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FiXiF toolbox : validated numerics for sound digital filter implementations

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Digital filter algorithms are ubiquitous for signal processing systems: from aerospace control to industrial telecommunications. In this work we develop approaches for improvement of the numerical behavior of digital filters with focus on the impact of accuracy of the computations. We exhibit the problems related to the use of finite precision during the whole process of filter implementation. We demonstrate on examples that in order to produce accurate implementations of digital filters, we must also provide accurate and validated design tools.

Usually, for implementations of digital filters in embedded systems we seek for the minimum possible precision of computations that will permit to meet the constraints on the output accuracy, speed and surface. Achieving this trade-off is complicated and existing approaches often require some level of fault tolerance [1]. In practice, the compromise between choices of arithmetic parameters and the cost of the system is sought manually; this process requires a significant effort and final implementations are far from optimal. Moreover, providing guarantees for each customized solution separately is time-consuming.

In this work, our goal is to provide a numerically validated and automatic tool for accurate linear filter implementations in software/hardware.

We present an overview of the FiXiF toolbox¹ [2] for the reliable code generation for linear filters/controllers in Fixed-Point Arithmetic. The stages of the code generator are illustrated in Figure 1.

FiXiF takes frequency- or time-domain representation of digital filters. The generator uses an internal representation SIF [3] that encompasses all possible algorithms for linear digital filters.

¹<https://github.com/fixif>

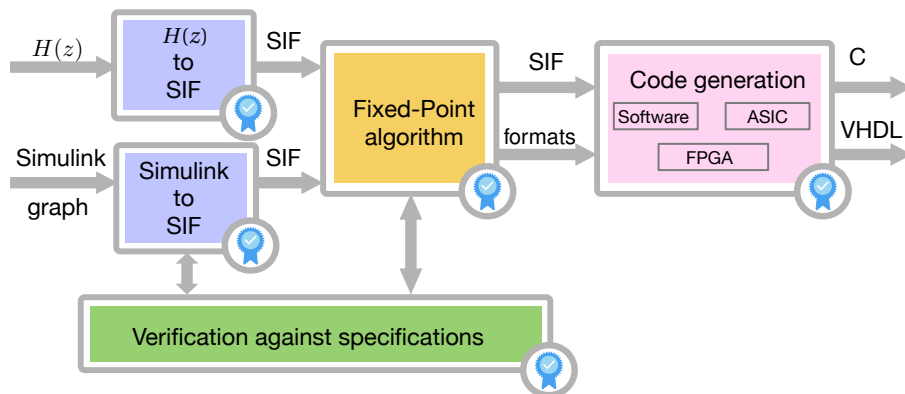


Figure 1: The workflow of the automatic filter code generator FiXiF.

The core of the generator is the validated and automatic Fixed-Point implementation for the *recursive* filter algorithms with *a priori* accuracy constraints. This process requires fine-grained error analysis of implemented filters and we demonstrate that purely Interval Arithmetic based approaches for filter analysis are not always efficient. Instead, we propose to use a worst-case error analysis and to couple it with Interval Arithmetic to ensure that the implementation process is validated itself.

We also provide a rigorous *a posteriori* verification of frequency specifications for the implemented filters, which is based on a combination of Interval and Rational Arithmetics. Finally, the code or hardware/software can be generated. It is guaranteed that the implementations will satisfy by construction *a priori* given accuracy constraints.

We demonstrate a complete workflow of the code generator on several real-life and artificial examples of linear digital filters.

References

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