



HAL
open science

Analysis of a castable refractory using the wedge splitting test and cohesive zone model

R Vargas, J. Neggers, R B Canto, J A Rodrigues, François Hild

► To cite this version:

R Vargas, J. Neggers, R B Canto, J A Rodrigues, François Hild. Analysis of a castable refractory using the wedge splitting test and cohesive zone model. *Journal of the European Ceramic Society*, 2019, 39 (13), pp.3903-3914. 10.1016/j.jeurceramsoc.2019.03.009 . hal-02055553

HAL Id: hal-02055553

<https://hal.science/hal-02055553v1>

Submitted on 4 Mar 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Analysis of a castable refractory using the wedge splitting test and cohesive zone model

R. Vargas^a, J. Neggers^b, R. B. Canto^a, J. A. Rodrigues^a, F. Hild^b

^a*Federal University of Sao Carlos (UFSCar)
Graduate Program in Materials Science and Engineering
Rodovia Washington Luis, km 235, 13565-905, São Carlos-SP, Brazil*
^b*Laboratoire de Mécanique et Technologie (LMT)
ENS Paris-Saclay, CNRS, Université Paris-Saclay
61 avenue du Président Wilson, 94235 Cachan, France*

Abstract

A cohesive zone approach is applied to the Wedge Splitting Test (WST) using the finite element code Abaqus to obtain the tensile strength, the fracture energy and insight about the crack wake region. A Finite Element Model Updating (FEMU) method, with a cost function based on the measured load (FEMU-F), is used to calibrate the sought parameters. Digital Image Correlation (DIC) provided the kinematic boundary conditions, and the images were also used to define the geometry for the finite element analysis. Besides the fracture energy analysis and the experimental load, gray level images and displacement fields are analyzed in order to validate the results. The cohesive region is active in the whole analyzed test as confirmed by estimates using the cohesive length.

Keywords: Cohesive zone model, digital image correlation, finite element model updating, wedge splitting test, castable refractory

1. Introduction

Refractory castables are ceramic materials with a fine matrix and coarser aggregates, which are utilized in transformation industries such as steel making and oil refineries [1, 2, 3]. Their main goal is to ensure functional properties

*Corresponding author
Email address: hild@lmt.ens-cachan.fr (F. Hild)

5 at high temperatures and corrosive environments, thereby calling for complex
6 processing of various materials [3]. In these environments with considerable
7 thermal shocks between processing cycles, it is not optimal to prevent crack
8 initiation [4]. The applied approach consists in tailoring the microstructure
9 with suitable compositions to make crack propagation difficult. The most im-
10 portant toughening mechanisms are extrinsic resulting from the interaction of
11 the crack with the microstructure. Some examples [5, 6] are crack branching,
12 microcrack formation to alleviate stresses at the crack tip, bridging and phase
13 transformations (*e.g.*, tetragonal to monoclinic zirconia transformation).

14 To study these toughening mechanisms, stable crack propagation tests may
15 be performed in laboratory conditions. The Wedge Splitting Test (WST) al-
16 lows such fracture tests to be conducted, even on quasi-brittle materials, by
17 decreasing the elastic energy stored in the testing machine thanks to a wedge
18 and cylinders to apply an opening (mode I) load [7, 8]. This test is commonly
19 used for obtaining the fracture energy of these materials, which is a key prop-
20 erty for understanding the thermal shock resistance of refractories [9, 10]. The
21 advantage of this test includes a high fracture surface area to specimen volume
22 ratio, which is needed for obtaining representative results if big aggregates are
23 used for toughening purposes [8].

24 The WST may be combined with Digital Image Correlation (DIC) for study-
25 ing crack propagation. DIC is based on tracking material points during the
26 loading of the sample [11, 12, 13]. It is a full-field measurement technique in-
27 stead of providing local data points obtained by, say, conventional extensometry.
28 Recently, several studies have reported on how to treat such results and obtain
29 further information from WSTs [14, 15, 16, 17, 18, 19].

30 If toughening mechanisms are activated during fracture, it is hard to define
31 a “binary” crack, with either a fully broken or fully intact material. A Fracture
32 Process Zone (FPZ) is usually defined, where some damage has already occurred
33 but some tractions between the crack surfaces remain [20, 21]. In that case, a
34 Cohesive Zone Model (CZM) can be used [22]. The CZMs define the traction-
35 separation law, which accounts for the fracture process. Several studies use

36 CZMs in Finite Element Analyses (FEA) for simulating fracture in quasi-brittle
37 materials [21, 23, 24, 25, 26].

38 The calibration of cohesive zone properties with DIC measurements was ad-
39 dressed in various studies, mostly for modeling composites and/or adhesives.
40 Measured displacements were used as Boundary Conditions (BCs) and inner
41 nodal displacements in the objective function to identify the cohesive prop-
42 erties of fiber-reinforced metallic laminate. Discussions about how to obtain
43 elastic and cohesive properties by minimizing the gap between measured and
44 calculated displacements were also reported for a fiber-reinforced cementitious
45 material [27], and for plastic and PMMA with adhesive [28]. Reference [29]
46 presents a sensitivity analysis in order to analyze the most relevant region for
47 identifying a CZM with full-field measurements. The sensitivity for the identi-
48 fication of cohesive parameters for an adhesive bonded structure is discussed in
49 Ref. [30]. The authors concluded that higher sensitivity for the cohesive strength
50 may be reached at pre-peak, and for the fracture energy with post-peak data.
51 Traction-separation laws could be accessed directly with the kinematics of a
52 Double Cantilever Beam test for composite materials [31]. The importance of
53 using load data to identify a mixed-mode CZM for a composite was highlighted
54 in another study [32]. Conversely, mixed-mode CZMs were calibrated without
55 the need for force data, only using the images of the experiment on a microelec-
56 tronic device [33].

57 Some studies also showed the feasibility of combining DIC and CZM for
58 other materials. In Ref. [34], a multiscale setup is introduced for analyzing a
59 photodegradable copolymer. Elastic and cohesive properties for concrete ma-
60 terials were identified with the Boundary Element Method coupled with DIC
61 measurements [35]. Failure in metals was modeled with a CZM, which was
62 calibrated with DIC data [36]. Micrometer-scale mechanisms in PMMA could
63 be related to a traction-separation law using images taken close to the crack
64 tip [37]. No study on castable refractories was found with such approaches.

65 In the present work, the parameters of a macroscale CZM for mode I fracture
66 are calibrated with a single WST by coupling DIC measurements, load data and

67 FEAs performed with the commercial code Abaqus [38] for a castable refractory.
68 First, the identification framework is introduced, then followed by the methods
69 and definition of the parameters to be calibrated. Last, the results are shown
70 and compared with previously reported data on different methodologies.

71 **2. Calibration procedure**

72 *2.1. Experiment*

73 The WST analyzed herein was performed on a class C, anti-erosive commer-
74 cial refractory, with ultra low cement content. The detailed chemical composi-
75 tion and heat treatment of the material are reported in Ref. [15]. Its processing
76 and microstructure may lead to an increasing R-curve behavior with weakly
77 bonded grains and initiated microcracks due to anisotropic phases and differ-
78 ential thermal expansions. The sample size was 100 mm in length, 100 mm in
79 height and 72.5 mm in thickness. The geometry is shown in Fig. 1 along with
80 the mesh introduced in Section 2.3. It is possible to see the sample and the
81 loading devices (*i.e.*, wedge, cylinders and blocks). Two grooves (*i.e.*, vertical
82 notches, evidenced in the right image in Figure 1) are machined on the two
83 opposite faces of the sample to reduce the local thickness and guide the crack
84 propagation vertically [19].

