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# Reconfigurable transfer line balancing problem: A new MIP approach and approximation hybrid algorithm.

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**ABSTRACT:** We consider the problem of balancing reconfigurable transfer lines. The problem is quite recent and motivated by the growing need of reconfigurability in the new INDUSTRY 4.0 context. The problem consists into allocating different tasks necessary to machine a single part to different workstations placed into a serial line. The workstations can contain multiple machines operating in parallel and the tasks allocated to a workstation should be sequenced since the machines considered are mono-spindle head CNC machines and setup times between operations are needed to perform tool changes. Besides precedence constraints between operations are considered. In this article we suggested a new MIP approach based on formulation presented by Andrés et al.(2008) for a resembling problem. We suggest effective pre-processing procedures and a new lower bound. We besides suggest a novel hybrid approach that approximate the optimal solution when the setup times are bounded by the completion times. Experimentation are performed on benchmark instances and the methods are compared with those suggested in the literature.

**KEYWORDS:** Transfer line balancing, Reconfigurability, Industry 4.0, Mixed integer programming, Hybrid algorithms, approximation algorithms

## 1 INTRODUCTION

New consuming trends, global competition and growing variety in demand in the actual economical context raises an important issue in transfer line design. In fact, shortening life cycle times imposes the consideration of reconfigurability in transfer line design. The modern transfer line should be easily and cost-effectively reconfigurable to address two different issues: the variability in production size and the variability in the product specifications. While the first one imposes a variation in cycle time, the second one is linked to the set of tasks involved.

To address this issue, Koren et al.(1999) suggested in late 1990s the novel concept of **Reconfigurable Manufacturing System** (RMS). A RMS could be seen as a serial line of workstations (corresponding to the stages in Figure.1). Each workstation is equipped by multiple machines operating in parallel. Part units are moved from a workstation to another thanks to a *conveyor*. The part is delivered then to the first available machine in a workstation from the *gantry*.

The RMS highly addresses the issue of production size variability. Indeed the ability to add or remove

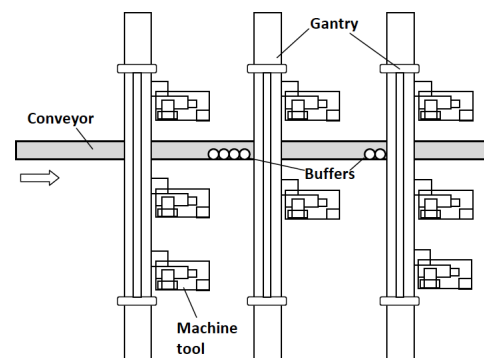


Figure 1 – Reconfigurable Manufacturing System: [Koren 2010]

a machine in a workstation allows monitoring the cycle time with high granularity which is referred to as **scalability** [Koren 2017]. Besides RMS offers a good trade off between productivity and flexibility while *Dedicated Manufacturing System* are highly productive but very poorly flexible and *Flexible Manufacturing System* highly flexible but very expensive and almost never profitable.

Despite being scalable and profitable, RMS could not

address the issue of variability in product specifications only if equipped with **mono-spindle head machines**. Indeed, those machines can perform a huge set of operations, each machine being equipped with a **tool magazine**. To perform an operation, a machine needs a specific tool. Thus, setup times between operations must be considered in addition to operation times in order to perform tool changing.

Once equipped with mono-spindle head machines, RMS addresses both the production size and product specifications variability issues: whenever one or both of these elements comes to change, the manufacturer can easily adjust the production by performing a *reconfiguration* of the system which can be seen as the process of balancing the transfer line taking into consideration setup times and multiple parallel machines at each workstation. Once this operation performed, machines are then added or removed from workstations if necessary, and machines are remotely configured to perform the new sequence of operations; RMS allowing to perform those two steps rapidly and cost-effectively. There is no need of physical machine reconfiguration, since all machines are equipped with the same tool magazine and can thereby perform the same set of operations.

The problem of balancing RMS appears then to be of strategic importance for the manufacturer. In section 2 we define this problem and introduce basic notations, a lower bound and an example are introduced in section 3 then in section 4 the related work is described. A new MIP approach is presented in section 5 and a novel approximation hybrid algorithm is then introduced in section 6. An experimental study was also performed, it is showing highly promising results. It is covered in the section 7 of the paper.

