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Endogenous Timing in the Presence of Non-Monotonicities

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Abstract

The aim of this paper is to generalize the endogenous timing game (ETG) proposed by Hamilton and Slutsky ((1990) *Games and Economic Behavior*, 2, 29 – 46) by allowing the payoff or the marginal payoff of a player to become non-monotonic with respect to the strategy of the opponent. We propose a taxonomy of the subgame perfect Nash equilibria (SPEs) based on the characteristics of the payoff functions proposed by Eaton ((2004) *Canadian Journal of Economics*, 37(4), 805 – 829). We determine under which conditions of the initial payoff functions commitment has a social value and when the simultaneous-move Nash equilibrium is *commitment robust* and discuss its Pareto-efficiency.

JEL classification: C72; D43; L13

Keywords: Endogenous Timing; Non-Monotonicities; Complements; Substitutes

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1. Introduction

Since Schelling (1960) commitment has been a prominent issue in economics. It is typically apprehended through dynamic games in which one player or a group of players has the opportunity to take an initial binding action, allowing to commit first. The subgame perfect equilibrium (SPE) of an endogenous timing game (ETG) provides a foundation for a self-enforcing sequence of moves in a two-player framework. ETG is a reply to the main weakness of Stackelberg equilibrium, which is the exogenous sequence of players' moves.¹ The seminal paper from Hamilton and Slutsky (1990) provides a simple formalization of ETG. In their *extended game with observable delay* the authors introduce in the basic game (e.g. a price competition game) a pre-play stage, in which both players determine the stage at which they choose their strategy in the continuation game (i.e., the basic game). The set of possible subgame perfect equilibria (SPEs) of the endogenous timing game (ETG) consists of the simultaneous-move Cournot-Nash equilibrium (NE) and both Stackelberg equilibria, with one of the two players leading. Interestingly, by endogeneizing the Stackelberg leadership in a duopoly game Hamilton and Slutsky (1990) highlight the trade-off between commitment and flexibility. By doing so they formalize the notion of *pure unconditional commitment* developed in Schelling (1960).

In the last two decades there has been a substantial increase in the number of studies that either refine the ETG model by Hamilton and Slutsky (1990) or apply it to several well-known frameworks. For instance, van Damme and Hurkens (1999) provide an ETG with *action commitment* and Amir and Grilo (1999) with *observable delay* applied to a quantity competition framework. van Damme and Hurkens (2004), Pastine and Pastine (2004), and Amir and Stepanova (2006) focus on price competition. Beyond Industrial Organization, ETG has been used to quota games (Syropoulos, 1994 and Raimondos-Møller and Woodland, 2000), charity donations (Romano and Yildirim, 2001), conflict (Siqueira and Sandler, 2006), tax competition (Kempf and Rota-Graziosi, 2010 and Eichner, 2013), contests (Hoffmann and Rota-Graziosi, 2012), and political economy (Mariani, 2013).

The majority of these works adopt two assumptions shared by Hamilton and Slutsky (1990), which limit the scope of the analysis: The monotonicity of the payoff function and of the marginal payoff function with respect to the opponent's strategy. The former determines whether an increase in one player's strategy increase or decrease the other player's payoff - that is following Eaton (2004)

¹This set-up has often been criticized in the literature (see Albæk, 1990, Robson, 1990, and Shapiro, 1989, p. 390). It was even argued that the Stackelberg equilibrium is a far too artificial concept to be observed in reality (see Friedman, 1983, p. 175).

terminology, whether a player regards her opponent's strategies as *plain complements* (PC) or *plain substitutes* (PS). The second assumption relies to the notion of *strategic substitutes* (SS) or *strategic complements* (SC) as coined by Bulow, Geanakoplos, and Klemperer (1985b). It identifies whether an increase in one player's strategy increases (SC) or decrease (SS) the other player's marginal payoff and consequently her (or his) own strategy.² These assumptions are consistent with the development of the literature on monotone comparative statics presented in Topkis (1998) and Vives (2001).

Even though monotonic payoff functions and monotonic best replies encompass the modeling specifications in various applications, there is a large body of literature that allows for exactly these non-monotonicities. Indeed, non-monotonic payoffs typically arise if multiple and opposing spillovers emerge in a game. In this case, a player may regard the rival's strategy as a PC or a PS contingent on the value of the opponent's strategy. This emerges, for example, in the R&D competition (d'Aspremont and Jacquemin, 1988, Amir, Amir, and Jin, 2000 or Tesoriere, 2008), contest (Dixit, 1987, Skaperdas, 1991), law enforcement (Bandyopadhyay, Pinto, and Wheeler, 2011), pollution control (Fischer, 2011), and tax competition (Wildasin, 1989, DePater and Myers, 1994). Non-monotonic marginal payoffs and consequently non-monotonic best responses emerge quite frequently in applied game theory literature: The same player may regard strategies as SS or SC contingent on the value of the other player's strategy.³ Even the standard Cournot duopoly may display the non monotonicity of best replies if the demand elasticity is constant (Bulow, Geanakoplos, and Klemperer, 1985a,b), if firms feature cubic cost functions (Furth, 1986), if firms benefit from the sales of their competitors because of a *buying habit* effect (Poston and Stewart, 1978, Shaffer, 1984) or from their R&D efforts due to a leakage of knowledge to all other firms (d'Aspremont and Jacquemin, 1988, Amir, Amir, and Jin, 2000, Adner and Zemsky, 2005, Tesoriere, 2008). In price competition, reaction functions may be non monotonic too (see Maskin and Tirole, 1988 on p. 589). Finally, hump-shaped best responses emerge in several frameworks such as contests (see for instance, Dixit, 1987 and Skaperdas, 1991), international trade and tariff policy (Syropoulos, 1994), etc.

²In Hamilton and Slutsky (1990) best-response functions are described as either *sloping down* or *sloping up* (see, for instance, THEOREM (V), p. 38 or THEOREM (VI), p. 41); this is generally considered to be the assumption of monotone best responses (see, for example, Amir, 1995 and Amir and Stepanova, 2006). Concerning the monotonicity of the payoff function, Hamilton and Slutsky (1990) make no specific assumptions. Amir (1995), however, shows that one main finding in Hamilton and Slutsky (1990) is invalid if payoffs are allowed to be non-monotonic with respect to the opponent's strategy and that the assumption of monotonic payoffs reestablishes the main findings. For an in-depth discussion of these topics, see section (3.2), page 13.

³Otherwise, the studied games would be either supermodular or submodular and a lot of results summarized in Topkis (1998) and Vives (2001) would apply.

In this paper, we propose a generalization of Hamilton and Slutsky (1990) by considering the non-monotonicities of the payoff and marginal payoff with respect to the opponent's strategy. We consider several possible structures of game (actually, 32) according to the nature of interactions: PC, PS, SC, and SS. We highlight the presence of a first-mover advantage or a second-mover incentive only depending on the plain and strategic properties. This allows us to determine the SPE of the ETG for the 32 cases we have. Subsequently, we deduce a taxonomy of endogenous timing based solely on the properties of the NE of the basic game. Furthermore, in the spirit of Rosenthal (1991) and van Damme and Hurkens (1996), we establish, under which conditions (i) the NE of the basic game is *commitment robust*, (ii) it is *viable*, and (iii) commitment has a social value. Such results lead us to discuss the appropriateness of a simultaneous timing structure given a particular framework, that is the plain and strategic properties of the basic game.

To the best of our knowledge, this paper is the first to offer a systematic analysis of the ETG of a two-player game with non monotonicity in a general setting - that is, without adherence to a particular framework. By doing this we take the approach adopted by Rosenthal (1991) and van Damme and Hurkens (1996), who define the notion of *commitment robust equilibria*, i.e. equilibria which resists alterations in the game's move structure. We also complete the works of Baik and Shogren (1992), Leininger (1993), and Hoffmann and Rota-Graziosi (2012), who determine the SPE of ETG in contests' game, which display some non-monotonicity. Finally, since we use several examples to consider specific characteristics of payoff functions, we also highlight the relevance of our findings within multiple frameworks. Moreover, we introduce a new sufficient condition (local monotonicity of the Stackelberg follower payoff function) to analyze ETGs in the presence of non monotonicity.⁴ We also pay close attention to the fact that non-monotonicity may lead to multiple SPEs or to an efficient NE.

The rest of this paper is organized as follows: Section 2 presents the basic model and introduces our main examples. Section 3 provides a comparison of the strategies in the three basic games (the simultaneous and the two sequential games). In section 4, we solve the endogenous timing game. In Section 5, we determine the viability and commitment robustness of the NE and the potential social value of commitment. Section 6 concludes.

⁴We highlight why weaker conditions (e.g. those introduced by von Stengel (2010) for symmetric games) are not appropriate when allowing asymmetries.

2. The model

First, we present our basic definitions and assumptions and introduce the three basic games.

Second, we introduce several examples that will guide us through the following sections.

2.1. Preliminaries

We consider a two-player normal form game $G \equiv \langle \Gamma, \mathcal{N}, (X_i, \Pi^i)_{i \in \mathcal{N}} \rangle$, where $\mathcal{N} = \{1, 2\}$ is the set of players; X_i , a non-empty and compact interval of the real line, is the set of strategies available to player i ; and $\Pi^i : \mathbf{X} \rightarrow \mathbb{R}$ is the payoff of player i , with $\mathbf{X} = X_i \times X_j$. The set of subgames is given by $\Gamma = \{\Gamma^N, \Gamma^{S_1}, \Gamma^{S_2}\}$, with Γ^N representing the static game, and Γ^{S_1} and Γ^{S_2} are the two-stage games with sequential moves (Stackelberg games), in which player 1 and player 2, respectively, leads. The payoff function $\Pi^i(\mathbf{x})$ is assumed to be continuous in both strategies and strictly quasi-concave in x_i , with $\mathbf{x} = x_i \times x_j$. We denote by $BR^i : X_j \rightarrow X_i$ the best reply map of player i , which, given the above assumptions, is non-empty, single-valued, and continuous (see Friedman, 1986). Depending on the underlying framework, the existence and uniqueness of NE and both Stackelberg equilibrium (SE) usually involve several restrictions on the payoff functions. To remain as general as possible, we make the following assumptions, which hold in many applications.

Assumption 1

1. The NE of the static game is interior and unique.
2. The payoff function of the leader **1** in the corresponding Stackelberg game, denoted by $L_{ii}^i(x_i)$, is strictly concave.

$$L_{ii}^i(x_i) \triangleq \frac{d^2 \Pi^i(x_i, BR^j(x_i))}{dx_i^2} < 0. \quad (1)$$

The consequence of assumption (1.1) is that our definitions of SC, SS, and PC, and PS are unique for each player (see definitions (1) and (2)).⁵

Lemma 1 (Concavity)

Given assumption (1), we have j

⁵Uniqueness of the NE prevails if

$$\begin{vmatrix} \Pi_{11}^1(\mathbf{x}) & \Pi_{12}^1(\mathbf{x}) \\ \Pi_{12}^2(\mathbf{x}) & \Pi_{22}^2(\mathbf{x}) \end{vmatrix} > 0 \quad \forall x_1, x_2 \text{ s.t. } \Pi_1^1(\mathbf{x}) = 0, \Pi_2^2(\mathbf{x}) = 0.$$

This condition is based on the *index theory* approach, which, in a two-player game, requires that the determinant of the Jacobian of the marginal payoffs be positive at any candidate equilibrium. Equivalently, the multiplied slope of both players' best responses has to be smaller than unity whenever $\Pi_i^i(\mathbf{x}) = \Pi_j^j(\mathbf{x}) = 0$, that is,

$$\frac{\Pi_{12}^1(\mathbf{x}^N) \Pi_{12}^2(\mathbf{x}^N)}{\Pi_{11}^1(\mathbf{x}^N) \Pi_{22}^2(\mathbf{x}^N)} < 1. \quad (2)$$

This is an extremely general approach to proving uniqueness of NEs (see Vives, 2001, p. 48). Now, one can invoke sufficient conditions on the primitives of a particular model to guarantee that inequality (2) holds. For example, in the case of constant elasticity of demand and constant marginal costs (see example (1) in this paper), it can be shown that $\varepsilon \leq 1$ is a sufficient assumption (see, for example, Février and Linnemer, 2004). Regarding contests (example (3) in this paper), the condition in (2) can be shown to hold for any fixed prize contests under the usual assumptions (see Dixit, 1987). In the case of endogenous prizes, uniqueness can be shown to exist via eq. (2) if the direct costs of effort are zero (see Skaperdas, 1992) or if players are sufficiently homogeneous (see Hoffmann and Rota-Graziosi, 2012).

1. $\text{sign } \Pi_{ij}^i(\mathbf{x}^N) = \text{sign } \Pi_{ij}^i(\mathbf{x}^{S_j})$,
2. $\text{sign } \Pi_j^i(\mathbf{x}^N) = \text{sign } \Pi_j^i(\mathbf{x}^{S_i})$.

$\forall (i, j) \in \{1, 2\}$ and $i \neq j$.

Proof. See APPENDIX A.1. ■

Lemma (??1) establishes that the sign of the slope of a player's best response function at the NE and at the Stackelberg equilibrium in which she follows are identical. It also guarantees that the sign of the cross effect of a player's payoff at the NE and at the Stackelberg equilibrium in which she leads are identical, that is, $\text{sign } \Pi_j^i(\mathbf{x}^N) = \text{sign } \Pi_j^i(\mathbf{x}^{S_i})$.⁶

A Nash equilibrium (NE) of the static game is defined by the following system of maximization programs

$$\begin{cases} x_i^N \triangleq \underset{x_i \in X_i}{\text{argmax}} \Pi^i(\mathbf{x}) & \text{given } x_j^N \\ x_j^N \triangleq \underset{x_j \in X_j}{\text{argmax}} \Pi^j(\mathbf{x}) & \text{given } x_i^N. \end{cases} \quad (3)$$

Since the strategy set is non-empty, compact, and convex and each player's payoff is continuous in both strategies and strictly quasi-concave in own strategy, Γ^N has at least one NE in pure strategies (see Debreu, 1952).

