

# Lois de comportement magneto-mécanique pour la modélisation des machines électriques

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## Préambule

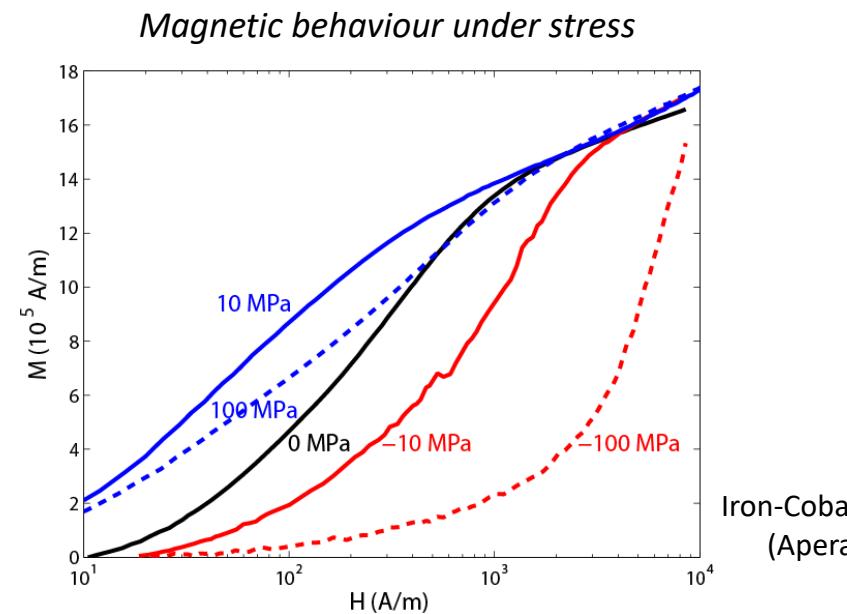
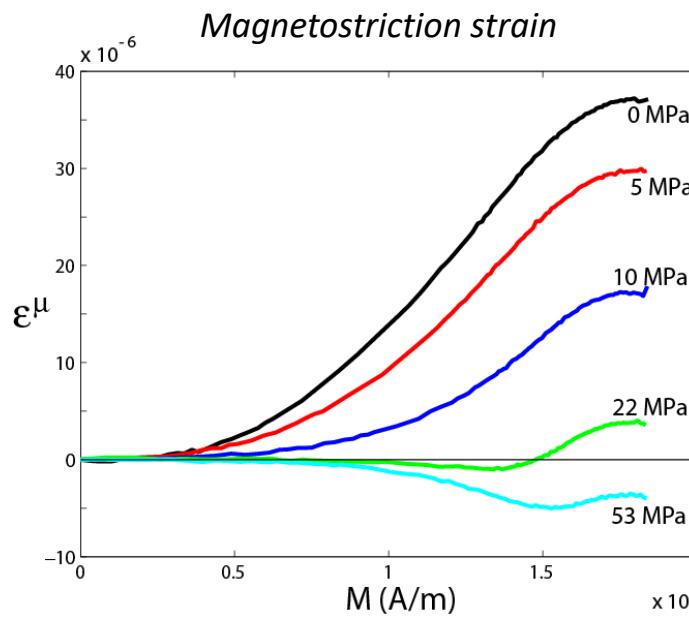
- Advanced constitutive laws for computational magnetics:  
the case of magneto-mechanical behaviour



- Compumag 2019, 15-19 juillet 2019, Paris



# Magnetic and mechanical behaviour are strongly coupled

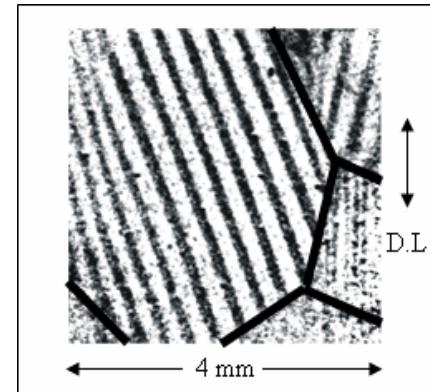
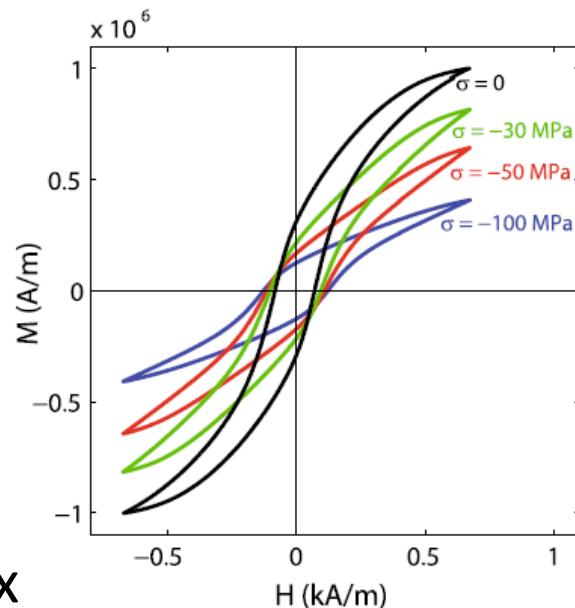


- ... sometimes it helps:
  - smart actuation systems, sound systems
  - force sensors
- ... sometimes not:
  - noise emitted by transformers
  - loss of efficiency of electrical machines (e.g. high speed)