## 2 PROBLEM DEFINITION

The instance of the optimization problem could be described by the following data:

- The set of operations, the corresponding times, setup times and precedence relations.
- A maximum number of workstations to be used placed serially.
- A maximum number of machines per workstation.
- A cycle time.
- A maximum number of operations to be allocated to a workstation.

The optimization problem consists then in finding an allocation of the operations to the workstations and

determining a number of machines per workstation while **minimizing** the number of machines used and respecting the following **constraints**:

- The sum of the times of the operations and the induced setup times of the sequence allocated to a workstation divided by the number of machines in that workstation must not exceed the cycle time.
- Precedence constraints must be respected: when an operation  $i$  precedes an operation  $j$ , the workstation to which the operation  $i$  is allocated must be less (placed before in the line) or as the workstation to which the operation  $j$  is allocated.
- The number of workstations must not exceed the maximum number of workstations.
- The number of operations allocated to a workstation must not exceed the maximum number of operations per workstation.
- The number of machines in a workstation must not exceed the maximum number of machines per workstation.

For the rest of the paper, we use the notations presented in Table.1.

<b>N</b>	Set of operations, indexed on $\{1, 2, \dots, n\}$ .
<b>S</b>	Set of workstations, indexed on $\{1, 2, \dots, s_{max}\}$ $s_{max}$ denoting the maximum number of workstations.
<b>P</b>	Set of couples $(i, j) \in N \times N$ such that $i$ precedes $j$ (also denoted: $i << j$ ).
<b>M</b>	Maximum number of operations to be allocated to a workstation.
<b>M'</b>	Maximum number of machines to be in a workstation.
<b>C</b>	Cycle time.
$t_i$	Completion time of operation $i$ .
$t_{i,j}$	Set-up time to be considered when operation $i$ is performed just before operation $j$ in some workstation

Table 1 – Table of notations.

## 3 RELATED WORK

This problem could be seen as an assembly line balancing problem. Those problems have been well studied in the literature; however those considering parallel machines or sequence dependent setup times have rarely been considered. The originality of the problems comes from the consideration of both elements. We can only list two papers dealing with this issue

with an exact approach: (Essafi et al., 2010) and (Borisovsky et al., 2012 & 2014).

Both approaches fail to solve the problem for medium to large scale instances. Essafi et al.(2010) suggest a MIP approach while Borisovsky et al.(2014) uses a set partitioning model coupled with a constraint generation algorithm.

The MIP approach of Essafi et al. (2010) lies on modelling the overall sequence constituted of the concatenation of the sequences of all the workstations :it uses the variables  $x_{i,q}$  ( $i$  for the operation and  $q$  for the position in the sequence). It can solve instances with 15 operations while the other approach (Borisovsky et al., 2014) can solve instances with up to 50 operations.

#### 4 EXAMPLE AND LOWER BOUND

We first introduce a new lower bound for the problem, we then draw an example and its optimal solution.

Taking into consideration the fact that for every workstation  $j$ , its workload time ( $W_j$ ) and its number of machines  $m_j$  should satisfy:

$$W_j \leq C.m_j$$

Then, the total workload  $W = \sum_{j \in S} W_j$  and the total number of machines  $m = \sum_{j \in S} m_j$  must satisfy:

$$W = \sum_{j \in S} W_j \leq \sum_{j \in S} C.m_j = C. \sum_{j \in S} m_j = C.m$$

i.e:

$$W \leq C.m \tag{1}$$

Let us now assume that  $n > s_{max}$ . Then the workload  $W$  must satisfy:

$$W \geq \sum_{i \in N} t_i + \lambda_{1+n-s_{max}} \tag{2}$$

where  $\lambda_{1+n-s_{max}}$  denotes the  $1 + n - s_{max}$  smallest setup times.