The taxonomy of the SPE(s) of the ETG that we establish rely on the nature of the interactions among players. We consider two degrees of interactions: *plain interaction* or interaction of the first degree (since it relies on the sign of the first derivative) and *strategic interaction* or interaction of the second degree. According to Eaton (2004), strategies are plain complements (substitutes) to a player if an increase in the strategy of her opponent increases (decreases) her own payoff. Plain complementarity (PC) is then equivalent to positive cross effects in the payoff functions. In contrast, plain substitutability (PS) indicates a negative relationship.⁷ We use the concept of plain

⁶Strict concavity of the Stackelberg leader's payoff function is a strong assumption. However, given both the non-monotonicity of the payoff and the marginal payoff function, it becomes necessary to obtain our results. Moreover, it is also assumed in various frameworks - as, for example, in Baye and Shin (1999), Ellingsen (1995), Fershtman and Weiss (1998), Gal-Or (1985), Hoffmann and Rota-Graziosi (2012), Morgan and Várdy (2013), Pastine and Pastine (2004), Romano and Yildirim (2005), and von Stengel (2010). In the case of quantity competition with constant marginal costs and linear demand, strict concavity of the leader's payoff function and interiority of the Stackelberg equilibrium follows easily if players are sufficiently homogeneous (see van Damme and Hurkens, 1999). The same holds in the case of price competition (see van Damme and Hurkens, 2004) and fixed prize contests (see Leininger, 1993 and Morgan, 2003). Notably, assumption (1) holds for all the examples introduced in the present paper. Note that existence, uniqueness, and interiority of the Stackelberg equilibrium in general are discussed in the literature on bilevel optimization, which is beyond the scope of this paper (see Colson, Marcotte, and Savard, 2007 for a recent survey).

⁷Bulow, Geanakoplos, and Klemperer (1985b) refer to these properties as *conventional* substitutes and complements. Note that the notion of plain interactions is very close to that of spillovers. However, the former is less ambiguous since it only states the sign of the first-order cross derivative of payoff functions, while spillovers may rely on derivatives of higher degree.

interaction in a more general way by allowing that the property of PC or PS does not hold for all strategy profiles. We define PC and PS by focusing only on the cross effects at $\mathbf{x}^N = (x_i^N, x_j^N)$

Definition 1 (Plain complements and plain substitutes)

Player i regards strategies as plain complements (substitutes) if player i 's payoff function increases (decreases) in player j 's strategy at the Cournot-Nash equilibrium, i.e., if

$$\Pi_j^i(\mathbf{x}^N) \equiv \left. \frac{\partial \Pi^i(\mathbf{x})}{\partial x_j} \right|_{\mathbf{x}=\mathbf{x}^N} > 0 \quad (< 0). \quad (4)$$

The second criterion for our taxonomy is based on the well-known concept of strategic interaction. According to Bulow, Geanakoplos, and Klemperer (1985b) strategies are strategic complements (SC) to a player if that player's marginal payoff increases in the opponent's strategy. If the opposite holds, strategies are strategic substitutes (SS) to i . Again, we will make a slight generalization.

Definition 2 (Strategic complements and strategic substitutes)

Player i regards strategies as strategic complements (substitutes) if player i 's marginal payoff increases (decreases) in player j 's strategy at the Cournot-Nash equilibrium, i.e., if

$$\Pi_{ij}^i(\mathbf{x}^N) \equiv \left. \frac{\partial^2 \Pi^i(\mathbf{x})}{\partial x_i \partial x_j} \right|_{\mathbf{x}=\mathbf{x}^N} > 0 \quad (< 0). \quad (5)$$

Definition (1) allows for games in which both players regard strategies as PS or as PC.⁸ Moreover, definition (1) allows for heterogeneous plain interactions: player i may regard strategies as PC while player j regards them as PS.⁹ Most importantly, we deviate from the usual assumption originating in the seminal works by Gal-Or (1985) and Amir (1995): that payoffs are monotonic with respect to the opponent's strategy.¹⁰

Definition (2) allows for games in which both players regard strategies as SC or as SS, or in which strategic heterogeneity prevails.¹¹ Most importantly, we also permit games in which the marginal payoff function becomes non-monotonic in the opponent's strategy, which leads to non-monotonic best responses. Since

$$BR_j^i(x_j) \triangleq \frac{dBR^i(x_j)}{dx_j} = -\frac{\Pi_{ij}^i(\mathbf{x})}{\Pi_{ii}^i(\mathbf{x})} \quad (6)$$

⁸The former case arises, for example, in the standard version of the Cournot duopoly (see van Damme and Hurkens, 1999) or in contests (see Dixit, 1987), while the latter arises in games of price competition (see van Damme and Hurkens, 2004), tax competition (see Wildasin, 1989), or the private provision of public goods (see Bergstrom, Blume, and Varian, 1986).

⁹This case emerges for example in the model of Singh and Vives (1984) (when one firm chooses quantities and the other chooses prices as basic strategies) or in Fudenberg and Tirole (1984).

¹⁰One of the exceptions is introduced by von Stengel (2010). See the discussion in Section (3.2).

¹¹The former case can be found in models of price competition, tax competition, the classical version of the Cournot duopoly, and the private provision of public goods. This last case can be found in Becker (1968), Singh and Vives (1984), Fudenberg and Tirole (1984), Tombak (2006), or Baliga and Sjöström (2012).

and $\Pi_{ii}^i(\mathbf{x}) < 0$ whenever $\Pi_i^i(\mathbf{x}) = 0$, the sign of the cross-partial derivative at the NE ($\Pi_{ij}^i(\mathbf{x}^N)$) determines the slope of the best response function at \mathbf{x}^N . Thus, we will say that player i regards strategies as SC (or SS) if the condition in (5) holds without restricting the value of $\Pi_{ij}^i(\mathbf{x})$ for all $\mathbf{x} \neq \mathbf{x}^N$.

Next, we turn to the two-stage game with sequential moves. The subgame perfect Nash equilibrium (SPE) of the sequential game in which player i leads and player j follows (Γ^{S_i}) is determined by backward induction. The follower's optimal strategy in stage 2 is given by her best response

$$BR^j(x_i) \triangleq \operatorname{argmax}_{x_j \in X_j} \Pi^j(\mathbf{x}), \quad (7)$$

so that the follower's payoff function, denoted by $F^j(x_i)$, becomes $F^j(x_i) \triangleq \Pi^j(x_i, BR^j(x_i))$. The leader's payoff maximizing strategy in stage 1 is then given by

$$x_i^L \triangleq \operatorname{argmax}_{x_i \in X_i} L^i(x_i), \quad (8)$$

with $L^i(x_i) \triangleq \Pi^i(x_i, BR^j(x_i))$ denoting the leader payoff. Thus, at the SPE of Γ^{S_i} we find that the follower's strategy is given by $x_j^F \triangleq BR^j(x_i^L)$ and we define the SPE strategy profile at Γ^{S_i} as $\mathbf{x}^{S_i} = (x_i^L, x_j^F)$.

2.2. Examples

In order to show that the generalizations introduced above are meaningful, we will now introduce several generic examples that will accompany us through the rest of the text and that all exhibit at least one non-monotonicity regarding either the payoff function or the marginal payoff function.¹² For example, in duopoly games with constant elasticity of demand, best-response functions are typically hump-shaped (see, for example, Bulow, Geanakoplos, and Klemperer, 1985a).

Example 1 (Cournot Duopoly with constant elasticity of demand)

Suppose we have two firms (1, 2) that compete in a Cournot duopoly with constant marginal costs and constant elasticity of demand. The demand is given by $P(\mathbf{x}) = (a(b(x_1 + x_2))^{-1})^\alpha$ with $\alpha \in (0, 1)$ and the marginal costs are given by $c_1 > 0$, $c_2 > 0$. It is obvious that firm i 's profit is a strictly monotonically decreasing function of the competitor's supply. The marginal profit $\Pi_i^i(\mathbf{x})$, however, is a non-monotonic function.

In the first symmetric variant of this example (example (1.1)), both players regard strategies as SS, that is, $\Pi_{12}^1(\mathbf{x}^N) = \Pi_{12}^2(\mathbf{x}^N) < 0$. This case is represented in Figure 1, where the bold and hump-shaped curves represent players' best-response functions and the dashed lines represent the iso-payoff curves at \mathbf{x}^N . The fact that players' iso-payoff curves going through \mathbf{x}^N are concave proves that firms regard supply as PS. Note also the grey surface in Figure 1, which represents the *Pareto-superior set* and which we define as follows.

¹²All the details of our examples can be found in Appendix B, pp. 30.

Definition 3 (Pareto-superior set)

The Pareto-superior set (\mathcal{P}^S) is the set of all strategy profiles \mathbf{x} such that each player's payoff at \mathbf{x} exceeds the payoff at the Cournot-Nash equilibrium. More formally,

$$\mathcal{P}^S \equiv \{\mathbf{x} \mid \Pi^i(\mathbf{x}) > \Pi^i(\mathbf{x}^N) \quad \forall i, j = 1, 2 \text{ and } i \neq j\}.$$

Both types of non-monotonicities emerge naturally in oligopoly games with cost spillovers, as, for example, in Monaco and Sabarwal (2011).

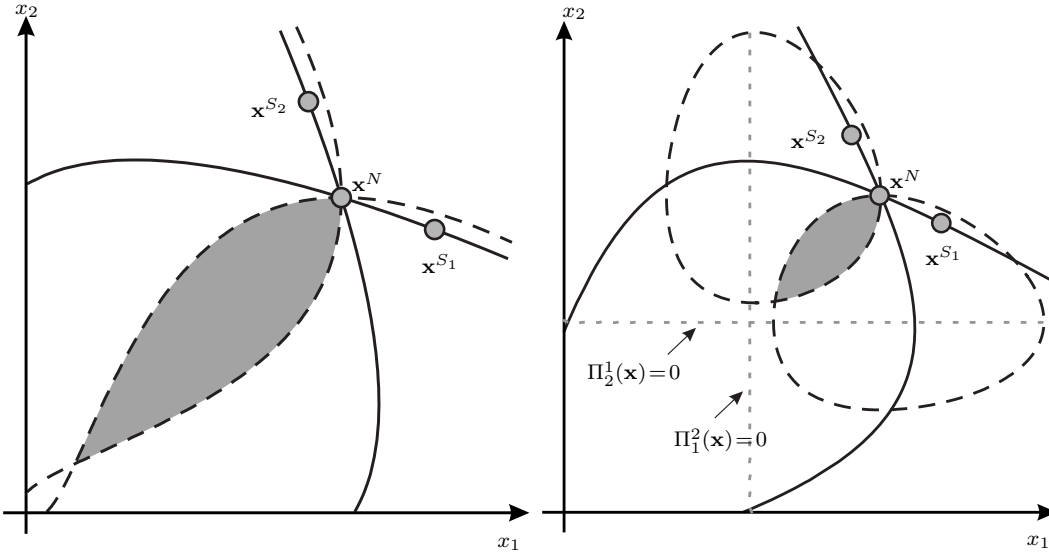


Figure 1
Cournot with CED
Example (1.1)

Figure 2
Cournot with cost spillovers
Example (2.1)

Example 2 (Cournot Duopoly with cost spillovers)

Two firms compete in a Cournot duopoly with a linear demand function and constant marginal costs. Spillover effects emerge as cost reductions. Suppose that the demand function is given by $P(\mathbf{x}) = a - b_1x_1 - b_2x_2$, with $a > 0$, $b_i > 0$. Because of the cost spillover effect, each firm's cost function is given by $C_i(x_i) = cx_i \exp(\gamma_i x_j)$, with $a > c > 0$ and $\gamma_i > 0$. Then, profits as well as the marginal profits become non-monotonic in the competitor's supply.

The first symmetric variant of this example (example 2.1) is shown in Figure 2, where the bold and hump-shaped curves again represent firms' non-monotonic best response functions. In this example, each firm experiences two opposing spillovers: (i) an increase in the rival's supply decreases, *ceteris paribus*, own profit because of the lower market price and (ii) an increase in own profit due to the cost spillover effect. $\Pi_{jj}^i(\mathbf{x}) < 0$ and $\lim_{x_j \rightarrow 0} \Pi_j^i(\mathbf{x}) > 0$ for $i \neq j$, so we find that to the left of the vertical dotted line (where $\Pi_1^2(\mathbf{x}) = 0$), the net spillover effect is positive (and firm 2's iso-profit curve is convex to the origin), while to its right the net effect is negative (and firm 2's iso-profit curve is concave). The symmetric argument applies to firm 1. Here, the horizontal dotted line

represents all the strategy profiles for which $\Pi_2^1(\mathbf{x}) = 0$. Since at \mathbf{x}^N both iso-payoff curves are concave, we find that both firms regard supply as PS.

Now, we turn to the contest literature. In the next example, which is based on Matros and Armanios (2009), the designer of a contest reimburses some of the loser's effort costs in order to increase the effort invested in the contest.

Example 3 (Contest with reimbursements)

Suppose two players (1,2) compete for a prize of value $V > 0$. The probability of winning for player i is given by a Tullock lottery contest success function, $p^i(x_1, x_2) = x_i(x_1 + x_2)^{-1}$, and the marginal costs of effort are $c_i > 0$ for player i . Assuming that the loser is reimbursed for a part α of her outlays and that $c_i > \alpha > 0$, we find that both players' payoff monotonically decreases in the contestant's effort while both marginal profits become non-monotonic.

