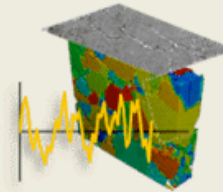


**SF2M**

**34<sup>èmes</sup> Journées  
de Printemps**

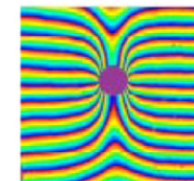
Paris, 27-28 Mai 2015



Microstructure – procédés –  
propriétés en fatigue

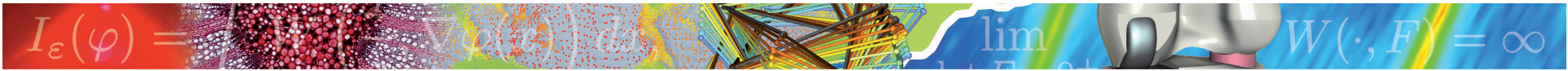
## Prise en compte des mécanismes dissipatifs et des effets de couplage thermomécanique, à l'échelle mésoscopique, pour la tenue en fatigue

**André Chrysochoos**  
**LMGC, Univ. Montpellier**  
**GDR 2519**



GDR CNRS 2519  
MESURES DE CHAMPS  
ET IDENTIFICATION EN  
MECANIQUE DES SOLIDES





# Goals

Observing kinematic and energetic effects accompanying the material transformations

Constructing energy balances – consistency of behavioral models

Quantitative imaging techniques – full-field measurements

IRT : since 1983 – DIC : since 1991 – DIC & IRT : since 2000

## DIC - Mechanics

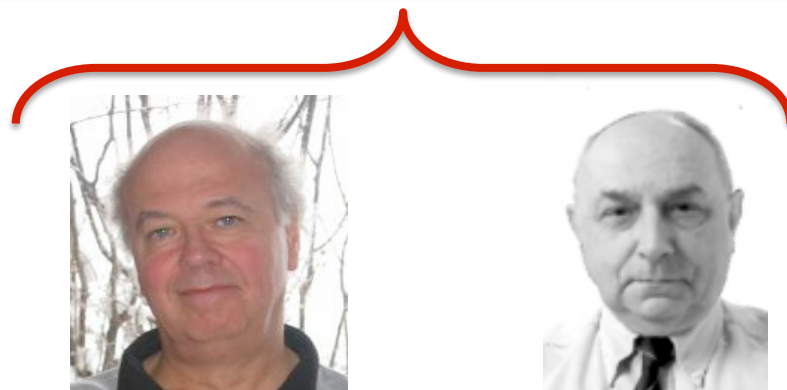
- displacement fields
- strain and strain rate
- stress
- deformation energy rate

## IRT - Thermodynamics

- absolute temperature
- heat source (via the heat equation )
- intrinsic dissipation
- coupling sources

# ThM approach to fatigue

Arcelor <b>Galtier's gang (95-08)</b>	ANR-Disfat <b>Favier's band (10-14)</b>	Durafip - Solvay <b>Robert's squad (12-...)</b>
Méthode thermique, aciers	Méso/micro, aciers + cuivre	Th-h-m... PA6.6 renforcé
LMT-Cachan	PIMM - Paris	ENSAM - PMB
LEMETA -Nancy	LSPM - Paris	Mines - Sofia
LPMM- Metz	LEME - Paris	UC - Louvain
LBMS - Brest	CETIM	LBMS - Brest
...	...	...



# Outline

## Theoretical background

- dissipation, energy storage
- thermomechanical (thm) coupling effects

## Experimental tools

- temperature fields
- displacement fields

## Focusing on cyclic loading

### Case study #1 : HCF of steel

- material vs. structure effects
- properties of intrinsic dissipation fields

### Case study #2 : LCF of rubber

- intrinsic dissipation vs. strong coupling + thermal dissipation

## Concluding comments



# Theoretical background

## Thm constitutive equations

Generalised standard material formalism

[Halphen & Nguyen, 75]

state variables	$\{T, \varepsilon, \alpha\}$
internal/free energy potential	$e(s, \varepsilon, \alpha) \quad \psi(T, \varepsilon, \alpha)$
state equations	$-s = \psi_{,T} \quad \sigma^r = \rho \psi_{,\varepsilon} \quad \mathbf{A}_\alpha = \rho \psi_{,\alpha}$
dissipation potential	$\varphi(q, \dot{\varepsilon}, \dot{\alpha}; T, \dots)$
evolution equations	$-\frac{\nabla T}{T} = \varphi_{,q} \quad \sigma^{ir} = \varphi_{,\dot{\varepsilon}} \quad X_{\dot{\alpha}} = \varphi_{,\dot{\alpha}}$

**Irreversibility**

Material degradation

Heat diffusion

$$d_1 = \sigma^{ir} : \dot{\varepsilon} + X_{\dot{\alpha}} \cdot \dot{\alpha}$$

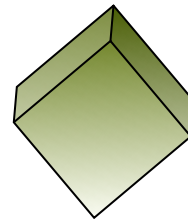
$$d_2 = \varphi_{,q} \cdot q = -\frac{\nabla T}{T} \cdot q$$

# Energy balance (I)

$$\begin{aligned}
 \dot{W}_{\text{def}} &= \sigma : \dot{\varepsilon} = \sigma^r : \dot{\varepsilon} + \sigma^{\text{ir}} : \dot{\varepsilon} \\
 &= \underbrace{\sigma^r : \dot{\varepsilon} + A_\alpha \cdot \dot{\alpha}}_{\dot{W}_e + \dot{W}_s} + d_1
 \end{aligned}$$

$\dot{W}_e$  : rate of elastic energy

$d_1$  : intrinsic dissipation



$\dot{W}_s$  : rate of stored energy



... incomplete balance !!

# Energy balance (II)

- rate of internal energy

$$\rho \dot{e} = \rho C \dot{T} + (\sigma^r : \dot{\varepsilon} + \mathbf{A} \cdot \dot{\alpha}) - (T \sigma^r_{,T} : \dot{\varepsilon} + T \mathbf{A}_{,T} \cdot \dot{\alpha})$$

$$= \rho C \dot{T} + \underbrace{w_e \dot{}}_{\text{« thc »}} + \underbrace{w_s \dot{}}_{\text{« thc »}} - \underbrace{w_{thc} \dot{}}_{\text{« thc »}}$$

« thc » = thermomechanical couplings


- heat equation

$$\rho C \dot{T} + \text{div} \mathbf{q} = \underbrace{\sigma^{ir} : \dot{\varepsilon} - \mathbf{A} \cdot \dot{\alpha}}_{d_1} + T \sigma^r_{,T} : \dot{\varepsilon} + T \mathbf{A}_{,T} \cdot \dot{\alpha} + r_e$$

- comments

C.1:  $C$  specific heat

C.2:  $q = -k \cdot \text{grad} T$

 kinematics required  
 $\downarrow$   
 C.3:  $\dot{T} = \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T$

# Experimental tools

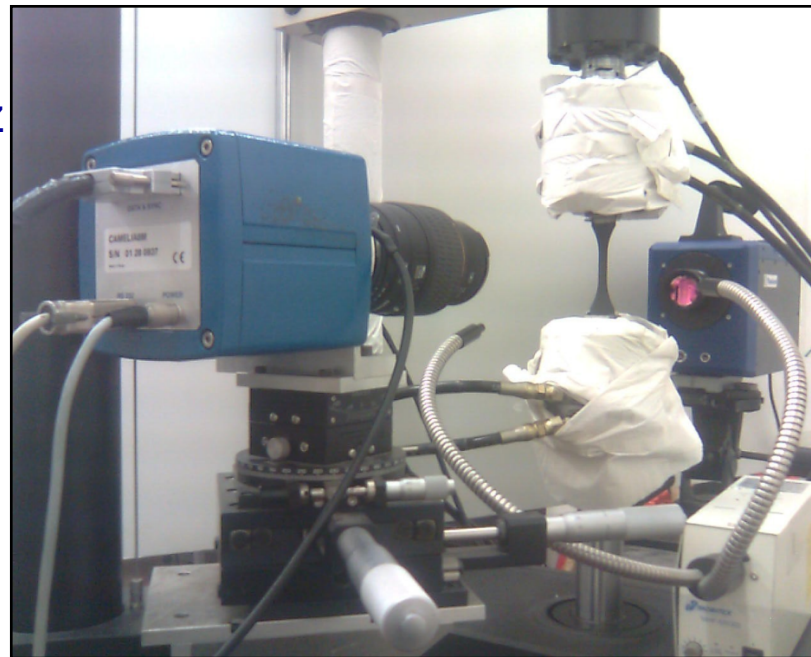
## Quantitative imaging – full field measurement system