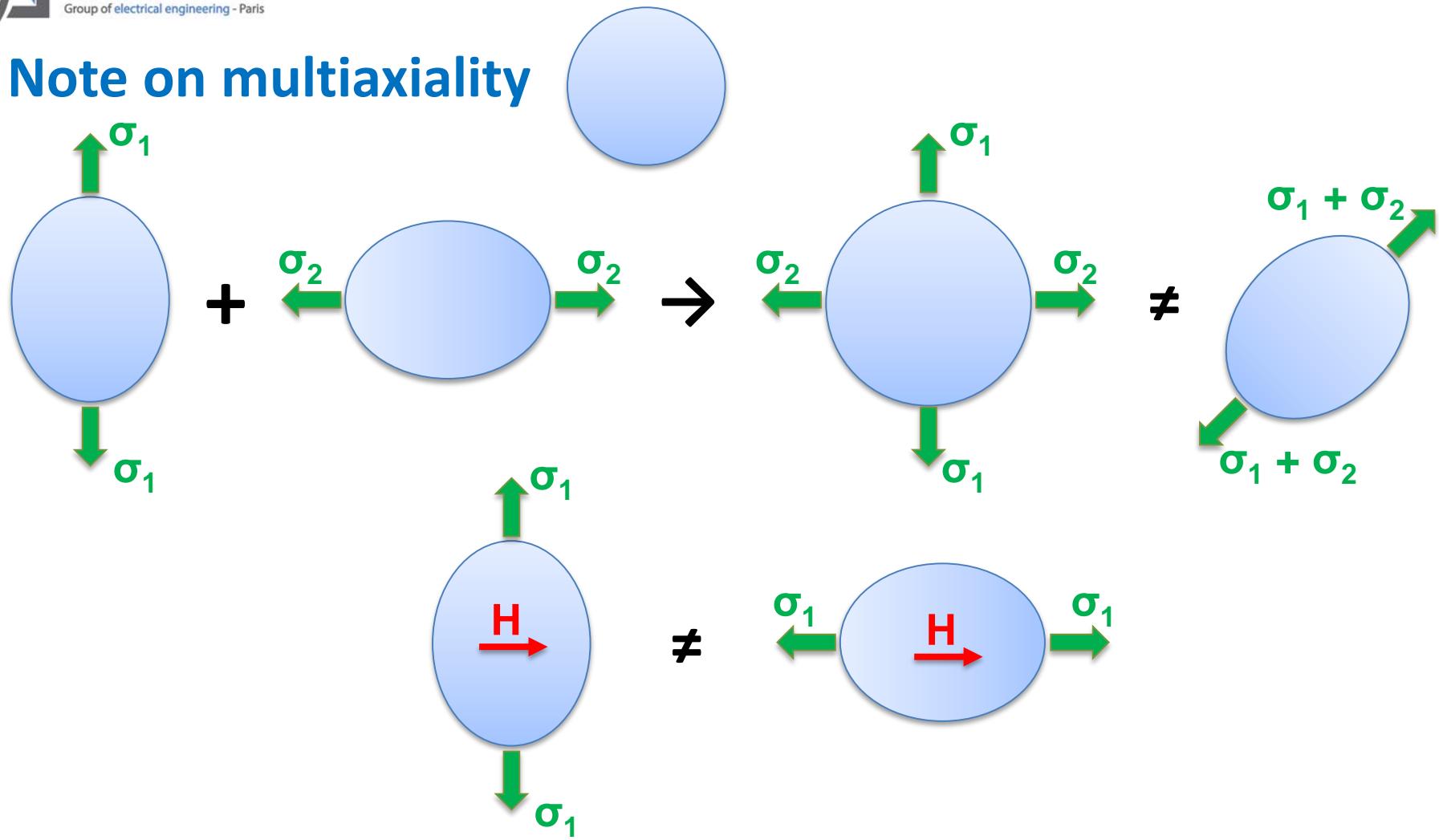


# A challenge for computational magnetics

- These effects can be very significant
- The behaviour is non-linear
  - Magnetic saturation
  - Hysteresis
  - Non linearity of the coupled effects
- The loading configurations are usually complex
  - Stress is multiaxial
  - Any relative orientation between stress and magnetic field is permitted
  - Possible dynamic loadings (rotating fields, etc...)
- The physical mechanisms are local
  - Standard phenomenological approaches
    - can't be predictive
    - require very complex identification procedures



## Note on multiaxiality



- Stress is a second order tensor (not a scalar!)
  - Only 3D approaches will be considered in the following

# Three families of 3D magneto-elastic approaches

- Equivalent stress (or strain) approaches
- Thermodynamic macroscopic approaches
- Multiscale approaches
- ... and their derivatives

## A. Equivalent stress approaches

- Principle :  
find a uniaxial stress which – applied along the direction of the magnetic field – would have the same effect as the real multiaxial stress
- Equivalent stresses can be defined from energy approaches
- This leads to fully multiaxial equivalent stresses
- Examples :
  - Equivalence in magneto-elastic energy

$$W_\sigma = \frac{1}{V} \int_V -\boldsymbol{\sigma} : \boldsymbol{\epsilon}^\mu dV$$

$$\sigma_{eq} = \frac{3}{2} {}^t \vec{h} \bar{\boldsymbol{\sigma}} \vec{h} - \frac{1}{2} \text{tr}(\bar{\boldsymbol{\sigma}}) = \frac{3}{2} {}^t \vec{h} \mathbf{s} \vec{h}$$

(isotropic materials)

$$\sigma_{eq} = \frac{3}{2} {}^t \vec{h} \mathbf{s} \vec{h} + \frac{\alpha}{2} ({}^t \vec{t}_1 \mathbf{s} \vec{t}_1 - {}^t \vec{t}_2 \mathbf{s} \vec{t}_2)$$

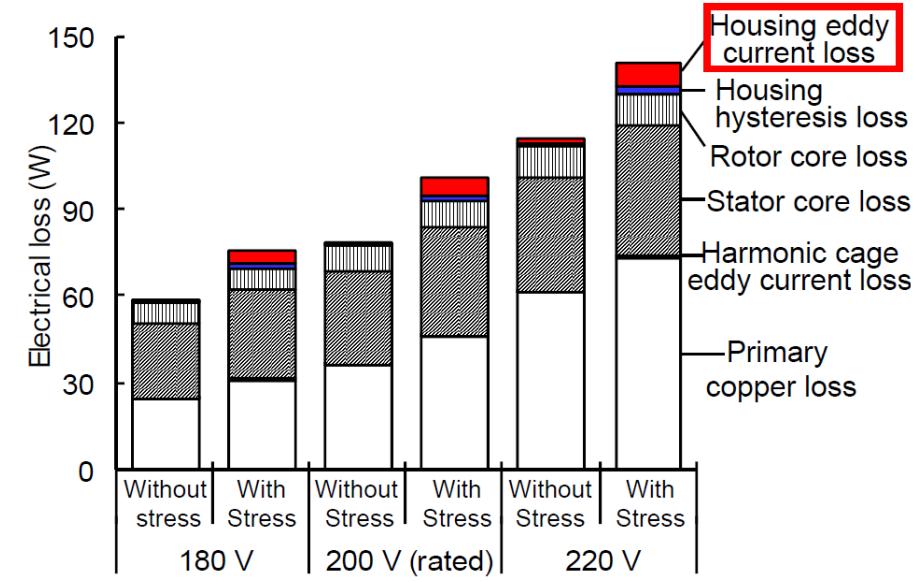
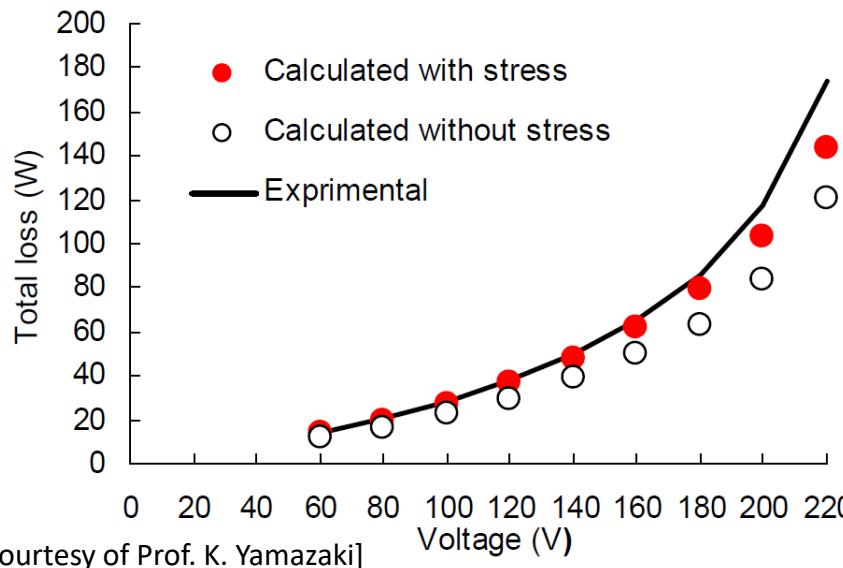
(orthotropic materials)

- Possible equivalence in magnetisation, magnetic susceptibility, magnetic losses, ....

