produces stable and very accurate results.

When solving FY equations in  $A \geq 3$  sector, it is of interest to apply SECS to hyperradius  $\rho$  only, for it allows a consistent treatment of the kinetic energy operators in different Jacobi coordinate sets. We have successfully realized this option in 3-body calculations, but it is worth noticing that the transformation of four radial Jacobi coordinates turned to be even a better choice. The last procedure slightly breaks the consistency of the kinetic energy in the different FY components. Nevertheless when working with radial Jacobi coordinates we are able to evaluate the transformed kinetic energy terms more accurately if the transformation is natural to these coordinates, which turns out to be a critical ingredient in the calculations requiring sharp regulators.

### 3. Results

In this work three different NN interactions have been considered: two realistic ones and a third one semi-realistic. Among the first, the I-N3LO chiral EFT potential of Entem and Machleidt [39] which was very successful in describing the  $n$ - $^3\text{H}$  elastic cross section near the resonance peak even without corresponding three nucleon forces [40], and the INOY non-local potential from [41] which provides an accurate description of  $A=2,3,4$  binding energies. The semi-realistic S-wave MT13 potential [42] was also considered which, despite its bare simplicity, describes well the gross properties of low energy few-nucleon systems.

We have first computed the  $J^\pi=2^-, 0^-, 1^-$  P-wave resonances of its closest isotope  $^4\text{H}$ , which dominate the low energy  $n$ - $^3\text{H}$  elastic cross section in the peak region and which are intimately related with the resonant states in  $^5\text{H}$ . The method used here is a direct calculation of the 4N resonant state by solving the FY equations with the techniques developed in [33, 43] supplied with



$J^\pi$	$2^-$		$1_1^-$		$0^-$		$1_2^-$	
	$E_R$	$\Gamma$	$E_R$	$\Gamma$	$E_R$	$\Gamma$	$E_R$	$\Gamma$
I-N3LO	1.15(5)	3.97(7)	0.90(3)	3.80(8)	0.78(15)	7.7(7)	0.2(2)	4.6(1)
INOY	1.31(4)	4.16(4)	0.96(7)	4.03(8)	0.80(15)	8.3(1)	0.2(2)	5.0(1)
MT13	1.08(3)	4.06(5)	1.08(3)	4.06(5)	1.08(3)	4.06(5)	0.88(9)	4.4(1)
RGM [20]	1.52	4.11	1.23	5.80	1.19	6.17	1.32	4.72
tn [10]	1.22	3.34	1.15	3.49	0.77	6.72	1.15	6.38
Exp. [44]	3.19	5.42	3.50	6.73	5.27	8.92	6.02	12.99

Table 1: Resonance parameters of  ${}^4\text{H}$ . Our results obtained with three different potentials (first three rows) are compared to previous calculations using a 2-body (t-n) potential [10] or an RGM microscopic calculation [20]. Experimental R-matrix analysis [44] are in the last line. All units are in MeV.

the corresponding asymptotically diverging boundary condition. Results are displayed in Table 1, where all values are given in MeV and energies are relative to  ${}^3\text{H}$  threshold. They are compared with previous calculations: the microscopic RGM 3-cluster model of [20] and the 2-body t-n model of [10]. As one can see, the three interactions we have considered in our *ab initio* calculations give close results, but are quite different from the RGM and t-n model calculations, especially for the  $1_2^-$  state which we predict as an almost subthreshold resonance (third E-quadrant. Notice the degeneracy of the first three states ( $2^-$ ,  $1_1^-$ ,  $0^-$ ) in the MT13 model. They all share total spin  $S=1$  and total angular momentum  $L=1$  without any coupling, due to the purely central and spin-spin terms of the interaction. The  $1_2^-$  state correspond to  $S=0$ . For completeness, we also listed the values provided by an R-matrix analysis of experimental cross sections [44]. Although for broad states only a qualitative comparison between S-matrix pole positions and R-matrix makes sense, we notice that in the present case the resonance parameters can differ by factors 2 to 5, what represents several MeV.