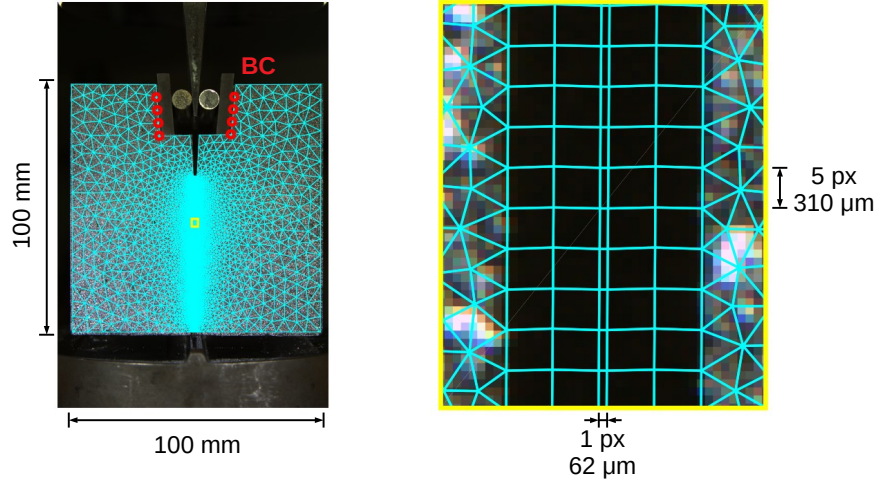


Figure 1: Sample geometry, with the FE mesh superimposed (left). The loading parts are visible at the top of the image. The red circles mark the nodes where the BCs are applied (see Section 2.3). The yellow box is zoomed (right), showing the mesh aligned with the groove edges. Triangular elements are used out of the groove, and Q4 quadrilaterals inside. The thin strip of elements in the middle of the groove shows the cohesive elements, which are collapsed to zero thickness for the present analyses.

85 The Young's modulus (E) and Poisson's ratio (ν) used for the investigated
 86 methods are equal to 17 GPa (measured by the bar resonance method [39]),
 87 and 0.2, respectively. The fracture energy calculated as the mean R-curve value
 88 (obtained by Integrated-DIC) is of the order of 68 J/m^2 [19]. The test was
 89 driven by setting the velocity of the machine actuator to $1.3 \text{ }\mu\text{m/s}$, and 313
 90 pictures (reference + 312) were taken for both faces of the specimen at a rate of
 91 one picture each 8 s. The images were simultaneously acquired with two Canon
 92 T5 cameras with 28–135 mm lenses, with the illumination provided by LEDs.
 93 The 14-bit images captured at a definition of 2601×1733 pixels are up-sampled
 94 to 16-bit images with a dynamic range of approximately 60,000 gray levels. The
 95 imaged physical size of one pixel was $62 \text{ }\mu\text{m}$. A random speckle pattern was
 96 sprayed onto the specimen surfaces to increase the image contrast and improve
 97 the DIC resolution.

98 The horizontal force versus the splitting displacement, averaged from DIC
 99 measurements on opposite sides of the groove, is shown in Figure 2. The red
 100 circles mark the envelope of the curve that will be used by the identification
 101 routine with always increasing opening displacements. Let us note that since
 102 the test was interrupted before final failure of the sample, only a lower bound
 103 to the work of fracture, and to the fracture energy can be obtained [19].

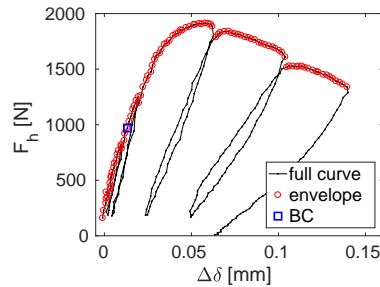


Figure 2: Horizontal force (*i.e.*, 5.715 times the vertical force) versus splitting displacement $\Delta\delta$ averaged on both sides of the sample. The images corresponding to the displacement envelope used in the identification are marked as red circles. Image no. 39 used for the BC_c parameter as explained in Section 2.6 is shown as a blue square.

104 Given the fact that the first picture was not acquired at zero load and a
 105 nonlinear model will be used, the displacement field accounting for this load
 106 offset is an additional unknown when calibrating constitutive parameters [40].
 107 This is in particular true for cohesive zone models [32]. Image no. 39, highlighted
 108 with a blue square, is chosen for the BC corrections (see Section 2.6) since it is
 109 considered to be in the linear elastic regime (*i.e.*, 50% of the maximum force)
 110 and with higher displacement levels than the first images, thus being less affected
 111 by acquisition noise. Further details on the same experiment can be found in
 112 Refs. [15, 19].

113 2.2. Digital Image Correlation

114 In global DIC, the displacement field \mathbf{u}_{DIC} is measured by considering that
 115 every pixel \mathbf{x} within the Region Of Interest (ROI) in the reference image f is
 116 present in the deformed (*i.e.*, in a loaded state) image g but has moved by

117 \mathbf{u}_{DIC} so that the displacement field globally minimizes the gap to gray level
 118 conservation

$$\phi^2 = \sum_{\text{ROI}} [f(\mathbf{x}) - g(\mathbf{x} + \mathbf{u}_{DIC}(\mathbf{x}))]^2, \quad (1)$$

119 which is the L2-norm of the gray level residuals $\rho(\mathbf{x})$. In order to ensure a
 120 good conditioning of this minimization and its robustness to noise, one more
 121 consideration is added to regularize the kinematics of a group of pixels, namely,
 122 it consists in expressing the sought displacement field as

$$\mathbf{u}_{DIC}(\mathbf{x}) = \sum_{i=1}^N v_i \Psi_i(\mathbf{x}), \quad (2)$$

123 in which v_i are the degrees of freedom, and Ψ selected vector fields. In such a
 124 framework, the measured displacements are obtained as

$$\{\mathbf{v}_{DIC}\} = \arg \min_{\{\mathbf{v}\}} \phi^2(\{\mathbf{v}\}), \quad (3)$$

125 where $\{\mathbf{v}_{DIC}\}$ is the column vector gathering all amplitudes v_i . A robust solution
 126 that works in most cases is choosing Ψ_i as finite element shape functions [41].
 127 In this paper, the DIC procedure is performed with 3-noded linear elements in
 128 a finite element discretization [42] and will be referred to as T3DIC.

129 In the method presented herein, the first step is to run T3DIC since it will
 130 provide the necessary Boundary Conditions (BC) as explained in Section 2.3,
 131 and also displacement fields that can be compared with FE results. The mesh
 132 used for T3DIC and one displacement field (for image no. 263, *i.e.*, the last of
 133 the envelope, see Figure 2) is shown in Figure 3. The average element length
 134 is 37 pixels. This relatively large element size is chosen in order to reduce
 135 uncertainties due to acquisition noise. Care was taken to properly get the con-
 136 tour of the sample for avoiding identification artifacts and fully exploiting the
 137 image contrast as shown in the zoomed yellow rectangle. The central grooves
 138 are designed to guide the crack propagation along the center plane. However,
 139 castable refractories are prone to crack branching. For this experiment, it was
 140 shown that no major side branches were formed and only a single macrocrack
 141 had propagated in the groove [19].