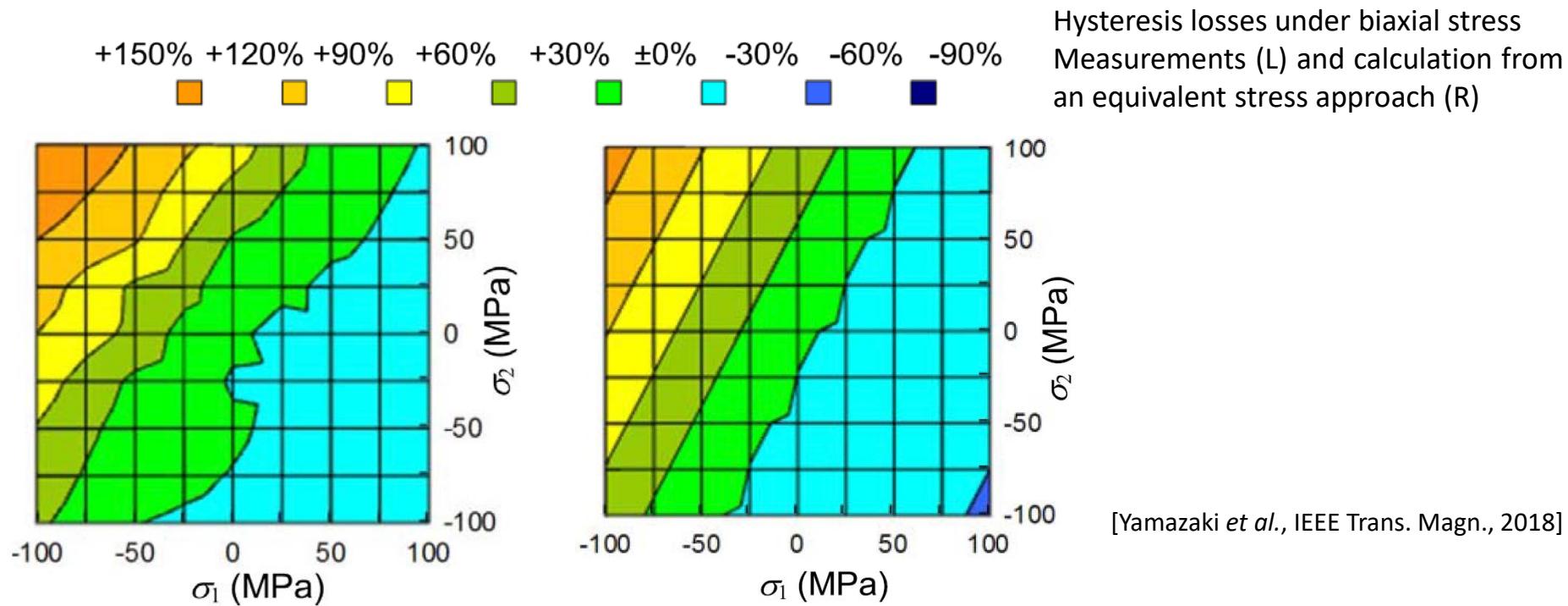
[Daniel and Hubert, J. Appl. Phys. 2009; Hubert and Daniel, J. Magn. Magn. Mater., 2011; Daniel, IEEE Trans. Magn., 2013; Rekik *et al.*, IEEE Trans. Magn., 2014; Daniel, IEEE Trans. Magn., 2017; Pavo *et al.*, IEEE Trans. Magn., 2019]

## A. Equivalent stress approaches

- How to use an equivalent stress :
  - Preliminary knowledge of unaxial coupled behaviour ( $H//\sigma$ )
  - Determination of the stress distribution in the structure
  - Calculation of the equivalent stress
  - Application of the 1D coupled constitutive equation
  - Loop until convergence if needed
- Example : effect of stress on losses in PMSM [Yamazaki and Aoki, 2016]



## A. Equivalent stress approaches



- Only a 1D model required (or measurements)
- Very straightforward implementation
- Accuracy can be poor (based on the relevance of the chosen equivalence)

## B. Thermodynamic macroscopic approaches

- Helmholtz Free Energy in terms of magneto-mechanical invariants

$$\rho\psi = \frac{1}{2}\lambda I_1^2 + 2GI_2 + \frac{1}{2} \sum_{i=0}^4 \left[ \frac{g_i(I_1)}{i+1} \left( \frac{I_4}{B_{ref}^2} \right) I_4 \right] + \frac{1}{2}\alpha_5 I_5 + \frac{1}{2}\alpha_6 I_6$$

$$I_1 = \text{tr}(\boldsymbol{\varepsilon})$$

$$I_5 = \mathbf{B} \cdot \boldsymbol{\varepsilon} \cdot \mathbf{B}$$

$$g_0 = \frac{3}{4}\alpha_0 \exp\left(\frac{3}{4}I_1\right) - \frac{1}{3}(\mu_0 - \alpha_5)$$

$$I_2 = \frac{1}{2} \text{tr}(\boldsymbol{\varepsilon}^2)$$

$$I_6 = \mathbf{B} \cdot \boldsymbol{\varepsilon}^2 \cdot \mathbf{B}$$

$$g_i = \frac{3(i+1)}{4}\alpha_i \exp\left(\frac{4(i+1)}{3}I_1\right)$$

$$I_4 = \mathbf{B} \cdot \mathbf{B}$$

- Cauchy stress tensor and Magnetization vector

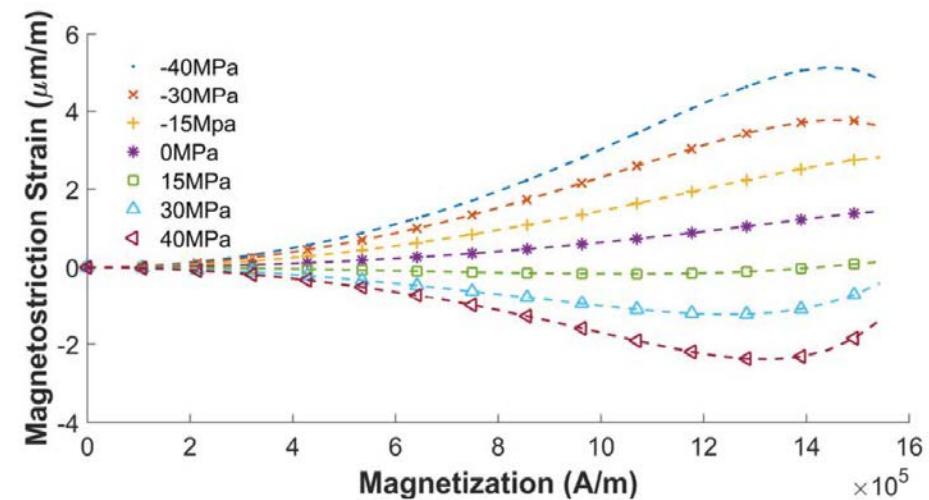
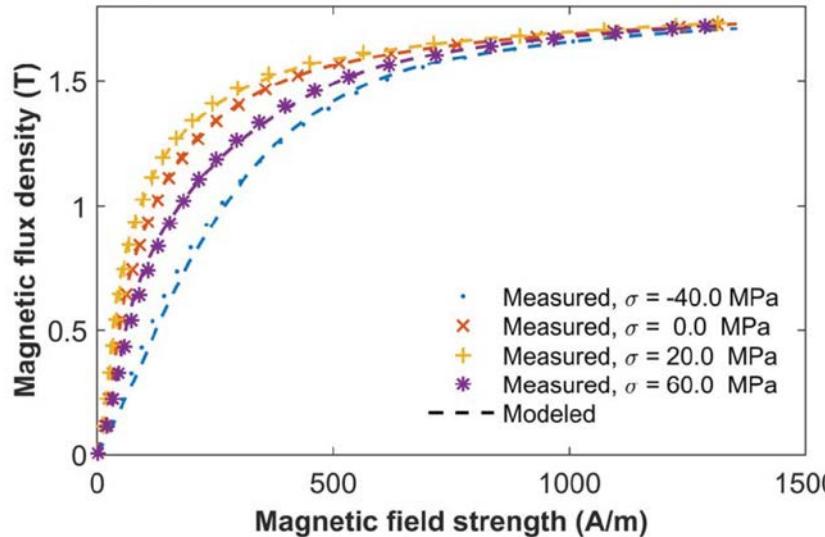
$$\psi_i = \frac{\partial\psi}{\partial I_i}$$

