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# Introduction of Baumgarte stabilisation schemas into the multibond graph representation

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**Keywords :** Bond graph, mechanical system, Lagrange equations, Lagrange multipliers, holonomic and non-holonomic constraint

## ABSTRACT

In this paper we propose to introduce the corrector schemas corresponding to the Baumgarte stabilisation into bond graph language. In a mechanical context, the Baumgarte stabilisation method aims at controlling the violation of constraints present in a system. This corresponds to a regulation process of the constraints with respect to the numerical perturbations. These perturbations stem from slight errors in initial conditions and/or the propagation of round-off errors occurring during integration.

In order to incorporate the Baumgarte stabilisation method we add a block diagram to the bond graph. It consists of a PID corrector in the case of holonomic constraint equations (figure A) and a PD corrector in the case of non-holonomic constraint equations (figure B).

First we present roughly the Baumgarte stabilisation theory. Then we propose to augment the multibond graph representation with a block diagram in which the stabilisation schemas, respectively for holonomic and non-holonomic constraint equations, are included. Coupling the block diagram to the multibond graph representation reveals to be an attractive way of synthesizing the use of the Baumgarte stabilisation method. Finally we illustrate this with the academic example of a pendulum. This underlines the role of this method in a strategy of modelisation with a view to simulation and the equation generation mechanism.

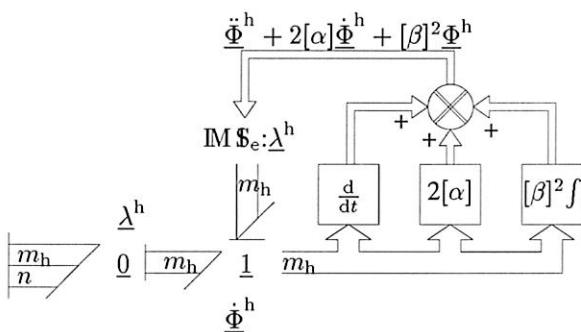


Figure A: Baumgarte stabilisation schema for holonomic constraints

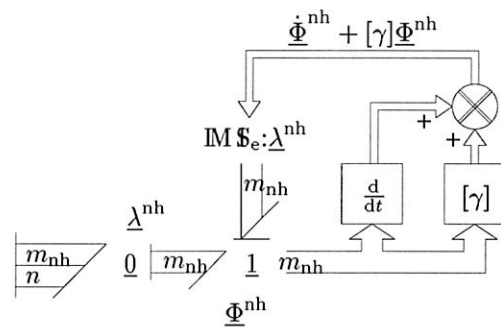


Figure B: Baumgarte stabilisation schema for non-holonomic constraints

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## 1. INTRODUCTION

Concerning the mechanical system simulation, the dynamic equations to be numerically resolved are most of the time differential-algebraic equations (DAE). This feature raises the problem of efficient simulation and depends on the simulation model index (see e.g. [12] or [13] in the bond graph framework).

Beside the use of numerical integration methods used for DAE, several solutions exist at the mathematical formulation stage of the system to circumvent the numerical problems attached to this type of equation. In the independant coordinate formulations we can use, for example, the extraction method (or coordinate partitioning) or the method based on a projection matrix [10], [9]. This necessitates finding a set of independant coordinates and expressing the dynamic equations in terms of these coordinates. The algebraic equations are then used to determine the dependant coordinates against the independant ones.

Concerning the dependant coordinate formulations, we can use the method again based on a projection matrix but also the Baumgarte stabilisation method or the penalty formulation method [1], [11], [9], [2] when Lagrange multipliers are used for setting up the equations of motion. The dependant coordinate formulations consist of resolving simultaneously both dynamic and algebraic equations.

In this paper we deal with the Baumgarte stabilisation method and the way it can be introduced into the multibond graph language. Multibond graph notation enables the juxtaposition of energy domains to be represented in bond graph formalism in a concise manner. This notation has been introduced by Bonderson [3] and it has been completely formalized by Breedveld [5] [6]. It is well suited for the three dimensional domain of mechanical systems.