The symmetric and first variant (Example 3.1) is shown in Figure 3, where both players regard strategies as SC, that is, $\Pi_{12}^1(\mathbf{x}^N) = \Pi_{12}^2(\mathbf{x}^N) > 0$.

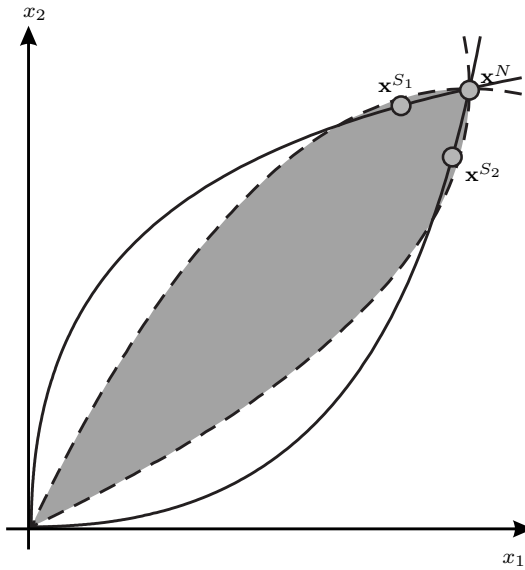


Figure 3
Contest with reimbursement
Example (3.1)

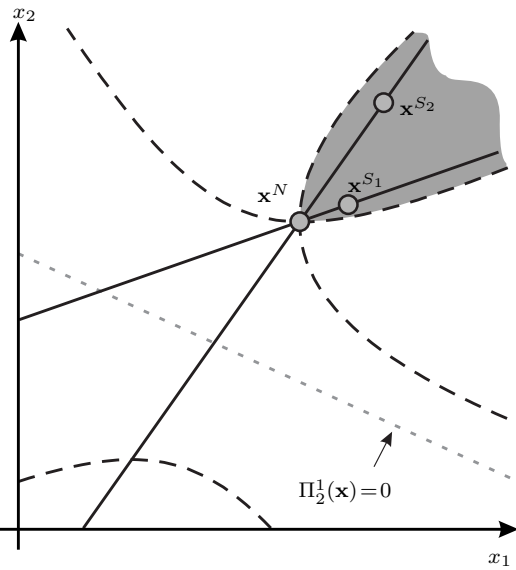


Figure 4
Asymmetric tax competition
Example (4.1)

Another non-monotonicity arises in the context of tax competition.

Example 4 (Tax competition)

We present a two-country version of the workhorse model of international tax competition proposed in Keen and Konrad (2013).¹³ Two countries (1 and 2) compete in tax rates, denoted by x_i , to

¹³Based on the same framework, Eichner (2013) presents a model of endogenous timing in tax competition that allows for opposite and aligned plain interactions, i.e. $\Pi_j^i(\mathbf{x}^N) \Pi_i^j(\mathbf{x}^N) \cong 0$.

attract capital, which is perfectly mobile between them and is fixed in total supply. The stock of capital, denoted by \bar{k} , is distributed identically between countries, and the share of the world's population in country i is given by σ_i , with $\sigma_1 + \sigma_2 = 1$. A single homogeneous good is produced in each country using inputs of labor and capital, and the representative resident in each country supplies one unit of labor. The production function, which is assumed to be concave in its two inputs and homogeneous of degree one, is represented in terms of the capital-labor ratio, which is denoted by k_i . We consider the following quadratic form:

$$f_i(k_i) = \left(a_i - \frac{b}{2} k_i \right) k_i,$$

with $a_i > 0$ and $b > 0$. The perfect mobility of capital involves the following capital market clearing conditions

$$f'_i(k_i) - x_i = f'_j(k_j) - x_j = r \quad (9)$$

$$k_i + k_j = \bar{k}, \quad (10)$$

where r is the net return of capital. The total income of the representative inhabitant in country i is $x_i = f_i(k_i) - f'_i(k_i)k_i + r\sigma_i\bar{k}$. Capital is taxed at the source and all government revenue is spent on the public good: $g_i = t_i k_i$. We assume a constant marginal rate of substitution between private and public consumption, which is the same across countries and equal to $1 + \varepsilon$. The objective function of each country is then given by

$$\Pi^i(\mathbf{x}) = f_i(k_i(\mathbf{x})) - (r(\mathbf{x}) + x_i)k_i(\mathbf{x}) + r(\mathbf{x})\bar{k}\sigma_i + (1 + \varepsilon)x_i k_i(\mathbf{x}), \quad (11)$$

with

$$r(\mathbf{x}) = -\bar{k}b + \sum_{j=1}^2 \sigma_j a_j - \sum_{j=1}^2 \sigma_j x_j,$$

$$k_i(\mathbf{x}) = \bar{k} + \frac{\sigma_j}{b}(a_i - a_j + x_2 - x_j).$$

Both countries' best responses are linear and increasing in the tax rate of the other country: tax rates are SC. However, the PC or PS property is not monotonic and depends in particular on the difference between the tax rates.

Figure 4 shows the first variant of Example 4, where the dotted line displays the set of strategy profiles for which the cross effect on country 1's welfare is zero, that is, $\Pi_2^1(\mathbf{x}) = 0$. Since $\Pi_2^1(\mathbf{x}) > 0$ and $\lim_{x_2 \rightarrow 0} \Pi_2^1(\mathbf{x}) < 0$, we find that country 1's welfare at \mathbf{x}^N can be represented by two distinct iso-payoff curves: one lying to the northeast of the dotted line (where the net spillover effect is positive and thus iso-welfare curves are convex) and one lying to the southwest of it (where the net spillover effect is negative and thus iso-welfare curves are concave).

3. A comparison of equilibrium strategies: First-mover (second-mover) advantage and incentive

In this section, we introduce the concepts of a first-mover (second-mover) advantage and incentive and identify the conditions leading to them. We start by determining the conditions, under which a Stackelberg leader increases or decreases her strategy with respect to the NE level.

3.1. A sufficient partial ranking of the equilibrium strategies in the three basic games

We consider the three basic games, denoted by Γ^N , Γ^{S_1} , and Γ^{S_2} , which respectively correspond to the static game and the two Stackelberg games. Given the optimizing behavior in the basic games (cf. eqs. (3), (7) and (8)), we establish partial rankings of the players' strategies at the different equilibria based on the terms SC or SS and PC or PS as in definitions (1) and (2).

Lemma 2 (Partial ranking of equilibrium strategies)

Given assumption (1), we find that player j

1. decreases her strategies at \mathbf{x}^{S_j} compared to \mathbf{x}^N , i.e. $x_j^L < x_j^N$, if $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$,
2. increases her strategies at \mathbf{x}^{S_j} compared to \mathbf{x}^N , i.e. $x_j^L > x_j^N$, if $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$,
3. does not alter her strategies at \mathbf{x}^{S_j} compared to \mathbf{x}^N , i.e. $x_j^L = x_j^N$, if $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) = 0$.

$\forall (i, j) \in \{1, 2\}$ and $i \neq j$.

Proof. See APPENDIX A.1. ■

This LEMMA compares the strategies chosen at \mathbf{x}^N with those chosen by the leader at the corresponding Stackelberg equilibrium (\mathbf{x}^{S_j}). The obtained rankings depend on the sign of the product of the first- and second-degree interactions at the NE. More precisely, the rankings result from the strict concavity of the objective function of the leader given in (1) and depend on the slope of the follower's best response and the sign of the cross effect for the leader. We consider first the case in which both players regard strategies as SS and PS (cf. example (1.1), figure (1)). Then, the leader, player j , will want to decrease the opponent's strategy since $\Pi_i^j(\mathbf{x}^N) < 0$. She will succeed in doing so by increasing her own strategy compared to the NE since strategies are SS for the opponent, that is, $\Pi_{ij}^i(\mathbf{x}^N) < 0$. We then find that $x_j^L > x_j^N$ and $x_i^F < x_i^N$ so that \mathbf{x}^{S_1} (\mathbf{x}^{S_2}) lies to the southwest (northeast) of \mathbf{x}^N . We proceed with the case in which both players regard strategies as SC and PS as in example (3.1) shown in Figure 3. Again, as the leader, player j will try to decrease the opponent's strategy since $\Pi_i^j(\mathbf{x}^N) < 0$, but now the latter regards strategies as SC, that is, $\Pi_{ij}^i(\mathbf{x}^N) > 0$. Thus, $x_j^L < x_j^N$, $x_i^F < x_i^N$ and both SPEs of the Stackelberg games lie to the south-west of \mathbf{x}^N .

Next, we turn to the cases, in which we have strategic heterogeneity or opposite plain interactions (see Appendix B). In example (1.2), firm 1 regards supply as SC, while firm 2 regards it as SS (cf. Figure 5). As the leader, player 1 (player 2) will thus increase (decrease) the supply compared to \mathbf{x}^N in order to decrease the competitor's supply (since still both firms still regard supply as PS) so that $x_1^L > x_1^N$ and $x_2^L < x_2^N$. Increasing the asymmetry between countries in the tax competition framework leads to example (4.2), where we find opposite plain interactions (cf. figure (6)). Here,

as the leader, player 1 (player 2) will try to decrease (increase) the opponent's strategy compared to \mathbf{x}^N . Since both countries regard strategies as SC, we have $x_1^L < x_1^N$ and $x_2^L > x_2^N$.

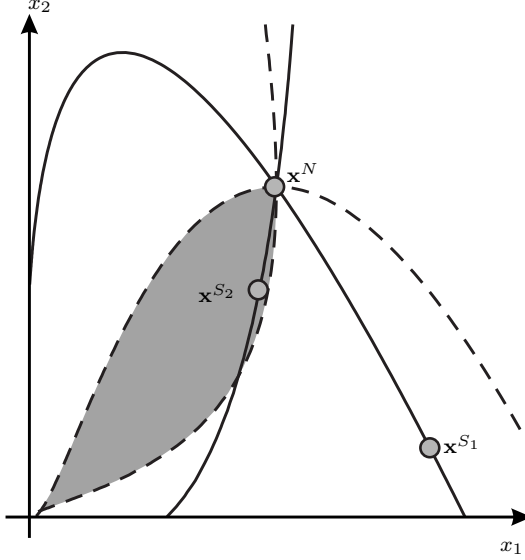


Figure 5
Cournot with CED
Example (1.2)

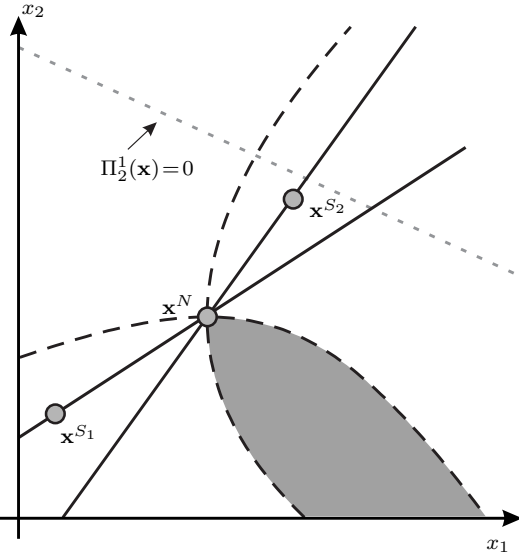


Figure 6
Asymmetric tax competition
Example (4.2)

Finally, we turn to those cases, in which at least one of the players does not move strategies away from the NE level (cf. LEMMA (1.3)). Such a case arises if the follower regards strategies as neither SC nor SS or if the leader regards strategies as neither PS nor PC. The former case can be represented by a variant of example (3). In example (3.2) shown in Figure 7, we find that $\Pi_{12}^1(\mathbf{x}^N) = 0 < \Pi_{12}^2(\mathbf{x}^N)$, so player 2 (1) regards effort as SC (as neither SC nor SS), while both regard efforts as PS. Consequently, player 1 decreases effort compared to the NE, while player 2, when becoming a leader, cannot improve upon his NE payoff, that is, $x_2^N = \operatorname{argmax}_{x_2} L^2(x_2)$. Finally, Figure 8 shows a variant of example (4) in which country 1 (2) regards taxes as neither PC nor PS (as PC), while both regard taxes as SC. Again, using lemma (2) we find that country 2 will increase taxes compared to the NE since it regards taxes as PS. For country 1, we find that $x_1^N = \operatorname{argmax}_{x_1} L^1(x_1)$, so $\mathbf{x}^N = \mathbf{x}^{S1}$.

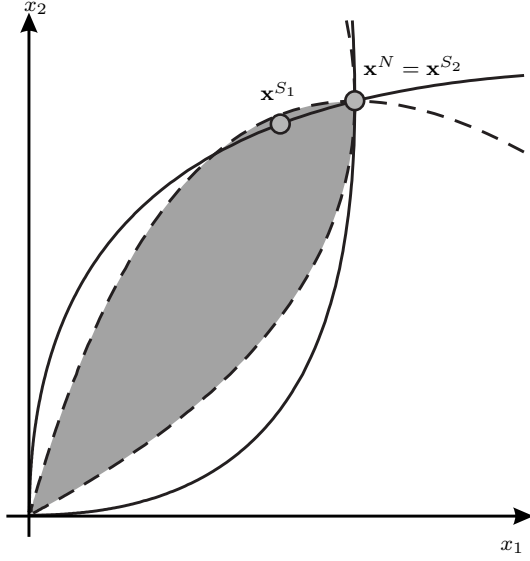


Figure 7
Contest with reimbursement
Example (3.2)

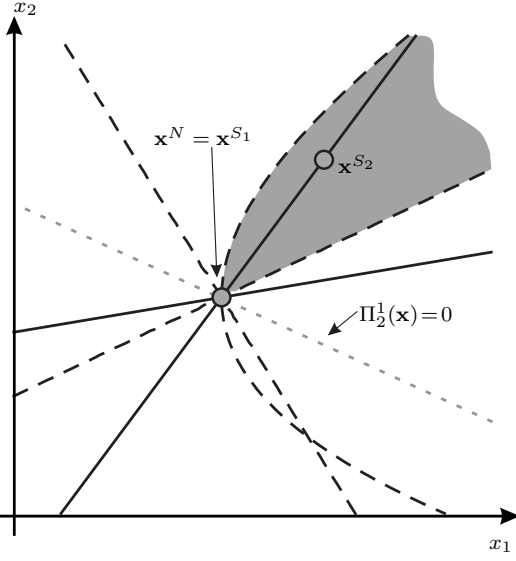


Figure 8
Asymmetric tax competition
Example (4.3)

3.2. First-mover advantage and second-mover incentive

We now compare the payoffs in the three basic games (Γ^N , Γ^{S_1} , and Γ^{S_2}), which will give us the opportunity to detect potential first-mover (second-mover) advantages or first-mover (second-mover) incentives. We define these notions as follows.