The states of  ${}^5\text{H}$  nucleus are genuine 3-body final state resonances, governed by a very complicated diverging behavior of the wave function asymptotes. In this case we are not able to directly impose the proper outgoing wave boundary conditions and our calculations for this system rely on the ACCC and the SECS methods explained in the previous section.

We illustrate in Fig. 3 the ACCC results for the  $J^\pi=1/2^+$  state with the I-N3LO Hamiltonian. The continuum extrapolation was based on 23 binding energies, starting from the triton threshold  $B({}^3\text{H}) = 7.81$  MeV up to 300 MeV. Notice that this  $B({}^3\text{H})$  value – obtained in a 5-body calculation with limited model space – differs only by 0.04 MeV from the fully converged 3-body calculations  $B({}^3\text{H}) = 7.85$  MeV. Though formally  $m+n+1$  eigenenergy entries suffice to determine coefficients of Padé approximant  $[m,n]$ , as suggested in ref. [18], we have employed much larger eigenenergy entry sets by determining the coefficient using least-square method. The last trick allows to minimize effect of eigenenergy set choice on the convergence of the Padé series as well as reduce the effect of numerical accuracy being finite. Two different parameterizations of  $V_5$  potential (5) were considered:  $\rho_0^2=78.4$  fm<sup>2</sup> and  $\rho_0^2=108.9$  fm<sup>2</sup>, both with  $p = 0$ .<sup>1</sup> They correspond respectively to the left- and right-

<sup>1</sup>In our previous work [23] parameterizations with different values of  $p$  were considered, providing results in agreement with the  $p = 0$  case; however its implementation requires a more important numerical effort.

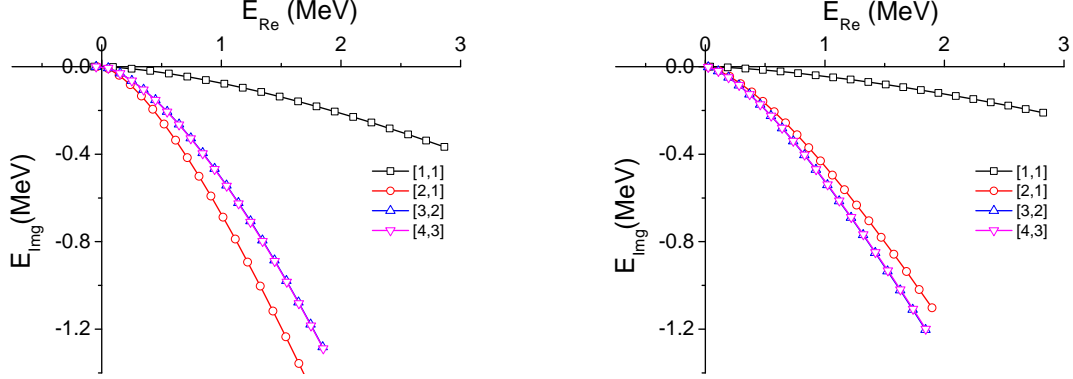


Figure 3: Resonance trajectories for  $J^\pi=1/2^+$  state of  $^5\text{H}$  relative to  $^3\text{H}$  threshold. Each trajectory is made by 20 equidistant values of the  $V_5$  strength parameter  $\lambda$  in eq. (5), starting from nearly threshold energy. The endpoint ( $\lambda = 0$ ) indicates the extrapolated physical values of the resonance predicted by the NN interaction. Left and right panels correspond to different parameterizations of  $V_5$  ( $\rho_0^2=78.4 \text{ fm}^2$  and  $\rho_0^2=108.9 \text{ fm}^2$  respectively, both with  $p = 0$ ). The convergence of the results with respect to order of Padé approximant  $[m,n]$  are distinguished by colored curves.

panel of this figure. In both cases, the Padé extrapolation nicely converges to the resonance energy  $E=1.85(1)-i1.25(5) \text{ MeV}$ , a result which is highly ensuring. Nevertheless the convergence relative to the order  $[m,n]$  of the Padé approximant should be considered with a grain of salt. Indeed, one could expect that by increasing the order of Padé approximant, the accuracy in the extrapolated energies should improve accordingly. However this expected improvement saturates at order  $[3,2]$ , when the extrapolated energies are reproduced within 100 keV. This numerical "saturation" is certainly due to the limited accuracy of the calculated 5-body energy inputs and introduce error bars in our results. In this context, increasing the order of the Padé approximants makes no sense and may even lead to numerical instabilities due to overfitting.

