Dynamic Pricing for Carriers in Physical Internet with Peak Demand Forecasting

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Abstract: This paper investigates a dynamic pricing problem for less-than-truckload (LTL) carriers during several auction periods in Physical Internet (PI), in consideration of peak demand forecasting. PI can be considered as the interconnection of logistics networks via open PI-hubs, in which there are plenty of requests to be transported. These requests continuously arrive at different auction periods with various quantity. Carriers can bid for these requests through participating several rounds of auction. In a dynamic environment, a major problem for the carrier is how to decide the bidding price to maximize his profit. Besides, to make better decision, when determining the bidding price in current period, carrier should forecast the possible requests in next periods, especially facing with peak/off-peak time periods. This paper proposes a dynamic pricing model considering the forecasted quantity of requests in the next auction periods to optimize the bidding price and maximize the total profit. A numerical study is conducted to evaluate the model and study how the future requests influence the current pricing decision, including the influence to the bidding price and profit.

Keywords: dynamic pricing, less-than-truckload, auction, demand forecasting, Physical Internet.

1. INTRODUCTION

In freight transportation, the dynamic pricing problem for multiple auction periods is the problem in which carriers prefer to participate several rounds of auction to obtain as much as transport requests and profit. It is an extension of the one period pricing problem. It involves determining the optimal bidding price for the known requests in one auction period to maximize the carrier's total expected profit. When participating several auction periods in a dynamic market, to decide the bidding price in current auction period, carrier should consider the possible arriving requests in the next periods, which will influence the future possible profit. For that, the quantity of the arriving requests in the next periods should be estimated, which generates the forecasting problem. The forecasting is particularly important when decision making arrives at peak/off-peak time periods, which represent the maximal and minimal demands during a day for example.

The related literature has also confirmed the vital role of dynamic pricing considering forecasting for revenue optimization (Pölt 1998). There have been a number of researches investigating how the forecasting could be used to improve the revenue for carriers in freight transport market. For example, Luo, Gao et al. (2015) investigate how to use dynamic forecasting to optimize the revenue in intermodal transportation. Helve (2015) studies the passenger forecasting in railway revenue management.

By following the literature, this paper introduces and investigates a dynamic pricing problem for less-than-truckload (LTL) carriers considering forecasting of the quantity of transport requests in next time periods in Physical Internet (PI), see the example in Fig. 1.. PI is a recent paradigm of logistics and transport. It can be considered as a network of logistics networks which are highly interconnected via open logistics hubs, i.e. pi-hubs (Montreuil 2011, Montreuil, Meller et al. 2013). In PI-hubs, shippers and carriers can both offer transport requests encapsulated in modular and standard PI-containers. These requests arrive at the hub in different time periods, which makes the quantity of requests in each period very dynamic (Ballot, Montreuil et al. 2014, Sarraj, Ballot et al. 2014, Qiao, Pan et al. 2016). At each period, the requests will be allocated to carriers, for example, using an auction mechanism to minimize the total cost; and carrier need to propose bidding prices to the requests to maximize his total profit. In order to further increase his profit, carriers will wish to participate into several auction periods. That means, in each period, the pricing decision should be made independently with considering the future possible requests.

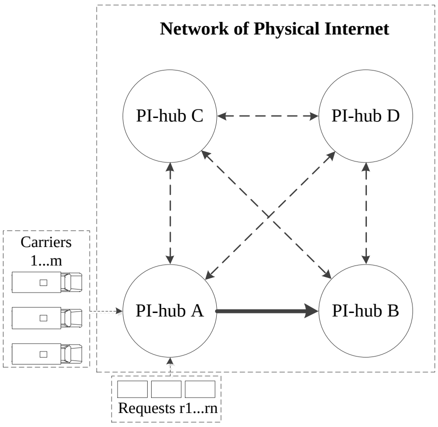

Fig. 1. Example of PI network with 4 PI-hubs.