- **CCD camera**
- max frame rate 20 Hz
- 1280×1024 pixels
- 13×13  $\mu\text{m}^2$
- 14 bits
- $\delta x \approx 0.1 \text{ mm}$   
(min 15  $\mu\text{m}$ )

### Refs

[Wattrisse et al.,  
Exp. Mech, 2001]

[Chryso et al.,  
IJES, 2000]

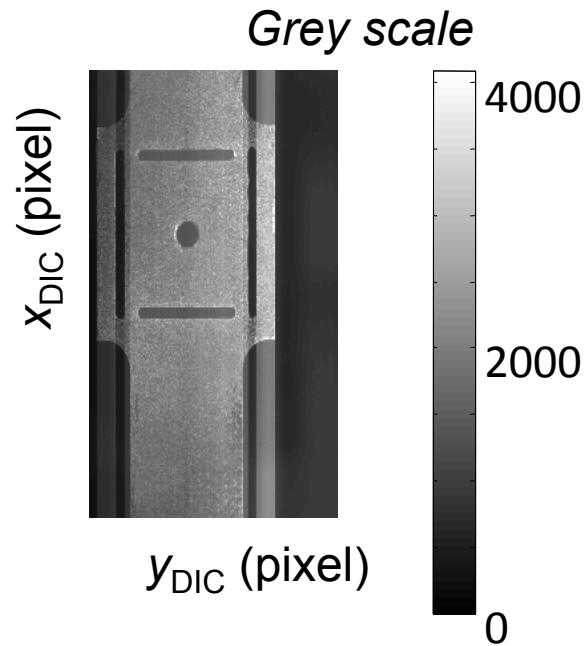


- **IRFPA camera**
- [3,5]  $\mu\text{M}$
- max frame rate 250 Hz
- 640×320 pixels
- 18×23  $\mu\text{m}^2$
- 14 bits
- $\delta x \approx 0.1 \text{ mm}$   
(min 25  $\mu\text{m}$ )
- $\delta T \approx 0.02 \text{ }^\circ\text{C}$

- **Hydraulic testing machine**
- load cell :  $\pm 25 \text{ kN}$
- frame :  $\pm 100 \text{ kN}$
- $\max(f_L) = 50 \text{ Hz}$  en  $R_s = -1$

# Combining DIC & IRT

Speckle image

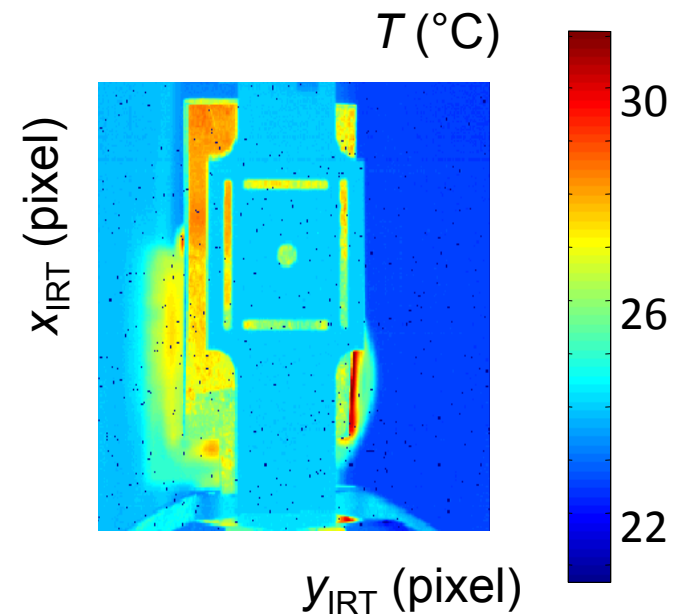


$$U(t_{DIC}, x_{DIC}, y_{DIC})$$

Map-to-map  
correspondence

$$T(t_{IRT}, x_{IRT}, y_{IRT})$$

IR image



[Chryso et al.,  
JoMMS, 2010]

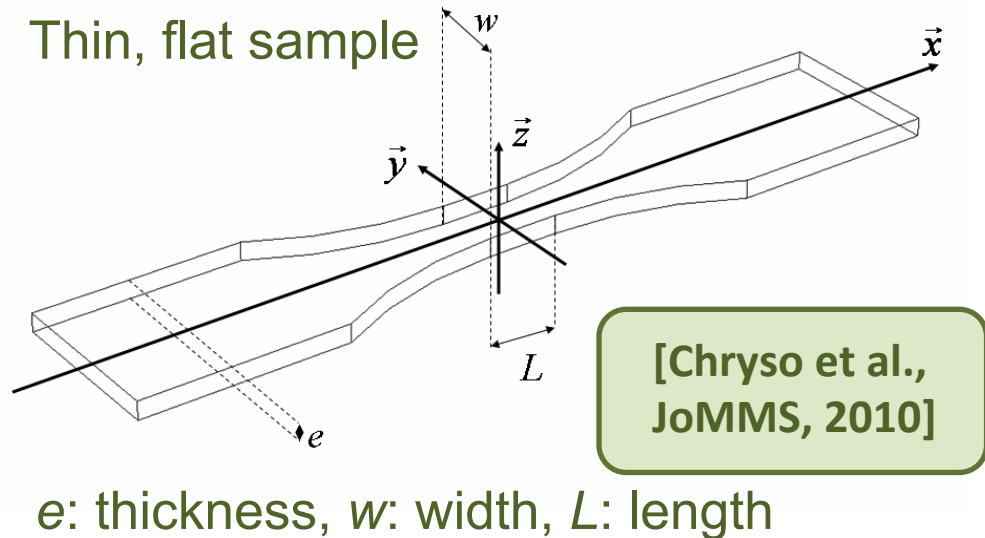
$t_{DIC}$   
( $x_{DIC}, y_{DIC}$ )

synchrocam  
target

$t_{IRT}$  (50  $\mu$ s)  
( $x_{IRT}, y_{IRT}$ ) ( $\approx 0.5$  pixel)



# Stress derivation



$$\sigma_{xx}(x,t) = \frac{F(t)}{S_0} \exp(\varepsilon_{xx}(x,t))$$

$$\sigma_{xy}(x,y,t) = -\sigma_{xx}(x,t) \frac{\partial \varepsilon_{xx}(x,t)}{\partial x} y$$

$$\sigma_{yy}(x,y,t) = \frac{\partial}{\partial x} \left( \frac{\sigma_{xx}(x,t)}{2} \frac{\partial \varepsilon_{xx}(x,t)}{\partial x} \right) \cdot \left( \frac{w(x,t)^2}{4} - y^2 \right)$$

① stress triaxiality neglected

plane stress

$$\begin{cases} \frac{\partial \sigma_{xx}(x,y,t)}{\partial x} + \frac{\partial \sigma_{xy}(x,y,t)}{\partial y} = 0 \\ \frac{\partial \sigma_{xy}(x,y,t)}{\partial x} + \frac{\partial \sigma_{yy}(x,y,t)}{\partial y} = 0 \end{cases}$$

② no volume variation

③ uniform distribution of tensile stress over a cross-section

④ no overall shear loading

⑤ no lateral stress

# Heat rate

Heat equation averaged over the sample thickness

$$\rho C \left( \frac{\partial \bar{\theta}}{\partial t} + v_x \frac{\partial \bar{\theta}}{\partial x} + v_y \frac{\partial \bar{\theta}}{\partial y} + \frac{\bar{\theta}}{\tau_{th}} \right) - k \left( \frac{\partial^2 \bar{\theta}}{\partial x^2} + \frac{\partial^2 \bar{\theta}}{\partial y^2} \right) = \bar{w}_h$$