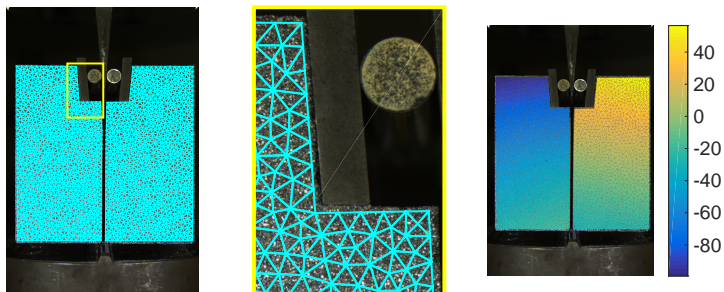


Figure 3: Sample geometry with superimposed T3DIC mesh (left). A zoom of the yellow rectangle is presented in the middle to show the contour of the mesh close to the loading plate. Horizontal displacement field expressed in μm for the last analyzed image.

142 2.3. Numerical model

143 The FEA is performed with the commercial code Abaqus [38]. The geometry
 144 is taken from the image for ensuring that the measurement by T3DIC is per-
 145 formed exactly on the same region. The mesh is generated with GMSH [43] and
 146 shown in Figure 1, in which the nodes where the Dirichlet BCs are prescribed
 147 are shown with red circles. The corner node is not considered due to higher
 148 measurement uncertainty [44]. Since the exact same contours are used for the
 149 DIC and FE meshes in this region, a linear interpolation is performed to get
 150 the BCs. The groove (see Figure 1) has a reduced out-of-plane thickness. The
 151 yellow box corresponds to a zoom in the groove to show the mesh details. In
 152 the middle of the groove, a single strip of zero thickness cohesive elements (200
 153 in the height) is added. Each face of the sample will be analyzed independently
 154 as a 2D model under plane strain hypothesis.

155 2.4. Identification strategy

156 The chosen identification scheme is based upon the Finite Element Model
 157 Updating (FEMU [45]) method. It is chosen to update the material parameters
 158 by reducing the difference between the calculated reaction force F_c and the
 159 experimentally measured force F_m . It is worth noting that the unload/reload

160 cycles are excluded from the identification since more complex CZMs would be
 161 necessary to accurately describe them [46]. Consequently, only the envelope
 162 of the curve is kept (*i.e.*, 100 out of the 312 images for which the crack is
 163 propagating, see Figure 2). It is chosen to have a continuous displacement of
 164 the actuator.

165 The identification methodology (*i.e.*, Newton-Raphson scheme) consists in
 166 a nonlinear least squares minimization of χ_F^2

$$\chi_F^2(\{\mathbf{p}\}) = \frac{1}{n_t \sigma_F^2} \sum_t (F_m(t) - F_c(t, \{\mathbf{p}\}))^2, \quad (4)$$

167 in which σ_F is the standard load uncertainty (on F_m), n_t the number of time
 168 steps, and F_c is the computed resultant of the reaction forces, which depends
 169 on unknown material parameters gathered in the column vector $\{\mathbf{p}\}$. If the
 170 only difference between the measured load levels $F_m(t)$ and $F_c(t, \{\mathbf{p}\})$ is acqui-
 171 sition noise, then χ_F will approach unity. Conversely, if there is a model error,
 172 then $\chi_F > 1$. By considering a given starting set of parameters $\{\mathbf{p}_n\}$ at itera-
 173 tion n , the minimization is performed by evaluating the correction $\{\delta\mathbf{p}\}$ on the
 174 linearized F_c

$$F_c(t, \{\mathbf{p}_n\} + \{\delta\mathbf{p}\}) \approx F_c(t, \{\mathbf{p}_n\}) + \frac{\partial F_c}{\partial \{\mathbf{p}\}}(t, \{\mathbf{p}_n\}) \{\delta\mathbf{p}\}, \quad (5)$$

175 about the current estimate $\{\mathbf{p}_n\}$ of the sought parameters. The minimized
 176 quantity then becomes

$$\frac{1}{n_t \sigma_F^2} \sum_t \left(F_m(t) - F_c(t, \{\mathbf{p}_n\}) - \frac{\partial F_c}{\partial \{\mathbf{p}\}}(t, \{\mathbf{p}_n\}) \{\delta\mathbf{p}\} \right)^2. \quad (6)$$

177 In Equation (6), the quantity to be minimized is quadratic in terms of $\{\delta\mathbf{p}\}$.
 178 Its minimization with respect to $\{\delta\mathbf{p}\}$ then leads to a linear system

$$[\mathbf{H}] \cdot \{\delta\mathbf{p}\} = \{\mathbf{h}\} \quad (7)$$

179 where $[\mathbf{H}]$ is the Hessian

$$[\mathbf{H}] = \sum_t \left(\frac{\partial F_c}{\partial \{\mathbf{p}\}}(t, \{\mathbf{p}_n\}) \right)^\top \frac{\partial F_c}{\partial \{\mathbf{p}\}}(t, \{\mathbf{p}_n\}) \quad (8)$$

180 and $\{\mathbf{h}\}$ the right hand member

$$\{\mathbf{h}\} = \sum_t (F_m(t) - F_c(t, \{\mathbf{p}_n\})) \frac{\partial F_c}{\partial \{\mathbf{p}\}}(t, \{\mathbf{p}_n\}). \quad (9)$$

181 Convergence is deemed successful when the root mean square (RMS) of the
182 relative variation of the parameters is less than 10^{-2} between two subsequent
183 iterations. The sensitivity fields $\frac{\partial F_c}{\partial \{\mathbf{p}\}}$ are computed via finite differences in which
184 the perturbation with respect to each parameter is set to 1%. The framework
185 of the identification methodology may be further discussed by the sensitivity
186 analysis presented in Section 3.1 using the Hessian $[\mathbf{H}]$.

187 2.5. Cohesive law

188 In this work, the selected cohesive law is the so-called PPR (Park, Paulino
189 and Roesler) model [47, 48]. It was considered since the built-in cohesive models
190 may give unrealistic responses for mixed mode propagations [49]. It is imple-
191 mented in Abaqus with a User ELeMent (UEL) subroutine¹ [48]. Apart from
192 the groove where the crack propagates and the cohesive model is implemented,
193 the remaining part of the specimen has a linear elastic behavior. The infor-
194 mation about mode II propagation or compressive damage was not considered,
195 but care was taken in the implementation so that it did not interfere with the
196 reported results. Figure 4 shows the two parameters to be calibrated for the
197 cohesive zone model used herein, namely, the cohesive strength σ_{max} , and the
198 fracture energy J_c .

¹https://paulino.ce.gatech.edu/PPR_tutorial.html

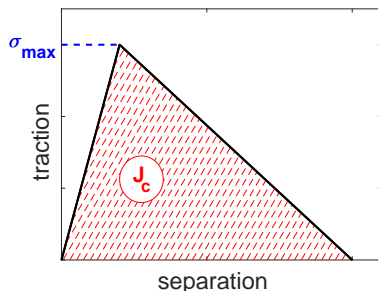


Figure 4: Schematic traction separation law highlighting the main parameters (σ_{max} and J_c) to be calibrated.

199 For the PPR model, the constitutive behavior for mode I includes the cohe-
 200 sive strength σ_{max} , and the fracture energy J_c . Two additional parameters are
 201 considered, namely, the initial slope λ_n and the shape parameter α . These last
 202 two parameters are chosen to be constant in the identification scheme. λ_n is
 203 kept equal to 0.005 for both cases [48], and considered as a small value within
 204 the stability limits [47]. The sensitivity to λ_n was tested and did not signifi-
 205 cantly affect the results. The parameter α however does change the softening
 206 response of the cohesive law. For the first case, its value is taken as 2 in order
 207 to approach a bilinear law [47] as shown in Figure 4. The second analyzed case
 208 considers $\alpha = 7$ to change the shape of the curve (Figure 11) to check if it
 209 better describes the considered test. Two built-in CZMs, namely, a bi-linear
 210 traction-separation law and the so-called Concrete Damaged Plasticity model,
 211 were also tested with parameters to replicate the studied case with $\alpha = 2$ and
 212 yielded very similar results [46]. For the sake of brevity, they are not discussed
 213 herein.