In considering the Baumgarte stabilisation method through bond graph language, the aim of this paper is to clearly show what is the issue of this technique and how it is implemented practically. First we present the Baumgarte stabilisation method

in the context of mechanical systems. Then we propose the introduction of the corresponding schemas in the multibond graph representation. Finally we illustrate this representation through the academic example of a pendulum. This shows how the dynamic equations integrating this technique are derived from the exploitation of the causal bond graph.

## 2. BAUMGARTE STABILISATION METHOD

We present here the method introduced by Baumgarte in the context of mechanical systems. Let us consider the following Lagrange equations in terms of generalized coordinates  $\underline{q}$  and Lagrange multipliers  $\underline{\lambda}^h$  relative to holonomic constraints and  $\underline{\lambda}^{nh}$  relative to non holonomic constraints:

$$\begin{cases} \underline{F}(\ddot{\underline{q}}, \dot{\underline{q}}, \underline{q}; t) = \underline{Q} + [\Phi_q^h]^T \underline{\lambda}^h + [\Phi_q^{nh}]^T \underline{\lambda}^{nh} \\ \underline{\Phi}^h(\underline{q}; t) = \underline{0} \\ \underline{\Phi}^{nh}(\dot{\underline{q}}, \underline{q}; t) = \underline{0} \end{cases} \quad (1)$$

where  $\underline{F}$  stems from the dynamic terms,  $\underline{Q}$  represents the generalized forces,  $[\Phi_q^h] = \left[ \frac{\partial \Phi^h}{\partial \underline{q}} \right]$  and  $[\Phi_q^{nh}] = \left[ \frac{\partial \Phi^{nh}}{\partial \underline{q}} \right]$ .

The numerical resolution of (1) consists of differentiating the constraint equations with respect to time in order to obtain constraint equations in terms of acceleration. These latter are resolved together with the dynamic equations in terms of  $\ddot{\underline{q}}$ ,  $\underline{\lambda}^h$  and  $\underline{\lambda}^{nh}$  and then  $\ddot{\underline{q}}$  is integrated twice to obtain  $\dot{\underline{q}}$  and  $\underline{q}$ .

Due to the derivation of the constraint equations, information is lost about velocities and positions. A slight error in initial conditions or the propagation of round-off errors occurring during integration makes the solution diverge from the ideal solution.



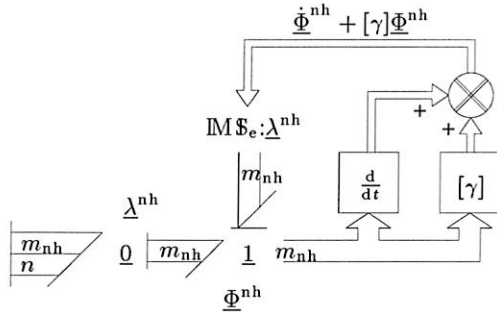


Figure 4: Baumgarte stabilisation schema for non holonomic constraints

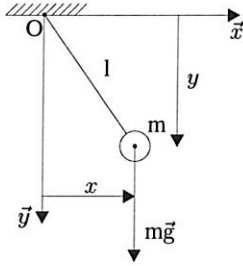


Figure 5: Sketch of a pendulum

action on these constraints (figure 4).

#### 4. ACADEMIC EXAMPLE: A PENDULUM

We will illustrate here the application of what has been presented previously when the Baumgarte stabilisation method is used on a very simple example. It shows the mechanism of equation generation from the causal bond graph. The figure 5 system is a pendulum which consist of a point mass  $m$  located at the end of a massless rigid link of length  $l$ . The pendulum is in rotation about the point  $O$ . We look at the system in terms of the absolute coordinates  $(x, y)$  of the point mass as the generalized coordinates.

