

FiXiF toolbox: validated numerics for sound digital filter implementations

Anastasia Volkova

▶ To cite this version:

Anastasia Volkova. FiXiF toolbox: validated numerics for sound digital filter implementations. 18th International Symposium on Scientific Computing, Computer Arithmetics and Verified Numerics., Sep 2018, Tokyo, Japan. hal-02064338

HAL Id: hal-02064338 https://hal.science/hal-02064338v1

Submitted on 11 Mar 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

FiXiF toolbox : validated numerics for sound digital filter implementations

Anastasia Volkova

Univ Lyon, Inria, CNRS, ENS de Lyon, Université Claude Bernard Lyon 1, LIP UMR 5668, F-69007 LYON, France anastasia.volkova@inria.fr

Keywords: floating-point arithmetic, interval arithmetic, validated linear filters, code generator

Digital filter algorithms are ubiquitous for signal processing systems: from aerospace control to industrial telecommunications. In this work we develop approaches for improvement of the numerical behavior of digital filters with focus on the impact of accuracy of the computations. We exhibit the problems related to the use of finite precision during the whole process of filter implementation. We demonstrate on examples that in order to produce accurate implementations of digital filters, we must also provide accurate and validated design tools.

Usually, for implementations of digital filters in embedded systems we seek for the minimum possible precision of computations that will permit to meet the constraints on the output accuracy, speed and surface. Achieving this trade-off is complicated and existing approaches often require some level of fault tolerance [1]. In practice, the compromise between choices of arithmetic parameters and the cost of the system is sought manually; this process requires a significant effort and final implementations are far from optimal. Moreover, providing guarantees for each customized solution separately is time-consuming.

In this work, our goal is to provide a numerically validated and automatic tool for accurate linear filter implementations in software/hardware.

We present an overview of the FiXiF toolbox¹ [2] for the reliable code generation for linear filters/controllers in Fixed-Point Arithmetic. The stages of the code generator are illustrated in Figure 1.

FiXiF takes frequency- or time-domain representation of digital filters. The generator uses an internal representation SIF [3] that encompasses all possible algorithms for linear digital filters.

¹https://github.com/fixif

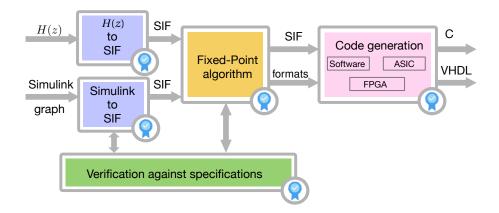


Figure 1: The workflow of the automatic filter code generator FiXiF.

The core of the generator is the validated and automatic Fixed-Point implementation for the *recursive* filter algorithms with a *priori* accuracy constraints. This process requires fine-grained error analysis of implemented filters and we demonstrate that purely Interval Arithmetic based approaches for filter analysis are not always efficient. Instead, we propose to use a worst-case error analysis and to couple it with Interval Arithmetic to ensure that the implementation process is validated itself.

We also provide a rigorous *a posteriori* verification of frequency specifications for the implemented filters, which is based on a combination of Interval and Rational Arithmetics. Finally, the code or hardware/software can be generated. It is guaranteed that the implementations will satisfy by construction *a priori* given accuracy constraints.

We demonstrate a complete workflow of the code generator on several real-life and artificial examples of linear digital filters.

References

- [1] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, 3rd ed. NJ, USA: Prentice Hall Press, 2009.
- [2] A. Volkova, "Towards reliable implementation of digital filters," Ph.D. dissertation, Sorbonne Universités, Sep 2017.
- [3] T. Hilaire, P. Chevrel, and J. F. Whidborne, "Finite Wordlength Controller Realizations using the Specialized Implicit Form," in *International Journal of Control*, vol. 2, no. 2, pp. 330–346, 2010.