214 2.6. Boundary condition correction

215 One additional parameter is related to the non-zero load associated with the
 216 acquisition of the first image (Figure 2). It calls for a BC correction [32, 40]
 217 and will be designated as BC_c . In the experiment, the reference image was
 218 taken with a pre-load in order to remove any slack in the loading configuration.

219 Thus, the reference image of the unloaded state is unknown, and all measured
 220 displacements are performed with respect to the pre-load configuration.

221 The parameter BC_c introduced herein thus has to correct the kinematics
 222 from the unloaded state to the pre-loaded state. It is chosen to define BC_c as
 223 a multiplicative scalar of the displacement field related to a specific time step
 224 in the elastic regime of the experiment, and add it to the displacement fields
 225 for all time steps. The logical choice would be to consider the displacements
 226 of the very first images but they are small and consequently more affected
 227 by acquisition noise. Image 39 (*i.e.*, the 24th of the envelope, see Figure 2),
 228 which corresponds to approximately half of the maximum load, is chosen as a
 229 compromise of remaining in the linear part of the load but not too close to the
 230 noisier beginning. The corrected displacement fields \mathbf{u}_{BC_c} read

$$\mathbf{u}_{BC_c} = (BC_c - 1) \cdot \mathbf{u}_{39}, \quad (10)$$

231 When BC_c is equal to 1, no correction is performed. It is expected that $BC_c > 1$
 232 for the correction of the reference state with an opening displacement field, *i.e.*, a
 233 fraction of the displacement field measured in image 39. In the case of $BC_c < 1$,
 234 a contraction displacement field would be considered in the correction.

235 2.6.1. Initial parameters

236 The properties used for initializing the identification scheme are listed in
 237 Table 1. The cohesive strength σ_{max} was selected as the maximum T-stress
 238 measured in Ref. [19] with the method that provided more trustworthy results
 239 for the T-stress (*i.e.*, FEMU). The initial fracture energy J_c corresponds to its
 240 estimate based upon Integrated-DIC results [19]. The last parameter, BC_c , has
 241 its initial value set to one (*i.e.*, no BC correction would be needed).

Table 1: Initial parameters for the identification scheme.

σ_{max} [MPa]	J_c [J/m ²]	BC_c [-]
2	68	1

242 3. Results

243 3.1. Sensitivity analysis

244 Before performing the calibration of material parameters, a sensitivity anal-
245 ysis is performed [50]. Only the case $\alpha = 2$ is reported since the sensitivities are
246 very close to those when $\alpha = 7$. The load sensitivities are defined as

$$S_F(t, \{\mathbf{p}_0\}) = \frac{\partial F_c}{\partial \{\mathbf{p}\}}(t, \{\mathbf{p}_0\}), \quad (11)$$

247 and approximated using a forward difference approach with a perturbation fac-
248 tor $\epsilon = 10^{-2}$ of each parameter. Figure 5 shows the computed load sensitivity
249 S_F , indicating the influence of each parameter as a function of time. The influ-
250 ence of the parameter BC_c is very important at the beginning of the test. The
251 peak influence of the cohesive strength σ_{max} occurs in the middle of the sequence
252 of images, which is related to the part of the test where the measured force is
253 high. The fracture energy J_c has higher sensitivity at the end of the test, which
254 is to be expected since the crack has propagated a significant distance [15, 19].
255 For all parameters, the load sensitivities are significant (in comparison with the
256 load uncertainty) for a one percent variation of each parameter. This result in-
257 dicates that the parameters are expected to be identifiable with the considered
258 test and identification procedure.

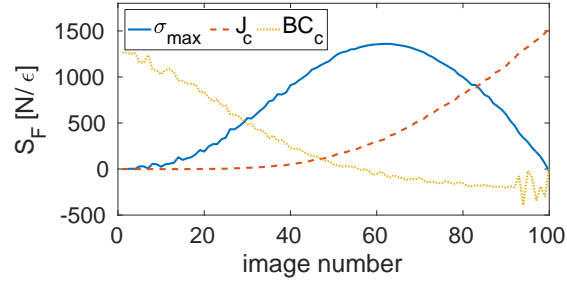


Figure 5: Force sensitivity to the parameters of the $\alpha = 2$ case. The blue solid line is the sensitivity to the cohesive strength σ_{max} , with a maximum sensitivity close to the middle of the test, *i.e.*, near the maximum splitting load. The red dashed line is related to the fracture energy J_c with a maximum sensitivity to the end of the test, after many elements are already damaged. The yellow dotted line corresponds to the BC correction BC_c , with maximum sensitivity at the beginning of the test where the displacements are very small.

259 Figure 6(a) shows the decimal logarithm of the values of the 3×3 Hessian
 260 ([**H**], see Equation (8)). The diagonal terms indicate the sensitivity to each
 261 property considered independently, and the off-diagonal members the cross in-
 262 fluences between parameters. In the case of fully independent parameters, only
 263 the diagonal terms would be different from zero. As expected from the previ-
 264 ous analysis, all parameters have very high sensitivities, and the conditioning
 265 of the system is very good (*i.e.*, less than 10). From this sensitivity analysis,
 266 it is confirmed that all parameters can be calibrated with the selected test and
 267 identification procedure.

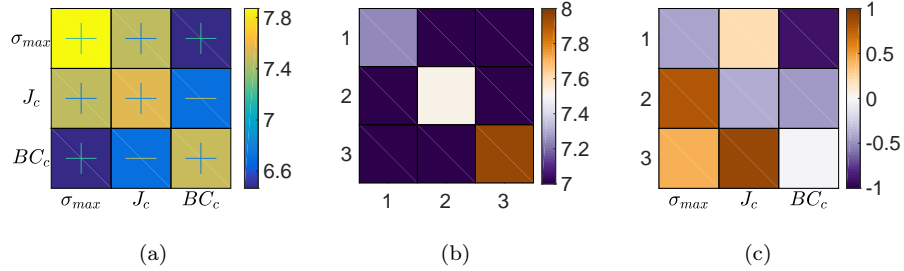


Figure 6: (a) Hessian of the identification procedure for the $\alpha = 2$ case shown as decimal logarithm. The diagonal terms show the sensitivity of each independent parameter. The off-diagonal terms show the cross influence between the parameters. (b) Decimal logarithm of the diagonalized Hessian. (c) Eigen column vectors associated with the diagonalization of the Hessian.

268 The decimal logarithm of the diagonalized Hessian is shown in Figure 6(b).
 269 Given the fact that the minimum eigen value of the Hessian is very high, there
 270 was no need for regularizing the Newton-Raphson scheme to ensure the definite-
 271 ness of $[\mathbf{H}]$. The first eigen value is dominant in BC_c and is almost independent
 272 of the other parameters. The second and the third eigen values are dominant in
 273 σ_{\max} and J_c , in the same order of magnitude, showing that they are more corre-
 274 lated. Such conclusion is drawn from the eigen vectors reported in Figure 6(c).

275 3.2. Calibration results

276 Figure 7 shows the experimental and computed resultant forces for the two
 277 analyzed cases, *i.e.*, with the PPR model and for $\alpha = 2$ and $\alpha = 7$. The
 278 identified parameters give a very good fit of the experimental curve, which is a
 279 first validation of the model. The differences between both cases are very small.