The pendulum described above has one degree of freedom. So there is a constraint equation between the generalized coordinates  $x$  and  $y$  of the form:

$$x^2 + y^2 = l^2 \quad (4)$$

This constraint equation is holonomic. The kinematic form of this constraint is simply given by differentiating with respect to time the equation (4):

$$2x\dot{x} + 2y\dot{y} = 0 \quad (5)$$

For the model and the generalized coordinates chosen, the figure 6 bond graph representation shows two storage elements of type I corresponding to the moving mass on the axes  $\dot{x}$  and  $\dot{y}$  and an effort source for the gravity action on the mass. The kinematic form (5) of the constraint equation and the associated  $\lambda$ -multiplier correspond respectively to the flow balance and the effort on the 0-junction. Moreover the modulated transformers ( $2x$  and  $2y$ ) require the presence of integrators from the 1-junctions corresponding to the flows  $\dot{x}$  and  $\dot{y}$ . The associated initial conditions are respectively  $x_0$  and  $y_0$ . Due to the constraint equation these initial conditions are not independent and must satisfy equation (4).

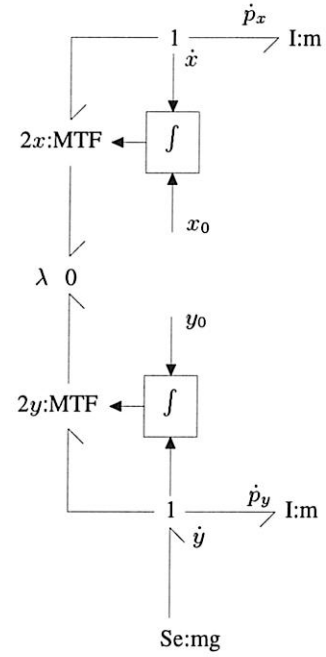


Figure 6: Bond graph representation of a pendulum

#### Classical Lagrange equations

We now proceed with the causality assignment by choosing both storage elements in preferential integral causality. Since they are not independent we use the Lagrange multiplier method to derive the system of equations and then install an artificial effort source to point out the presence of the  $\lambda$ -multiplier figure 7 on the 0-junction. On the signal path modulating the effort source  $\lambda$ , we install a differentiation block to emphasize the mechanism of the entire set of equation generation.

The corresponding Lagrange equations (6) are derived by expressing (i) the accelerations  $\ddot{x}$  and  $\ddot{y}$  versus the efforts  $\dot{p}_x$  and  $\dot{p}_y$  and (ii) these latter efforts in terms of  $\lambda$ , the generalized coordinates. These relations are obtained along the causal path in the figure 7 causal bond graph.

$$\begin{cases} \ddot{x} = \frac{1}{m}\dot{p}_x = -\frac{2x}{m}\lambda \\ \ddot{y} = \frac{1}{m}\dot{p}_y = g - \frac{2y}{m}\lambda \end{cases} \quad (6)$$

We now express the input signal  $\ddot{\Phi}$  of the modulated source. It results from the differentiation with respect to time of the kinematic constraint  $\dot{\Phi}$  and is expressed in terms of both the generalized coordinates and velocities (equation 7). The expression of  $\ddot{\Phi}$  is obtained by following the algebraic relations in terms of flow along the causal paths on the figure 7 causal bond graph.

$$\ddot{\Phi} = \frac{d}{dt}(2x\dot{x} + 2y\dot{y}) = 2x\ddot{x} + 2y\ddot{y} + 2\dot{x}^2 + 2\dot{y}^2 \quad (7)$$

The effort  $\lambda$  imposed by the (artificial) modulated effort source is such that the constraint equation (in acceleration form)  $\ddot{\Phi}=0$  must be verified (equation 8). In this equation the accelerations are replaced by their expressions (6) in order to obtain an implicit algebraic equation in terms of  $\lambda$ . This equation must then be resolved to get  $\lambda$  in terms of the generalized coordinates and velocities. All the symbolic operations described in this paragraph characterize the modulated effort source of figure 7.

