Lois de comportement magneto-mécanique pour la modélisation des machines électriques

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Préambule

- Advanced constitutive laws for computational magnetics: the case of magneto-mechanical behaviour

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Magnetic and mechanical behaviour are strongly coupled

- ... sometimes it helps:
  - smart actuation systems, sound systems
  - force sensors

- ... sometimes not:
  - noise emitted by transformers
  - loss of efficiency of electrical machines (e.g. high speed)
A challenge for computational magnetics

- These effects can be very significant
- The behaviour is non-linear
  - Magnetic saturation
  - Hysteresis
  - Non linearity of the coupled effects
- The loading configurations are usually complex
  - Stress is multiaxial
  - Any relative orientation between stress and magnetic field is permitted
  - Possible dynamic loadings (rotating fields, etc...)
- The physical mechanisms are local

➤ Standard phenomenological approaches
  - can’t be predictive
  - require very complex identification procedures
Note on multiaxiality

- Stress is a second order tensor (not a scalar!)

➤ Only 3D approaches will be considered in the following
Three families of 3D magneto-elastic approaches

- Equivalent stress (or strain) approaches
- Thermodynamic macroscopic approaches
- Multiscale approaches
- ... and their derivatives
A. Equivalent stress approaches

- Principle:
  find a uniaxial stress which – applied along the direction of the magnetic field – would have the same effect as the real multiaxial stress

- Equivalent stresses can be defined from energy approaches

- This leads to fully multiaxial equivalent stresses

- Examples:
  - Equivalence in magneto-elastic energy
    \[
    W_\sigma = \frac{1}{V} \int_V -\sigma : \varepsilon^\mu dV
    \]
    (isotropic materials)

  - Equivalence in magneto-elastic energy
    \[
    \sigma_{eq} = \frac{3}{2} \mathbf{t} \mathbf{h} \mathbf{\sigma} \mathbf{h} - \frac{1}{2} \text{tr}(\mathbf{\sigma}) = \frac{3}{2} \mathbf{t} \mathbf{h} \mathbf{s} \mathbf{h}
    \]
    (orthotropic materials)

  - Possible equivalence in magnetisation, magnetic susceptibility, magnetic losses, ....

A. Equivalent stress approaches

- How to use an equivalent stress:
  - Preliminary knowledge of unaxial coupled behaviour ($H/\sigma$)
  - Determination of the stress distribution in the structure
  - Calculation of the equivalent stress
  - Application of the 1D coupled constitutive equation
  - Loop until convergence if needed

- Example: effect of stress on losses in PMSM [Yamazaki and Aoki, 2016]
A. Equivalent stress approaches

- Only a 1D model required (or measurements)
- Very straightforward implementation
- Accuracy can be poor (based on the relevance of the chosen equivalence)

Hysteresis losses under biaxial stress
Measurements (L) and calculation from an equivalent stress approach (R)

B. Thermodynamic macroscopic approaches

- Helmholtz Free Energy in terms of magneto-mechanical invariants

\[ \rho \psi = \frac{1}{2} \lambda I_1^2 + 2GI_2 + \frac{1}{2} \sum_{i=0}^{4} \left[ \frac{g_i(I_1)}{i + 1} \left( \frac{I_4}{B_{ref}^2} \right) I_4 \right] + \frac{1}{2} \alpha_5 I_5 + \frac{1}{2} \alpha_6 I_6 \]

\[ I_1 = \text{tr}(\varepsilon) \quad I_5 = B \cdot \varepsilon \cdot B \]
\[ I_2 = \frac{1}{2} \text{tr}(\varepsilon^2) \quad I_6 = B \cdot \varepsilon^2 \cdot B \]
\[ I_4 = B \cdot B \]

\[ g_0 = \frac{3}{4} \alpha_0 \exp\left(\frac{3}{4} I_1\right) - \frac{1}{3} (\mu_0 - \alpha_5) \]
\[ g_i = \frac{3(i + 1)}{4} \alpha_i \exp\left(\frac{4(i + 1)}{3} I_1\right) \]