The aim of this paper is to study how to decide the bidding price for the transport requests at a PI-hub in current auction period considering the possible arriving requests in next auction periods. The dynamic pricing problem in one auction period in PI has been studied in (Qiao, Pan et al. 2016), in which the optimal dynamic pricing decision is made to maximize the total profit. In this paper, we extend the one-period situation to multi-periods. This paper aims to povide decision making models for carrier's dynamic pricing decision in multiple auction periods at a PI-hub. Another objective is to investigate how the forecasting, at peak/off-peak time periods in particular, would influence the current pricing decision-making.

This paper is organised as follows. After the introduction, section 2 presents a brief literature review of the related research in order to identify the research gap and research interest. Section 3 describes the dynamic pricing problem in several auction periods in PI, which is formulated in section 4. A computational study and the results are presented in section 5. Finally, section 6 concludes the contributions of this work and points out some research prospects.

2. LITERATURE REVIEW

Two research problems in the freight transport literatures are related to this paper and will be discussed in this section: dynamic pricing in road freight transport, and forecasting problem in transport related industries.

2.1 Dynamic pricing in road freight transport

Dynamic pricing aims to determine various prices for different customers and demands to maximize revenue over time. Firms always use dynamic pricing to respond to market fluctuations and demand uncertainty (Chiang, Chen et al. 2006, Talluri and Van Ryzin 2006).

However, in the road freight transport, dynamic pricing has not been applied widely. Only a few relevant papers focusing on pricing decisions can be found in the TL transport sector, and even fewer in the LTL sector. In the TL transport, many references are focus on the opportunity cost, which is used to describe the influence of current decisions (bidding price) on the future status. For example, opportunity costs can describe the loss in expected future revenue due to fulfilling a new request. Figliozzi, Mahmassani et al. (2006) present a method to calculate the opportunity cost in sequential TL requests auctions. The opportunity costs in a dynamic routing problem is considered. After this study, Figliozzi, Mahmassani et al. (2007) investigate the carrier pricing strategy for the dynamic vehicle routing problem. Similarly, the pricing strategy considering opportunity cost to decide whether to accept a new request in current task sequence is studied in (Mes, Heijden et al. 2006) and opportunity costs could also be used in scheduling decisions (Mes and Van der Heijden 2007). In the LTL pricing, reference (Douma, Schuur et al. 2006) presents how to determine the price of one-leg transport requests dynamically to maximize carrier profit according to the remaining capacity and the remaining waiting time. In (Qiao, Pan et al. 2016), the authors investigate the dynamic pricing problem for LTL requests in one auction period. The optimal bidding price for the transport request in Physical Internet is dynamically decided based on carrier's remaining capacity and the quantity of requests in the current period.

We can find that the dynamic pricing strategy is not studied a lot in the road freight industry. Especially considering several auction periods, the research about the dynamic pricing is very rare. Thus, the dynamic pricing problem during multiple auction periods in PI is novel. The most important reason is that in PI, the environment is very stochastic and dynamic, which will make the pricing more complex than in the traditional industries.

2.2 Forecasting problem in transport related industries

Forecasting is an important part in Revenue Management, especially in a dynamic market. The performance of other RM components depends on the quality of forecasting. For example, the capacity allocation, overbooking, and pricing decisions are made based on the forecasting result of the future demand, market price, and so on. As presented in Talluri and Van Ryzin (2006), the forecasting results are used as the input of optimization models, which aim to make the optimal RM decisions, such as pricing and capacity control.