Direct estimate of heat sources using noisy and discrete thermal data

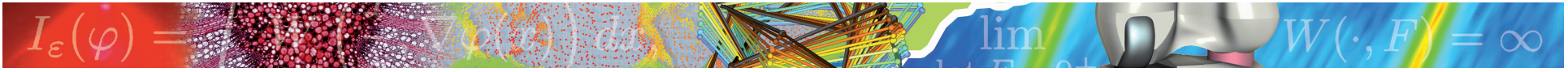
## *Thermal noise*

- uniform power spectrum
- Gaussian probability distribution

**Refs:**  
 IJES : 2000  
 EXP-MECH : 2007  
 JoMMS : 2010  
 EXP-MECH : 2014

## *Estimate of the partial derivative operators*

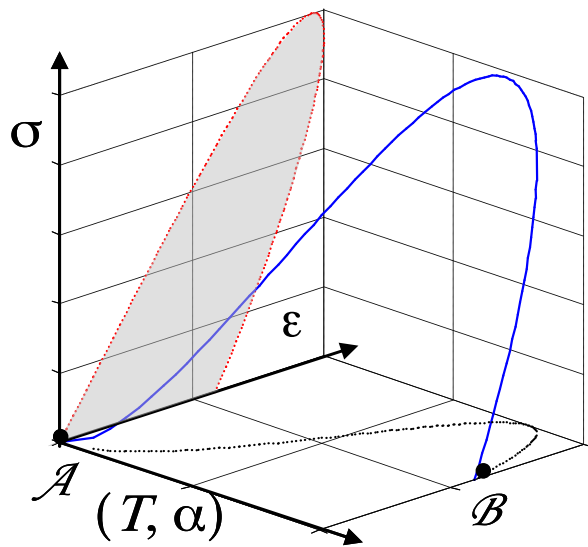
- thermal data projection onto spectral solutions (1995)
- periodic expansion and convolutive filtering by DFT (2000)
- local approximation of  $\theta$  using l.sq. fitting (2004)
- POD: pre-filtering of thermal fields (2013)



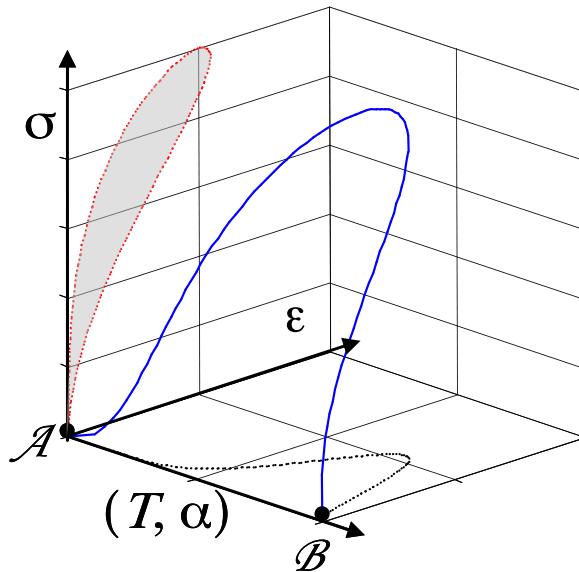
# Focusing on cyclic tests

Monochromatic uniaxial  
loading

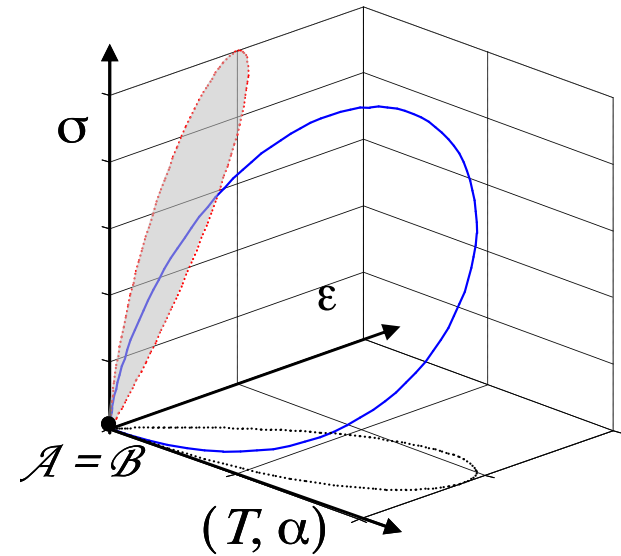
# Focus on a load-unload cycle



(i)  $\mathcal{A} \neq \mathcal{B}$



(ii)  $\varepsilon_{\mathcal{A}} = \varepsilon_{\mathcal{B}}$



(iii)  $\mathcal{A} = \mathcal{B}$

(i) 
$$W_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} \sigma : \dot{\varepsilon} dt = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1 dt + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} (\rho \dot{e} - \rho C \dot{T} + w_{\text{thc}} \dot{\phantom{t}}) dt$$

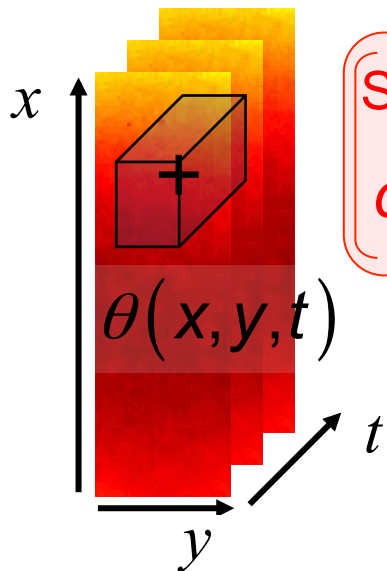
(ii) Hysteresis loop :  $W_{\text{def}} = A_h$  ( for uniaxial loading)



(iii) Load-unload cycle = thermodynamic cycle  $W_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1 dt + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} w_{\text{thc}} \dot{\phantom{t}} dt$

# IR image processing

$$\rho C \left( \frac{\partial \bar{\theta}}{\partial t} + \frac{\bar{\theta}}{\tau_{th}} - \frac{k}{\rho C} \left( \frac{\partial^2 \bar{\theta}}{\partial x^2} + \frac{\partial^2 \bar{\theta}}{\partial y^2} \right) \right) = \bar{d}_1 + \bar{s}_{the}$$



Slow evolution of mean dissipation

$$\tilde{d}_1 = \int_{\text{cycle}} f_L \bar{d}_1 d\tau$$

In phase with loading

$$\tilde{w}_{the} = \int_{\text{cycle}} f_L \bar{s}_{the} d\tau$$

Linear PDE + Linear BC

$$\bar{\theta} = \bar{\theta}_d + \bar{\theta}_{the}$$

Local approximation function

$$\theta_{\text{app}}(x, y, t) = \underbrace{p_1(x, y) \cos(2\pi f_L t) + p_2(x, y) \sin(2\pi f_L t)}_{\text{periodic response}} + \underbrace{p_3(x, y) t + p_4(x, y)}_{\text{drift}}$$

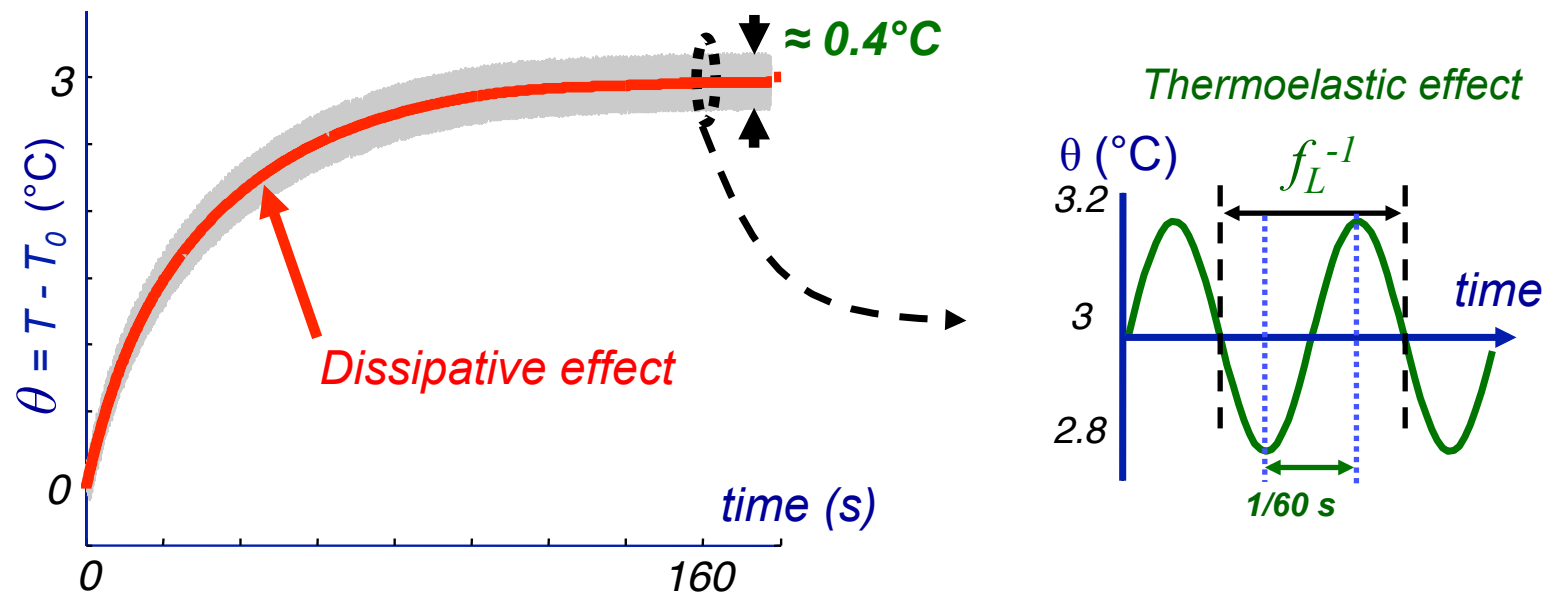
$p_i(x, y), i=1, \dots, 4$ , are 2nd order polynomials of  $x$  and  $y$