The extraction of the same resonance parameters by means of SECS, provide the value  $E=1.85(10)-i1.20(5)$ , where errors correspond to the variations of the rotation angle  $\theta \in [30,45]^\circ$  and the range parameter  $r_0 \in [13,16] \text{ fm}$  with  $n=3,4$ .

A similar study was performed for the INOY potential. ACCC results, based on  $V_5$  parameters  $\rho_0^2=108.9 \text{ fm}^2$  were obtained. A Padé extrapolation was constructed from 28  $B(^5\text{H})$  binding energies in the interval  $[8.40,300] \text{ MeV}$ . They converge to the value  $E=1.73(5)-i1.19(9) \text{ MeV}$  while SECS provides this resonance at  $E=1.8(1)-i1.2(1) \text{ MeV}$ .

Results for the  $J^\pi=1/2^+$  ground state of  $^5\text{H}$ , with different interactions<sup>2</sup> and the two different methods (ACCC and SECS) used to access complex energies, are summarized in Table 2. As one can see, both methods provide resonance positions and widths compatible within the estimated errors. The width of this state turns out to be almost independent of the employed interaction model – including the semi-realistic MT13 – around a value  $\Gamma = 2.4(2) \text{ MeV}$ ; its real part seems to be less stable. In particular for MT13 potential real energy of the resonant state seems to be slightly smaller than ones provided by the realistic Hamiltonians. Apparently this might also be the reason for

<sup>2</sup>MT13 results with ACCC method were presented in [23]

larger deviation between the ACCC and SECS predictions – one has increasing difficulty to calculate accurately resonant states once  $E_i/E_R$  ratio increases. For  $J=3/2^+$  state we were able to get stable results only for MT13 potential and using SECS. As it was the case for  ${}^4\text{H}$ , the  $J=3/2^+$  and  $J=5/2^+$  states resulting from  $L=2$  and total spin  $S=1/2$  coupling, are degenerate due to the absence of spin-orbit and tensor terms within this model.

	$J=1/2^+$		$J=3/2^+$		$J=5/2^+$	
	$E_R$	$\Gamma$	$E_R$	$\Gamma$	$E_R$	$\Gamma$
N3LO <sup>(ACCC)</sup>	1.8(1)	2.4(2)				
<sup>(SECS)</sup>	1.9(2)	2.4(2)				
INOY <sup>(ACCC)</sup>	1.7(1)	2.4(2)				
<sup>(SECS)</sup>	1.8(1)	2.4(2)				
MT13 <sup>(ACCC)</sup>	1.4(1)	2.4(2)				
<sup>(SECS)</sup>	1.7(2)	2.4(2)	2.5(2)	3.8(4)	2.5(2)	3.8(4)
[6] tnn	2.75(25)	3.5(5)	6.6(3)	8	4.8(2)	5
[15] RGM	3.1(1)	2.5(15)				
[20] RGM	1.59	2.48	3.0	4.8	2.9	4.1
[10] tnn	1.57	1.53	3.25	3.89	2.82	2.51
[21] RGM	1.39	1.60	2.11	2.87	2.10	3.14
[19] RGM	1.9 (2)	0.6(2)	4(1)	3(1)		
Exp. [2]	1.7 $\pm$ 0.3	1.9 $\pm$ 0.4				
Exp. [4]	2.4 $\pm$ 0.3	5.3 $\pm$ 0.4				

Table 2: Resonance parameters for the lowest  ${}^5\text{H}$  states. Our results, using different interactions and different methods to access the continuum, are in the first three rows. They are compared to previous calculations using a t-n-n 3-body model [6, 10] or a microscopic RGM 3-cluster [15, 20, 21, 19] description. The most recent experimental findings are in the last rows [2, 4]. All units are in MeV.