Indeed, (2) is true because the workload is composed of the operations times ( $\sum_{i \in N} t_i$ ) and the induced setup times. And since  $n > s_{max}$ , there must be at least  $n - s_{max}$  operations that are "not alone" at the workstation they are affected to. Any solution must then consider at least  $n - s_{max} + 1$  setup times. (+1 because a sequence of  $k$  operations induce exactly  $k$  setup times)

Then, we have from (1) and (2):

$$\sum_{i \in N} t_i + \lambda_{1+n-s_{max}} \leq C.m$$

From this equation we deduce a new lower bound for the number of machines used if  $n > s_{max}$ :

$$z_{lb} = \left\lceil \frac{\sum_{i \in N} t_i + \lambda_{1+n-s_{max}}}{C} \right\rceil \tag{3}$$

If  $n \leq s_{max}$ , the classical lower bound for SALBP is still available :

$$z_{lb} = \left\lceil \frac{\sum_{i \in N} t_i}{C} \right\rceil$$

Let us now give the example of a small instance, compute the lower bound and give an optimal solution.

The instance is described by the following data:

- The part requires the execution of 7 operations numbered from 1 to 7 ( $n = 7$ ).
- At most 3 stations can be used. ( $s_{max} = 3$ ).
- Precedence constraints are given by:

$$P = \{(1, 3), (2, 3), (3, 4), (4, 5), (5, 6), (5, 7)\}$$

represented in Figure.2.

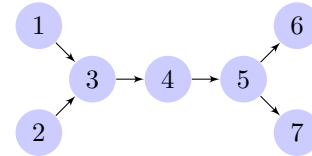


Figure 2 – Precedence graph.

- $M = 3$ , Maximum number of operations that could be allocated to a workstation.
- $M' = 3$ , Maximum number of machines that could be hosted by a workstation.
- Completion times are represented in Table.2:

$i$	1	2	3	4	5	6	7
$t_i$	1.5	1	3.5	1.5	2.5	3	1

Table 2 – Operations times.

- Setup times are represented in Table.3.
- $C = 2.5$ , cycle time.

$t_{i,j}$	$j = 1$	2	3	4	5	6	7
$i = 1$	X	0.5	10	10	10	10	10
2	10	X	0.5	10	10	10	10
3	0.5	10	X	10	10	10	10
4	10	10	10	X	0.5	10	10
5	10	10	10	0.5	X	10	10
6	10	10	10	10	10	X	0.5
7	10	10	10	10	10	0.5	X

Table 3 – Setup times.

Computing the lower bound (equation 3) gives:  $z_{lb} = 7$ .

We describe a realizable solution by allocating to the first workstation the sequence 1, 2, 3 and 3 machines, to the second workstation the sequence 4, 5 and 2 machines and to the third workstation the sequence 6, 7 and 2 machines.

The solution is realizable since the workload divided by the number of machines for each station does not exceed the cycle time:

For station 1:  $(t_1 + t_2 + t_3 + t_{1,2} + t_{2,3} + t_{3,1})/3 = 2.5$

For station 2:  $(t_4 + t_5 + t_{4,5} + t_{5,4})/2 = 2.5$

For station 3:  $(t_6 + t_7 + t_{6,7} + t_{7,6})/2 = 2.5$

Since the solution is realizable and its cost equals the lower bound, it is optimal.

## 5 A MIP FORMULATION AND PRE-PROCESSING PROCEDURES

In this section, we describe a new MIP approach based on a formulation for the sequence-dependent setup times assembly line balancing problem introduced by Andrés et al.(2008) that has not been experimented and does not consider parallel machines in workstations. Besides we suggest novel pre-processing procedures that computes:

- $e_i$ : The earliest station to which operation  $i$  could be affected.
- $l_i$ : The latest station to which operation  $i$  could be affected.
- $M_j$ : The maximum number of operations that could be affected to workstation  $j$ .

We first give the pre-processing algorithms, we then describe the MIP approach for our problem.

While  $x$  denotes the earliest station to ensure that the workload of  $i$  and all its predecessors is done, the condition  $x < y$  captures the fact that  $x$  stations are

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### Algorithm 1 $e_i(i \in N$ : opération)

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$P_i := \{i\} \cup \{j \in N; (j, i) \in P\}$  ( $i$  and its predecessors).

$$x := \left\lceil \frac{\sum_{i \in P_i} t_i}{C.M'} \right\rceil$$

$$y := \left\lceil \frac{\sum_{i \in P_i} t_i + \lambda_1 + |P_i| - x}{C.M'} \right\rceil$$

**while**  $x < y$  **do**

$$x := x + 1$$

$$y := \left\lceil \frac{\sum_{i \in P_i} t_i + \lambda_1 + |P_i| - x}{C.M'} \right\rceil$$

**end while**

**Return**  $x$

---

not enough to ensure that the induced setup times are also processed. The second algorithm computing the latest workstation is quite similar to the first one but considering the successors instead of predecessors and counting from  $s_{max}$  in stead of 1.