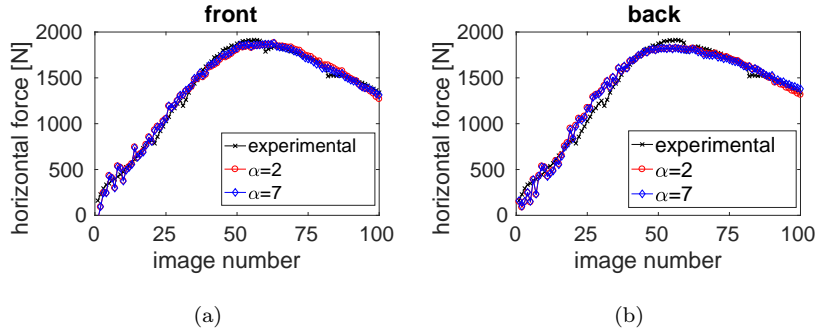


Figure 7: Experimental and computed reaction forces on the converged state for the two analyzed cases, for the front (a) and back (b) faces of the experiment.

280 For easier comparison, the difference between the calculated and experimen-
 281 tal forces, which have been normalized by the standard load uncertainty², are
 282 shown in Figure 8. Although some oscillations are seen, the mean value is plot-
 283 ted in dashed lines, showing that on average the error was of the order of twice
 284 the acquisition noise. This level is sufficiently small [32] to validate both cases.
 285 For the front face, $\alpha = 7$ provided slightly better results (RMS error of 1.5 as
 286 opposed to 1.8 when $\alpha = 2$), and for the back face $\alpha = 2$ was a bit better
 287 (RMS error of 2.0 against 2.1). Overall, considering only the force residuals,
 288 it is concluded that $\alpha = 7$ is (a bit) more suitable for the test studied herein.
 289 However, a bilinear model should not be excluded since its performance is also
 290 very good.

²The load uncertainty is equal to 0.1 % of the 5 kN load cell capacity, *i.e.*, 5 N for the vertical force, namely, of the order of 30 N for the horizontal force.

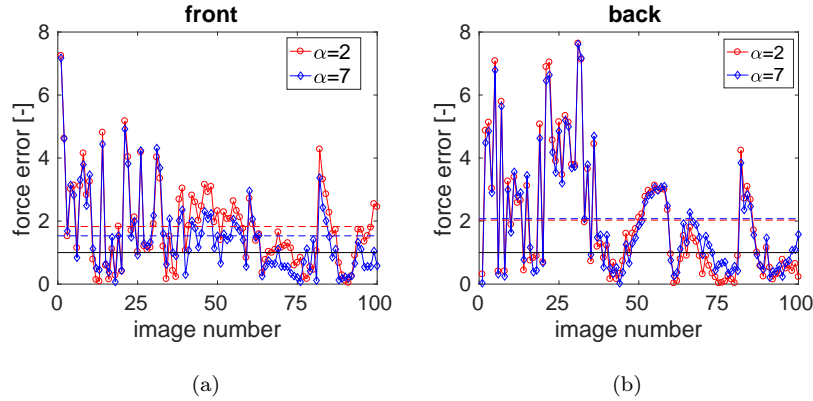


Figure 8: Absolute reaction force residuals for the two cases normalized by the standard load uncertainty (solid lines), for both faces of the sample. The temporal average is shown as dashed lines.

291 The displacement fields were not used in the identification routine except
 292 at very few points as BCs in the FEA. The measured and computed displace-
 293 ment fields can thus be compared at any other location provided the computed
 294 displacement field corresponding to the reference image cancels out. For the
 295 calibrated parameters, the relative displacements (with respect to the pre-load
 296 configuration) are computed and then compared to the T3DIC kinematics at
 297 each time step of the test. These corrected FE results (whose mesh is shown in
 298 Figure 1) are interpolated onto the T3DIC mesh (Figure 3) and the RMS differ-
 299 ence normalized by the standard displacement uncertainty is shown in Figure 9.
 300 The differences between front and back faces are mostly related to experimental
 301 inaccuracies, which are higher for the back face. The increasing trend shows
 302 that the numerical assumptions (*i.e.*, the constitutive law) are less true as the
 303 test goes on. However, the average error is of the order of 2.5 times the displace-
 304 ment uncertainty for the front face, and about 1.5 times for the back face. This
 305 level is sufficiently low [32] to give confidence in the model considered herein.
 306 The differences of the mean value between both cases is negligible (*i.e.*, of the
 307 order of 10^{-3}).

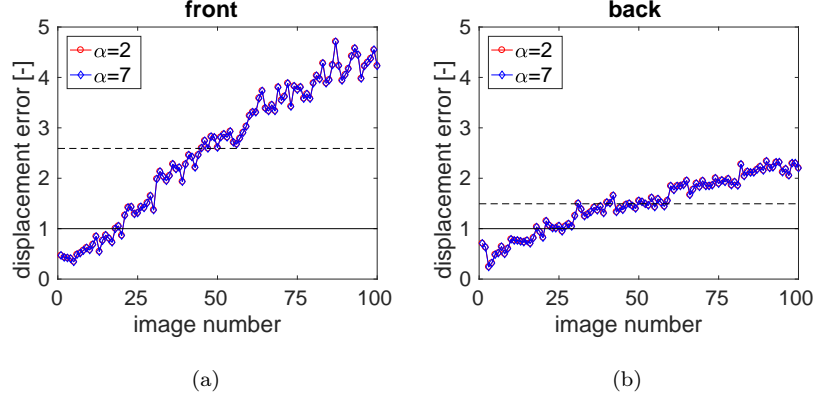


Figure 9: RMS residual between computed and experimental displacements normalized by the T3DIC displacement uncertainty (solid lines). The temporal average is shown as dashed lines.

308 With the proposed framework, the gray level residuals from the FE results
 309 can also be checked. This is possible because measured displacements were
 310 prescribed as BCs in the numerical model, and the computed displacement
 311 fields were corrected to account for the fact that the reference configuration
 312 corresponds to the pre-loaded sample. The gray level residuals read

$$\rho_{FEA}(\mathbf{x}, t) = f(\mathbf{x}) - g_t(\mathbf{x} + \mathbf{u}_{FEA}(\mathbf{x}, t)) \quad (12)$$

313 where \mathbf{u}_{FEA} is the computed displacement field, after taking out the pre-load
 314 kinematics related to BC_c , interpolated onto the T3DIC mesh. The same frame-
 315 work used for performing T3DIC may then be used to evaluate the gray level
 316 residuals. The RMS level of $\rho_{FEA}(\mathbf{x}, t)$ performed over all pixel location \mathbf{x} of
 317 the ROI normalized by the corresponding T3DIC residual $\rho_{DIC}(\mathbf{x}, t)$

$$\frac{\text{rms}_{\text{ROI}}[\rho_{FEA}(\mathbf{x}, t)]}{\text{rms}_{\text{ROI}}[\rho_{DIC}(\mathbf{x}, t)]} \quad (13)$$

318 for each image is shown in Figure 10, where

$$\rho_{DIC}(\mathbf{x}, t) = f(\mathbf{x}) - g_t(\mathbf{x} + \mathbf{u}_{DIC}(\mathbf{x}, t)) \quad (14)$$

319 The former is only 50 to 60 % higher than the latter in which no hypothesis
 320 was made on the constitutive behavior. This observation further validates the

321 overall trends of both cases. As reported for the displacement residual, the mean
 322 value of the normalized gray level residual between both cases is of the order of
 323 10^{-3} .