$$\boldsymbol{\sigma} = \rho \sum_{i=1, i \neq 3}^6 \psi_i \frac{\partial I_i}{\partial \boldsymbol{\varepsilon}}$$

$$\mathbf{M} = \rho \sum_{i=1, i \neq 3}^6 \psi_i \frac{\partial I_i}{\partial \mathbf{B}}$$

[Courtesy of Prof. A. Belahcen; Fonteyn *et al.*, IEEE Trans. Magn., 2010; Rasilo *et al.*, IEEE Trans. Magn., 2016]

## B. Thermodynamic macroscopic approaches

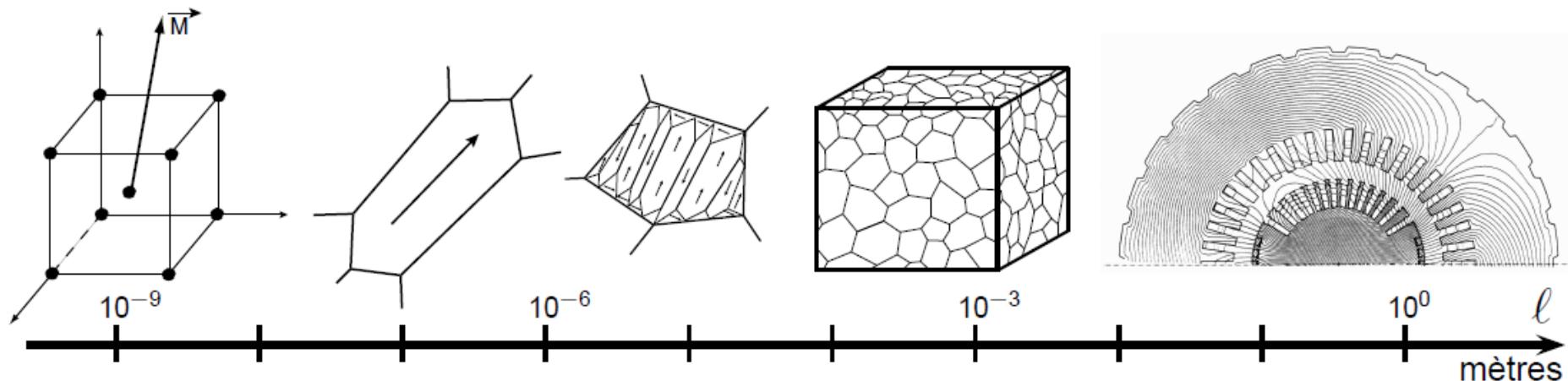


[Aydin *et al.*, IEEE Trans. Magn., 2016 ]

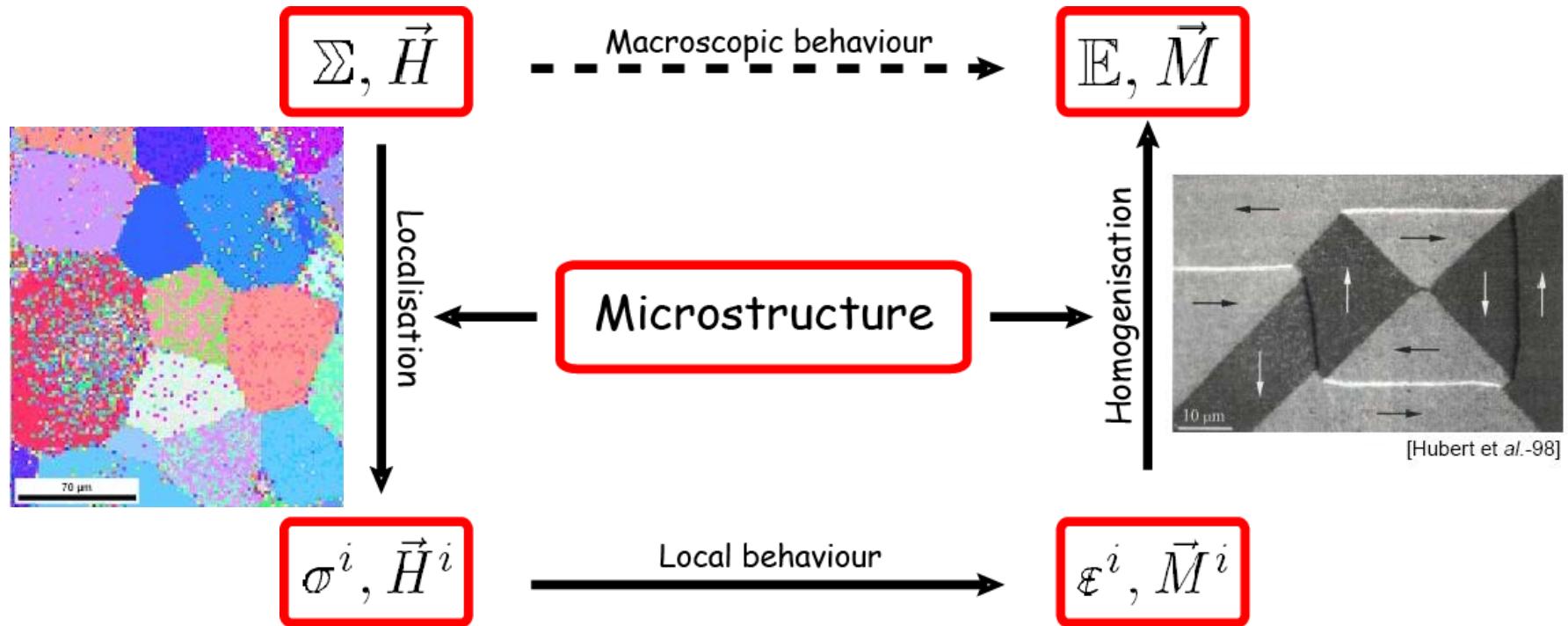
- Good accuracy, fully 3D, efficient numerical implementation
- Descriptive model (no prediction out of the identification range)
- Identification procedures can be complex  
(high number of material parameters)

## C. Multiscale approaches

- Pioneering works by Néel's phase model (1944)
- Further developed by Armstrong (Terfenol-D, 1997), Buirom *et al.* (Fe-Si polycrystals, 1999)
- A recent renewed interest : Daniel *et al.* (2004, 2008, 2014), Hubert *et al.* (2008), van den Berg *et al.* (2010), Vanoost *et al.* (2016), Ito *et al.* (2016), Martin *et al.* (2019),...