# Thermal and calorific effects

HCF test on DP 600 steel

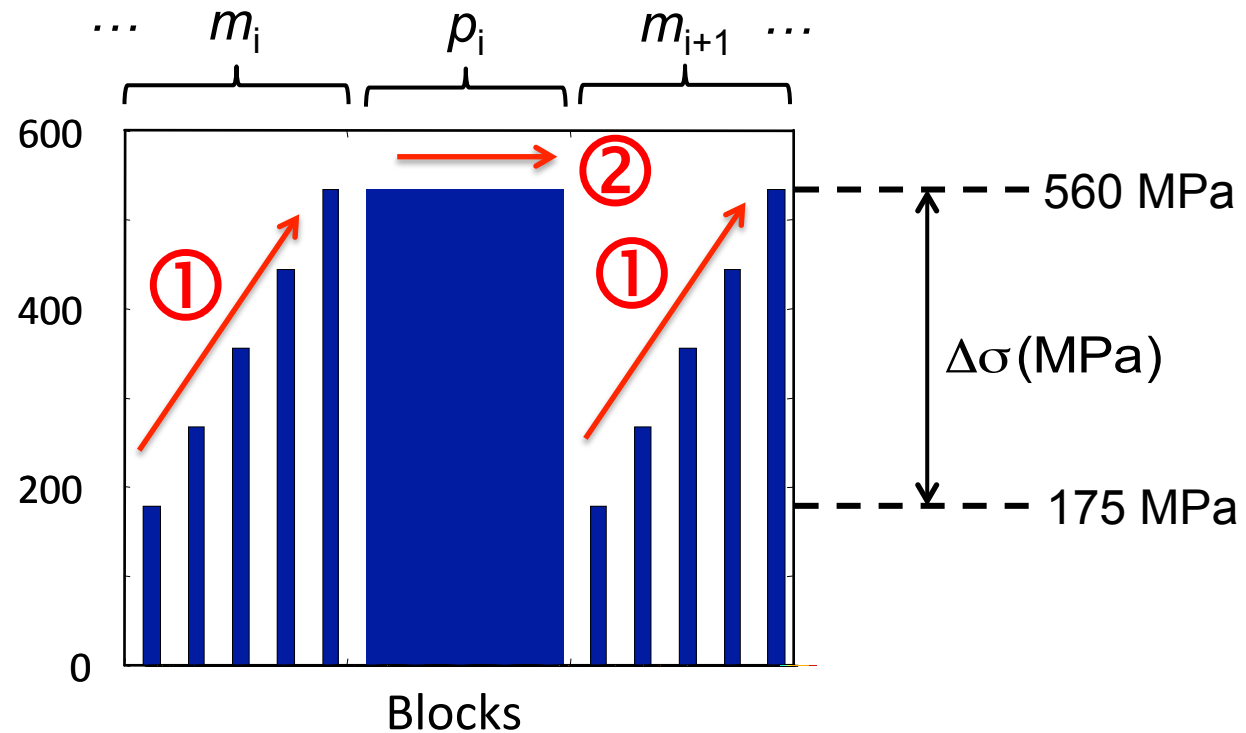
$\Delta\sigma = 500 \text{ MPa}$ ,  $R_\sigma = \sigma_{min} / \sigma_{max} = -1$  et  $f_L = 30 \text{ Hz}$



$0.4^\circ\text{C}$  every  $1/60 \text{ s}$   $\longrightarrow$   $\frac{\Delta s_{the}}{\rho C} \approx 75^\circ\text{C}\cdot\text{s}^{-1}$   
 $3^\circ\text{C}$  within  $160 \text{ s}$   $\longrightarrow$   $\frac{\tilde{d}_1}{\rho C} \approx 0.1^\circ\text{C}\cdot\text{s}^{-1}$

# Fatigue of steels

[Boulanger, PhD 2004]  
 [Berthel, PhD 2008]  
 [Blanche, PhD 2012]