Definition 4 (First-mover (second-mover) advantage and second-mover incentive)

1. Player i has a **first-mover advantage (FMA)** if her equilibrium payoff in the Stackelberg game, in which she leads (Γ^{S_i}), is larger than in the Stackelberg game, in which she follows (Γ^{S_j}) or, more formally, if

$$\Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^{S_j}).$$

In the complementary case, that is, if

$$\Pi^i(\mathbf{x}^{S_i}) < \Pi^i(\mathbf{x}^{S_j}),$$

player i is said to have a **second-mover advantage (SMA)**.

2. Player i has a **second-mover incentive (SMI)** if her equilibrium payoff in the Stackelberg game, in which she follows (Γ^{S_j}) is at least as large as in the static game (Γ^N) or, more formally, if

$$\Pi^i(\mathbf{x}^{S_j}) \geq \Pi^i(\mathbf{x}^N). \quad (12)$$

In case $\Pi^i(\mathbf{x}^{S_j}) > \Pi^i(\mathbf{x}^N)$, player i has a **strong form of second-mover incentive (SSMI)**; if $\Pi^i(\mathbf{x}^{S_j}) = \Pi^i(\mathbf{x}^N)$, player i is said to have a **weak form of second-mover incentive (WSMI)**.

3. Player i has a **first-mover incentive (FMI)** if her equilibrium payoff in the Stackelberg game in which she leads (Γ^{S_i}) is at least as large as in the static game (Γ^N) or, more formally, if

$$\Pi^i(\mathbf{x}^{S_i}) \geq \Pi^i(\mathbf{x}^N). \quad (13)$$

In the case of $\Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^N)$, player i has a **strong form of first-mover incentive (SFMI)**; if $\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N)$, player i is said to have a **weak form of first-mover incentive (WFMI)**.

It is clear from the definition of the strategies at the NE and at the Stackelberg equilibrium that a player always has a first-mover incentive since she can always choose $x^L = x^N$ (see Shapiro, 1989).

Before we resolve the endogenous timing game, we discuss two topics of importance, which only emerge in the presence of non-monotonic payoffs.

3.3. Local monotonicity of the Stackelberg follower payoff

In order to establish a strong or weak form of the first- or second-mover incentive, we have to compare the Stackelberg follower's payoff $F^j(x_j^L)$ with the NE payoff $\Pi^j(\mathbf{x}^N)$ of the same player. An additional assumption is thus made, which stipulates local monotonicity of the Stackelberg followers' payoff. The reason for including such an assumption is presented below. As mentioned in the introduction, Amir (1995) highlights the fact that one of the main theorems of Hamilton and Slutsky (1990) may be invalid since no restrictions were imposed on the effect of the competitor's strategy on each player's payoff ($\Pi_j^i(\mathbf{x})$). Hence, the non-monotonicity of $\Pi^i(\mathbf{x})$ with respect to x_j was not explicitly ruled out. The subsequent consequences of this were illustrated by a non-generic example, which we have reproduced in Appendix B (see Example 5, p. 32) and depicted in graphic form (see Figure 9).

Here, player 1's follower payoff is non-monotonic, which leads to a surprising result: although player 1 regards strategies as PS ($\Pi_2^1(\mathbf{x}^N) < 0$) and $x_2^L < x_2^N$, we find that he is worse off at \mathbf{x}^{S_2} compared to \mathbf{x}^N since $\mathbf{x}^{S_2} \notin \mathcal{P}^S$. Amir (1995) stresses that the assumption of the monotonicity of $\Pi^i(\mathbf{x})$ with respect to x_j will resolve this issue since this property also implies the monotonicity of the follower's payoff: $F_j^i(x_j) \neq 0 \forall x_j \in X_j$ (see LEMMA 2 in von Stengel (2010)). However, as von Stengel (2010) emphasizes, the opposite does not hold.

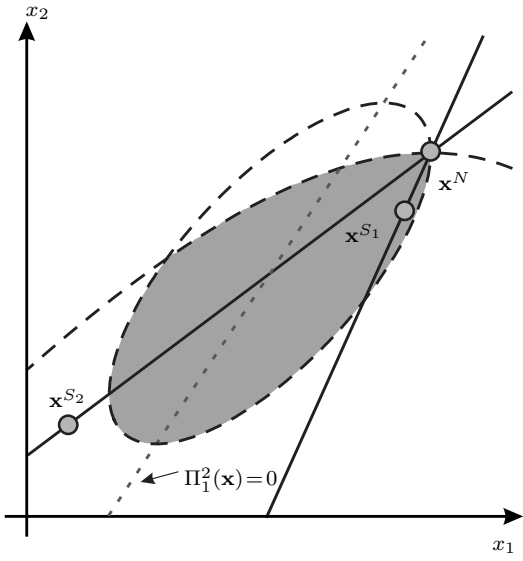


Figure 9
Amir (1995)
Example (5.1)

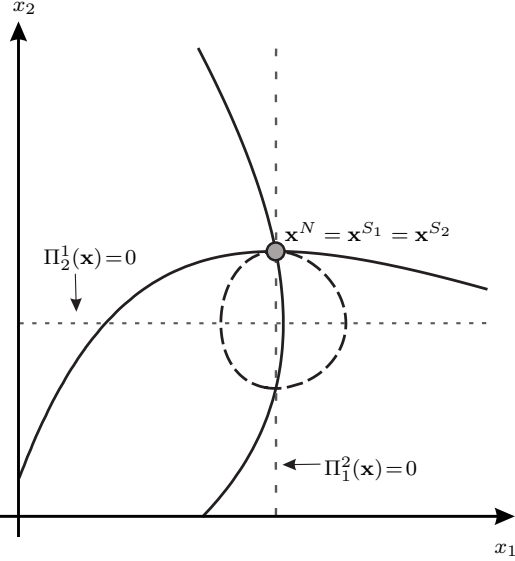


Figure 10
Cournot with cost spillovers
Example (2.2)

Although the assumption made by von Stengel (2010), namely the monotonicity of $F^i(x_j)$, is a strictly weaker assumption than stipulating the monotonicity of $\Pi^i(\mathbf{x})$ with respect to x_j , it has two shortcomings. Graphically speaking, the assumption of the monotonicity of $F^i(x_j)$ requires that the best-response function of player i never crosses the (implicit) function $\Pi_j^i(\mathbf{x}) = 0$. Thus, in order to be able to apply this assumption, information is required about the location of $BR^i(x_j)$ for the whole strategy space. Second, the non-monotonicity of $\Pi^i(\mathbf{x})$ and the monotonicity of $F^i(x_j)$ seems to be a rare case, especially if the best responses are allowed to be non-monotonic and players are allowed to be heterogeneous (as in the present paper).¹⁴ In all of the examples that have been presented thus far either $F^i(x_j)$ and $\Pi^i(\mathbf{x})$ are both non-monotonic with respect to x_j or both are monotonic. For this reason, we introduce a slightly different assumption for the remainder of this paper: the *local monotonicity* of $F^i(x_j)$.

Assumption 2

Each player's Stackelberg follower payoff $F^i(x_j)$ is locally monotonic with respect to x_j - that is, either $F_j^i(x_j) \geq 0 \forall x_j \in [a, b]$ or $F_j^i(x_j) \leq 0 \forall x_j \in [a, b]$ where $a = \min\{x_j^N, x_j^L\}$ and $b = \max\{x_j^N, x_j^L\}$.

¹⁴Note that von Stengel (2010) only considers symmetric games.

Application of the envelope theorem leads to

$$\begin{aligned}
F_j^i(x_j^N) &\triangleq \frac{d\Pi^i(BR^i(x_j), x_j)}{dx_j} \Big|_{x_j=x_j^N} \\
&= \frac{\partial \Pi^i(BR^i(x_j), x_j)}{\partial x_i} \frac{dBR^i(x_j)}{dx_j} \Big|_{x_j=x_j^N} + \frac{\partial \Pi^i(BR^i(x_j), x_j)}{\partial x_j} \Big|_{x_j=x_j^N} = \Pi_j^i(\mathbf{x}^N),
\end{aligned} \tag{14}$$

and to $F_j^i(x_j^L) = \Pi_j^i(\mathbf{x}^{S_j})$. Hence, the local monotonicity of the follower payoff $F^i(x_j)$ implies the following condition:

$$\Pi_j^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^{S_j}) \geq 0. \tag{15}$$

If the monotonicity of $F_j^i(\mathbf{x})$ prevails, the opposite also holds - that is, if condition (15) holds, $F^i(x_j)$ is locally monotonic.¹⁵

Assumption (2) is naturally fulfilled in cases of unique spillovers (see Examples 1 and 3) since $\Pi^i(\mathbf{x})$ is monotonic. This may no longer be the case in the presence of multiple and opposing spillovers as, for instance, in Example 2 and 4. In the first variant of the *Cournot duopoly with cost spillovers* (example (2.1) represented in figure (2)) the iso-payoff curves for both firms are represented by concentric circles. To the north (south) of $\Pi_2^1(\mathbf{x}) = 0$ firm 1's iso-payoff curves are concave (convex); to the east (west) of $\Pi_1^2(\mathbf{x}) = 0$ firm 2's iso-payoff curves are concave (convex). Given definition (1), we find that both firms regard supply as PS and that assumption (2) is fulfilled since \mathbf{x}^N , \mathbf{x}^{S_1} , and \mathbf{x}^{S_2} lie in the same vicinity so that condition (15) holds. In the second variant (Example 2.2, shown in Figure 10), condition (15) obviously holds since $\mathbf{x}^N = \mathbf{x}^{S_1} = \mathbf{x}^{S_2}$.

In the first variant of the *tax competition game* (Example 4.1 shown in Figure 4), country 1's NE payoff can be represented by two different iso-payoff curves: one lying to the northeast of $\Pi_1^2(\mathbf{x}) = 0$ and the other lying to the southwest. The former passes through \mathbf{x}^N , so by definition (1), country 1 regards taxes as PC. Since \mathbf{x}^{S_2} also lies to the northeast of $\Pi_1^2(\mathbf{x}) = 0$, we conclude that assumption (2) is fulfilled. In the second variant (Example 4.2 shown in Figure 6), we find that condition (15) holds since \mathbf{x}^N and \mathbf{x}^{S_2} both lie to the southwest of $\Pi_2^1(\mathbf{x}) = 0$, so $\{\Pi_2^1(\mathbf{x}^N), \Pi_2^1(\mathbf{x}^{S_2})\} \in \mathbb{R}_-$. Finally, in the third variant (Example 4.3 shown in Figure 8), we find that assumption (2) is satisfied for country 1 since $\Pi_2^1(\mathbf{x}^N) = 0$.

The next topic deals with the NE as a critical point.

¹⁵Given the monotonicity of $F_j^i(x_j)$, we find that $F_j^i(x_j) \neq 0 \forall x_j \in X_j$. It is easy to check that for all the examples presented here, the monotonicity of $F_j^i(\mathbf{x})$ does hold.

3.4. The Nash equilibrium as a critical point

In the presence of non-monotonic payoffs, we may find that $\Pi^i(\mathbf{x})$ has a critical point, that is,

$$\nabla \Pi^i(\mathbf{x}^N) \triangleq \left(\frac{d\Pi^i(\mathbf{x}^N)}{d\mathbf{x}} \right)^T = \mathbf{0}.$$

Then, \mathbf{x}^N is either a global maximum of $\Pi^i(\mathbf{x})$ or a saddle point. It is the former if the determinant of the Hessian matrix at \mathbf{x}^N , $\det(\nabla^2 \Pi^i(\mathbf{x}^N))$, is positive, while it is the latter when $\det(\nabla^2 \Pi^i(\mathbf{x}^N))$ is negative.¹⁶ We state the following LEMMA.

Lemma 3

If and only if $\Pi_{jj}^i(\mathbf{x}^N) = 0$ we have $\nabla \Pi^i(\mathbf{x}^N) = \mathbf{0}$. Then, \mathbf{x}^N is a saddle point of $\Pi^i(\mathbf{x})$ if $\Pi_{jj}^i(\mathbf{x}^N) \geq 0$.

Proof. See APPENDIX A.2. ■

Thus, if $\Pi_j^i(\mathbf{x}^N) = 0$ and $\Pi_{jj}^i(\mathbf{x}) > 0$, as for player $i = 1$ in the third variant of the tax competition game (Example 4.3 shown in Figure 8), we find that

$$x_j^N = \operatorname{argmin}_{x_j} F^i(x_j) \quad \text{and} \quad x_i^N = \operatorname{argmax}_{x_i} L^i(x_i) \quad (16)$$

because of the strict concavity of $L^i(x_i)$ and the local monotonicity of $F^i(x_j)$. Consequently, any $x_j^L \neq x_j^N$ leads to $\Pi^i(\mathbf{x}^{S_j}) = F^i(x_j^L) > F^i(x_j^N) = \Pi^i(\mathbf{x}^N)$.