In the road traffic sector, the forecasting method has been applied widely. For instance, Chen and Grant-Muller (2001) propose and discuss the potential application of neural networks algorithm in the short-term traffic flow forecasting of motorway. Chrobok, Kaumann et al. (2004) investigate two methods of short-term traffic forecasting, which are based on 2 years of real data. The application of forecasting in freight transport is also significant, because the freight transport demand fluctuate frequently over space and time (Garrido and Mahmassani 2000). Chow, Yang et al. (2010) review the current freight forecasting models and advances. Moreover, the authors present the future development of forecasting with data using. Petri, Fusco et al. (2014) propose a new freight demand forecasting model driven by data and based on Bayesian Network. In Fite, Don Taylor et al. (2002), the freight demand forecasting in truckload (TL) industry is discussed. The air cargo demand forecasting is investigated by Suryani, Chou et al. (2012), in which a system dynamics simulation model is developed. Nuzzolo and Comi (2014) present the demand forecasting in urban freight and propose a mixed modelling approach comprising quantity, delivery and vehicle. The forecasting problem in Physical Internet has been studied in Qiao, Pan et al. (2018), in which the influence of forecasting to the request selection and pricing in PI is investigated. In addition, the forecasting in Qiao, Pan et al. (2018) refers to the estimation of the quantity of the requests in the next hubs, which helps carrier improve the request selection decision. While in this paper, the quantity of requests in the next time periods is forecasted to improve carrier's current pricing decision.

All these researches show that an appropriate forecasting method could improve the operational efficiency in the transport system and help the actor in the system to produce effective RM decisions, e.g. the pricing decisions.

3. PROBLEM DEFINITION

This paper considers the dynamic pricing problem with multi-periods. To simplify, we assume that all requests are homogenous, which means they have the same O-D pair and uniform size. As shown in Fig. 2, the waiting time of a carrier in one PI-hub can be divided into *m* periods, e.g. $t_1, t_2, \ldots, t_{m-1}, t_m$. Each period can be considered as an auction period, and carrier bid for all requests arriving during that period. Carrier could forecast the possible request numbers during each period, e.g. $n_1, n_2, \ldots, n_{m-1}, n_m$. At the first period, a carrier will give a bidding price p_{1d} according to his full capacity *D*, the known requests number n_1, and the possible requests number of the following periods. Because the future request number could influence the carrier's current pricing decision. For example, if the request number in the next period is huge, the carrier might want to save the capacity for the next auction period, as we know that more requests bring more profit from (Qiao, Pan et al. 2016).

It is also worth noting that from period t_2, the remaining capacity could be less than *D*, and it is decided by the auction results in the previous periods. In other words, the remaining capacity in the latter periods rely on how many requests carrier got in the previous periods. In each period from t_2, carrier will have *D* kinds of possible remaining capacity, i.e. *1, 2, …, D*. Carrier need to decide the bidding price according to the remaining capacity, so there will be *D* possible prices in each period, e.g. $p_{21}, p_{22}, \ldots, p_{m1}$ in period t_2.

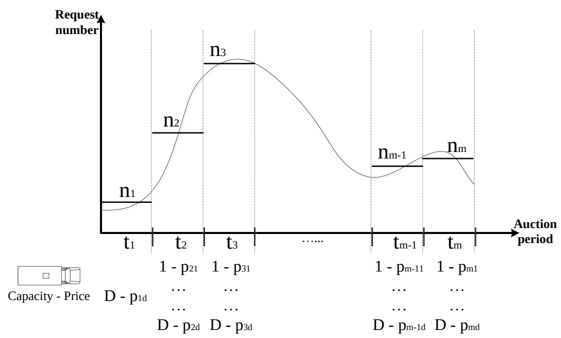

Fig. 2. Auction periods division in the PI-hub.

In this context, we aim to give pricing guidance for decision making according to the carrier's remaining capacity, quantity of the forecasted requests, and the auction period the carrier is in. In addition, to simplify the problem, we adopt the unique price strategy, i.e. carrier submits one unique optimal price to bid for all requests in each auction period.

4. MODEL FORMULATION

4.1 Notations

Parameters:

r[t]: requests remaining in auction period *t*. As presented in chapter 3, we assume that a vehicle can bid n times at most if there are n requests during the auction period, so *r[t] = n, n-1, ···, 1*.

p(x): the probability of winning with a given bid price x in an auction. We have $p(x) = e^{-(\frac{x}{\lambda})^k}, with\ \lambda = 1, k = 5$.