Our results are compared with previous calculations. Refs [6] and [10] are based on a 3-body t-n-n approximation with an adjusted t-n two-body potential and an ad-hoc 3-body force, while Refs. [15], [20], [21] and [19] attempt a microscopic calculation assuming a  ${}^3\text{H}+n+n$  cluster wave function, properly antisymmetrized. Notice that these model calculations display much larger fluctuations among them than our *ab initio* results with different potentials.

It is worth noticing that the lowest resonant state of  ${}^4\text{H}$  is significantly wider, i.e. has larger width, than the  ${}^5\text{H}$  one. This suggest that the  ${}^4\text{n}$  system is less repulsive than the  ${}^3\text{n}$  one, and contributes in this way to stabilize the  ${}^5\text{H}$  isotope with respect to  ${}^4\text{H}$ . If this *nn* "pairing tendency" is pursued with  $6n$  one can indeed expect that  ${}^7\text{H}$  would be a narrower resonant state than  ${}^5\text{H}$  in agreement with GANIL's 2007 findings, where a ground state of  ${}^7\text{H}$  is predicted with the astonishing resonance parameters  $E_R=0.57$  MeV and  $\Gamma=0.09$  MeV. Conclusions from the recent measurement performed at RIKEN with SAMURAI multineutron detector are expected with interest.

For completeness we have also added in Table 2, the last two experimental results from Refs. [2] and [4]. The results of [2] are fully compatible with our calculations, although, as we have already emphasized, any direct comparison between experiments and theory is not very meaningful, apart

from denoting the existence of the same S-matrix pole.

In order to allow the easiest comparison of the ensemble of results of Tables 1 and 2 we have plotted in Fig. 4 the corresponding complex energy values  $E = E_R + iE_I$  (we recall that  $E_I = -\frac{\Gamma}{2}$ ). Left panel corresponds to the three lowest  ${}^4\text{H}$  states and right panel to the  $J=1/2$  one in  ${}^5\text{H}$ . Our *ab initio* results are denoted by filled black dots ( $\bullet$ ). Those based on a microscopic 3-cluster calculations the same NN interaction are denoted by colored squares and those based on a 3-body t-n-n description by triangle up symbols. The energy scales have been taken the same in both figures to compare the proximity of  ${}^5\text{H}$  and  ${}^4\text{H}$  resonances to the physical (real energy) axis and their potential impact on physical observables. The relative stability of  ${}^5\text{H}$  with respect to  ${}^4\text{H}$  is clearly manifested. Notice that by comparing only the  $E_R$  values of these two isotopes, that is forgetting their imaginary part, we could have reached an opposite conclusion. As one can see in this figure, there is quite a large dispersion of the results, even among the microscopic approaches. The positions extracted from the experimental data [44, 2, 4] are indicated by filled green circles.

In the  ${}^4\text{H}$  case, the R-matrix analysis systematically predicts resonances parameters larger than our theoretical values which, as well recognized in ref.[44], is quite a common feature when dealing with broad resonant states. This trend is a clear consequence of two approaches having different goals: the S-matrix approach tries to determine the analytical properties of the underlying Hamiltonian, whereas the R-matrix analysis attempts to describe the experimental observables with the smallest number of parameters. In the  ${}^5\text{H}$  case, since the resonant states turn to be narrower and more pronounced, one might expect a better agreement between the theoretically calculated S-matrix pole positions and the values extracted from the experimental data. Indeed the results from [2] agrees well with our calculated S-matrix poles, while those of [4] provide significantly larger values - similar to the mismatch provided by the R-matrix analysis in the  ${}^4\text{H}$  case.