If, on the other hand, $\Pi_j^i(\mathbf{x}^N) = 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$, as for player $i = 2$ in the second variant of the Cournot duopoly with cost spillovers (Example 2.2 shown in Figure 10), we find that

$$x_j^N = \operatorname{argmax}_{x_j} F^i(x_j) \quad \text{and} \quad x_i^N = \operatorname{argmax}_{x_i} L^i(x_i), \quad (17)$$

and any $x_j^L \neq x_j^N$ will lead to $\Pi^i(\mathbf{x}^{S_j}) = F^i(x_j^L) < F^i(x_j^N) = \Pi^i(\mathbf{x}^N)$. Then, the NE is Pareto efficient, a case which has been discussed in Ogawa and Wildasin (2009) and Eichner and Runkel (2012), among others.¹⁷ Thus, whether (16) or (17) applies will make an immense difference for i 's evaluation of any $\mathbf{x}^{S_j} \neq \mathbf{x}^N$: Does player i have a SMI or not? To answer this question we turn to the final LEMMA before the central THEOREM.

Lemma 4

Given assumptions (1) and (2), we find the following. Player i has

¹⁶Note that $\Pi_{ii}^i(\mathbf{x}^N) < 0$.

¹⁷Here, we refer to the discussion in the introduction.

1. *no second-mover incentive (no SMI) and a first-mover advantage (FMA), that is, $\Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^N) > \Pi^i(\mathbf{x}^{S_j})$, if either*

$$\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) < 0 \text{ or}$$

$$\Pi_j^i(\mathbf{x}^N) = 0, \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \neq 0 \text{ and } \det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0;$$

2. *a strong form of second-mover incentive (SSMI), that is $\Pi^i(\mathbf{x}^{S_j}) > \Pi^i(\mathbf{x}^N)$, if either*

$$\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) > 0 \text{ or}$$

$$\Pi_j^i(\mathbf{x}^N) = 0, \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \neq 0 \text{ and } \det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0;$$

3. *a weak-form of second-mover incentive (WSMI), that is, $\Pi^i(\mathbf{x}^{S_j}) = \Pi^i(\mathbf{x}^N)$, if*

$$\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) = 0;$$

4. *a strong form of first-mover incentive (SFMI), that is, $\Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^N)$, if*

$$\Pi_{ij}^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) \neq 0;$$

5. *a weak form of first-mover incentive (WFMI), that is, $\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N)$, if*

$$\Pi_{ij}^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) = 0.$$

Proof. See APPENDIX A.3. ■

The intuition is straightforward. In Example 1.1, both firms have no SMI and a FMA (cf. figure (1)). As a leader, player j will increase supply in order to decrease the opponent's supply ($\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) > 0$), and this is not to the benefit of firm i since it also regards strategies as PS ($\Pi_j^i(\mathbf{x}^N) < 0$). Thus, both firms gain (compared to the NE) by becoming a Stackelberg leader and suffer (compared to the NE) by becoming a follower. The opposite holds in Example 3.1, which is shown in Figure 3. Here, as a leader, player j will always decrease effort compared to the NE since she regards efforts as PS and the competitor regards efforts as SC ($\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) < 0$). This, however, is to the benefit of the competitor since she also regards efforts as PS ($\Pi_j^i(\mathbf{x}^N) < 0$). Consequently, both players have a strong form of SMI. Generally, if a player has a SSMI, we know that the Stackelberg equilibrium in which she follows is an element of the Pareto-superior set since Stackelberg follower payoffs are locally monotonic. If a player i has a SSMI we know that $\mathbf{x}^{S_j} \neq \mathbf{x}^N$, and therefore player j must have a SFMI which leads unambiguously to $\mathbf{x}^{S_j} \in \mathcal{P}^S$. Obviously, since both players have a SSMI in Example 3.1, both Stackelberg equilibria are an element of \mathcal{P}^S . The same holds in Example 4.1, where both countries have a SSMI, which can be seen in figure (4). Since none of the firms have a SSMI in Examples 1.1 and 2.1, we find that neither \mathbf{x}^{S_1} nor \mathbf{x}^{S_2} are elements of the Pareto-superior set (see Figures 1 and 2).

Note that once we know whether we have opposite or aligned plain interactions, we only need to know whether a player regards strategies as SC or SS in order to identify a FMA or a SSMI. For instance in Example 1.2 (cf. Figure 5), both firms regard supply as PS (aligned plain interactions). Because firm 1 (2) regards supply as SS (SC), it has a SSMI (FMA and no SMI). The case of opposite plain interactions can be found in Example 4.2 (cf. Figure 6), where we have strategic homogeneity. Country 1 tries to reduce and country 2 to increase the competitor's tax rate since the former regards taxes as PS and the latter regards them as PC. Since both countries regard taxes as SC, country 1 decreases its tax rate (which harms 2), while country 2 increases its tax rate (which harms country 1). Consequently, both countries have a FMA and no SMI.

A less complex case arises if either $\Pi_j^i(\mathbf{x}^N)$ or $\Pi_{ij}^i(\mathbf{x}^N)$ becomes zero. In Example 3.2 (cf. Figure 7), contestant 1 has a WSMI and contestant 2 has a WFMI since $\Pi_{12}^1(\mathbf{x}^N) = 0$ so that $\mathbf{x}^N = \mathbf{x}^{S_2}$. Figure 8 shows that a WSMI and a WFMI also emerge if one of the countries (here, country 1) regards taxes as neither PS nor PC. Then, $\mathbf{x}^N = \mathbf{x}^{S_1}$. Moreover, $\Pi_{22}^1(\mathbf{x}) > 0$ and country 1 has a SSMI since $x_2^N = \underset{x_2}{\operatorname{argmin}} F^1(x_2)$ and $x_2^I \neq x_2^N$ (cf. LEMMA (3)). In Example 2.2, we find that $\Pi_1^2(\mathbf{x}^N) = 0$, and consequently, $\mathbf{x}^N = \mathbf{x}^{S_2}$ (see Figure 10). However, in this case, $x_1^N = \underset{x_1}{\operatorname{argmax}} L^2(x_1)$, so the NE becomes Pareto efficient, that is, $\mathcal{P}^S = \emptyset$. Interestingly, we also find in this example that $\mathbf{x}^N = \mathbf{x}^{S_1}$ since $\Pi_{12}^2(\mathbf{x}^N) = 0$. Thus, both players have a WSMI and a WFMI.

4. Resolving the endogenous timing game

The issue of endogenous timing is examined according to the extended game proposed by Hamilton and Slutsky (1990). This game, denoted by $\tilde{\Gamma}$, allows players to non-cooperatively and simultaneously choose their timing decisions in a pre-play stage. The strategy set for player i in the pre-play stage is thus $d_i = \{e, l\}$, where e and l indicate decision making in the first and second stages of the basic game, respectively. Each player's decision concerning d_i is subsequently announced, and the players then choose their strategy for the basic game according to the timing decision to which they have committed. If both players choose their strategy early, so that $\mathbf{d} = (d_i, d_j) = (e, e)$, or late, so that $\mathbf{d} = (l, l)$, we find that players choose strategies \mathbf{x}^N in the SPE of the extended game. If player i chooses to move early and player j chooses to move late ($\mathbf{d} = (e, l)$), we find that both players choose \mathbf{x}^{S_i} at the SPE of the extended game. The extended game ($\tilde{\Gamma}$) thus has the following reduced normal form:

Under our assumptions, the solution of the timing game can be directly deduced from the nature of

		Player 2	
		e	l
Player 1	e	$\Pi^1(\mathbf{x}^N), \Pi^2(\mathbf{x}^N)$	$\Pi^1(\mathbf{x}^{S_1}), \Pi^2(\mathbf{x}^{S_1})$
	l	$\Pi^1(\mathbf{x}^{S_2}), \Pi^2(\mathbf{x}^{S_2})$	$\Pi^1(\mathbf{x}^N), \Pi^2(\mathbf{x}^N)$

Table 1
Normal form representation of the extended game

the interactions among players at the Cournot-Nash equilibrium. The (nonempty) set of SPEs of the extended game is denoted by $\mathcal{E} = \{\mathbf{d}, \mathbf{x}^\lambda\}$, with $\lambda = \{N, S_i, S_j\}$. Here, if \mathbf{d} implies a sequential choice of strategies, \mathbf{x}^λ must be subgame perfect. We split the main THEOREM, which shows that the SPE(s) is (are) contingent on the nature of plain and strategic interactions.

Theorem 1.A

Under ASSUMPTIONS (1) and (2) we find a unique subgame perfect equilibrium of the extended game under the following conditions. The SPE of $\tilde{\Gamma}$ equals

1. *the Cournot-Nash equilibrium, that is, $\mathcal{E} = \{(e, e), \mathbf{x}^N\}$, if either*
 1. $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$ and $\Pi_{ij}^i(\mathbf{x}^N) \leq \Pi_{ij}^j(\mathbf{x}^N) < 0$ or
 2. $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_{ij}^i(\mathbf{x}^N) \geq \Pi_{ij}^j(\mathbf{x}^N) > 0$;
2. *the Stackelberg equilibrium in which player i leads, that is, $\mathcal{E} = \{(e, l), \mathbf{x}^{S_i}\}$, if either*
 1. $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_{ij}^i(\mathbf{x}^N) > 0 > \Pi_{ij}^j(\mathbf{x}^N)$ or
 2. $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$ and $\Pi_{ij}^j(\mathbf{x}^N) > 0 > \Pi_{ij}^i(\mathbf{x}^N)$.

Proof. See APPENDIX A.4. ■

THEOREM (1.A.1) shows the case in which both players have a SFMI and no SMI. This case emerges if we have an aligned (opposed) plain interaction, and both players regard strategies as SS (SC), cf. LEMMA (4.1) and (4.4). Then, both players have a dominant strategy to move *early* so that $d_1 = d_2 = e$ in equilibrium. The case with aligned plain interaction (THEOREM (1.A.1.1)) emerges in Examples 1.1 and 2.1, shown in Figures 1 and 2, while the case with an opposite plain interaction (THEOREM (1.A.1.2)) emerges in Example 4.2, shown in Figure 6. In THEOREM (1.A.2), we have a unique SPE of $\tilde{\Gamma}$ with sequential moves. Again, player i has a dominant strategy to move *early* since he has a SFMI and no SMI, but now player j 's best response is to move *late* since he has a SSMI (cf. LEMMA (4.2)). This case is represented in Example 1.2, Figure 5 for $i = 2, j = 1$. Next, we turn to those cases in which $\tilde{\Gamma}$ has multiple SPEs.

Theorem 1.B

Under ASSUMPTIONS (1) and (2), we find multiple subgame perfect equilibria of the extended game under the following conditions. The SPEs of $\tilde{\Gamma}$ consists of the following.

1. **Both Stackelberg equilibria**, that is, $\mathcal{E} = \{(e, l), \mathbf{x}^{S_i}\} \cup \{(l, e), \mathbf{x}^{S_j}\}$, if either
 1. $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$ and $\Pi_{ij}^i(\mathbf{x}^N) \geq \Pi_{ij}^j(\mathbf{x}^N) > 0$;
 2. $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_{ij}^i(\mathbf{x}^N) \leq \Pi_{ij}^j(\mathbf{x}^N) < 0$;
 3. $\Pi_j^i(\mathbf{x}^N) = 0$, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) \neq 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0$;
 4. $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) > 0$ and $\Pi_{ij}^j(\mathbf{x}^N) = 0$.
2. **The Stackelberg equilibrium in which player i leads and the Cournot-Nash equilibrium**, that is, $\mathcal{E} = \{(e, e), \mathbf{x}^N\} \cup \{(e, l), \mathbf{x}^{S_i}\}$ if either
 1. $\Pi_j^i(\mathbf{x}^N) = 0$, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) \neq 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$;
 2. $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_{ij}^j(\mathbf{x}^N) = 0$.
3. **All equilibria in all basic games**, that is, $\mathcal{E} = \{(e, l), \mathbf{x}^{S_i}\} \cup \{(l, e), \mathbf{x}^{S_j}\} \cup \{(e, e), \mathbf{x}^N\} \cup \{(l, l), \mathbf{x}^N\}$ if $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) = 0$ and $\Pi_i^i(\mathbf{x}^N) \cdot \Pi_j^j(\mathbf{x}^N) = 0$.