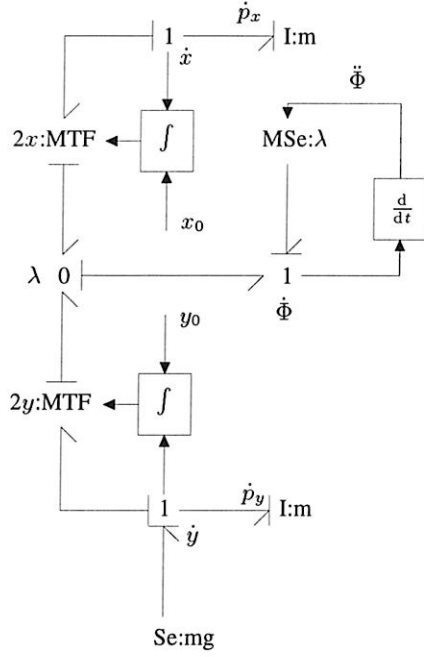


Figure 7: Causal bond graph representation when using  $\lambda$ -multiplier method

$$\begin{aligned}
 2x\ddot{x} + 2y\ddot{y} + 2\dot{x}^2 + 2\dot{y}^2 &= 0 \\
 \Leftrightarrow 2x\left(-\frac{2x}{m}\lambda\right) + 2y\left(g - \frac{2y}{m}\lambda\right) + 2\dot{x}^2 + 2\dot{y}^2 &= 0 \\
 \Leftrightarrow \lambda = \frac{m}{2x^2 + 2y^2} (yg + \dot{x}^2 + \dot{y}^2) & \quad (8)
 \end{aligned}$$

Finally we re-introduce the expression found for  $\lambda$  (equation 8) in the dynamic equations (6) to find a set of explicit ordinary differential equations of second order, in terms of the generalised coordinates  $x$  and  $y$  (equations 9).

$$\begin{cases} \ddot{x} = -\frac{x}{x^2+y^2} (yg + \dot{x}^2 + \dot{y}^2) \\ \ddot{y} = g - \frac{y}{x^2+y^2} (yg + \dot{x}^2 + \dot{y}^2) \end{cases} \quad (9)$$

### Lagrange equations using Baumgarte stabilisation

The introduction of the Baumgarte stabilisation method leads to the causal bond graph representation in figure 8. The differentiation block of figure 7 has been replaced by the corrector schema of the method. The input of the modulated effort source is now identical to the first member of equation (2). We mention the initial condition  $-l^2$  indicated for the integrator block of the corrector. This corresponds to the constant term in the equation (4) geometric constraint. It will be the subject of further discussion in the conclusion below.

The set of equations (6) and (7) are the same when derived from the causal bond graph of figure 8. Only the input of the modulated effort source has changed. This effort source is now characterized by the implicit equation (10). It is resolved to obtain  $\lambda$  in terms of the generalized coordinates and velocities (equation 11). We note here the importance of having indicated the initial condition  $-l^2$ .

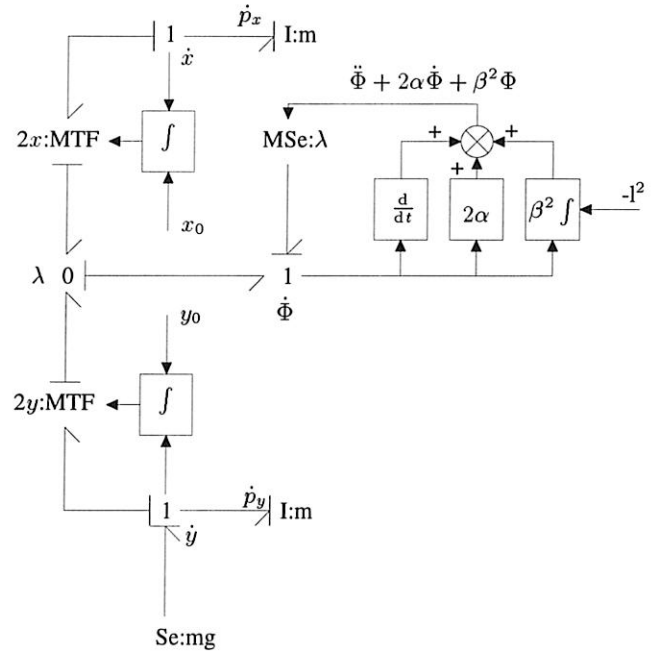


Figure 8: Causal bond graph representation when using the Baumgarte stabilisation method