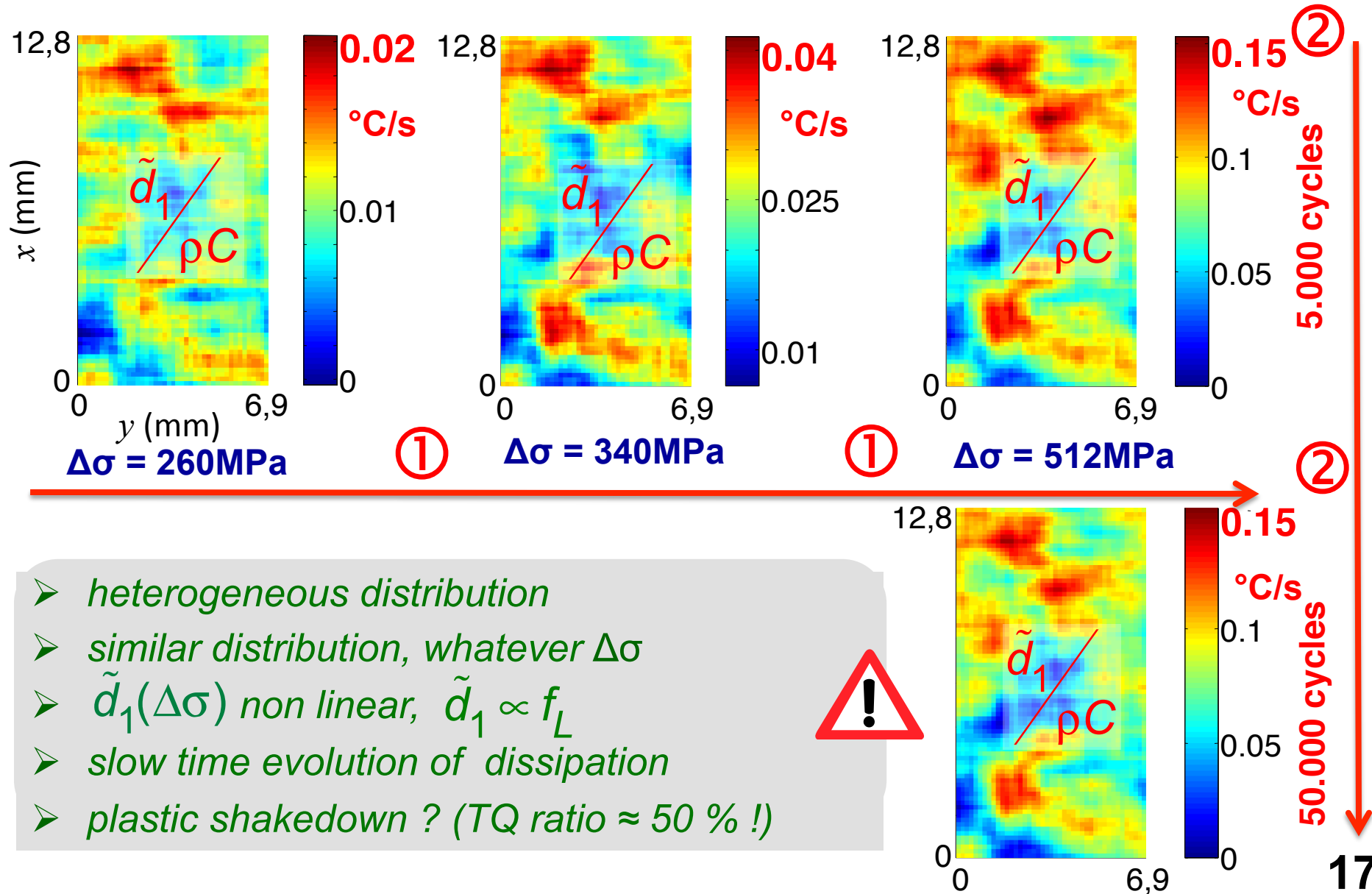


- ①  $m_i$ : series of “mini” cycle blocks (3000 cycles) at different stress ranges:  
**energy balance at “constant fatigue state”**
- ②  $p_i$ : large blocks (100 000 cycles) at constant stress range:  
**energy balance evolution induced by fatigue mechanisms**

highest stress range  $\approx$  fatigue limit

# Dissipation properties (I)

$f_L = 30\text{Hz}$  and  $R_\sigma = -1$



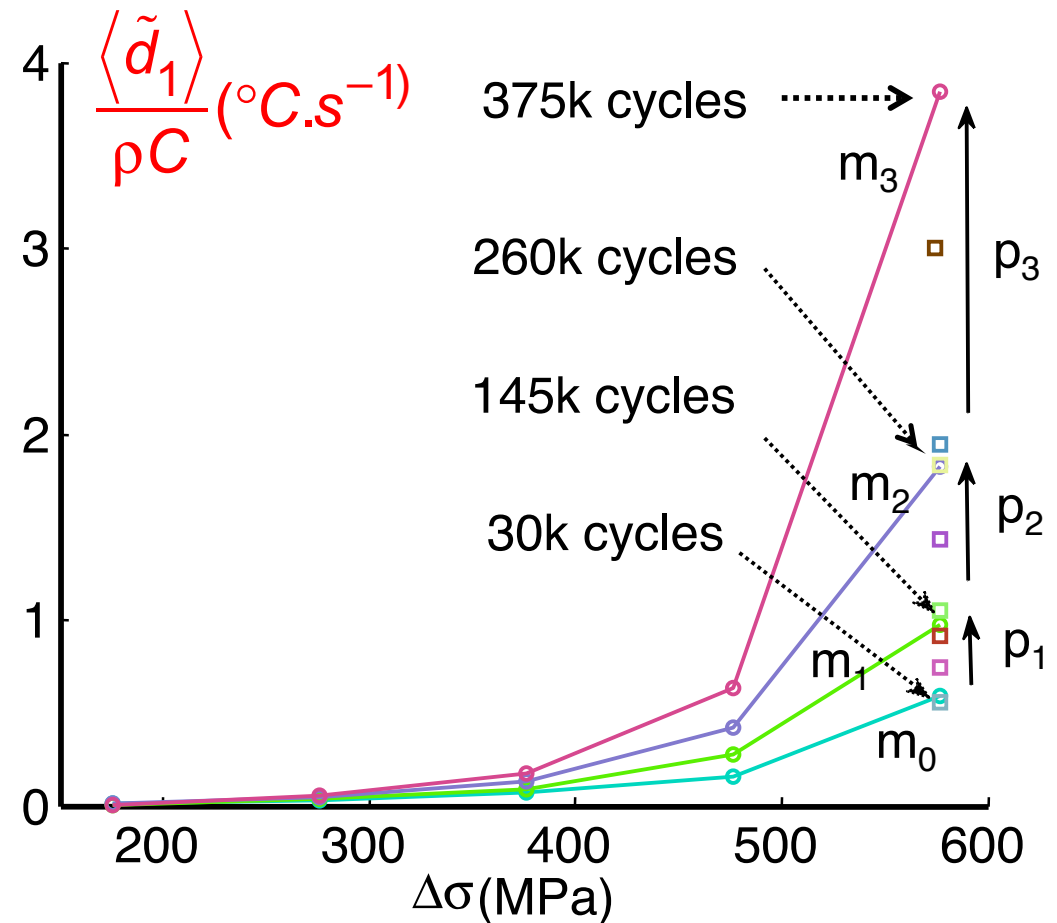
- *heterogeneous distribution*
- *similar distribution, whatever  $\Delta\sigma$*
- *$\tilde{d}_1(\Delta\sigma)$  non linear,  $\tilde{d}_1 \propto f_L$*
- *slow time evolution of dissipation*
- *plastic shakedown ? (TQ ratio  $\approx 50\%$  !)*

# Dissipation properties (II)

## Interpretation of curves

$m_i$  = dissipation induced by activated “micro-defects” at constant fatigue state for different stress ranges

$p_i$  = dissipation drift at constant stress range, reflecting a slow evolution of the fatigue state



➤ energy safeguard: kinetics of fatigue progress

# Case study #2 : back to thm couplings

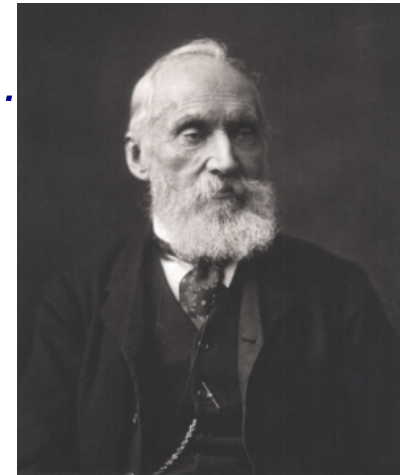
The most simplistic non-adiabatic thermoelastic (the) model...

$$\begin{cases} \varepsilon = \frac{\sigma}{E} + \lambda_{th}(T - T_0) \\ \dot{T} + \frac{T - T_0}{\tau_{th}} = -\frac{E\lambda_{th}T\dot{\varepsilon}}{\rho C} \end{cases}$$

$\longleftrightarrow$

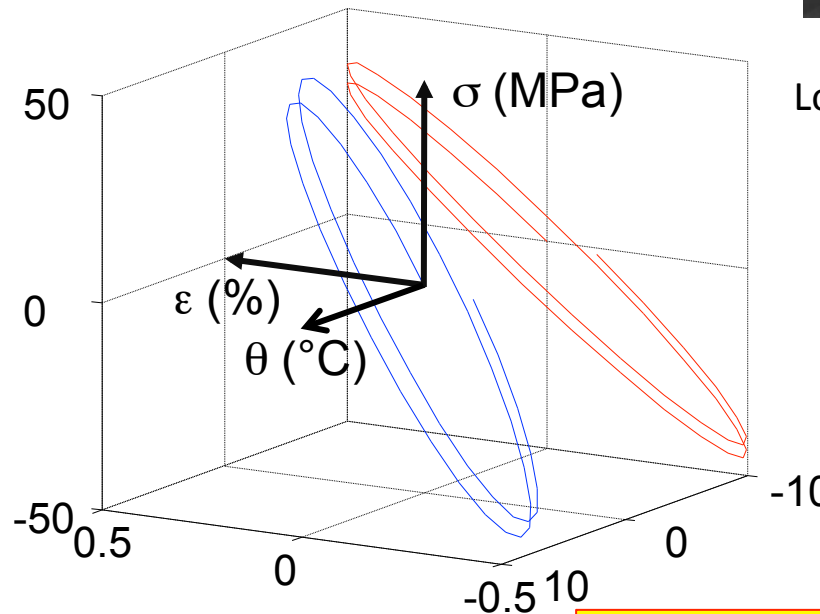
« 0D » approach

linear heat losses



William Thomson  
Lord Kelvin (1824-1907)  
Zener 1937

$E = 1000 \text{ MPa}$   
 $\rho = 1000 \text{ kg.m}^{-3}$   
 $C = 1000 \text{ J.kg}^{-1}.\text{K}^{-1}$   
 $\lambda_{th} = 50 \cdot 10^{-5} \text{ K}^{-1} \times \mathbf{100}$   
 $\tau_{th} = 30 \text{ s}$   
 $T_0 = 294 \text{ K}$



Toward a  
stabilised  
cycle...