- Cauchy stress tensor and Magnetization vector

\[ \sigma = \rho \sum_{i=1, i \neq 3}^{6} \psi_i \frac{\partial I_i}{\partial \varepsilon} \quad M = \rho \sum_{i=1, i \neq 3}^{6} \psi_i \frac{\partial I_i}{\partial B} \]

[Courtesy of Prof. A. Belahcen; Fonteyn et al., IEEE Trans. Magn., 2010; Rasilo et al., IEEE Trans. Magn., 2016]
B. Thermodynamic macroscopic approaches

- Good accuracy, fully 3D, efficient numerical implementation
- Descriptive model (no prediction out of the identification range)
- Identification procedures can be complex
  (high number of material parameters)

[Aydin et al., IEEE Trans. Magn., 2016]
C. Multiscale approaches

- Pioneering works by Néel’s phase model (1944)
- Further developed by Armstrong (Terfenol-D, 1997), Buiron et al. (Fe-Si polycrystals, 1999)
- A recent renewed interest : Daniel et al. (2004, 2008, 2014), Hubert et al. (2008), van den Berg et al. (2010), Vanoost et al. (2016), Ito et al. (2016), Martin et al. (2019),...
C. Multiscale approaches

\[ \Sigma, \vec{H} \rightarrow \text{Macroscopic behaviour} \rightarrow \vec{E}, \vec{M} \]

\[ \sigma^i, \vec{H}^i \rightarrow \text{Localisation} \rightarrow \text{Microstructure} \]

\[ \varepsilon^i, \vec{M}^i \rightarrow \text{Homogenisation} \]

- Scale transition rules
- Local constitutive laws

[Hubert et al.-98]
C. Multiscale approaches

• Magnetic materials: three-scale approach

- Single crystal
- Magnetic domain
- Polycrystal
C. Multiscale approaches

- Scale transition rules
  - Define the local loading (stress, magnetic field) depending on:
    - Macroscopic loading
    - Heterogeneities
    - Material behaviour (+ evolving microstructure)

- Local constitutive laws
  - Local energetic equilibrium (free energy)
  - Statistic description of microstructure (e.g. volume fraction)
  - Evolution laws for internal variables

- Homogenization
  - Appropriate averaging operations
C. **Multiscale approaches**: focus on local constitutive law

- Potential energy of a magnetic domain
  \[ W_\alpha = W_\alpha^{\text{mag}} + W_\alpha^{\text{an}} + W_\alpha^{\sigma} + W_\alpha^{\text{conf}} \]
  - Magnetostatic energy : \[ W_\alpha^{\text{mag}} = -\mu_0 \vec{H}_\alpha \cdot \vec{M}_\alpha \]
  - Magnetocrystalline anisotropy energy : \[ W_\alpha^{\text{an}} = K_1 (\gamma_1^2 \gamma_2^2 + \gamma_2^2 \gamma_3^2 + \gamma_3^2 \gamma_1^2) \]
  - Elastic energy : \[ W_\alpha^{\sigma} = \frac{1}{2} \sigma_\alpha : \mathbf{C}^{-1} : \sigma_\alpha \]
  - Configuration energy : \[ W_\alpha^{\text{conf}} \]

- Choice of internal variables : volume fractions \( f_\alpha \)
  \[ f_\alpha = \frac{\exp (-A_s \cdot W_\alpha)}{\sum_\alpha \exp (-A_s \cdot W_\alpha)} \quad A_s = \frac{3 \chi^0}{\mu_0 M_s^2} \]

- Magnetisation and magnetostriction at the grain scale
  \[ \vec{M}^I = \langle \vec{M}_\alpha \rangle = \sum_\alpha f_\alpha \cdot \vec{M}_\alpha \]
  \[ \varepsilon_\alpha^I = \langle \varepsilon_\alpha^I \rangle = \sum_\alpha f_\alpha \cdot \varepsilon_\alpha^I \]
C. Multiscale approaches: material parameters

• Single crystal parameters
  - Elastic constants (cubic symmetry): \((C_{11}, C_{12}, C_{44})\)
  - Magnetic constants: \(M_S, K_1, K_2\)
  - Magneto-elastic constants (cubic symmetry): \((\lambda_{100}, \lambda_{111})\)