$$\begin{aligned}
 \ddot{\Phi} + 2\alpha\dot{\Phi} + \beta^2\Phi &= 0 \\
 \Leftrightarrow 2x\left(-\frac{2x}{m}\lambda\right) + 2y\left(g - \frac{2y}{m}\lambda\right) + 2\dot{x}^2 + 2\dot{y}^2 + \\
 2\alpha(2x\dot{x} + 2y\dot{y}) + \beta^2(x^2 + y^2 - l^2) &= 0 \quad (10) \\
 \Leftrightarrow \lambda = \frac{m}{2x^2 + 2y^2} [yg + \dot{x}^2 + \dot{y}^2 + 2\alpha(x\dot{x} + y\dot{y}) + \\
 \beta^2(x^2 + y^2 - l^2)/2] & \quad (11)
 \end{aligned}$$

Finally the set of dynamic equations using the Baumgarte stabilisation method is obtained by substituting  $\lambda$  in the equations (6) with its expression previously obtained in the equation (11):

$$\begin{cases} \ddot{x} = -\frac{x}{x^2+y^2} [yg + \dot{x}^2 + \dot{y}^2 + 2\alpha(x\dot{x} + y\dot{y}) + \\ \beta^2(x^2 + y^2 - l^2)/2] \\ \ddot{y} = g - \frac{y}{x^2+y^2} [yg + \dot{x}^2 + \dot{y}^2 + 2\alpha(x\dot{x} + y\dot{y}) + \\ \beta^2(x^2 + y^2 - l^2)/2] \end{cases} \quad (12)$$

## 5. CONCLUSION

This paper has no more pretention than presenting the Baumgarte stabilisation method through the framework of bond graph language. Bond graph has revealed itself to be an attractive formalism for understanding the issue of this method and the practical implementation of the mechanism of equation generation.

The Baumgarte stabilisation method has been presented in a mechanical context with the Lagrange equation formalism. There is absolutely no restriction for a generalisation in other domains. In fact the pluridisciplinarity feature of the bond graph language enables the Baumgarte stabilisation method to be applied in other physical fields. Moreover the formulation can be entirely dualised in the sense of power variables. In this way the corrector schemas (figure 3 or 4) can modulate a multiport flow source. This multiport flow source would then be attached to a 1-junction array (with the go-between of an O-junction array)

corresponding to effort constraint equations. The transdisciplinarity and the duality of the Lagrange equation formalism using the  $\lambda$ -multiplier method have been pointed out by Van Dijk [13]. The Baumgarte stabilisation method presented here is just an extension of these formulations. Also the Baumgarte stabilisation method may be used in combination with the Hamilton equation formalism using the Lagrange multiplier method [8]. In both cases (Lagrange or Hamilton equation formalism) the type of constraint equations i.e. holonomic or non holonomic must be identified in order to apply the appropriate stabilisation schema i.e. respectively figure 3 or 4. The constraint equation type can be investigated through the transformers associated to the transformations of generalized velocities to obtain the kinematic constraints (figure 1). The method of finding out the type has been presented in [8].

The introduction of the Baumgarte stabilisation method in the bond graph language shows the nature of this method i.e. a signal point of view. This is to be distinguished from the nature of bond graph formalism itself which has an energetic point of view. Moreover we mentioned in the introduction the penalty formulations. Together with the method based on the singular perturbation formulation (see [15], [14] in the bond graph context) and in contrast to the Baumgarte stabilisation method, they belong to the category of energetic methods for mathematically reformulating the system of equations. The application of these methods results in an augmentation of the bond graph representation with power bonds and bond graph elements instead of a block diagram [8]. These energetic methods consist in relaxing the constraints in the system by adding fast dynamic modes.