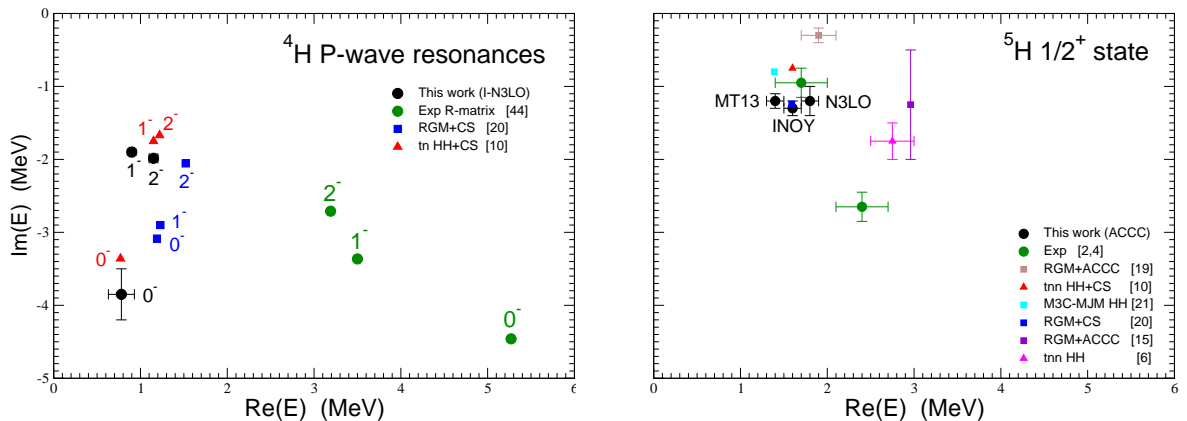


Figure 4: Complex energy values of the three lowest states of  ${}^4\text{H}$  (left panel) and to the  $J^\pi=1/2^+$  one of  ${}^5\text{H}$  (right panel). Energy scales in both figures are the same. Our results are denoted by filled black dots and experimental values in green ones. Those based on microscopic 3-cluster calculations are denoted by squares and those based on a 3-body t-n-n description by triangle up. Corresponding references are indicated in brackets.

To conclude this section we would like to point out that the methods we have used in this work to compute the resonant position of  ${}^5\text{H}$  are strictly the same as those used in our previous studies

on the  $3n$  and  $4n$  systems [32, 33, 34, 35, 36] where we refuted the existence of any experimentally meaningful resonant state.

#### 4. Conclusion

We have presented the first *ab initio* calculation to determine the positions of  ${}^5\text{H}$  resonant states. This has been achieved by solving 5-nucleon Faddeev-Yakubovsky equations in configuration space based on realistic nucleon-nucleon interaction models.

Two independent methods were used to locate the resonance positions in the complex energy plane: a variant of the smooth exterior complex scaling method and the analytic continuation on the coupling constant. The results show a remarkable stability with respect to the different tested interactions and support the recent experimental findings [2, 4], although the direct comparison is biased due to the reaction mechanism involved in each experiment and to the indirect way the resonance parameters are extracted.

The resonance parameters of the  $J^\pi=2^-, 0^-, 1^-$  states in  ${}^4\text{H}$ , which dominate the low-energy  $n$ - ${}^3\text{H}$  elastic cross section, have been also computed and found to be slightly wider than the ones for  ${}^5\text{H}$  ( $\Gamma_{4H} \approx 4$  MeV for  $\Gamma_{5H} \approx 2.5$  MeV) advocating presence of additional attraction of the  $4n$  with respect to  $3n$  system. In view of that, any attempt to reproduce the  ${}^7\text{H}$  narrow state would be of the highest interest.

These *ab initio* methods, recently applied to disprove the existence of any 3- and 4-neutron resonance [32, 33, 34, 35, 36], are found to be very efficient to compute the resonance parameters when they exist, as it is the case in  ${}^4\text{H}$  and  ${}^5\text{H}$  isotopes.

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