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G rard Meunier, Jean-Michel Guichon, Olivier Chadebec, Bertrand Bannwarth, Laurent Kr henb hl, et al.. Unstructured–PEEC Method for Thin Electromagnetic Media. *Compumag* 2019, Jul 2019, Paris, France. hal-02064483

HAL Id: hal-02064483

<https://hal.science/hal-02064483v1>

Submitted on 10 Nov 2022

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Unstructured–PEEC Method for Thin Electromagnetic Media

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An unstructured-PEEC method for modelling electromagnetic thin region is proposed. Two coupled circuits representations are used for solving both electric and/or magnetic effects in thin regions discretized by a finite element surface mesh. Dynamic effects across the sheet are modeled by equivalent complex conductivity and reluctivity. Non simply connected regions are treated with fundamental branch independent loop matrixes coming from the circuit representation. The formulation enables the computation of eddy current losses and can be coupled with external circuits, PEEC-1D cables or coil thanks to the circuit representation.

Index Terms—Volume Integral Formulation, Unstructured–PEEC, Thin regions, Losses.

I. INTRODUCTION

A major interest of using the Volume Integral Method for 3D magnetic and electric fields analysis is that the air region does not need to be meshed. Moreover, solving Maxwell's equations by Green's function integral methods has known a renewed interest in recent years with the development of efficient matrix compression algorithms (FMM, ACA), which greatly improve memory storage and resolution time of fully dense matrix systems. In this work, we propose a new formulation dedicated to thin electromagnetic regions. It is an extension of a previous volume integral approach based on 3D facet interpolations of the current density and of the magnetic flux density [1].

II. FORMULATION

A. Volume Integral Method

Volume Integral Method (VIM) is based on the integral solution of Maxwell Equations. Electric field \mathbf{E} and magnetic field \mathbf{H} are expressed with integrals depending on Green's functions, and current density \mathbf{J} and magnetization \mathbf{M} over both conductive and magnetic media Ω_J and Ω_M (Ω_J and Ω_M can be totally or partially the same region). In the frequency domain, by neglecting propagation effects, and thanks to Lorentz gauge, we have:

$$\begin{aligned} \mathbf{E}(P) &= -j\omega \frac{\mu_0}{4\pi} \left(\iint_{\Omega_J} \frac{\mathbf{J}}{r} d\Omega + \iint_{\Omega_M} \mathbf{M} \wedge \mathbf{grad} \frac{1}{r} d\Omega \right) - \mathbf{grad}V \\ \mathbf{H}(P) &= \frac{1}{4\pi} \iint_{\Omega_J} \mathbf{J} \wedge \mathbf{grad} \frac{1}{r} d\Omega - \mathbf{grad}\phi \\ V(P) &= \frac{1}{4\pi\epsilon_0} \iint_{\Omega_J} \mathbf{J} \cdot \mathbf{grad} \frac{1}{r} d\Omega \\ \phi(P) &= \frac{1}{4\pi} \iint_{\Omega_M} \mathbf{M} \cdot \mathbf{grad} \frac{1}{r} d\Omega \end{aligned} \quad (1)$$

In these expressions r is the distance between observation point P and integration point in media. Volume Integral Equations are then obtained by matching expression of \mathbf{E} and \mathbf{H} with constitutive relationships $\mathbf{E}(\mathbf{J})$ and $\mathbf{H}(\mathbf{M})$ inside the

media. Different formulations can be obtained by discretizing the regions and by choosing adequate unknowns and interpolation shape function. The use of 2-form Whitney face interpolation with the use of current density \mathbf{J} and magnetic flux density \mathbf{B} leads to the unstructured PEEC method [1].