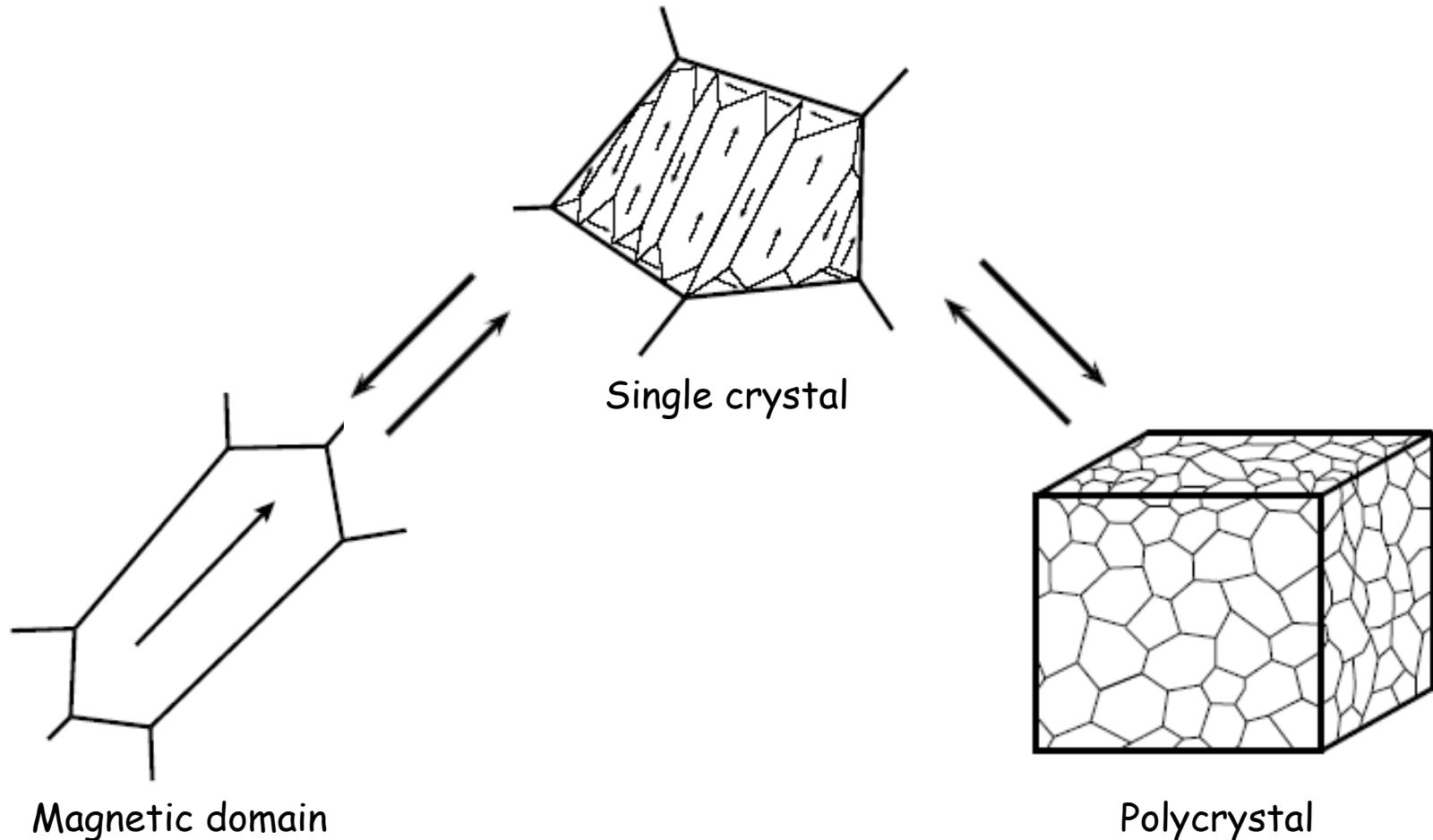
## C. Multiscale approaches



- ★ **Scale transition rules**
- ★ **Local constitutive laws**

## C. Multiscale approaches

- Magnetic materials : three-scale approach



## C. Multiscale approaches

- Scale transition rules
  - Define the local loading (stress, magnetic field) depending on :
    - Macroscopic loading
    - Heterogeneities
    - Material behaviour (+ evolving microstructure)
- Local constitutive laws
  - Local energetic equilibrium (free energy)
  - Statistic description of microstructure (e.g. volume fraction)
  - Evolution laws for internal variables
- Homogenization
  - Appropriate averaging operations

## C. Multiscale approaches: focus on local constitutive law

- Potential energy of a magnetic domain

$$W_\alpha = W_\alpha^{mag} + W_\alpha^{an} + W_\alpha^\sigma + W_\alpha^{conf}$$

- Magnetostatic energy :  $W_\alpha^{mag} = -\mu_0 \vec{H}_\alpha \cdot \vec{M}_\alpha$
- Magnetocrystalline anisotropy energy :  $W_\alpha^{an} = K_1(\gamma_1^2 \gamma_2^2 + \gamma_2^2 \gamma_3^2 + \gamma_3^2 \gamma_1^2)$
- Elastic energy :  $W_\alpha^\sigma = \frac{1}{2} \boldsymbol{\sigma}_\alpha : \mathbf{C}^{-1} : \boldsymbol{\sigma}_\alpha$
- Configuration energy :  $W_\alpha^{conf}$
- Choice of internal variables : volume fractions  $f_\alpha$