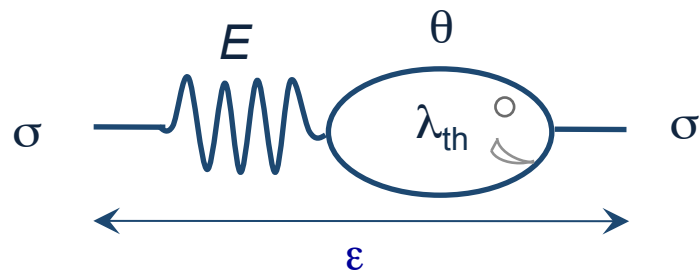
Thm couplings + thermal dissipation

$$\tilde{w}_{def} = A_h = \tilde{w}_{the}$$



# Thm couplings vs. viscosity

*Thermoelastic coupling (i.e.  $d_1=0$ )*  
*strong coupling framework*



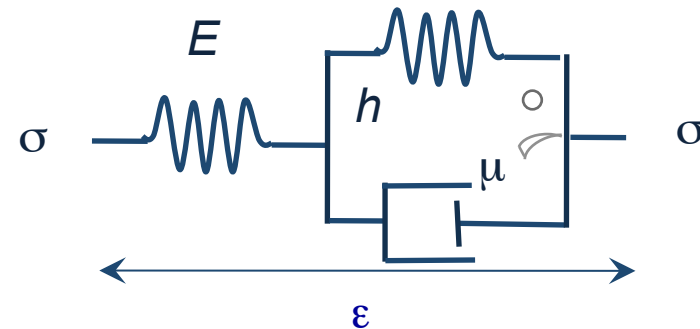
state variables  $(\theta, \varepsilon)$

$$\begin{cases} \varepsilon = \frac{\sigma}{E} + \lambda_{th} \theta \\ \dot{\theta} + \frac{\theta}{\tau_{th}} = -\frac{E\lambda_{th}(T_0 + \theta)\dot{\varepsilon}}{\rho_0 C_0} \end{cases}$$

rheological equation

$$\sigma + \tau_{th} \dot{\sigma} \approx E\varepsilon + E\tau_{th}(1 + \chi)\dot{\varepsilon}$$

*Viscous dissipation*  
*weak coupling framework*



state variables  $(\varepsilon, \varepsilon_v)$

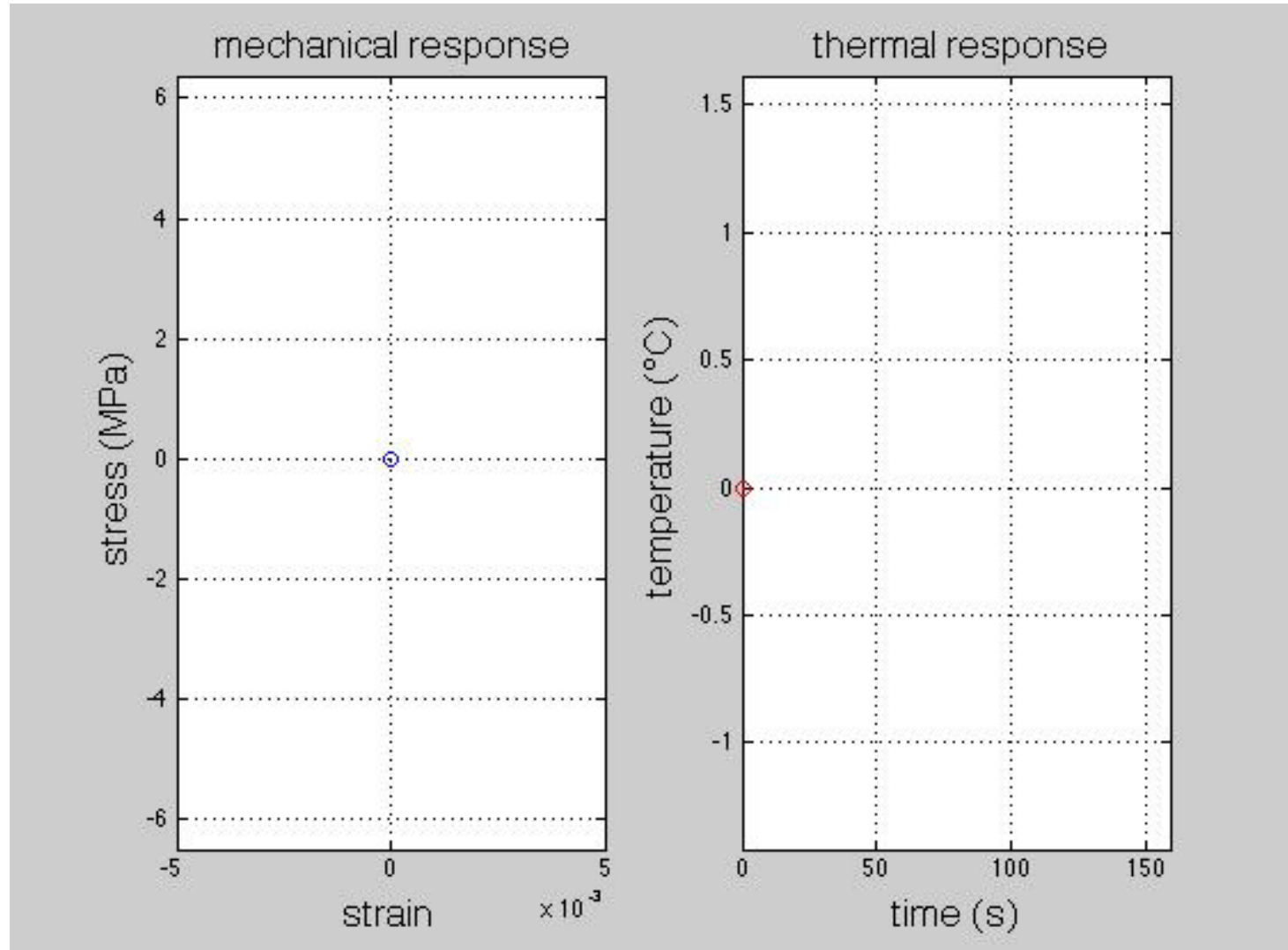
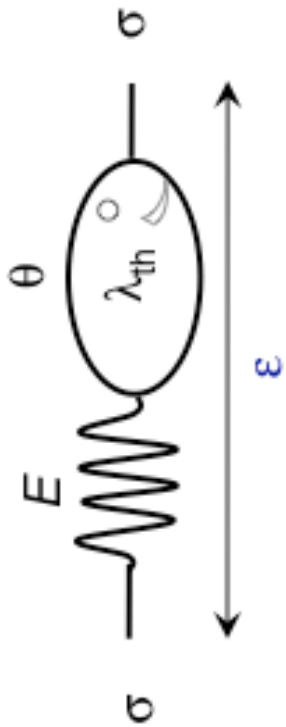
$$\begin{cases} \sigma = E(\varepsilon - \varepsilon_v) \\ \sigma = h\varepsilon_v + \mu \dot{\varepsilon}_v \end{cases}$$