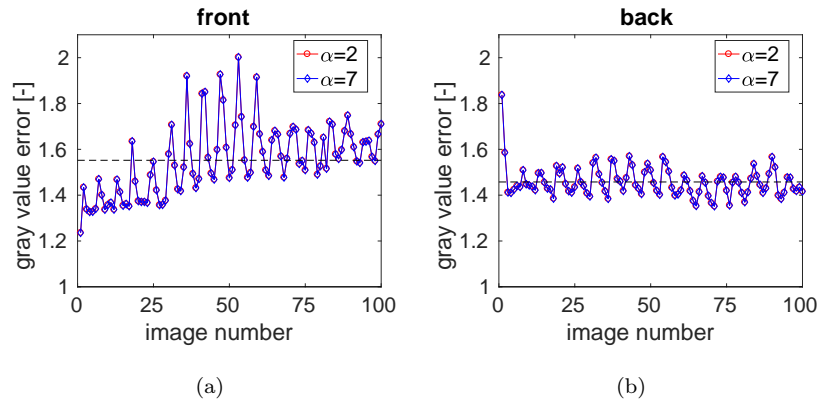


Figure 10: Gray level residuals using FEA kinematics (ρ_{FEA}) normalized by the gray level residuals obtained from T3DIC (ρ). The temporal average is shown as dashed lines.

324 Figure 11 shows the traction vs. separation responses for each calibrated
 325 model. The curves represent the response of the most damaged element, *i.e.*,
 326 the closest element to the pre-crack, showing that no element was fully dam-
 327 aged (*i.e.*, the maximum level is equal to ≈ 0.85 and 0.75 when $\alpha = 2$ and
 328 $\alpha = 7$, respectively). It is worth remembering that complete propagation was
 329 not achieved since the experiment was performed until 70 % of the peak load
 330 (Figure 2).

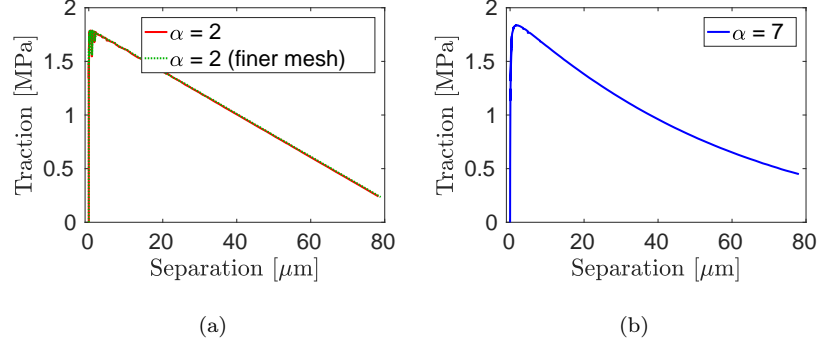


Figure 11: Traction vs. separation responses of the most damaged element for the two cases calibrated with the front face data. The response for the mesh sensitivity analysis (discussed at the end of this section) is shown when $\alpha = 2$, where the main difference at the end of the curve is of the order of 0.01 MPa.

331 Table 2 gathers the calibrated parameters for each studied case when initial-
 332 ized with the set given in Table 1. The traction vs. separation curves (Figure 11)
 333 are similar for both cases, close to a bilinear response when $\alpha = 2$, and with a
 334 nonlinear softening when $\alpha = 7$. They all converged after at most five iterations
 335 of the FEMU-F procedure. It is possible to see that σ_{max} varies in the range
 336 1.70 ± 0.15 MPa for the two cases; J_c has a difference of almost 20 % from
 337 $\alpha = 2$ to $\alpha = 7$, with a smaller difference if the same case is considered and
 338 both faces are compared. The BC_c difference highlights that the wedge was
 339 not fully aligned. It is possible to conclude that the wedge was applying more
 340 force on the back face of the specimen at the beginning of the test. It is worth
 341 noting that the cohesive strengths σ_{max} are of the same order of magnitude as
 342 the maximum T-stresses reported in Ref. [19].

Table 2: Converged parameters on the identification scheme for the two studied cases.

face	α	σ_{max} [MPa]	J_c [J/m ²]	BC_c [-]
front	2	1.79	82	1.314
	7	1.84	100	1.320
back	2	1.58	89	0.897
	7	1.59	115	0.906

343 Although the mesh is finer than classical guidelines for cohesive elements [51,
 344 52, 53, 54] (*i.e.*, 200 elements in the total length of the propagation path, which
 345 will be shown to be less than the process zone length in Section 3.3.2), a mesh
 346 sensitivity analysis was performed with a finer mesh having more than 3 times
 347 the number of cohesive elements (*i.e.*, 620). When the subsequent FEMU-F
 348 procedure was initialized with the parameters calibrated with the coarser mesh,
 349 only one iteration was needed to reach convergence (*i.e.*, the parameter differ-
 350 ences were less than 0.5 %). This observation proves that the two solutions are
 351 very close, which is confirmed in Figure 11(a) in terms of the traction separa-
 352 tion law. From this analysis it is concluded that mesh convergence was achieved
 353 with the 200-cohesive element mesh. All the results of the next section were also
 354 checked for the two mesh densities and no tangible differences were observed.
 355 For the sake of brevity, they will not be presented.

356 3.3. Discussion

357 3.3.1. Fracture energy

358 In order to better understand the simulated fracture behavior, one last anal-
 359 ysis is proposed. The displacements of the nodes of each cohesive element on
 360 the identification analysis are applied as BCs in a zip-like model, namely, only
 361 using the cohesive elements. With this approach, it is ensured that the same ex-
 362 perimental kinematics is applied. It follows that the reaction forces at each node
 363 can be extracted. From the reaction forces, the tractions in each element and
 364 each time step are available. Using the traction / separation of each element,
 365 the dissipated energy can be computed. The vertical (mode II) displacements

366 and corresponding reaction forces are insignificant and are thus not accounted
 367 for hereafter.

368 The mode I tractions T_I are shown in Figure 12 for the front face. The
 369 image number on the x-axis is related to the time step for the envelope images
 370 (Figure 2). The y-axis is the vertical position of each node along the groove,
 371 *i.e.*, $y = 0$ is the node closest to the pre-crack and $y = 60$ mm the one at the
 372 bottom of the sample. A compressive zone develops after image no. 40, which
 373 hinders the propagation but does not stop the crack. Even though the color
 374 bar is fixed from -2 to 2 MPa for easier visualization, the minimum level is
 375 approximately -9 MPa. Similar figures are generated for the back face, which
 376 are reported in Appendix A (Figure 17).

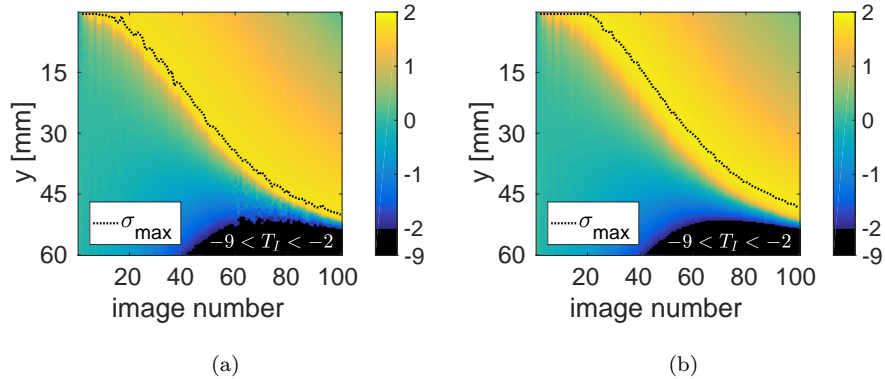


Figure 12: Normal traction (expressed in MPa) history for the front face when $\alpha = 2$ (a) and $\alpha = 7$ (b). The black dotted line shows the location for which the cohesive strength σ_{max} was reached.