Proof. See APPENDIX A.4. ■

THEOREMS (1.B.1.1) and (1.B.1.2) show that both Stackelberg equilibria turn out as the SPEs of $\tilde{\Gamma}$ if both players have a SFMI and a SSMI (cf. LEMMAS (4.2) and (4.4)). This only emerges in cases in which we have aligned (opposed) interactions and both players regard strategies as SC (SS). In Example 3.1, we thus find that both Stackelberg equilibria are SPEs of $\tilde{\Gamma}$ (cf. Figure 3). In addition, $\{(e, l), \mathbf{x}^{S_i}\}$ and $\{(l, e), \mathbf{x}^{S_j}\}$ are both SPEs of $\tilde{\Gamma}$ if the conditions of THEOREMS (1.B.1.3) and (1.B.1.4) apply, that is, if player i has a WFMI and a SSMI while player j has a WSMI and SFMI. Here, player j has a SFMI since $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) \neq 0$ (cf. LEMMA (4.4)). In THEOREM (1.B.1.3), player i has a SSMI since $x_j^N = \underset{x_j}{\operatorname{argmin}} F^i(x_j)$ (note that $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0$). In THEOREM (1.B.1.4), player i has a SSMI since either $x_j^L < x_j^N$ and $\Pi_j^i(\mathbf{x}^N) < 0$ or $x_j^L > x_j^N$ and $\Pi_j^i(\mathbf{x}^N) > 0$, both of which lead to $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) > 0$ (cf. LEMMA (4.2)). We thus conclude that any unilateral deviation from $\{(l, e), \mathbf{x}^{S_j}\}$ in the pre-play stage will make players worse off. We also find in THEOREMS (1.B.1.3) and (1.B.1.4) that $x_i^N = \underset{x_i}{\operatorname{argmax}} L^i(x_i)$, so that $\mathbf{x}^{S_i} = \mathbf{x}^N$. In THEOREM (1.B.1.3) because $\Pi_j^i(\mathbf{x}^N) = 0$ and in THEOREM (1.B.1.4) because $\Pi_{ij}^i(\mathbf{x}^N) = 0$. Thus, any unilateral deviation from $\{(e, l), \mathbf{x}^{S_i}\}$ in the pre-play stage will lead to the payoff equivalent NE, and hence $\{(e, l), \mathbf{x}^{S_i}\}$ constitutes the other SPE of $\tilde{\Gamma}$. The conditions of THEOREM (1.B.1.3) emerge in Example 4.3 for $i = 1, j = 2$ (cf. Figure 8) and those of THEOREM (1.B.1.4) in Example 3.2 for $i = 1, j = 2$ (cf. Figure 7).

Strictly speaking, the last two cases involve one pseudo-Stackelberg equilibrium as a SPE of $\tilde{\Gamma}$ since one of the Stackelberg equilibria is identical to the NE in terms of the equilibrium strategy profiles of the basic game. A pseudo-Stackelberg equilibrium also emerges as a SPE of $\tilde{\Gamma}$ according to THEOREM (1.B.2). In THEOREM (1.B.2.1), we find that player i again has a WFMI (since

	$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$
$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$(e, l)^* \cup (l, e)^*$	$(e, l)^* \cup (l, e)^*$	(e, e)
$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	(e, e)	(e, e)	$(e, l)^* \cup (l, e)^*$
$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	$(l, e)^*$	$(l, e)^*$	$(e, l)^*$
$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) = 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$
$\Pi_{ij}^i(\mathbf{x}^N) = 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	$(e, e) \cup (l, e)^\circ$	$(e, e) \cup (l, e)^\circ$	$(e, l)^* \cup (l, e)^\circ$
$\Pi_{ij}^i(\mathbf{x}^N) = 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$(e, l)^* \cup (l, e)^\circ$	$(e, l)^* \cup (l, e)^\circ$	$(e, e) \cup (l, e)^\circ$
$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$(e, l)^*$	$(e, l)^*$	$(l, e)^*$
$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) = 0$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (l, e)^*$

Table 2

Equilibrium strategy choice at the pre-play stage for $\Pi_{ij}^i(\mathbf{x}^N) \neq 0$. An asterisc (circle) indicates that the SPE of $\tilde{\Gamma}$ Pareto-dominates the NE of the static game Γ^N (is in weakly dominated strategies).

$\Pi_{ij}^i(\mathbf{x}^N) = 0$), player j has a WSMI, and $\mathbf{x}^{S_i} = \mathbf{x}^N$. However, now $x_j^N = \operatorname{argmax}_{x_j} F^i(x_j)$ (note that $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$) so that any $x_j^L \neq x_j^N$ is to the disadvantage of player i . Since player j has a SFMI ($\Pi_{ij}^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) \neq 0$), player i thus has no SMI, and therefore, a dominant strategy in the pre-play stage: $d_i = e$. Given this and the fact that player j has a WSMI, we find that $\mathcal{E} = \{(e, e), \mathbf{x}^N\} \cup \{(e, l), \mathbf{x}^{S_i}\}$, with $\mathbf{x}^N = \mathbf{x}^{S_i}$. The same SPEs emerge in THEOREM (1.B.2.2), where player j regards strategies neither as SC nor as SS so that he has a WSMI and player i has a WFMI ($\mathbf{x}^{S_i} = \mathbf{x}^N$). In addition, i has no SMI and j has a SFMI (since $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) < 0$), so both $\{(e, e), \mathbf{x}^N\}$ and $\{(e, l), \mathbf{x}^{S_i}\}$ constitute SPEs of $\tilde{\Gamma}$.

In the last case (THEOREM (1.B.3)), we find that any strategy profile \mathbf{d} constitutes a SPE of $\tilde{\Gamma}$ since $\mathbf{x}^N = \mathbf{x}^{S_i} = \mathbf{x}^{S_j}$. This case emerges in Example 2.2, shown in Figure 10.

5. Discussion

We will now discuss some of the obtained results. First, we state the following corollary.

	$\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0$ $\Pi_j^i(\mathbf{x}^N) = 0$ $\Pi_i^j(\mathbf{x}^N) < 0$	$\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0$ $\Pi_j^i(\mathbf{x}^N) = 0$ $\Pi_i^j(\mathbf{x}^N) > 0$	$\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$ $\Pi_j^i(\mathbf{x}^N) = 0$ $\Pi_i^j(\mathbf{x}^N) < 0$	$\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$ $\Pi_j^i(\mathbf{x}^N) = 0$ $\Pi_i^j(\mathbf{x}^N) > 0$
$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$
$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$
$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$
$\Pi_{ij}^i(\mathbf{x}^N) > 0$ $\Pi_{ij}^j(\mathbf{x}^N) = 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$
$\Pi_{ij}^i(\mathbf{x}^N) = 0$ $\Pi_{ij}^j(\mathbf{x}^N) < 0$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$
$\Pi_{ij}^i(\mathbf{x}^N) = 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$	$(e, e) \cup (l, l) \cup$ $(l, e) \cup (e, l)$
$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) > 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$
$\Pi_{ij}^i(\mathbf{x}^N) < 0$ $\Pi_{ij}^j(\mathbf{x}^N) = 0$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (l, e)^*$	$(e, l)^\circ \cup (e, e)$	$(e, l)^\circ \cup (e, e)$

Table 3

Equilibrium strategy choice at the pre-play stage for $\Pi_j^i(\mathbf{x}^N) = 0$. An asterisc (circle) indicates that the SPE of $\tilde{\Gamma}$ Pareto-dominates the NE of the static game Γ^N (is in weakly dominated strategies).

Corollary 1

Under ASSUMPTIONS (1) and (2), we find the following.

1. If the SPE of $\tilde{\Gamma}$ is unique, then $\Pi_{12}^1(\mathbf{x}^N) \cdot \Pi_{12}^2(\mathbf{x}^N) \cdot \Pi_2^1(\mathbf{x}^N) \cdot \Pi_1^2(\mathbf{x}^N) \neq 0$.
2. If and only if $\Pi_{12}^1(\mathbf{x}^N) \cdot \Pi_{12}^2(\mathbf{x}^N) \cdot \Pi_2^1(\mathbf{x}^N) \cdot \Pi_1^2(\mathbf{x}^N) = 0$ then
 1. $\mathcal{E} \subseteq \{(e, l), \mathbf{x}^{S_1}\} \cup \{(l, e), \mathbf{x}^{S_2}\}$,
 2. $\exists \{\mathbf{d}, \mathbf{x}^{S_i}\} \in \mathcal{E}$ s.t. $\mathbf{x}^{S_i} = \mathbf{x}^N$.

First, note that we can find a unique SPE of the extended game only if each player regards strategies as either SC or SS and as either PC or PS, that is, only if $\Pi_{ij}^i(\mathbf{x}) \neq 0$ and $\Pi_{ij}^j(\mathbf{x}) \neq 0$ for $\{i, j\} \in \{1, 2\}$ and $i \neq j$. Otherwise, we will have at least two SPEs of $\tilde{\Gamma}$. If so, we find that (i) at least one of the SPEs of $\tilde{\Gamma}$ shows sequential moves in the basic game and (ii) at least one of those sequential-move SPEs is payoff equivalent to the NE. However, we can not conclude that if $\mathbf{x}^N = \mathbf{x}^{S_i}$, both are elements of \mathcal{E} . For instance, in THEOREMS (1.B.1.3) and (1.B.1.4), we find that $\mathbf{x}^N = \mathbf{x}^{S_i}$ and neither $\{(e, e), \mathbf{x}^N\}$ nor $\{(l, l), \mathbf{x}^N\}$ is a SPE of $\tilde{\Gamma}$ (but $\{(e, l), \mathbf{x}^{S_i}\}$ and $\{(l, e), \mathbf{x}^{S_j}\}$

are). Basically, because of the non-monotonicity of the payoff or marginal payoff function, the sequential choice of strategies in the basic game does not lead to a strategy choice in a Stackelberg equilibrium that is different from that in the Cournot-Nash equilibrium.

Second, note that thus far, we have not covered the problems that arise in the presence of multiple Nash equilibria in the pre-play stage, as have been shown to exist in THEOREM (1.B). The literature has suggested several refinements in order to solve these problems. One very intuitive concept is that of Pareto dominance: a NE is more likely to occur if it dominates all other NEs in a Pareto sense. Given our setup, we then find that the Pareto-dominated equilibria are (at least for one player) in weakly dominated strategies.¹⁸ Next, we discuss the viability of the NE and analyze when the NE is a commitment-robust equilibrium (CRE). The CRE was introduced by Rosenthal (1991) and revised by van Damme and Hurkens (1996). According to Rosenthal (1991), a CRE emerges if the strategies in the NE are also a SPE of each sequential-move game given that (i) the Stackelberg leader can commit to a mixed strategy and (ii) the Stackelberg follower cannot observe the realization of the mixed strategy prior to the own strategy choice.¹⁹ A different definition of a CRE was used by van Damme and Hurkens (1996) because they rule out fractional commitment. In their definition a CRE emerges if no player has an incentive to move first.²⁰ Given our framework, particularly the fact that we rule out mixed strategy choices, the two definitions coincide.

Definition 5 (CRE, Rosenthal (1991), van Damme and Hurkens (1996))

The Cournot Nash Equilibrium of the simultaneous move game (NE) is a commitment robust equilibrium (CRE) if and only if each player has a weak form of first-mover incentive (WFMI), that is,

$$\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N) \quad \forall i \in \{1, 2\}.$$

Then, we also must have $\mathbf{x}^N = \mathbf{x}^{S_1} = \mathbf{x}^{S_2}$.

Thus, a CRE emerges only in those cases discussed in THEOREM (1.B.3). In line with van Damme and Hurkens (1996), we now ask which of the NE are *viable* given that players have the ability to commit.²¹ A NE is *viable* in our terms if the strategy choice of the basic game in each SPE of $\tilde{\Gamma}$ coincides with \mathbf{x}^N . In this case, the strategy choice in the NE survives if the basic game is extended in a manner that allows for endogenous commitment. Therefore, the viability of the NE is a weaker

¹⁸All these equilibria are labeled in tables (2) and (3) by a circle.

¹⁹van Damme and Hurkens (1996) point out that, in the words of Schelling (1960) this bears comparison with *fractional commitment*, which is contrary to the *pure unconditional commitment* discussed so far.

²⁰In our terms each player has only a WFMI.

²¹However, we use a different framework since we rule out the use of mixed strategies and since we use a different model of endogenous timing. To be precise, van Damme and Hurkens (1996) apply the *action commitment* model of Hamilton and Slutsky (1990).

concept than commitment robustness. Consequently, a CRE is always viable, but the converse does not hold. If a NE is not viable, then there is at least one SPE of $\tilde{\Gamma}$ with sequential play that is an element of the Pareto-superior set, that is, there is an SPE in which commitment has a social value. Regarding the Pareto-superior set, we find the following: if $\mathbf{x}^{S_i} \notin \mathcal{P}^S$ and $\mathbf{x}^{S_i} \neq \mathbf{x}^N \Rightarrow \mathbf{x}^{S_i} \notin \mathcal{E}$ - that is, if the Stackelberg equilibrium in which i leads is not an element of \mathcal{P}^S and $x_i^L \neq x_i^N$ then this Stackelberg equilibrium cannot be a SPE of the extended game. However, if $\mathbf{x}^{S_i} \in \mathcal{P}^S \Rightarrow \mathbf{x}^{S_i} \in \mathcal{E}$. We say that the Stackelberg equilibrium is *dominant* if any SPE of $\tilde{\Gamma}$ shows sequential moves and hence, each SPE Pareto-dominates the NE.

Tables (2) and (3) show the equilibrium strategies at the pre-play stage for all permutations of the relevant parameters.²² If a SPE of $\tilde{\Gamma}$ Pareto-dominates the NE, then it is marked with an asterisk. Thus, the Stackelberg equilibrium is *dominant* if any strategy choice in a cell of tables (2) or (3) has an asterisk. The complementary case (no asterisk) arises if the NE is viable. We state the following corollary.

Corollary 2

Under ASSUMPTIONS (1) and (2), we find the following.