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### Algorithm 2 $l_i(i \in N$ : opération)

---

$S_i := \{i\} \cup \{j \in N; (i, j) \in S\}$  ( $i$  and its successors).

$$x := \left\lceil \frac{\sum_{i \in S_i} t_i}{C.M'} \right\rceil$$

$$y := \left\lceil \frac{\sum_{i \in S_i} t_i + \lambda_1 + |S_i| - x}{C.M'} \right\rceil$$

**while**  $x < y$  **do**

$$x := x + 1$$

$$y := \left\lceil \frac{\sum_{i \in S_i} t_i + \lambda_1 + |S_i| - x}{C.M_0} \right\rceil$$

**end while**

**Return**  $s_{max} - x + 1$

---

Once having computed  $e_i$  and  $l_i$ , we could review the maximum number of operations that could be affected to any workstation  $j$  as:  $M_j = \text{Min}(M, |\{i, e_i \leq j \leq l_i, i \in N\}|)$ .

Another pre-processing procedure is run to remove redundant precedence constraints :  $(i, j) \in P$  is removed if there exists a non trivial path between  $i$  and  $j$  in the precedence graph.

We could now describe the MIP approach. This approach is based in modelling the workstations sequences. It uses the following binary **variables**:

- 

$$x_{i,j,s} = \begin{cases} 1 & \text{If operation } i \text{ is affected to workstation } j \text{ in the } s^{th} \text{ position of its sequence.} \\ 0 & \text{If not.} \end{cases}$$

- 

$$y_j = \begin{cases} 1 & \text{If at least one operation is} \\ & \text{affected to workstation } j \\ 0 & \text{If not.} \end{cases}$$

- 

$$z_{i,j,k} = \begin{cases} 1 & \text{If the operation } i \text{ is processed} \\ & \text{just before operation } j \text{ at} \\ & \text{workstation } k. \\ 0 & \text{If not.} \end{cases}$$

- 

$$w_{i,j} = \begin{cases} 1 & \text{If the operation } i \text{ is affected to} \\ & \text{the last position of workstation } j. \\ 0 & \text{If not.} \end{cases}$$

- 

$$v_{j,k} = \begin{cases} 1 & \text{If } k \text{ machines are affected to} \\ & \text{workstation } j. \\ 0 & \text{If not.} \end{cases}$$

We consider the **objective** of minimizing the number of machines used:  $Min \sum_{j \in S} \sum_{k=1}^{M'} k.v_{j,k}$  under the **constraints**:

$$\sum_{j=e(i)}^{l(i)} \sum_{s=1}^{M_j} x_{i,j,s} = 1, \forall i \in N \quad (4)$$

This set of constraints ensures that every operation is affected to exactly one workstation at a unique position of its sequence.

$$\sum_{i \in N, e(i) \leq j \leq l(i)} x_{i,j,s} \leq 1, \forall j \in S, s = 1, \dots, M_j \quad (5)$$

This set of constraints ensures that at most one operation is affected to each position of the sequences of the workstations.

$$\sum_{i \in N, e(i) \leq j \leq l(i)} x_{i,j,s+1} \leq \sum_{i \in N, e(i) \leq j \leq l(i)} x_{i,j,s} \quad (6)$$

$\forall j \in S, s = 1, \dots, m_j - 1$

This set of constraints ensures that no position  $s + 1$  in any workstation is taken by any operation unless the position  $s$  is also taken by some operation.

$$\sum_{k=1}^{M'} v_{j,k} = y_j, \forall j \in S \quad (7)$$

This set of constraints ensures that only one number of machines is chosen for every used workstation.

$$y_{j+1} \leq y_j, \forall j = 1, \dots, s_{max} - 1 \quad (8)$$

This set of constraints ensures that no workstation is used unless its precedent workstation is also used.

$$\sum_{j=e(i)}^{l(i)} \sum_{s=1}^{M_j} (M.(j-1) + s)x_{i,j,s} \leq$$

$$\sum_{j=e(i')}^{l(i')} \sum_{s=1}^{M_j} (M.(j-1) + s)x_{i',j,s}, \forall (i, i') \in P \quad (9)$$