377 It is worth noting that σ_{max} was not reached for the last elements since the
 378 test was not performed until final failure. For the same reason, non-vanishing
 379 tractions are still observed in the first elements. The region where the energy is
 380 being dissipated is large (*i.e.*, many damaged cohesive elements), with remaining
 381 cohesion even close to the pre-crack. Only the elements closest to the initial
 382 notch experience small traction levels, which indicates that their damage level is
 383 high. To determine the fractured surface, it is possible to consider an equivalent

384 damage variable for any CZM. For each node, the damage parameter is defined
 385 as

$$D = 1 - \frac{\sigma_I}{\sigma_{max}} \quad \text{when } 0 < \llbracket u_I \rrbracket < \delta_c \quad (15)$$

386 where σ_I is the mode I traction (whose spacetime history is shown in Figures 12
 387 and 17), $\llbracket u_I \rrbracket$ the mode I opening displacement, δ_c the maximum separation,
 388 and σ_{max} the cohesive strength. The damage variable history calculated with
 389 Equation (15) for the front face and both analyzed α is shown in Figure 13.
 390 Although the material is quasi-brittle, Figure 13 shows that the damage grows
 391 slowly. About two thirds of the sample have been damaged at the end of the
 392 reported test. Some cohesion remains along the whole propagation path (as
 393 already discussed above). However, there is a ligament in which no damage at
 394 all has occurred. The same trends are observed for the back face, as reported
 395 in Appendix A (Figure 18).

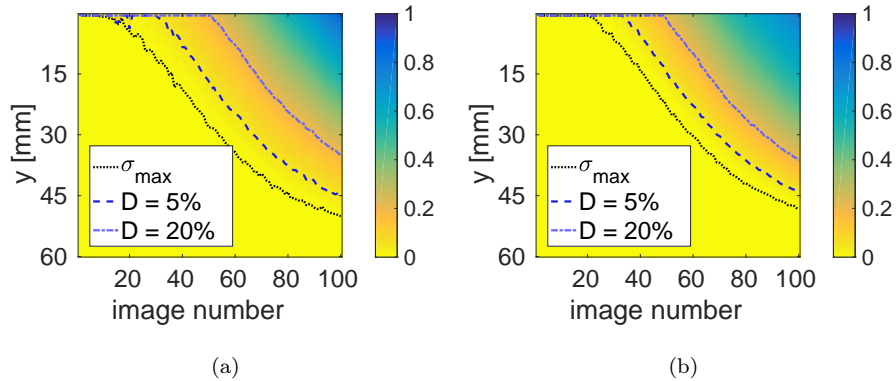


Figure 13: Damage history for the front face when $\alpha = 2$ (a) and $\alpha = 7$ (b). The black dotted line shows the location for which the damage variables starts to grow (*i.e.*, when the cohesive strength σ_{max} was reached).

396 The elementary fracture energy corresponds to the area under the cohesive
 397 response. It is obtained by integrating the opening displacement vs. traction
 398 responses of each cohesive element. At a given step, the dissipated energy is
 399 calculated by removing the elastic energy from the total work of each element.
 400 Figure 14 shows the dissipated energy for each cohesive element of the front face

401 of the analyzed cases. The maximum possible dissipated energy is equal to J_c
 402 (*i.e.*, 82 J/m² for $\alpha = 2$ and 100 J/m² for $\alpha = 7$). Not a single element reached
 403 this value, since the maximum level is found equal to 71 J/m² and 64 J/m²,
 404 respectively. The same tendencies are observed for the back face (Figure 19).

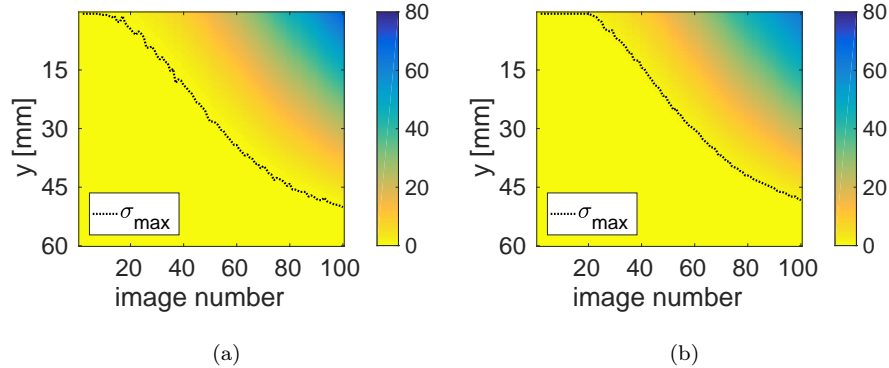


Figure 14: Elementary dissipated energy (expressed in J/m²) for the front face, with $\alpha = 2$ (a) and $\alpha = 7$ (b). The black dotted line shows the location for which the cohesive strength σ_{max} was reached.

405 Let us consider the cracked surface as the damaged area, *i.e.*, the region
 406 from the first to the last cohesive elements that reached σ_{max} . The total dis-
 407 sipated energy is calculated by multiplying the elementary dissipated energy
 408 (see Figure 14) by the element area, and summing the contributions of every
 409 element at each time step. The total dissipated energy is shown as a function
 410 of the “crack” length (as defined above) in Figure 15. An exponential interpo-
 411 lation describes the observed trends, which means that as the damaged zone
 412 grows, a bigger energy increment is needed to further propagation. Overall, the
 413 crack propagated a little farther and dissipated more energy in the back face,
 414 as seen in Figure 15. This result was expected from the conclusions analyzing
 415 the BC_c parameter (*i.e.*, tilted wedge applying more force on the back side).
 416 When $\alpha = 2$ case, the crack propagated a little farther and more energy was
 417 dissipated on both faces of the sample.

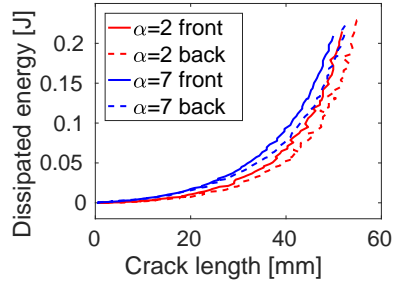


Figure 15: Dissipated energy in the analyzed test for the studied parameters. Solid lines represents the front face results, and the back face ones are in dashed lines.

418 To evaluate the global fracture energy, \mathcal{G}_c , let us consider E_{diss} as the total
 419 dissipated energy in the specimen during crack propagation

$$\mathcal{G}_c = \frac{\partial E_{\text{diss}}}{\partial A} \quad (16)$$

420 where the derivative describes how much energy is dissipated for each unitary
 421 increment of cracked area. Before taking the derivative, an exponential fit is
 422 considered to suppress the amplification of measurement uncertainties. The
 423 corresponding results are shown in Figure 16. Both cases lead to consistent
 424 results with small differences on the curvature, maximum crack length, and
 425 \mathcal{G}_c level for a given face. Longer cracks are seen at the back of the specimen
 426 when the results of the two faces are compared. It is worth noting that the
 427 curve reported in Figure 16 is not physically allowed to start from $\mathcal{G}_c = 0$, since
 428 there is a minimum energy to break chemical bounds. However, this works aims
 429 to analyze propagating macrocracks, and at this scale, the resolution was not
 430 sufficient to check for the very beginning of this curve.