Pw(r,x,i): the probability that carrier could win *i* requests with a bid price *x* when facing *r* requests in total. We assume the winning request number in one auction period is distributed following a Binomial Distribution.

D: the actual capacity of a vehicle.

c: the cost of fulfilling a request.

T: the number of auction periods carrier will participate.

(d,t): the vehicle status, defined according to the remaining capacity *d* when carrier is in auction period *t*.

V(d,r,x): the expected maximum profit in one auction period with *d* remaining capacity, *r* requests and a bidding price *x*.

VT(d,t): the expected maximum profit when carrier is in status *(d,t)*.

X: the set of bid prices, i.e. range of prices to be tested in the model, and *X* = [0, 2] here.

Decision Variable:

x_{dt}: bid price given by the carrier for a request at state *(d,t)*. In particular, the optimal bid price for each state determined by the model is noted as x_{dt}*.

4.2 Model

Same with the problem of dynamic pricing in Physical Internet discussed in (Qiao, Pan et al. 2016), the pricing problem considering several auction periods also concerns sequential auctions, i.e. the decision in the present status will affect the future status. Moreover, the pricing decision will influence the capacity remained for the next auction periods. Thus, we propose the following dynamic programming (DP) model to solve this problem:

Equation (1) is used to calculate the expected maximum profit of status *(d,r,x)* in auction period *t*.

$$V(d, r[t], x) = p(x) \cdot [x - c + V(d-1, r[t]+1, x)] + (1 - p(x)) \cdot V(d, r[t]+1, x), \quad r[t] = 1,2, \dots n-1, n \tag{1}$$

Equation (2) presents the probability to win i requests in one period.

$$Pw(r, x, i) = \binom{r}{i} p(x)^i (1 - p(x))^{(r-i)}, \; i = 0,1, \dots, r-1, r \tag{2}$$

The expected maximum profit in the state of *(d,t)* could be calculated through (3).

$$VT(d, t) = \max_{x_{dt} \in X} \left[V(d, r[t], x_{dt}) + \sum_{i=0}^{i=d} Pw(r[t], x_{dt}, i) \cdot VT(d-i, t+1) \right], \; r = 1, \dots n-1, n \tag{3}$$

Boundary conditions are given by (4) and (5).

$$V(d, r[t], x) = 0, \text{if } d \leq 0 \; OR \; r[t] \geq n+1 \tag{4}$$

$$VT(d, t) = 0, \text{if } d \leq 0 \; OR \; t \geq T+1 \tag{5}$$

Then the optimal bidding price $x_{dt}*$ for the state of (d,t) can be found through (6).

$$x_{dt}^* = arg \max_{x_{dt} \in X} \left[V(d, r[t], x_{dt}) + \sum_{i=0}^{i=d} Pw(r[t], x_{dt}, i) \cdot VT(d-i, t+1) \right], \; r = 1, \dots n-1, n \tag{6}$$

Function (1) is a recursive function that calculate the carrier's expected maximum profit when they bid for *r[t]* requests using price x with a remaining capacity of d. Function (2) is used to calculate the probability that the carrier wins i requests in one auction period based on a Binomial Distribution. Function (3) is a recursive function based on the auction period to calculate the maximum profit when carrier is in auction period t and with remaining capacity d. According to the boundary condition (4) and (5), the profit will be 0 when capacity is sold out, or there are no requests, or the waiting time is finish. The expected maximum profit in the whole waiting time will be *VT (D, 1)*. Functions (6) presents the optimal bidding price $x_{dt}*$.

5. NUMERICAL STUDY

A numerical study is designed to evaluate the performance of the proposed models. We qualitatively investigate the influence of the quantity of request in the following auction periods to the current pricing decision. All the experiments in this paper were run on Mathematica 10.4 under Windows 10 on a DELL of Model Inspiron 15 (5000) with 16 GB of RAM.