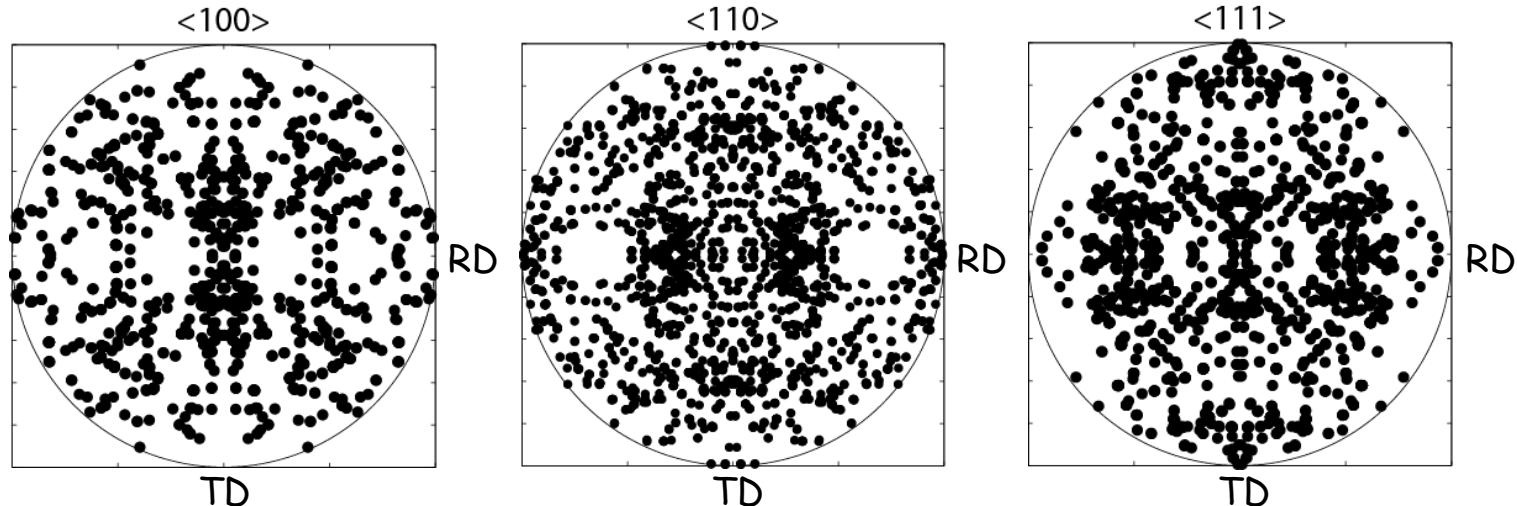
$$f_\alpha = \frac{\exp(-A_s \cdot W_\alpha)}{\sum_\alpha \exp(-A_s \cdot W_\alpha)} \quad A_s = \frac{3\chi^0}{\mu_0 M_s^2}$$

- Magnetisation and magnetostriction at the grain scale

$$\vec{M}^I = \langle \vec{M}_\alpha \rangle = \sum_\alpha f_\alpha \cdot \vec{M}_\alpha \quad \varepsilon_I^\mu = \langle \varepsilon_\alpha^\mu \rangle = \sum_\alpha f_\alpha \cdot \varepsilon_\alpha^\mu$$

## C. Multiscale approaches: material parameters

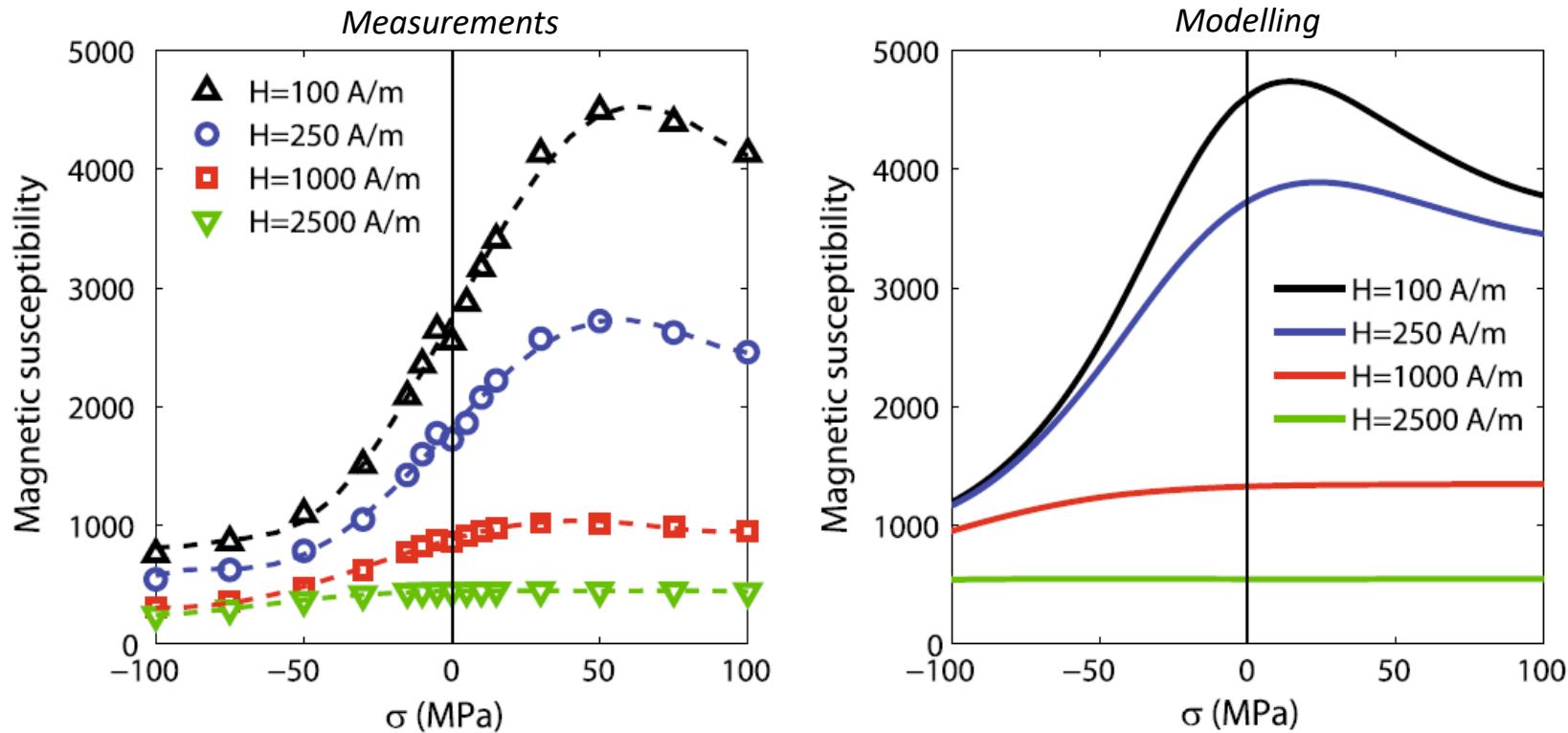
- Single crystal parameters
  - Elastic constants (cubic symmetry) : ( $C_{11}$ ,  $C_{12}$ ,  $C_{44}$ )
  - Magnetic constants :  $M_s$ ,  $K_1$ ,  $K_2$
  - Magneto-elastic constants (cubic symmetry) : ( $\lambda_{100}$ ,  $\lambda_{111}$ )
- Texture data (EBSD measurements)



- Macroscopic parameter  $A_s = \frac{3\chi^0}{\mu_0 M_s^2}$

## C. Multiscale approaches

- Identification of the (physical) parameters on a limited set of experiments
- Prediction in various and independent loading configuration