B. Thin regions model

In the case of thin regions, we consider that the evaluation of electric and magnetic fields outside of the media can be obtained by using the average current and magnetisation \mathbf{J}_m , \mathbf{M}_m flowing the thin regions:

$$\begin{aligned} \mathbf{E}(P) &\approx -j\omega \frac{\mu_0}{4\pi} \left(\int_{\Gamma_J} e \frac{\mathbf{J}_m}{r} d\Gamma + \int_{\Gamma_M} e \mathbf{M}_m \wedge \mathbf{grad} \frac{1}{r} d\Gamma \right) \\ &\quad - \mathbf{grad}V \\ \mathbf{H}(P) &\approx \frac{1}{4\pi} \int_{\Gamma_J} e \mathbf{J}_m \wedge \mathbf{grad} \frac{1}{r} d\Gamma - \mathbf{grad}\phi \end{aligned} \quad (2)$$

where e is the thickness of thin regions. Equivalent material behavior laws which take into account dynamic effects across the thickness, can be used. Following [2], these equivalent properties link average surface electric and magnetic fields \mathbf{E}_{sm} , \mathbf{H}_{sm} to average current and flux densities \mathbf{J}_m , \mathbf{B}_m :

$$\mathbf{E}_{sm} = \frac{\mathbf{J}_m}{G\sigma} \quad \mathbf{H}_{sm} = \frac{\mathbf{B}_m}{G\mu} \quad G = \frac{th \left((1+j) \frac{e}{\delta} \right)}{(1+j) \frac{e}{\delta}} \quad (3)$$

where σ is the conductivity, $\mu = \mu_0 \mu_r$ is the permeability and δ is the skin depth. Integral equations are then obtained by matching expression of (2) on the external surfaces of the thin region with constitutive relationships (3). It leads to surface integral equations with \mathbf{J}_m and \mathbf{B}_m as unknowns.

C. Unstructured PEEC model

Regions Γ_J and Γ_M are discretized by surface finite element meshes (composed of triangles or quadrilaterals for instance) on which \mathbf{J}_m and \mathbf{B}_m are interpolated by using 2D first order surface facet elements:

$$\mathbf{J}_m = \frac{1}{e} \sum_j \mathbf{w}_j I_{sj} \quad \mathbf{B}_m = \frac{1}{e} \sum_j \mathbf{w}_j \Phi_{sj}$$

I_{sj} and Φ_{sj} are respectively the current and the magnetic flux flowing the 2D facets on Γ . Applying a Galerkin projection on facet functions w_i of equations (1), we get a circuit representation (capacitive effects are not represented here):

$$\begin{cases} \Delta V_s \\ \Delta \phi_s \\ \Delta \phi_e \end{cases} = \begin{pmatrix} R + j\omega L & j\omega C & 0 \\ C' & Y & 0 \\ 0 & 0 & P \end{pmatrix} \begin{cases} I_s \\ \phi_s \\ \phi_e \end{cases} \quad (4)$$

$$R_{ij} = \int_{\Gamma_j} \frac{1}{e} \frac{\mathbf{w}_i \mathbf{w}_j}{G\sigma} d\Omega \quad L_{ij} = \frac{\mu_0}{4\pi} \int_{\Gamma_j} \mathbf{w}_i \int_{\Gamma_j} \frac{\mathbf{w}_j}{r} d\Gamma d\Gamma$$

$$Y_{ij} = \int_{\Gamma_M} \frac{1}{e} \frac{\mathbf{w}_i \mathbf{w}_j}{G\mu} d\Omega$$

$$C_{ij} = \frac{1}{4\pi} \int_{\Gamma_j} \mathbf{w}_i \int_{\Gamma_M} \frac{\mu_r - 1}{\mu_r} \mathbf{w}_j \wedge \mathbf{grad} \frac{1}{r} d\Gamma d\Gamma$$

$$C'_{ij} = \frac{1}{4\pi} \int_{\Gamma_M} \mathbf{w}_i \int_{\Gamma_j} \mathbf{w}_j \wedge \mathbf{grad} \frac{1}{r} d\Gamma d\Gamma$$

$$P_{kl} = \frac{1}{4\pi\mu_0} \int_{\Gamma_{M_k}} \frac{1}{s_k} \int_{\Gamma_{M_l}} \frac{\mu_r - 1}{\mu_r} \frac{1}{s_l} \frac{1}{r} d\Gamma d\Gamma$$

In (4), ΔV_s and $\Delta \phi_s$ represent the differences of electric and magnetic potentials between two adjacent elements. ϕ_e are the magnetic fluxes going outside the elements and $\Delta \phi_e$ are the differences of magnetic potentials between elements and the infinite [1]. The solution of (4) can be obtained using a circuit solver based on an independent loops search technique. Fundamental circuit equations to be solved are:

$$[M_J]\{\Delta V_s\} = 0 \quad [M_M]\begin{cases} \Delta \phi_s \\ \Delta \phi_e \end{cases} = 0$$

where $[M_J]$ and $[M_M]$ are the branch-fundamental independent loop matrixes for electric and magnetic circuit representations respectively. Let us notice that with such a formulation, non-simply connected domain can be treated naturally (see an example on Figure2).