1. The NE is viable, that is, no SPE of the extended game $\tilde{\Gamma}$ is an element of \mathcal{P}^S , that is if
 1. $\Pi_{ij}^i(\mathbf{x}^N) \geq \Pi_{ij}^j(\mathbf{x}^N) > 0$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$,
 2. $\Pi_{ij}^i(\mathbf{x}^N) \leq \Pi_{ij}^j(\mathbf{x}^N) < 0$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$,
 3. $\Pi_{ij}^i(\mathbf{x}^N) > 0 = \Pi_{ij}^j(\mathbf{x}^N)$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$,
 4. $\Pi_{ij}^i(\mathbf{x}^N) < 0 = \Pi_{ij}^j(\mathbf{x}^N)$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$,
 5. $\Pi_j^i(\mathbf{x}^N) = 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$,
 6. $\Pi_{ij}^i(\mathbf{x}^N) = \Pi_j^i(\mathbf{x}^N) = 0$.
2. The Stackelberg equilibrium is dominant, that is each SPE of the extended game $\tilde{\Gamma}$ is an element of \mathcal{P}^S , if
 1. $\Pi_{ij}^i(\mathbf{x}^N) \geq \Pi_{ij}^j(\mathbf{x}^N) > 0$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^i(\mathbf{x}^N) > 0$,
 2. $\Pi_{ij}^i(\mathbf{x}^N) \leq \Pi_{ij}^j(\mathbf{x}^N) < 0$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^i(\mathbf{x}^N) < 0$,
 3. $\Pi_{ij}^i(\mathbf{x}^N) > 0 > \Pi_{ij}^j(\mathbf{x}^N)$ and $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^i(\mathbf{x}^N) \neq 0$.

Proof. Immediate ■

6. Conclusion

We generalize the model of Hamilton and Slutsky (1990) by allowing the non-monotonicity of the payoff and the marginal payoff function. Assuming the uniqueness of the NE, the concavity

²²We omit the trivial cases in which $\Pi_{ij}^i(\mathbf{x}^N) = \Pi_{ij}^j(\mathbf{x}^N) = 0$ or $\Pi_j^i(\mathbf{x}^N) = \Pi_i^j(\mathbf{x}^N) = 0$ where we always find $\mathbf{x}^{S_i} = \mathbf{x}^N = \mathbf{x}^{S_j}$ so that the equilibrium strategy choice in the pre-play stage is always $(l, e) \cup (e, l) \cup (e, e) \cup (l, l)$.

of the leader payoff function, and the local monotonicity of the follower payoff, we propose a taxonomy of the endogenous timing game based solely on the properties of the Cournot-Nash equilibrium of the basic (static) game (denoted by NE). We consider two dimensions to identify the nature of the basic game: the plain property defined by Eaton (2004) and the strategic property given by Bulow, Geanakoplos, and Klemperer (1985b). Given non-monotonicities and potential asymmetries among players, the two previous criteria make it possible to distinguish 32 different basic games. We establish which plain and strategic properties involve a strong or weak first- or second-mover incentive and advantage. This allows us to solve the endogenous timing game for all cases. We then deduce under which conditions (for which combination of plain and strategic properties at the NE) the NE is commitment robust (in the sense of Rosenthal, 1991) and viable (in the sense of van Damme and Hurkens, 1996). We also highlight when commitment has a social value that allows players to reach a SPE that Pareto-dominates the NE. In particular, we show that committing to play early or late is socially desirable when both players display plain and strategic complementarities or substitutabilities or when the strategic property differs among them regardless of the plain property (as long as the latter is determined).

Our analysis is relevant in many applied fields. Several examples in industrial organization (Cournot and Bertrand duopoly) or public economics (tax competition and prize contest) illustrate our results. The main limits are the assumptions of the uniqueness of a NE of the static game and the concavity of the leader's payoff function. Beyond these constraints, we contribute to the questioning of the relevance of the Nash equilibrium in line with Schelling (1960), who emphasized the crucial role of commitments.²³

²³Myerson 2009, p. 28 writes: "Thus, Strategy of Conflict demonstrated both the importance of non-cooperative equilibrium analysis and the inadequacy of doing it only in the normal form."

Appendix A. Proofs

Appendix A.1. Proof of Lemma 2

We define

$$\Lambda^i(x_i) \equiv \Pi_i^i(x_i, BR^j(x_i)) + \Pi_j^i(x_i, BR^j(x_i)) \frac{dBR^j(x_i)}{dx_i}, \quad (\text{A.1})$$

so that $\Lambda^i(x_i)$ corresponds to the first derivative of the leader's payoff function. Due to the definition of the x_i^L and x_i^N we know that $\Lambda^i(x_i^L) = 0$ and

$$\Lambda^i(x_i^N) = \Pi_j^i(x_i^N, BR^j(x_i^N)) \frac{dBR^j(x_i)}{dx_i} \Big|_{x_i=x_i^N} = \Pi_j^i(\mathbf{x}^N) \left(-\frac{\Pi_{ij}^j(\mathbf{x}^N)}{\Pi_{ii}^i(\mathbf{x}^N)} \right). \quad (\text{A.2})$$

Since $\Pi^i(x_i, BR^j(x_i))$ is strictly concave and $\Pi_{ii}^i(\mathbf{x}^N) < 0$ we now can distinguish between the following cases:

1. If $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) > 0$, then $\Lambda^i(x_i^N) > 0 = \Lambda^i(x_i^L)$ so that $x_i^L > x_i^N$.
2. If $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) < 0$, then $\Lambda^i(x_i^N) < 0 = \Lambda^i(x_i^L)$ so that $x_i^L < x_i^N$.
3. If $\Pi_j^i(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) = 0$, then $\Lambda^i(x_i^N) = 0 = \Lambda^i(x_i^L)$ so that $x_i^L = x_i^N$.

Appendix A.2. Proof of LEMMA 3

The Hesse matrix of $\Pi^i(\mathbf{x})$ evaluated at \mathbf{x}^N is given by

$$\nabla^2 \Pi^i(\mathbf{x}^N) \triangleq \left(\frac{d^2 \Pi^i(\mathbf{x}^N)}{d\mathbf{x}^T d\mathbf{x}} \right)^T = \begin{bmatrix} \Pi_{ii}^i(\mathbf{x}^N) & \Pi_{ij}^i(\mathbf{x}^N) \\ \Pi_{ji}^i(\mathbf{x}^N) & \Pi_{jj}^i(\mathbf{x}^N) \end{bmatrix} \quad (\text{A.3})$$

It is obvious that

$$\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0 \Leftrightarrow \Pi_{ii}^i(\mathbf{x}^N) \Pi_{jj}^i(\mathbf{x}^N) < \left(\Pi_{ij}^i(\mathbf{x}^N) \right)^2 \quad (\text{A.4})$$

if $\Pi_{jj}^i(\mathbf{x}^N) \geq 0$ since $\Pi_{ii}^i(\mathbf{x}^N) < 0$.

Appendix A.3. Proof of LEMMA 4

- Given definition (4) we know that $\Pi^i(\mathbf{x}^{S_j}) > \Pi^i(\mathbf{x}^N)$ if player i has a SSMI, and that $\Pi^i(\mathbf{x}^{S_j}) = \Pi^i(\mathbf{x}^N)$ if player i has a WSMI. Generally, in case of a SMI we find that

$$\Pi^i(\mathbf{x}^N) = F^i(x_j^N) \triangleq \Pi^i(BR^i(x_j^N), x_j^N) \leq \Pi^i(BR^i(x_j^L), x_j^L) \triangleq F^i(x_j^L) = \Pi^i(\mathbf{x}^{S_j}). \quad (\text{A.5})$$

Given the local monotonicity of $F^i(x_j)$ the weak inequality in eq. (A.5) becomes strict if either

1. $x_j^N > x_j^L$ and $\Pi_j^i(\mathbf{x}^N) < 0 \Leftrightarrow \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_j^i(\mathbf{x}^N) < 0$, or
2. $x_j^N < x_j^L$ and $\Pi_j^i(\mathbf{x}^N) > 0 \Leftrightarrow \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$ and $\Pi_j^i(\mathbf{x}^N) > 0$,
3. $x_j^N \neq x_j^L$ and $\Pi_j^i(\mathbf{x}^N) = 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0$.

Note, that in the last case $x_j^N = \underset{x_j}{\operatorname{argmin}} F^i(x_j)$. The weak inequalities in eq. (A.5) become equalities if $x_j^N = x_j^L$. This is the case if $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) = 0$.

- Given definition (4) we know that player i has no SMI if $\Pi^i(\mathbf{x}^{S_j}) < \Pi^i(\mathbf{x}^N)$.

$$F^i(x_j^N) \equiv \Pi^i(BR^i(x_j^N), x_j^N) > \Pi^i(BR^i(x_j^L), x_j^L) \equiv F^i(x_j^L). \quad (\text{A.6})$$

The condition in eq. (A.6) holds if either

1. $x_j^N < x_j^L$ and $\Pi_j^i(\mathbf{x}^N) < 0 \Leftrightarrow \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$ and $\Pi_j^i(\mathbf{x}^N) < 0$, or
2. $x_j^N > x_j^L$ and $\Pi_j^i(\mathbf{x}^N) > 0 \Leftrightarrow \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_j^i(\mathbf{x}^N) > 0$, or
3. $x_j^N \neq x_j^L$ and $\Pi_j^i(\mathbf{x}^N) = 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$.

Note that $x_j^N = \operatorname{argmax}_{x_j} F^i(x_j)$ in the last case.

- By the definition of the Stackelberg and the Cournot-Nash equilibrium we know that the following always holds:

$$\Pi^i(\mathbf{x}^{S_i}) \equiv \max_{x_i} \Pi^i(x_i, BR^j(x_i)) \geq \Pi^i(x_i^N, BR^j(x_i^N)) = \Pi^i(\mathbf{x}^N), \quad (\text{A.7})$$

which means that every player has a FMI. The weak inequality in eq. (A.7) becomes strict if $x_i^N \neq x_i^L$, which emerges if $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \neq 0$. The weak inequality in eq. (A.7) becomes an equality if $x_i^N = x_i^L$, which emerges if $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) = 0$.

Appendix A.4. Proof of THEOREM 1.A and 1.B

1. If $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) > 0$ holds for both players, then both players have a SSMI and a SFMI:

$$\Pi^i(\mathbf{x}^{S_j}) > \Pi^i(\mathbf{x}^N) \text{ and } \Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^N) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_i}) > \Pi^j(\mathbf{x}^N) \text{ and } \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N), \quad (\text{A.8})$$

so that (e, l) and (l, e) are the SPE of the timing game (cf. THEOREM (1.B.1.1) and (1.B.1.2)).

2. If $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ holds for both players, then both players have a SFMI and no SMI:

$$\Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^N) > \Pi^i(\mathbf{x}^{S_j}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N) > \Pi^j(\mathbf{x}^{S_i}), \quad (\text{A.9})$$

so that each player has a dominant strategy in the timing game ($d_i = d_j = e$) and we have a unique SPE of $\tilde{\Gamma}$ with (e, e) (cf. THEOREM (1.A.1.1) and (1.A.1.2)).

3. If $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) < 0$ and $\Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_i^j(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) > 0$, then player i has a SFMI and no SMI while player j has a SFMI and a SSMI:

$$\Pi^i(\mathbf{x}^{S_i}) > \Pi^i(\mathbf{x}^N) > \Pi^i(\mathbf{x}^{S_j}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_i}) > \Pi^j(\mathbf{x}^N) \text{ and } \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N), \quad (\text{A.10})$$

so player i has a dominant strategy ($d_i = e$) and player j 's best response to this is ($d_j = l$), so that at the SPE of $\tilde{\Gamma}$ we have (e, l) (cf. THEOREM (1.A.2.1) and (1.A.2.2)).

4. If $\Pi_j^i(\mathbf{x}^N) = 0$ then $\mathbf{x}^N = \mathbf{x}^{S_i}$ and player i has a WFMI and player j a WSMI.

1. If, in addition, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) \neq 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) > 0$ then player j has a SFMI, and player i no SMI since $x_j^N = \operatorname{argmax}_{x_j} F^i(x_j)$ (see eq. (17)), or more formally,

$$\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N) > \Pi^i(\mathbf{x}^{S_j}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N) = \Pi^j(\mathbf{x}^{S_i}). \quad (\text{A.11})$$

Player i has a dominant strategy in the timing game ($d_i = e$) and player j is indifferent between $d_j = e$ and $d_j = l$ so that (e, e) and (e, l) are SPE of the timing game (cf. THEOREM (1.B.2.1)).

2. If, in addition, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) \neq 0$ and $\det(\nabla^2 \Pi^i(\mathbf{x}^N)) < 0$ then player j has a SFMI, and player i a SSMI since $x_j^N = \operatorname{argmin}_{x_j} F^i(x_j)$ (see eq. (16)), or more formally,

$$\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N) < \Pi^i(\mathbf{x}^{S_j}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N) = \Pi^j(\mathbf{x}^{S_i}). \quad (\text{A.12})$$

So (e, l) and (l, e) are SPE of the timing game (cf. THEOREM (1.B.1.3)), where the latter is an element of \mathcal{P}^S but the former is not.

3. If, in addition, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^j(\mathbf{x}^N) = 0$, then player j has a WFMI, and player i a WSMI. Then we find that

$$\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N) = \Pi^i(\mathbf{x}^{S_j}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_j}) = \Pi^j(\mathbf{x}^N) = \Pi^j(\mathbf{x}^{S_i}) \quad (\text{A.13})$$

and we have four pure strategy equilibria in the timing game (cf. THEOREM (1.B.3)).

5. If $\Pi_{ij}^j(\mathbf{x}^N) = 0$ then $\mathbf{x}^N = \mathbf{x}^{S_i}$ and player i has a WFMI and player j a WSMI.

1. If, in addition, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) < 0$ then player j has a SFMI and player i has no SMI:

$$\Pi^i(\mathbf{x}^{S_i}) = \Pi^i(\mathbf{x}^N) > \Pi^i(\mathbf{x}^{S_j}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N) = \Pi^j(\mathbf{x}^{S_i}). \quad (\text{A.14})$$

So player i has a dominant strategy ($d_i = e$) and player j is indifferent between all pure strategies in the timing game. Hence, (e, e) and (e, l) are SPE of the timing game (cf. THEOREM (1.B.2.2)).