Finally we would like to point out some important observations resulting from the introduction of the Baumgarte stabilisation method in bond graph language. Equation generation has showed the necessity of symbolically manipulating the relations obtained from causal bond graph exploitation. We suggest that a serious effort must be made to integrate symbolic methods in software accepting bond graph for the model input. Furthermore we have noticed the need for information not displayed *a priori* in bond graph representation. This information is the integrated forms of the kinematic constraints in the holonomic case. This generally results in the loss of an integration constant. In the example of the pendulum (section 4) this constant is  $-l^2$  and appears neither in the figure 6 bond graph nor in the figure 7 bond graph. This information is only present in the figure 8 bond graph due to the integrator of the stabilisation schema. However outside the Baumgarte stabilisation context nothing gives this information in bond graph representation. This is important because it shows that the initial conditions  $x_0$  and  $y_0$  cannot be chosen independently. We alert bond graph modellers to this observation and propose to take this feature into account in the bond graph representation, for example, as follows. On the 0 junction corresponding to a kinematic constraint we can attach an arrow with the initial condition for the geometric constraint as it is realized for integrators (figure 9). But the question is still open and something better can possibly be found.

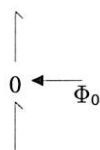


Figure 9: Initial condition for a geometric constraint

## REFERENCES

- [1] BAUMGARTE, J. Stabilization of constraints and integrals of motion in dynamical systems. In *Proceedings of the NATO ASI on Computer Aided Analysis and Optimization of Mechanical Systems Dynamics, Iowa city/USA, August 1-12, 1983*, pages 1–16, 1984.
- [2] BAYO, E., JIMENEZ, J. M., SERNA, M. A., BASTERO, J. M. Penalty based Hamiltonian equations for the dynamic analysis of constrained mechanical systems. *Journal of Mechanism and Machine Theory*, 29(5):pages 725–737, 1994.
- [3] BONDERSOHN, L. S. Vector bond graphs applied to one-dimensional distributed systems. *Journal of Dynamic Systems, Measurement, and Control*, 97(1):pages 75–82, 1975.
- [4] BOS, A. M. *Modelling multibody systems in terms of multibond graphs*. Ph.D. Thesis : Electrical engineering, University of Twente, Enschede, Netherlands, 1986.
- [5] BREEDVELD, P. C. Proposition for an unambiguous vector bond graph notation. *Journal of Dynamic Systems, Measurement, and Control*, 104(3):pages 267–270, 1982.
- [6] BREEDVELD, P. C. A definition of multibond graph language. In *Proceedings of the 11th International Association for Mathematics and Computers in Simulation (IMACS) World Congress on Scientific Computation, August 5-9 1985, Oslo, Norway*, volume 4, pages 325–328, 1985.
- [7] BRIX, F., ALIRAND, M. Bond graph representation of the manipulation of constraint equations in multibody software. In *Proceedings CISS'94, First Joint Conference of International Simulation Societies, Zürich*, pages 649–653, 1994.
- [8] FAVRE W. *Contribution à la représentation bond graph des systèmes mécaniques multicorps*. Ph. D. thesis, Institut National des Sciences Appliquées de Lyon, 1997.
- [9] GARCÍA de JALÓN, J., BAYO, E. *Kinematic and dynamic simulation of multibody systems. The real time challenge*. New York : Springer Verlag, 1994.
- [10] NIKRAVESH, P. E. *Computer-aided analysis of mechanical systems*. New Jersey : Prentice Hall, Englewood Cliffs, 1988.
- [11] OSTERMEYER, G. On Baumgarte stabilization for differential algebraic equations. In *Real-time integration methods for mechanical system simulation, series F: Computer and Systems Sciences*, volume 69, pages 193–207, 1991.
- [12] PETZOLD, L. Methods and software for differential-algebraic systems. In *Real-time integration methods for mechanical system simulation, series F: Computer and Systems Sciences*, volume 69, pages 127–140, 1991.
- [13] VAN DIJK, J. *On the role of bond graph causality in modelling mechatronic systems*. Ph.D. Thesis : Electrical engineering, University of Twente, Enschede, Netherlands, 1994.
- [14] ZEID, A., OVERHOLT, J. L. Singularly perturbed bond graph models for simulation of multibody systems. *Journal of Dynamic Systems, Measurement, and Control*, 117(3):pages 401–410, 1995.
- [15] ZEID, A., OVERHOLT, J. L. Singularly perturbed formulation: Explicit modeling of multibody systems. *Journal of the Franklin Institute*, 332B(1):pages 21–45, 1995.