Capacitive effects in vacuum can be considered by adding in (4) a capacitive matrix, similar to $[P]$. Moreover, the formulation can be naturally coupled with volume regions, non-meshed inductor coils and external circuits, connected or not to the meshed regions. In order to get a good accuracy, integrals of Green's kernels of L , Y , C and P matrixes are computed with the use of analytical integration techniques for the self-interactions [3]. Finally, this formulation has been developed with the possibility of using matrix compression technique.

III. RESULTS

We first present an academic problem constituted of a magnetic and conductive thin sphere (diameter 1m, thickness 1mm, $\mu_r = 1000$, $\sigma = 5E+5$) placed in a field produced by a cylinder coil. The results are compared with a reference solution obtained with an axi-symmetrical FEM using a mesh ensuring the convergence of the solution at any frequency. Figure 1 compares eddy current losses obtained with both simulations. In the case of the unstructured-PEEC Method, losses are computed by using the equivalent material properties [2]:

$$Losses = e \int_r \text{Real} \left\{ \frac{\mathbf{J}_m \overline{\mathbf{J}_m}}{G\sigma} - j\omega \frac{\mathbf{B}_m \overline{\mathbf{B}_m}}{G\mu} \right\} d\Gamma$$

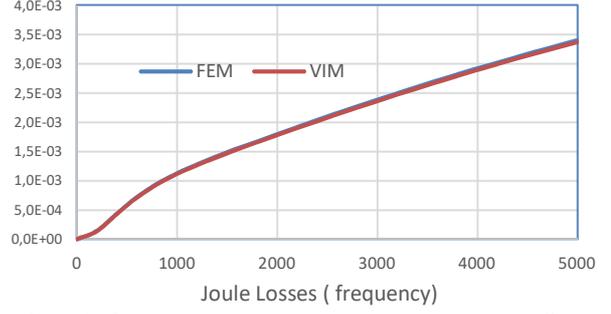


Figure1 : Losses in the magnetic and conductive thin sphere. Comparison between VIM and FEM method for various frequencies.

Results obtained are very satisfactory (difference lower than 1% at any frequencies), given the difficulty of accurately calculating eddy current losses. We also test the formulation for computing eddy currents and magnetic flux density in a body car. The mesh contains about 18.000 triangles and matrix compression techniques (FMM and HCA) has been used for the solving (Fig. 2).

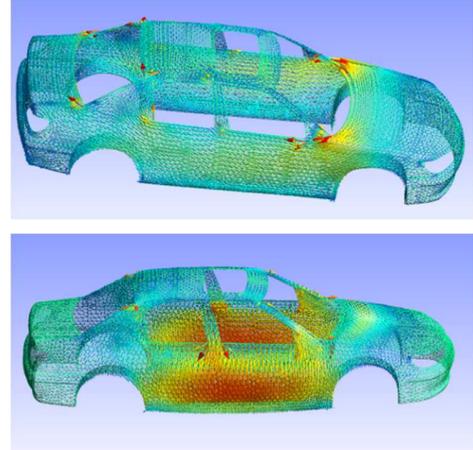


Figure 2: Eddy currents (top) and magnetic flux density (bottom) on a body car

IV. CONCLUSION

The new formulation enables the modeling of thin conductive and/or magnetic regions in order to simulate efficiently various devices with only a surface mesh. The computational effort is considerably reduced in comparison with volume approaches. Equivalent material properties enable the computation of eddy currents losses with a good accuracy at any frequency. The formulation has been validated on various devices from academic examples to more complex problems.

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