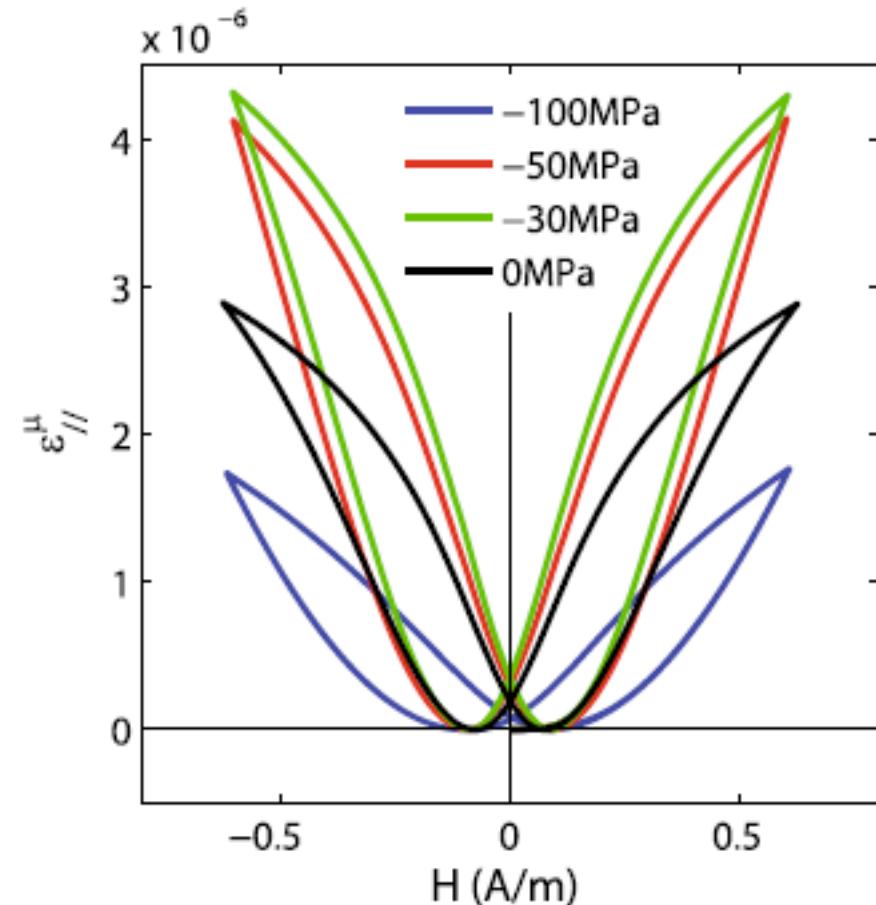
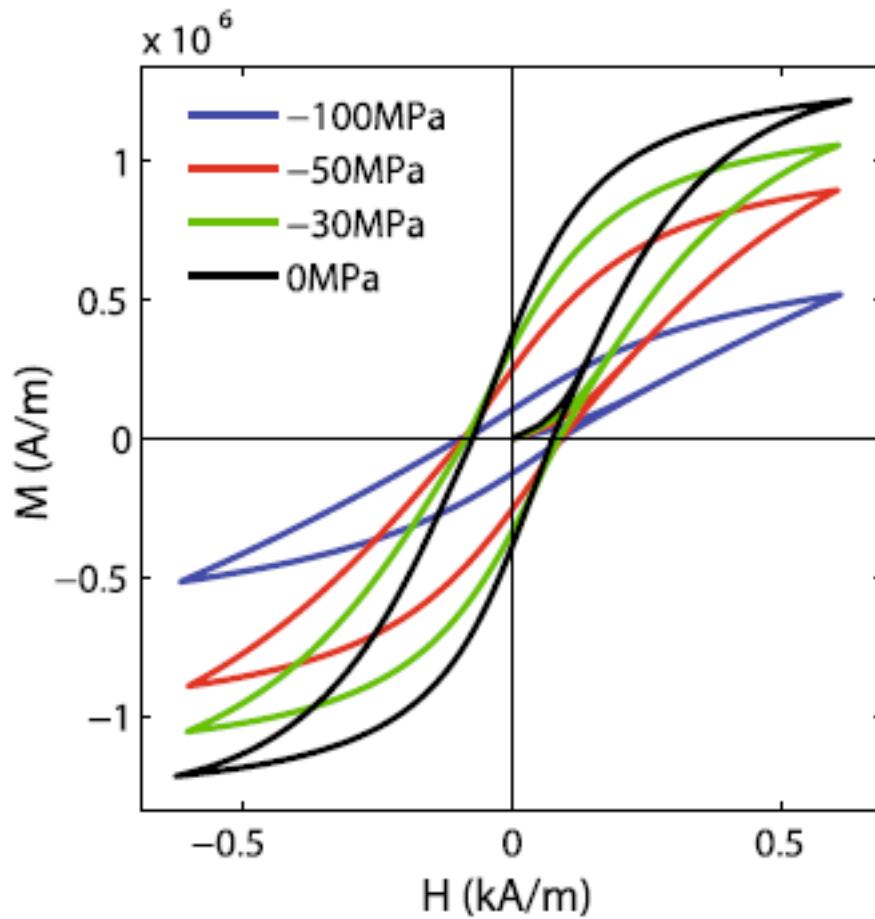


[Daniel et al., Arch. Appl. Mech., 84, 2014]

## C. Multiscale approaches: note on hysteresis

- Phenomenological estimate (Hauser 1994)

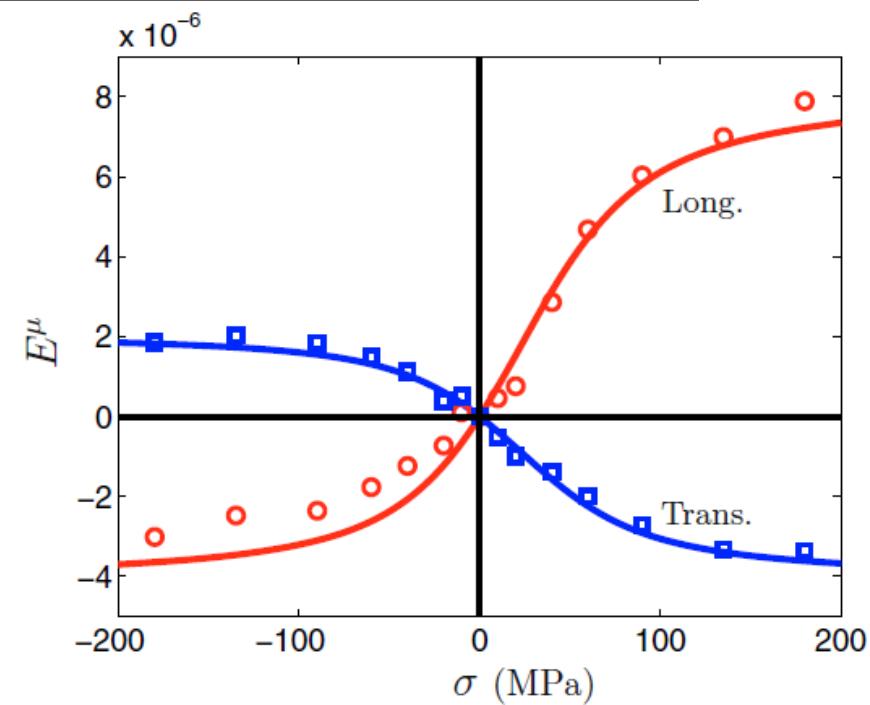
$$H = H_{an} \pm [1 - \alpha \exp(\beta |M_{an} - M_{reb}|)](\gamma + \delta |H_{an}|)$$