2. If, in addition, $\Pi_i^j(\mathbf{x}^N) \cdot \Pi_{ij}^i(\mathbf{x}^N) \cdot \Pi_j^i(\mathbf{x}^N) > 0$ then player j has a SFMI and player i a SSMI:

$$\Pi^i(\mathbf{x}^{S_j}) > \Pi^i(\mathbf{x}^N) = \Pi^i(\mathbf{x}^{S_i}) \quad \text{and} \quad \Pi^j(\mathbf{x}^{S_j}) > \Pi^j(\mathbf{x}^N) = \Pi^j(\mathbf{x}^{S_i}). \quad (\text{A.15})$$

So (e, l) and (l, e) are SPE of the timing game (cf. THEOREM (1.B.1.4)), where the latter is an element of \mathcal{P}^S but the former is not.

Appendix B. Examples

Example 1 - Cournot Duopoly with constant elasticity of demand

Suppose we have two firms (1,2) that compete in a Cournot duopoly with constant marginal costs and constant elasticity of demand. The demand is given by $P(\mathbf{x}) = (a(b(x_1 + x_2))^{-1})^\alpha$ with $\alpha \in (0, 1)$, $b > 0$ and the marginal costs are given by $c_1 > 0$, $c_2 > 0$. The profit of firm 1 is thus

$$\Pi^i(\mathbf{x}) = \left(\left(\frac{a}{b(x_1 + x_2)} \right)^\alpha - c_i \right) x_i. \quad (\text{B.1})$$

It follows that

$$\Pi_j^i(\mathbf{x}) = -\alpha x_i \left(\frac{a}{b} \right)^\alpha (x_i + x_j)^{-1-\alpha} \quad (\text{B.2})$$

which is negative $\forall \mathbf{x} \geq \mathbf{0}$, and

$$\Pi_{ij}^i(\mathbf{x}) = \alpha \left(\frac{a}{b} \right)^\alpha (x_i + x_j)^{-2-\alpha} (\alpha x_i - x_j) \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} 0 \Leftrightarrow \alpha x_i \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} x_j. \quad (\text{B.3})$$

Example 1.1 - Figure 1, page 8

For $a = 10$, $c_1 = c_2 = 6$, $b = 2$, $\alpha = \frac{4}{5}$ we find

- $\mathbf{x}^N \approx (0.1406, 0.1406)$,
- $\Pi_{12}^1(\mathbf{x}^N) = \Pi_{12}^2(\mathbf{x}^N) \approx -2.8453$,
- $\mathbf{x}^{S_i} \approx (0.1617, 0.1381)$.

Example 1.2 - Figure 5, page 12

For $a = 10$, $c_1 = 6$, $c_2 = 8$, $b = 2$, $\alpha = \frac{4}{5}$ we find

- $\mathbf{x}^N \approx (0.1408, 0.0911)$,
- $\Pi_{12}^1(\mathbf{x}^N) \approx 3.7374 > 0 > -11.7871 \approx \Pi_{12}^2(\mathbf{x}^N)$,
- $\mathbf{x}^{S_1} \approx (0.2216, 0.0664)$ and $\mathbf{x}^{S_2} \approx (0.0811, 0.1395)$.

Example 2 - Cournot Duopoly with cost spillovers

Suppose we have two firms (1,2) that compete in a Cournot duopoly with a linear demand function and constant marginal costs. Spillover effects emerge as cost reduction. Suppose the demand function is given by $P(\mathbf{x}) = a - b_1 x_1 - b_2 x_2$, with $a > 0$, $b_1 > 0$ and $b_2 > 0$, and the cost function of firm i by $C_i(x_i) = c_i x_i \exp(-\gamma_i x_j)$, with $a > c_i > 0$ and $\gamma_i > 0$. The profit is thus

$$\Pi^i(\mathbf{x}) = (a - b_1 x_1 - b_2 x_2 - c_i \exp(-\gamma_i x_j)) x_i. \quad (\text{B.4})$$

It follows that

$$\Pi_j^i(\mathbf{x}) = (-b_j + c_i \gamma_i \exp(-\gamma_i x_j)) x_i \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} 0 \Leftrightarrow x_j \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} \frac{1}{\gamma_i} \log \left(\frac{b_j}{c_i \gamma_i} \right) \quad (\text{B.5})$$

and

$$\Pi_{ij}^i(\mathbf{x}) = -b_j + c_i \gamma_i \exp(-\gamma_i x_j) \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} 0 \Leftrightarrow x_j \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} \frac{1}{\gamma_i} \log \left(\frac{b_j}{c_i \gamma_i} \right). \quad (\text{B.6})$$

Hence, a firm either regards supply as PS and SS or as PC and SC.

Example 2.1 - Figure 2, page 8

For $a = 10$, $c_1 = c_2 = 7$, $b_1 = b_2 = 1$, $\gamma_1 = \gamma_2 = 1$ we find

- $\mathbf{x}^N \approx (3.2422, 3.2422)$,
- $\Pi_2^1(\mathbf{x}^N) = \Pi_1^2(\mathbf{x}^N) \approx -2.3552$,

- $\Pi_{12}^1(\mathbf{x}^N) = \Pi_{12}^2(\mathbf{x}^N) \approx -0.7264$,
- $\mathbf{x}^{S_i} \approx (3.8625, 2.9952)$.

Example 2.2 - Figure 10, page 15

For $a = 10$, $c_1 \approx 6.8489$, $c_2 = 9$, $b_1 = 1$, $b_2 = 3$, $\gamma_1 = \gamma_2 = 1$ we find

- $\mathbf{x}^N \approx (2.1973, 1.1338)$,
- $\Pi_2^1(\mathbf{x}^N) \approx -1.7489$, $\Pi_1^2(\mathbf{x}^N) \approx 0$,
- $\Pi_{12}^1(\mathbf{x}^N) \approx -0.7959$, $\Pi_{12}^2(\mathbf{x}^N) \approx 0$,
- $\mathbf{x}^{S_1} \approx (2.1973, 1.1338)$, $\mathbf{x}^{S_2} \approx (1.1338, 2.1973)$.

Example 3 - Contest with reimbursements

Suppose two players (1, 2) compete for a prize of value $V > 0$. The probability of winning for player i is given by a Tullock lottery contest success function,

$$p^i(x_1, x_2) = \begin{cases} \frac{x_i}{x_1 + x_2} & \text{for } \mathbf{x} > \mathbf{0}, \\ \frac{1}{2} & \text{for } \mathbf{x} = \mathbf{0}, \end{cases}$$

and the marginal costs of effort are $c_i > 0$. Moreover, assume that the loser is reimbursed for a part α of her outlays and that $c_i > \alpha > 0$. The payoff of player i becomes

$$\Pi^i(\mathbf{x}) = \frac{x_i}{x_i + x_j} (V - x_i) + \frac{x_j}{x_i + x_j} (-x_i + \alpha x_i) \quad (\text{B.7})$$

so that

$$\Pi_j^i(\mathbf{x}) = -\frac{x_i(V - \alpha x_i)}{(x_i + x_j)^2}. \quad (\text{B.8})$$

Note that

$$\Pi^i(\mathbf{x}) \geq 0 \Leftrightarrow x_i \leq \frac{V - c_i x_j + \alpha x_j}{c_i} \equiv h_i(x_j)$$

and that

$$h_i'(x_j) = \alpha - c_i < 0.$$

Consequently,

$$0 = \operatorname{argmax}_{x_j \geq 0} h_i(x_j)$$

and the strategy space of player i becomes $X_i = [0, \bar{x}_i]$, with $\bar{x}_i \triangleq h_i(0) = \frac{V}{c_i}$, since any $x_i > \bar{x}_i$ is a strictly dominated strategy. This holds for both players. Turning to eq. (B.8) we find that $\Pi_j^i(\mathbf{x}) \geq 0$ iff $x_i \geq \frac{V}{\alpha} > \frac{V}{c} \triangleq \bar{x}_i$. Since $\mathbf{x}^N \in [0, \bar{x}_i] \times [0, \bar{x}_j]$ we find that $\Pi_j^i(\mathbf{x}^N) < 0$ and both players regard efforts as PS.

Turning to the cross-partial derivative we find that

$$\Pi_{ij}^i(\mathbf{x}) = \frac{V(x_i + x_j) + 2x_i x_j \alpha}{(x_i + x_j)^3} \begin{cases} \leq \\ > \end{cases} 0 \Leftrightarrow x_i \begin{cases} \leq \\ > \end{cases} \frac{V x_j}{V + 2 x_j \alpha}. \quad (\text{B.9})$$

Example 3.1 - Figure 3, page 9

For $\alpha = 0.9$, $c_1 = c_2 = 1$, $V = 10$ we find

- $\mathbf{x}^N \approx (3.2258, 3.2258)$,
- $\Pi_{12}^1(\mathbf{x}^N) = \Pi_{12}^2(\mathbf{x}^N) \approx 0.06975$,
- $\mathbf{x}^{S_i} \approx (2.7222, 3.0994)$.

Example 3.2 - Figure 7, page 13

For $\alpha = 0.5$, $c_1 = 1.3064$, $c_2 = 1$, $V = 10$ we find

- $\mathbf{x}^N \approx (2.1434, 2.7280)$,
- $\Pi_{12}^1(\mathbf{x}^N) = 0 < 0.1012 \approx \Pi_{12}^2(\mathbf{x}^N)$,
- $\mathbf{x}^{S_1} \approx (1.6524, 2.5772)$ and $\mathbf{x}^{S_2} \approx (2.7280, 2.1434)$.

Example 4 - Tax competition

For $a_1 = 10$, $b = 10$, $\sigma_1 = \sigma_2 = 0.5$, and $\varepsilon = 0.1$ we find that

$$\Pi_{12}^1(\mathbf{x}) = \Pi_{12}^2(\mathbf{x}) = 0.03, \quad (\text{B.10})$$

and

$$\Pi_2^1(\mathbf{x}) = \frac{1}{4} - \frac{5a_2 + 6x_1 + 5x_2}{200} \quad \text{and} \quad \Pi_1^2(\mathbf{x}) = -\frac{1}{4} + \frac{5a_2 + 5x_1 + 6x_2}{200}. \quad (\text{B.11})$$

Hence, both countries regard taxes as SC and may regard taxes as either PC or PS.

Example 4.1 - Figure 4, page 9

For $a_2 = 13$ and $\bar{k} = 1$ we find

- $\mathbf{x}^N \approx (1.0356, 2.6008)$,
- $\Pi_2^1(\mathbf{x}^N) \approx 0.0211$ and $\Pi_1^2(\mathbf{x}^N) \approx 0.1789$,
- $\mathbf{x}^{S_1} \approx (1.1582, 2.6441)$ and $\mathbf{x}^{S_2} \approx (3.6409, 1.4027)$.

Example 4.2 - Figure 6, page 12

For $a_2 = 38.99$, and $\bar{k} = 5$ we find

- $\mathbf{x}^N \approx (1.5283, 16.6535)$,
- $\Pi_2^1(\mathbf{x}^N) \approx -0.2626$ and $\Pi_1^2(\mathbf{x}^N) \approx 1.2626$,
- $\mathbf{x}^{S_1} \approx (0.0019, 16.1148)$ and $\mathbf{x}^{S_2} \approx (23.9936, 4.1189)$.

Example 4.3 - Figure 8, page 13

For $a_2 = \frac{1670}{121}$ and $\bar{k} = 1$ we find

- $\mathbf{x}^N \approx (0.8265, 2.8099)$,
- $\Pi_2^1(\mathbf{x}^N) \approx 0$ and $\Pi_1^2(\mathbf{x}^N) \approx 0.2$,
- $\mathbf{x}^{S_1} \approx (0.8265, 2.8099)$ and $\mathbf{x}^{S_2} \approx (3.9726, 1.2368)$.

Example 5 - Amir (1995)

The payoffs of two players are given by

$$\Pi^1(\mathbf{x}) = -\frac{1}{2}x_1^2 + \frac{1}{4}x_1x_2 - x_2, \quad (\text{B.12})$$

$$\Pi^2(\mathbf{x}) = -\frac{1}{2}x_2^2 + x_1x_2 + \alpha x_2 - x_1^2, \quad (\text{B.13})$$

with $\{\alpha, x, y\} \in \mathbb{R}$. We find

$$\Pi_2^1(\mathbf{x}) = -1 + \frac{x_1}{4} \quad \text{and} \quad \Pi_{12}^1(\mathbf{x}) = \frac{1}{4}, \quad (\text{B.14})$$

$$\Pi_1^2(\mathbf{x}) = -2x_1 + x_2 \quad \text{and} \quad \Pi_{12}^2(\mathbf{x}) = 1. \quad (\text{B.15})$$

Example 5.1 - Figure 9, page 15

For $\alpha = -1$ we find

- $\mathbf{x}^N = (-\frac{1}{3}, -\frac{4}{3})$,
- $\mathbf{x}^{S_1} \approx (-\frac{5}{2}, -\frac{7}{2})$ and $\mathbf{x}^{S_2} \approx (-\frac{8}{5}, -\frac{2}{5})$,
- $\Pi_2^1(\mathbf{x}^N) = -\frac{13}{12}$ and $\Pi_2^1(\mathbf{x}^{S_2}) = -\frac{11}{10}$,
- $\Pi_1^2(\mathbf{x}^N) = -\frac{2}{3}$ and $\Pi_1^2(\mathbf{x}^{S_1}) = \frac{3}{2}$.

Player 2's follower payoff $\Pi^2(x_1) = \frac{1-x_1(2+x_1)}{2}$ is not only non-monotonic, but also locally non-monotonic. Here, it leads to $\Pi^2(\mathbf{x}^N) = \frac{7}{9} > -\frac{1}{8} = \Pi^2(\mathbf{x}^{S_1})$ although player 2 regards strategies as PS and SC, and $x_1^L < x_1^N$.

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