rheological equation

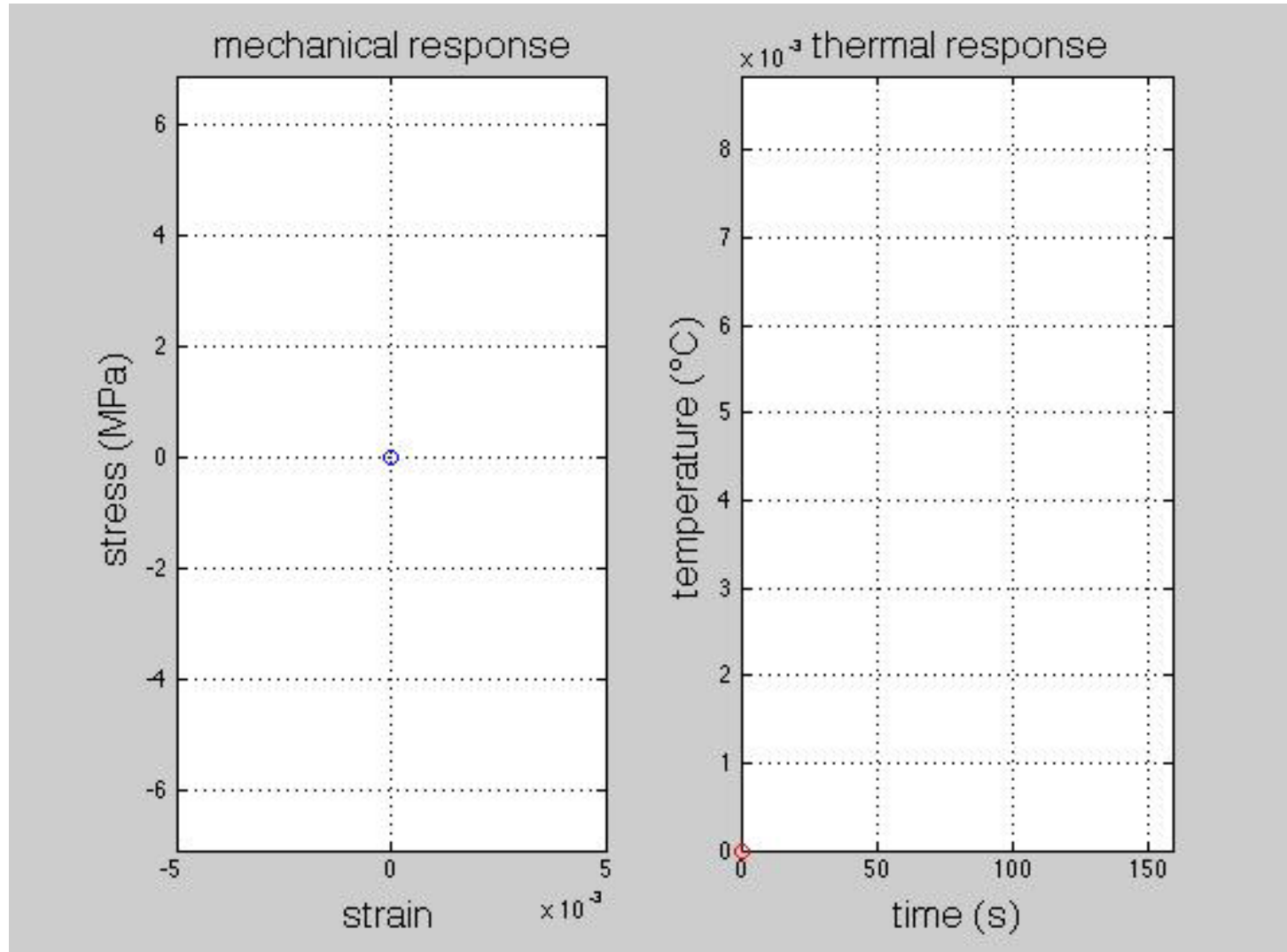
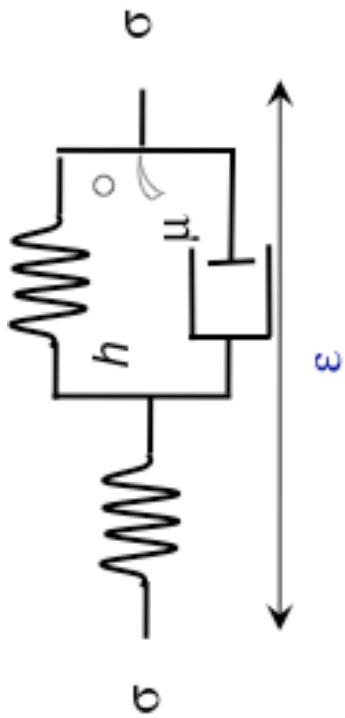
$$\sigma + \frac{\mu}{E+h} \dot{\sigma} = \frac{Eh}{E+h} \varepsilon + \frac{E\mu}{E+h} \dot{\varepsilon}$$



# Thermoelastic coupling effects



# Viscous dissipative effects

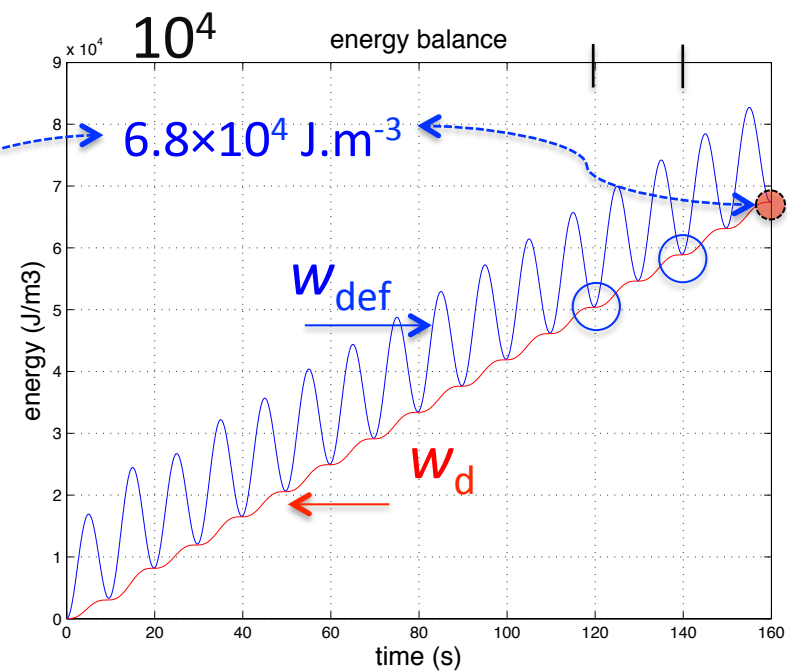
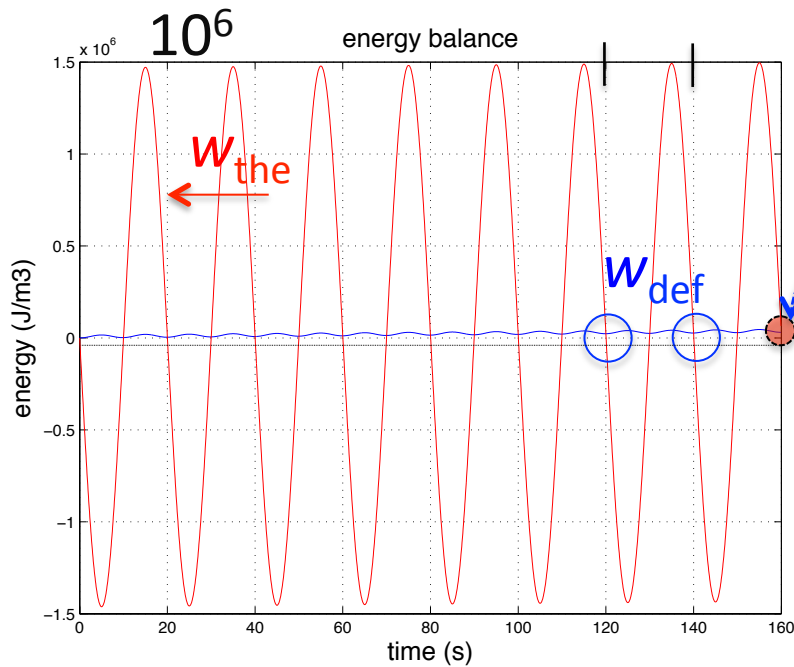


# Energy balance : comparison

## Thermoelasticity

vs.