[Daniel et al., Arch. Appl. Mech., 84, 2014]

## C. Multiscale approaches

- Fully 3D
  - Description of complex magneto-mechanical effects :
    - Multiaxiality effects
    - $\Delta E$  effect
    - Villari effect
    - Anisotropy effects
  - Parameters with clear physical significance
  - Prediction out of the identification tests range
  - Less accurate than a brute fitting
  - Computation time can hinder numerical implementation
- **Exploitation strategies required for computational magnetics**



[Daniel and Hubert, Eur. J. Phys. Appl. Phys., 2009]

## D. Exploitation strategies: numerical testing machine

- Identification of multiscale models from a limited number of experiments
- Use of multiscale model as a numerical testing machine to identify the parameters of macroscopic models (compatible with numerical computation)
  - Thermodynamic approaches (Azoum *et al.* 2006, Fonteyn *et al.* 2006, 2010, Rasilo *et al.* 2016, 2019 ...)
  - Phenomenological approaches (Zeze et al. 2012, 2013, ...)
- Combine the advantages of thermodynamic and multiscale approaches
- Choose the test suited to the model identification rather than use the available experimental tests

## D. Exploitation strategies: simplified multiscale approaches

- Equivalent single crystal approach [Daniel *et al.*, IEEE Trans. Magn., 2015; ...]
  - Principle: Consider the material as a – fictitious – single crystal with properties equivalent – at macroscopic level – to the real material
    - Heterogeneity effects not included
    - Main mechanisms still described
    - Computational cost significantly reduced
  - Example of a switched reluctance motor
    - Material considered as a collection of magnetic domains

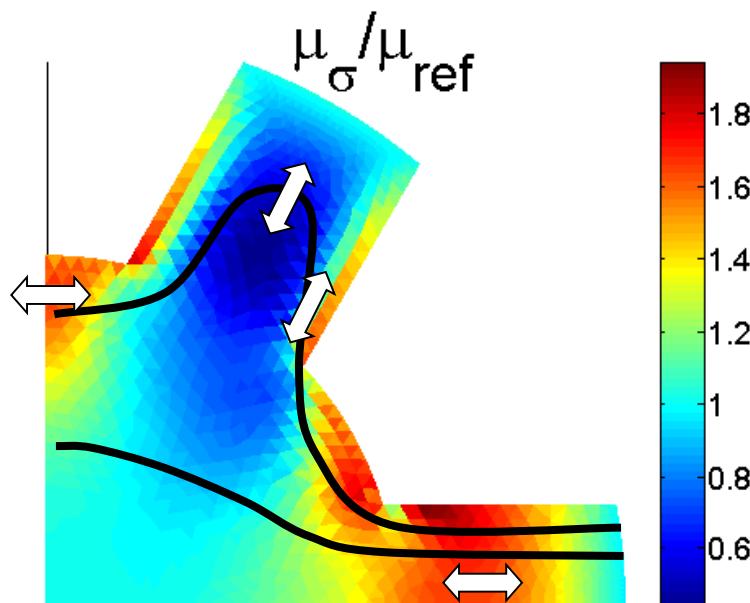
$$W_k = -\mu_0 H M_s \mathbf{x} \cdot \mathbf{u}_k - \boldsymbol{\sigma} : \boldsymbol{\varepsilon}_k^\mu \quad f_k = \frac{\exp(-A_s W_k)}{\int_k \exp(-A_s W_k)}$$

- Coupled with a J-A model for hysteresis effects

## D. Exploitation strategies: simplified multiscale approaches

- Example: Effect of stress on a switched reluctance motor

Effect of inertial stress  
on the local permeability

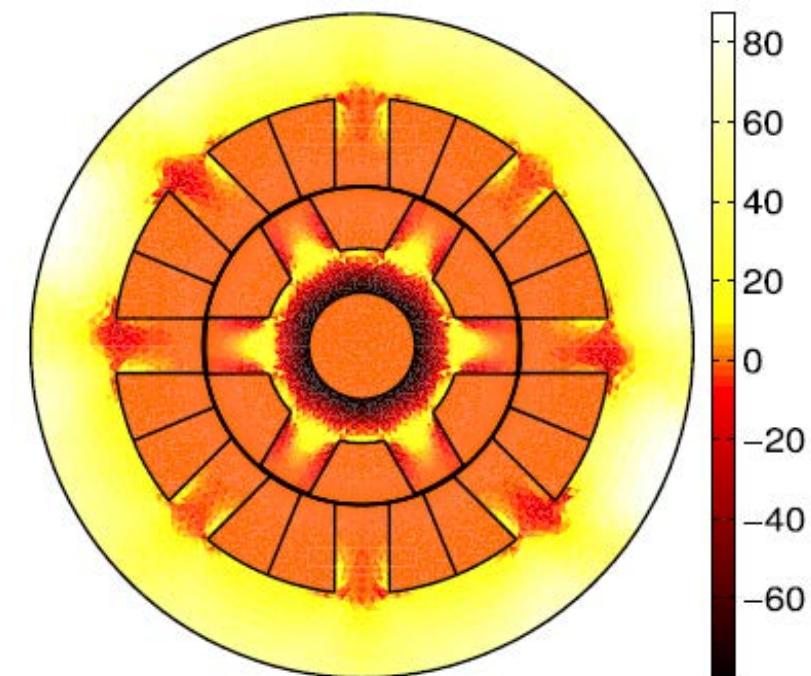


↔ Maximum stress  
— Magnetic field lines

N = 30krpm, R = 34mm

[Bernard *et al.*, IEEE Trans. Magn., 2011]

Effect (%) of shrink fitting  
on the local hysteresis losses



R = 34mm,  $\delta R = 2.5\mu\text{m}$   
[Bernard and Daniel, IEEE Trans. Magn., 2015]

## D. Exploitation strategies: simplified multiscale approaches

- Compact 3D models
- Possible analytical expression for the constitutive equations

$$\mathbf{M} = \frac{\mathbf{A}_x \sinh(\kappa H)}{\mathbf{A}_x \cosh(\kappa H) + \mathbf{A}_y + \mathbf{A}_z} M_s \mathbf{x} \quad [\text{Daniel, IEEE Trans. Magn., 2013}]$$

$$\lambda = \lambda_s \left( 1 - \frac{3(\mathbf{A}_y + \mathbf{A}_z)}{2(\mathbf{A}_x \cosh(\kappa H) + \mathbf{A}_y + \mathbf{A}_z)} \right) \quad [\text{Daniel, EPJ Appl. Phys., 2018}]$$

- Implementation into numerical analysis tools
- Simplifying assumption to be cautiously defined
- Loss of information compared to full multiscale approach

## Conclusion

- Coupled problems such as **magneto-mechanical problems** are a great challenge in computational electromagnetics
- In addition to other challenges in the uncoupled case (non linearity hysteresis, anisotropy, frequency effects ...) they **require specific constitutive equations**
- **Multiscale approaches**, based on the description of physical mechanisms at the appropriate scale are very promising
- Their development require interaction between the **Magnetism and Magnetic materials** community and the **computational electromagnetics** community so that the constitutive equations can be both **physically relevant** and **practically implementable** in numerical tools for structural analysis

# Acknowledgment

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