We designed a numerical experiment to investigate how the forecasting of future requests can affect the current pricing decision and the total revenue. In this experiment, we assume the quantity of forecasted future requests to be a static number. Three auctions periods with three different request quantities are considered, which represent the demands level of off-peak, normal, peak period that are (50, 100, 150) respectively. Moreover, in order to evaluate the influence of the quantity of future request, there will be 6 scenarios according to the different sequence of the three levels of demands, which is presented in Table 1. The model developed in Section 4 is applied in each scenario to determine the optimal price and expected maximum revenue. In the experiment, the vehicle capacity is assumed 20 requests, and transportation cost to 0.5€/ unit.

Table 1. Input data and scenarios

Scenario	Request quantity in each Auction Period (AP)		
	AP1	AP2	AP3
S1	50	100	150
S2	50	150	100
S3	100	50	150
S4	100	150	50
S5	150	50	100
S6	150	100	50

5.1 Results of optimal dynamic pricing and profit: with or without forecasting

Firstly, based on these 6 scenarios, we compared the expected profit in each scenario based on two strategies, i.e. with or without considering forecasting. Fig. 3 compares the profit calculated by the model for each scenario under the two strategies. Several observations are notable here. Frist, it is easy to find that, in all scenario, the strategy considering forecasting always generates higher profit than the strategy without considering forecasting. Moreover, when considering forecasting, the profit between each scenario is very similar, since the total quantity of requests in each scenario is the same. However, the profit of each scenario is very different when not considering forecast. Second, the profit difference between the two strategies varies according to the periodic sequence of demand level. The biggest difference appears in Scenario 1, in which the level of demands increase from period to period. Conversely, Scenario 6 generates the smallest difference, in which the level of demands decreases. Third, under the strategy of without considering forecasting, the profit depends strongly on the demand level in the actual auction period.

The first results provide some important implications to carriers when making pricing decision in a dynamic market. First, it shows clearly the importance of forecasting to maximize the profit. Second, forecasting will also help to mitigate the impact of the variance of demand level on profit, e.g. considering off-peak and peak period. That means the profit for the carriers without considering forecasting would be strongly period-dependent.

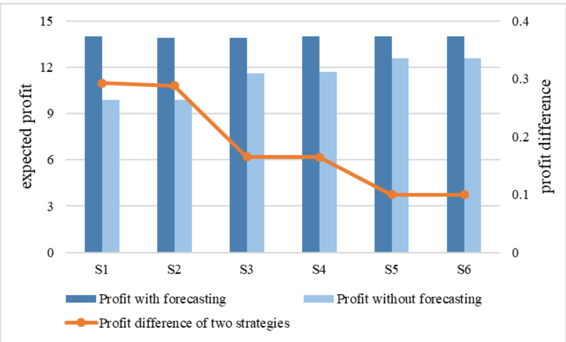

Fig. 3. Expected profit for each scenario with and without forecasting.

Besides, as presented before, we adopt unique price strategy here, which means there only one biding price for one pair of request quantity and remaining capacity, i.e. one price *x* for one status *(d,t)*. Table 2. presents only the part of the results (scenario 1 and scenario 6) obtained based on model proposed because of the lack of space. The results shows the optimal bidding price corresponding to each possible status *(d,t)* when considering forecasting. For example, in scenario 1, in the first period t=1 in which carrier has full capacity of d=20, the optimal price calculated by the model is 1.21. While in the second auction period, for example if there are 15 units capacity left after the first period, the status will be (*d=15, t=2*), and the corresponding optimal price is 1.22. Consequently, the model can help carrier determine optimal price for every situation in all periods.