• Texture data (EBSD measurements)

• Macroscopic parameter
  \[ A_s = \frac{3\chi^0}{\mu_0 M_s^2} \]
C. Multiscale approaches

- Identification of the (physical) parameters on a limited set of experiments
- Prediction in various and independent loading configuration

[Daniel et al., Arch. Appl. Mech., 84, 2014]
C. Multiscale approaches: note on hysteresis

- Phenomenological estimate (Hauser 1994)
  \[ H = H_{an} \pm \left[ 1 - \alpha \exp(\beta |M_{an} - M_{reb}|) \right] (\gamma + \delta |H_{an}|) \]

[Daniel et al., Arch. Appl. Mech., 84, 2014]
C. Multiscale approaches

• Fully 3D
• Description of complex magneto-mechanical effects:
  • Multiaxiality effects
  • ΔE effect
  • Villari effect
  • Anisotropy effects
• Parameters with clear physical significance
• Prediction out of the identification tests range
• Less accurate than a brute fitting
• Computation time can hinder numerical implementation
  ➢ Exploitation strategies required for computational magnetics

D. **Exploitation strategies:** numerical testing machine

- Identification of multiscale models from a limited number of experiments
- Use of multiscale model as a numerical testing machine to identify the parameters of macroscopic models (compatible with numerical computation)
  - Phenomenological approaches (Zeze *et al.* 2012, 2013, …)

- Combine the advantages of thermodynamic and multiscale approaches
- Choose the test suited to the model identification rather than use the available experimental tests
D. Exploitation strategies: simplified multiscale approaches

• Equivalent single crystal approach [Daniel et al., IEEE Trans. Magn., 2015; ...]
  • Principle: Consider the material as a – fictitious – single crystal with properties equivalent – at macroscopic level – to the real material
    ➢ Heterogeneity effects not included
    ➢ Main mechanisms still described
    ➢ Computational cost significantly reduced

• Example of a switched reluctance motor
  • Material considered as a collection of magnetic domains

\[
W_k = -\mu_0 H M_s \mathbf{x} \cdot \mathbf{u}_k - \sigma : \varepsilon^\mu_k \quad \quad f_k = \frac{\exp(-A_s W_k)}{\int_k \exp(-A_s W_k)}
\]

• Coupled with a J-A model for hysteresis effects
D. Exploitation strategies: simplified multiscale approaches

- Example: Effect of stress on a switched reluctance motor

Effect of inertial stress on the local permeability

Effect (%) of shrink fitting on the local hysteresis losses

$\frac{\mu}{\mu_{\text{ref}}} \times 100$

$N = 30 \text{krpm}, R = 34 \text{mm}$


$R = 34 \text{mm}, \delta R = 2.5 \text{µm}$

D. Exploitation strategies: simplified multiscale approaches

- Compact 3D models
- Possible analytical expression for the constitutive equations

\[
\mathbf{M} = \frac{A_x \sinh(\kappa H)}{A_x \cosh(\kappa H) + A_y + A_z} \quad M_s \mathbf{x}
\]

\[
\lambda = \lambda_s \left( 1 - \frac{3(A_y + A_z)}{2(A_x \cosh(\kappa H) + A_y + A_z)} \right)
\]

- Implementation into numerical analysis tools
- Simplifying assumption to be cautiously defined
- Loss of information compared to full multiscale approach


[Daniel, EPJ Appl. Phys., 2018]
Conclusion

• Coupled problems such as **magneto-mechanical problems** are a great challenge in computational electromagnetics

• In addition to other challenges in the uncoupled case (non linearity hysteresis, anisotropy, frequency effects ...) they **require specific constitutive equations**

• **Multiscale approaches**, based on the description of physical mechanisms at the appropriate scale are very promising

• Their development require interaction between the **Magnetism and Magnetic materials** community and the **computational electromagnetics** community so that the constitutive equations can be both **physically relevant** and **practically implementable** in numerical tools for structural analysis
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