## Viscoelasticity



$$\Delta W_{the} \gg \Delta W_{def}$$

$\Delta =$  ranges

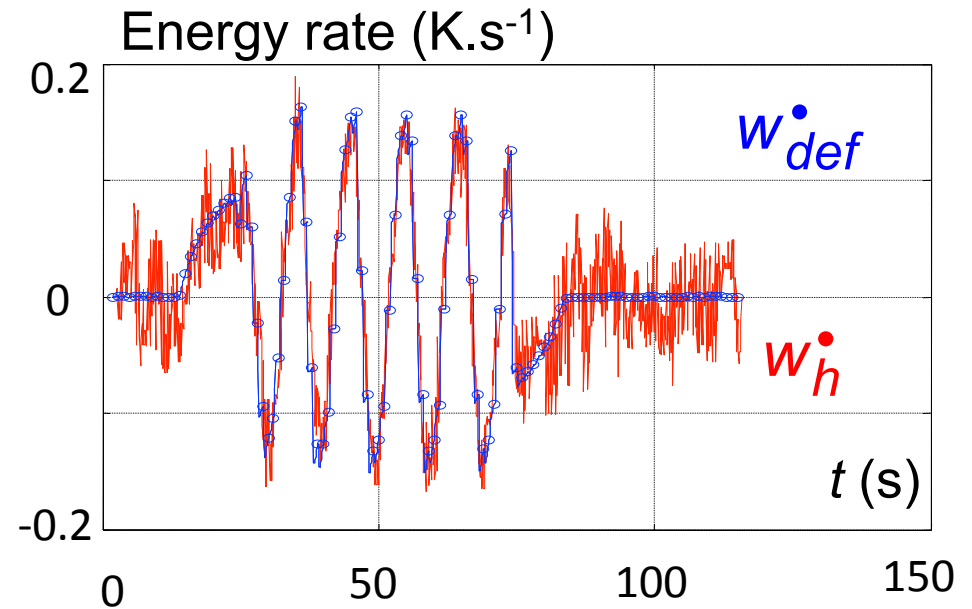
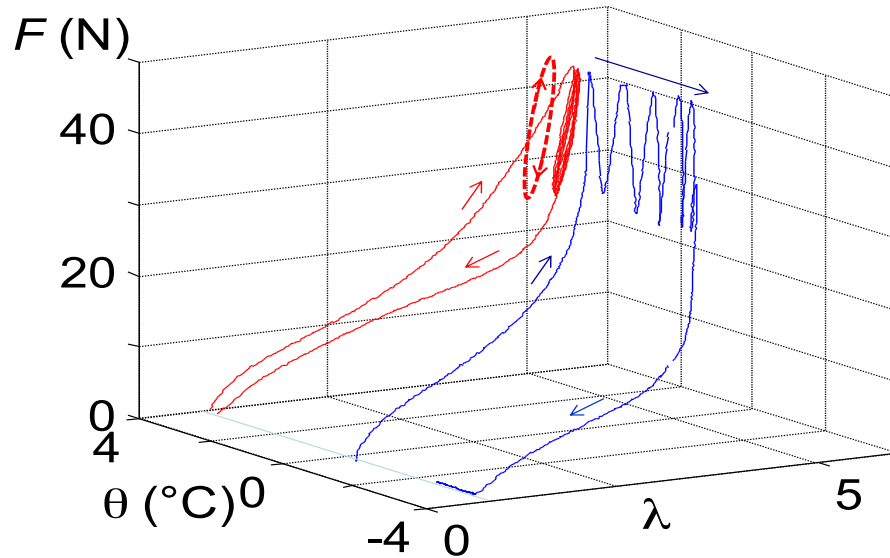
$$W_d \leq W_{def}$$

$$W_{the} = W_{def}$$

$\bigcirc = \varepsilon \nu \delta \sigma \phi \chi \psi \chi \lambda \varepsilon$

$$W_d = W_{def}$$

# Entropic elasticity of natural rubber



Gough (1805) – Joule (1857) – Treloar (1960) – ...

$e(s, \varepsilon) = e_{nr}(T)$       perfect gas analogy

$\psi_{nr}(T, \varepsilon) = TK_1(\varepsilon) + K_2(T)$

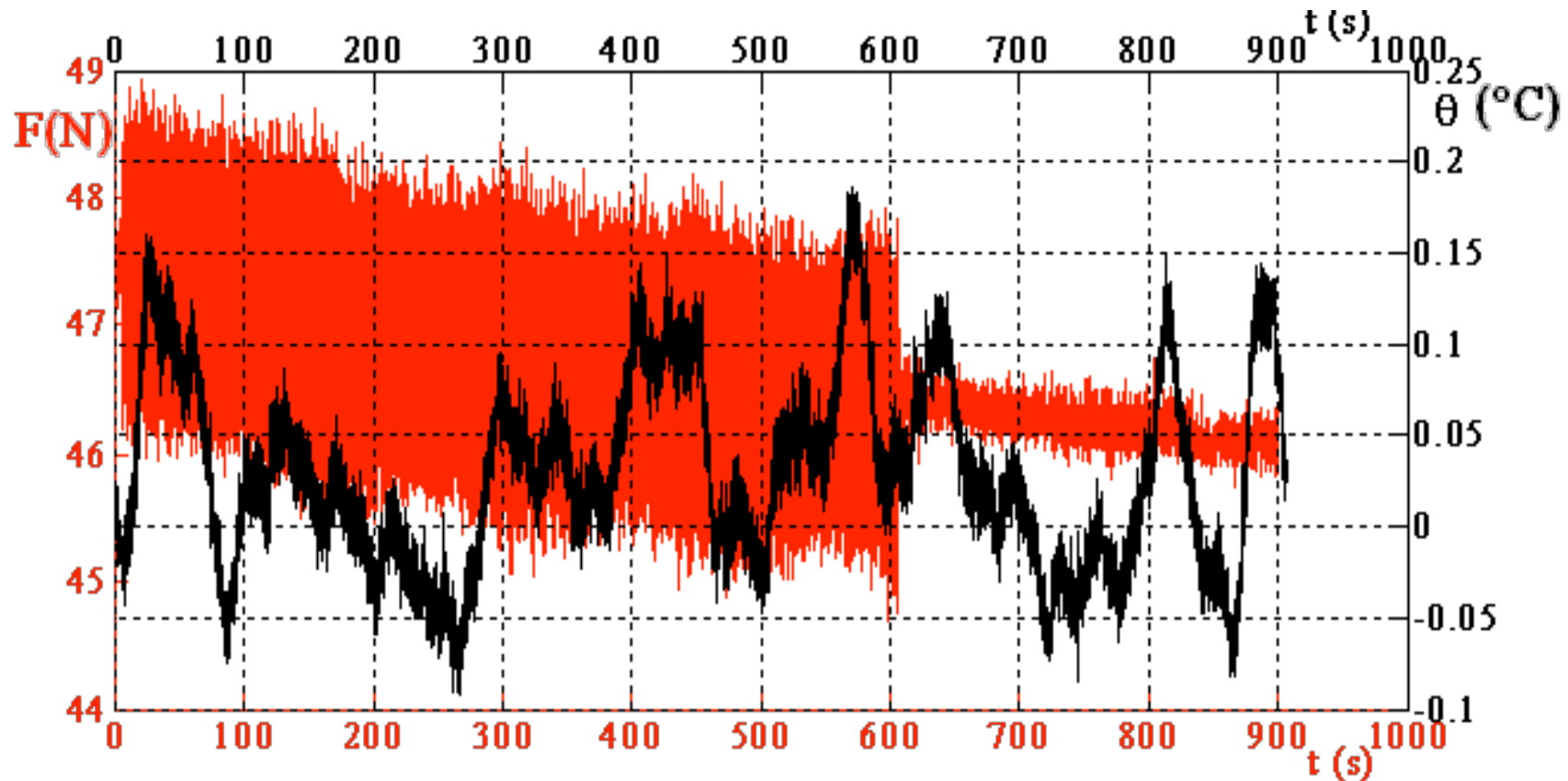
$w_{def}^\bullet = w_h^\bullet$

- [Saurel, PhD 99]
- [Honorat, PhD 06]
- [Caborgan, PhD 11]



# Thermal signature of NR

Undetectable self heating! Adiabatic process!  
 Thermal noise only! [Caborgan, PhD 11]



Essai [fL= 60 Hz/  $\lambda_{\text{moy}} = 5,27$ /  $B = \pm 0,5$  mm]

# Concluding comments

- ❑ Full-field measurements  
Material vs. Structure effects
- ❑ Temperature, the 1<sup>st</sup> state variable ...  
thermal effects vs. calorimetric effects  
not totally intrinsic (heat diffusion)
- ❑ Heat sources of different nature  
Thm coupling sources : thermo-sensitivity  
dissipation sources : irreversibility of material deformation
- ❑ Energy balance and constitutive equations  
stored energy / state laws  
dissipated energy / evolution law
- ❑ Rate-dependent behaviour  
[ $d_1$  (viscosity)] vs. [thm coupling +  $d_2$  (heat diffusion)]





Merci pour votre attention !