Table 2. Optimal bidding prices for each status with forecasting (example of Scenario 1 and 6)

Capacity	Optimal bidding price					
	Scenario 1			Scenario 6		
	50	100	150	150	100	50
1	\	1.37	1.3	\	1.34	1.22
2	\	1.34	1.28	\	1.31	1.19
3	\	1.33	1.26	\	1.29	1.17
4	\	1.31	1.25	\	1.27	1.15
5	\	1.3	1.23	\	1.26	1.14
6	\	1.29	1.22	\	1.25	1.12
7	\	1.28	1.21	\	1.24	1.11
8	\	1.27	1.2	\	1.23	1.09
9	\	1.26	1.2	\	1.22	1.08
10	\	1.25	1.19	\	1.21	1.07
11	\	1.24	1.18	\	1.2	1.06
12	\	1.24	1.17	\	1.19	1.05
13	\	1.23	1.17	\	1.19	1.04
14	\	1.23	1.16	\	1.18	1.03
15	\	1.22	1.16	\	1.17	1.02
16	\	1.21	1.15	\	1.17	1.01
17	\	1.21	1.14	\	1.16	1
18	\	1.2	1.14	\	1.16	0.99
19	\	1.2	1.13	\	1.15	0.98
20	1.21	1.19	1.13	1.22	1.14	0.98

5.2 Impact of the quantity of future demand

From the results above, we found that the quantity of future request could influence the pricing decisions in the current auction period. To further investigate that how the quantity of the future requests influence the current pricing decision, we designed another experiment. We assume the quantity of request in current auction period is 100 and the vehicle capacity is 20. The quantity of the request in the next auction period varied independently over a given range, which is from 0 to 200 with 5 increments. Therefore, that in total 40 instances have been studied and the optimal bidding price for requests in current auction period is presented as the curve shown in Fig. 4.

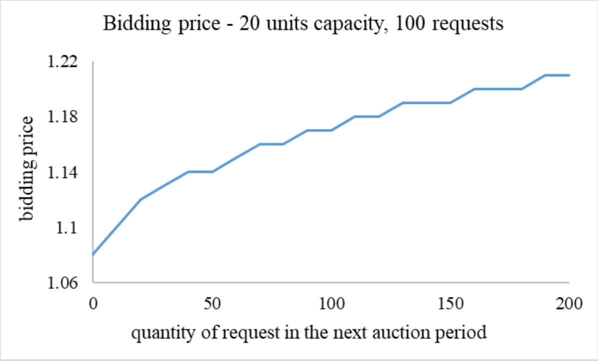

Fig. 4. Optimal bidding price for current 100 requests.

Several conclusions can be obtained from the figure. First, if the transport cost is constant, the current bidding price increases when the quantity of request in the future increases. This is because the carrier should save more capacity for the next auction periods. In particular, when there are no requests arriving in the future, the bidding price is the lowest. The reason is that the carrier should sell the capacity out as soon as possible to increase the fill rate. Second, the function of optimal price (axis *y*) to quantity of future request (axis *x*) is concave rather than linear. It means optimal price variation is more sensitive when future request quality is less and close to carrier's capacity.

6. CONCLUSIONS

This paper presents and investigates the dynamic pricing problem for LTL carriers in several auction periods in Physical Internet. We extend the dynamic pricing problem in one auction period to several auction periods. Consequently, carrier need to forecast the quantity of request in the next periods to optimize the pricing decision. The influence of the future requests to the current pricing decision is studied.

The work in this paper contributes to the research of dynamic pricing considering forecasting in Physical Internet. We develop a multi-periods dynamic pricing model, for carriers who would like to participate several auction periods in a hub to maximize his profit. This model could help them to decide the optimal bidding price according to the upcoming requests.

However, due to the lack of appropriate real data, we did not study the method to do the forecasting. We assumed that forecasting has been done beforehand and is used as input to the model. How to do the forecasting, as well as the impact of forecasting error, should be a future work in the next steps. Besides, the study can also be extended with consideration of variable transport cost. As suggested in some works, transport service can be considered as perishable product of which the value decreases over time. Considering such assumption, it is foreseeable that carrier will adopt different dynamic pricing strategies.

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