This set of constraints ensure that precedence constraints are satisfied.

$$\sum_{i \in N, e(i) \leq j \leq l(i)} \sum_{s=1}^{M_j} t_i.x_{i,j,s} +$$

$$\sum_{i, i' \in N^2; i \neq i', e(i) \leq j \leq l(i), e(i') \leq j \leq l(i')} t_{i,i'}.z_{i,i',j} \leq$$

$$C. \sum_{k=1}^{M'} k.v_{j,k}, \forall j \in S \quad (10)$$

This set of constraints ensure that the cycle time is not exceeded in any workstation.

$$x_{i,k,s} + x_{i',k,s+1} \leq 1 + z_{i,i',k}, \forall i, i' \in N^2, i \neq i',$$

$$k \in \{e(i), \dots, l(i)\} \cap \{e(i'), \dots, l(i')\}, s = 1, \dots, m_k - 1 \quad (11)$$

This set of constraints ensures that if operation  $i'$  is followed by operation  $i$  at station  $k$  then  $z_{i,i',k}$  is put to 1.

$$x_{i,j,s} - \sum_{i' \in N; i' \neq i, e(i') \leq j \leq l(i')} x_{i',j,s+1} \leq w_{i,j}, \forall i \in N,$$

$$\forall j \in \{e(i), \dots, l(i)\}, s = 1, \dots, m_j - 1 \quad (12)$$

$$x_{i,j,M_j} \leq w_{i,j}, \forall i \in N, \forall j \in \{e(i), \dots, l(i)\} \quad (13)$$

The constraints (13) and (14) ensure that  $w_{i,j}$  is put to one whenever operation  $i$  is positioned in the last occupied position of workstation  $j$ .

$$w_{i,j} + x_{i',j,1} \leq 1 + z_{i,i',j}, \forall i \in N, i' \in N, i \neq i',$$

$$j \in \{e(i), \dots, l(i)\} \cap \{e(i'), \dots, l(i')\} \quad (14)$$

This constraint ensures that if operation  $i$  is positioned in the last occupied position of workstation  $j$  and operation  $i'$  positioned in the first position of workstation  $j$  then  $z_{i,i',j} = 1$  and consequently the setup time  $t_{i,i'}$  is considered in (10).

The MIP is experimented in Section.7.

## 6 AN HYBRID APPROACH

Let's now describe a novel hybrid approach for our problem. Besides, we show that the algorithm is a 2-approximation if we assume that:

$$t_{i,j} \leq t_k, \forall i, j, k \in N \quad (15)$$

which seems to be a very reasonable and realistic assumption while considering industrial instances. (see section.7)

The algorithm consists of two steps. The first consists of solving a MIP while the second one is an exact dynamic programming algorithm for solving the ATSP:

- *Step 1:* Perform a parallel line balancing without taking setup times into consideration using a MIP approach.
- *Step 2:* For each workstation, perform a sequencing of the operations using a dynamic programming algorithm then if the number of machines in the workstations is insufficient to fulfill the cycle time, add the necessary ones. If the maximum number of machines is violated, then we return to step 1 by adding a constraint that forbids the assignment of the set of operations to the workstation. This step is described in more details in section 6.2.

The first subsection is devoted to the first step and the second subsection is devoted to the second step. We refer to this method as "BFSL" (*Balance First, Sequence Last*)

Let us now remark that the solution outputted by the algorithm is feasible and its overall cost ( $c$ ) is given by the cost of the solution outputted by the step1 ( $c_1$ ) plus the number of machines added in step 2 ( $m$ ). i.e

$$c = c_1 + m$$

besides we have  $c_1 \leq c^*$  where  $c^*$  denotes the optimal solution of the RMS balancing problem (because  $c_1$  does not take setup times into consideration).

And thanks to (15) we have  $m \leq c_1$  (because the number of setup times for each workstation is less or equal to the number of operations and then the workload involved by the setup times in each workstation is less or equal to the workload involved by the operations times). Those two inequations ( $c_1 \leq c^*$ ,  $m \leq c_1$ ) finally give:

$$c \leq 2.c^*$$

which shows the approximation ratio.

### 6.1 Step 1: Balancing without setup times

In this step we are concerned with balancing the RMS but without taking into consideration setup-times. This is done by the following MIP that is derived from the precedent one by removing unnecessary variables and constraints. We use the following **variables**:

$$x_{i,j} = \begin{cases} 1 & \text{If operation } i \text{ is affected to} \\ & \text{workstation } j. \\ 0 & \text{If not.} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{If at least one operation is} \\ & \text{affected to workstation } j \\ 0 & \text{If not.} \end{cases}$$

$$v_{j,k} = \begin{cases} 1 & \text{If } k \text{ machines are affected to} \\ & \text{workstation } j. \\ 0 & \text{If not.} \end{cases}$$

We consider the **objective** of minimizing the number of machines used:  $Min \sum_{j \in S} \sum_{k=1}^{M'} k.v_{j,k}$

under the **constraints**:

$$\sum_{j=e(i)}^{l(i)} x_{i,j} = 1, \forall i \in N \quad (16)$$

This set of constraints ensures that every operation is affected to exactly one workstation.

$$\sum_{k=1}^{M'} v_{j,k} = y_j, \forall j \in S \quad (17)$$

This set of constraints ensures that only one number of machines is chosen for every used workstation.

$$\sum_{i \in N, e(i) \leq j \leq l(i)} x_{i,j} \leq M_j, \forall j \in S \quad (18)$$

This set of constraints ensures that the maximum number of operations to be allocated to a workstation is respected.

$$y_{j+1} \leq y_j, \forall j = 1, \dots, s_{max} - 1 \quad (19)$$

This set of constraints ensures that no workstation is used unless its precedent workstation is also used.

$$\sum_{j=e(i)}^{l(i)} j.x_{i,j} \leq \sum_{j=e(i')}^{l(i')} j.x_{i',j}, \forall (i, i') \in P \quad (20)$$

This set of constraints ensures that precedence constraints are satisfied.

$$\sum_{i \in N, e(i) \leq j \leq l(i)} t_i.x_{i,j} \leq C. \sum_{k=1}^{M'} k.v_{j,k}, \forall j \in S \quad (21)$$

This set of constraints ensures that the cycle time is not exceeded in any workstation.

The above MIP introduces far less variables and constraints than the first MIP. Experimentation shows that it can solve instances with very large sets of operations.

## 6.2 Step 2: Sequencing operations in every workstation and adding necessary machines

From step 1, we are given the affectation of operations to the workstations with a number of machines at each workstation. We are now concerned with sequencing the operations in every workstation and since the setup times were not considered in step 1 we may be obliged to add some machines at some workstations to fill the cycle time constraint.

The sequencing problem is an ATSP where operations represent cities and set-up times distances between cities. This operation is performed with an exact dynamic programming algorithm introduced in Held and Karp (1962). We compute then the workload for every workstation and determine easily the number of machines to be added. If more machines than the maximum authorized is required then we return to step 1 and a constraint forbidding the allocation of this set of operations to the workstation is added to the linear model.

## 7 EXPERIMENTAL RESULTS

We describe in this section the experimentation being held on a 16Go RAM computer with JAVA 8 and CPLEX (v12.7.0). Three sets of instances are considered, provided by Borisovsky et al.(2014) and described to be of industrial importance. However those instances consider additional constraints: accessibility, inclusion and exclusion. Those constraints are taken into consideration by adding the necessary linear constraints. Experimentation are summarized in Table.4.  $z^*$  and  $z$  denote respectively the optimal solution and the lower bound. "MIP sol." denote the solution outputted by CPLEX after "MIP time" seconds. If the optimal solution is not found after 10,000 seconds, CPLEX is stopped and the actual best known solution is taken with its corresponding duality gap.  $d$  denotes the density of the precedence graph while  $d_s$  denotes the Scholl density:

$$d_s = \frac{2 \cdot 100 \cdot \sum_{i \in N} |P_i|}{n \cdot (n-1)} \text{ where } P_i \text{ denotes the predecessors of } i.$$

"BFSL" and "BFSL time" denote respectively the solution and the time of the hybrid approach.

First set of instances (p14-10) have a Scholl density

in [5,15]. 6 instances out of 10 have been solved to optimum. With the MIP from Essafi et al.(2010) only 4 out of 10 instances have been solved to optimum with a time limit of 10,000 seconds. Second set of instances (p14-25) have a Scholl density in [15,25]. All instances have been solved to optimum. With the MIP from Essafi et al.(2010) only 5 out of 10 instances have been solved to optimum with a time limit of 10,000 seconds. Third set of instances (p14-40) have a Scholl density in [25,40]. All instances have been solved to optimum. With the MIP from Essafi et al.(2010) only 8 out of 10 instances have been solved to optimum with a time limit of 10,000 seconds.

The MIP was able to solve instances with 20 operations within 10,000 seconds. All the instances were solved by the set partitioning model, besides the set partitioning model was able to solve instances with 50 operations.

## 8 CONCLUSION AND PERSPECTIVES

We can make some remarks from the experimentation:

- The MIP is more efficient with instances having bigger Scholl density.
- The lower bound that we present is on average at 15% of the optimal solution.
- The hybrid approach gives medium results and is very fast. The approximation ratio is satisfied. A posterior local search improvement step would be an interesting perspective.
- The MIP that we present is more efficient than the one presented in Essafi et al.(2010) and less efficient than the algorithm presented in Borisovsky et al.(2014).

We have presented in this paper a new MIP, and a novel hybrid approximation algorithm and held experimentation on both Benchmark and randomly generated instances. The results are quite promising. However, we could take many directions as a continuation of this research:

- Improvement of the BFSL algorithm with posterior local search improvement algorithms for example.
- The use of polyhedral approaches lying on the MIP.
- Research could be done to show a better approximation ratio if there exists some  $\lambda \in [0, 1]$  s.t:

$$t_{i,j} \leq \lambda \cdot t_k, \forall i, j, k \in N$$



Instance	$d$	$d_s$	$z^*$	$z_{lb}$	MIP sol.	MIP time	MIP Gap	BFSL Sol.	BFSL time
p14-10-1	7.69	8.76	11	9	11	4512	0	14	0.78
p14-10-2	7.69	9.89	10	8	10	9558	0	11	0.69
p14-10-4	7.69	9.89	-	9	10	10000	17.04	12	1.93
p14-10-5	7.69	9.89	-	8	11	10000	26.06	13	0.80
p14-10-6	7.69	9.89	-	8	11	10000	25.29	13	0.80
p14-10-7	7.69	9.89	9	8	9	1482	0	10	1.52
p14-10-8	8.79	12.08	11	9	11	9491.69	0	13	0.43
p14-10-9	8.79	9.89	10	9	10	1021	0	12	0.17
p14-10-10	7.69	8.79	9	8	9	13428.47	0	13	4.48
p14-25-1	15.38	16.48	10	8	10	7336	0	13	1.91
p14-25-2	14.28	21.97	10	8	10	406	0	12	1.18
p14-25-3	10.98	21.97	10	8	10	585	0	12	2.07
p14-25-4	10.98	18.68	10	9	10	1543	0	11	0.52
p14-25-5	14.28	19.78	11	8	11	511	0	13	3.86
p14-25-6	15.38	25.27	10	8	10	498	0	10	0.96
p14-25-7	14.28	18.68	10	9	10	2772	0	12	1.82
p14-25-8	13.18	20.87	9	9	9	1822	0	13	0.22
p14-25-9	13.18	23.07	10	8	10	636	0	14	1.18
p14-25-10	12.08	17.58	10	8	10	2658	0	11	1.38
p14-40-1	19.78	36.26	11	8	11	229	0	13	1.31
p14-40-2	21.98	29.67	9	7	9	1778.32	0	12	0.93
p14-40-3	17.58	24.17	10	8	10	4006	0	12	0.41
p14-40-4	18.68	29.67	10	8	10	322	0	11	0.72
p14-40-5	14.28	26.37	10	8	10	838	0	12	1.78
p14-40-6	17.58	38.46	10	8	10	104	0	11	1.22
p14-40-7	17.58	26.37	9	7	9	1368	0	11	0.39
p14-40-8	19.78	26.37	9	8	9	491	0	11	0.40
p14-40-9	16.48	27.47	10	8	10	333.97	0	12	1.32
p14-40-10	16.48	29.67	9	8	9	409	0	10	0.41

Table 4 – Experimentation with benchmark instances.

- It will be more relevant to compare the hybrid approach with the approximate methods rather than the exact methods.
- Studying the problem in an uncertain context is a must to fill with INDUSTRY 4.0 requirements.

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