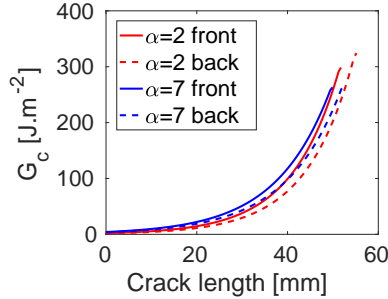


Figure 16: Global fracture energy \mathcal{G}_c predicted by the studied CZMs, which are calculated from the exponential fit of the total dissipated energy. The results from the front face are in solid lines and for the back face in dashed lines.

431 To check these results and compare them with earlier estimates [15, 19], the
 432 mean level $\bar{\mathcal{G}}_c$ is calculated

$$\bar{\mathcal{G}}_c = \frac{1}{\Delta a_{\max}} \int_0^{\Delta a_{\max}} \mathcal{G}_c(a) da \quad (17)$$

433 where Δa_{\max} denotes the maximum length of the damaged area. The values
 434 of $\bar{\mathcal{G}}_c$ are reported in Table 3. A good agreement is observed with the values
 435 reported in Refs. [15, 19] when integrated DIC is considered. The values are
 436 farther apart with FEMU-U [19] (*i.e.*, with a cost function using displacement
 437 measurements) is used instead. With the present case, it is not possible to
 438 clearly distinguish which α is better since both yielded very low residuals and
 439 consistent results.

440 Table 3 compares the fracture energy predicted the proposed approach with
 441 two other independent methods applied to the same experiment. Although the
 442 methodologies reported in Refs. [15, 19] do not use CZMs, the energetic approach
 443 allows such comparisons. The fracture energies obtained with the PPR model
 444 are very close for the front and back faces, while those reported before had more
 445 significant differences. It is believed that it may related to the BC_c parameter,
 446 which corrects for the unknown fully unloaded state and was not accounted for
 447 with the other methods [15, 19]. For the finer mesh and $\alpha = 2$, the average
 448 fracture energy is found equal to 62.2 J/m², which is very close to the level

449 found with the coarser mesh (Table 3). This observation further confirms the
 450 quasi mesh independence of the results reported herein.

Table 3: Average fracture energy \bar{G}_c expressed in J/m² for the two CZMs applied to the back and front faces separately. These predictions are compared with earlier results obtained by two independent approaches (*i.e.*, integrated DIC [15] and FEMU-U [19])

model	front face	back face
PPR ($\alpha = 2$)	61.3	62.4
PPR ($\alpha = 7$)	63.5	62.2
IDIC [#]	84	52
FEMU-U [#]	162	97

[#] according to Refs. [15, 19]

451 In order to improve the proposed methodology, the identification could be
 452 coupled for both faces, or even based on 3D simulations. Implementation of
 453 cohesive elements with similar approaches for cases where the crack bifurcates
 454 on the surfaces is also an interesting outlook. Last, 4D analyses via in-situ
 455 tests in x-ray tomographs may elucidate further the crack paths and fracture
 456 mechanisms taking place in such materials.

457 3.3.2. FPZ size

458 Since nonzero tractions are still predicted even close to the precrack at the
 459 end of the test, a final discussion about the FPZ size and its relationship with the
 460 material is discussed. With the identified parameters, the so-called Hillerborg
 461 size ℓ_H [20] is evaluated

$$\ell_H = \frac{EJ_c}{\sigma_{\max}^2} \quad (18)$$

462 and reported in Table 4. The length of the process zone is generally a fraction
 463 of ℓ_H [55]. In the present case, the FPZ length is of the order of ca. 50 mm (see
 464 Figures 12 to 16, for instance), which is one order of magnitude smaller than
 465 ℓ_H .

Table 4: ℓ_H for the studied cases in millimeters.

model	front face	back face
PPR ($\alpha = 2$)	434	604
PPR ($\alpha = 7$)	500	773

466 The estimate of the FPZ length leads to the conclusion that the FPZ was
 467 not yet fully developed, which is proven by the remaining stresses close to the
 468 pre-crack. This also explains the increasing energy release rate curve [56]. To
 469 further analyze the FPZ, the test should be continued until the end of the
 470 propagation, or in the case of materials with high ℓ_H as the one studied herein,
 471 a longer sample would be preferable.

472 The obtained fracture parameters and the FPZ length are consistent with
 473 the material microstructure and its processing. The analyzed composition is
 474 suitable for fluidized catalytic cracking units that operate in temperature ranges
 475 of 550 to 800 °C. Such materials are not sintered in situ as other refractories, and
 476 thus, their resistance strongly depends on the hydraulic bindings and packing
 477 of the raw material [57]. As this specimen was fired at low temperature (*i.e.*,
 478 500 °C [19]), it is expected to have aggregates weakly bonded to the matrix
 479 and microcracks related to the anisotropy of the phases [57]. The initiation of
 480 a crack is easy in such materials, which explains the low level of the cohesive
 481 strength σ_{max} . Since the specimen cannot store much elastic energy prior to
 482 crack initiation (*i.e.*, the elastic energy is proportional to σ_{max}^2), the latter is
 483 easy and occurs very early on, and subsequent crack branches and bridging
 484 are possible, thereby dissipating more energy through friction and leading to a
 485 considerably higher J_c . In the present case, it is believed that crack bridging
 486 is the most likely mechanism since no branches were detected macroscopically
 487 on the investigated faces [19]. All these effects result in a large FPZ that in the
 488 case investigated herein spans over all the propagation path.

489 4. Conclusions

490 A FEMU-F methodology was applied to calibrate cohesive properties, *i.e.*,
491 the cohesive strength (σ_{max}) and the fracture energy (J_c), of the so-called PPR
492 model. Only the reaction forces were considered in the cost function to be min-
493 imized. By using the geometry seen on digital images to build the FE mesh,
494 T3DIC results were directly used as boundary conditions to drive the simula-
495 tions. The same region being used in T3DIC and FE analyses also allowed the
496 experimental displacements to be compared with the simulated kinematics. The
497 Hessian matrix was directly analyzed to infer the conditioning and sensitivity of
498 the identification scheme to the sought parameters. It was also proposed to add
499 a third parameter related to the correction of the applied boundary conditions.

500 The two studied softening regimes of the cohesive law resulted in similar
501 material properties. Both σ_{max} and J_c were identified in the range of castable
502 refractories, with values close to identifications obtained via integrated DIC [15,
503 19]. The cohesive strengths were very close on both analyzed faces of the sample,
504 while some deviation of the order of 20 % was reported for the fracture energy.
505 However, the dissipated energy was similar for both sides of the sample. The
506 parameters identified for the cohesive law allowed the Hillerborg length to be
507 calculated. When coupled with the traction space-time history of the cohesive
508 elements, it gives insight into FPZ length. The boundary condition corrections
509 were significantly different for both analyzed faces, thereby emphasizing that
510 the wedge was slightly tilted at pre-load. It is worth noting that the thickness
511 of the specimen combined with the presence of aggregates on the composition
512 makes it difficult to perfectly align the wedge. However, the proposed approach
513 showed robustness to tackle this misalignment.

514 The present study shows the feasibility of modeling crack propagation of
515 castable refractories tested in the WST with cohesive elements. Cohesive pa-
516 rameters were calibrated with good correlations to previously reported fracture
517 parameters for the same experiment. The residuals in force, displacement and
518 gray level were very close to the noise level, which validates the methodology

519 *and* the investigated model. Given the fact that damage did not reach its maxi-
520 mum value explains why an ever increasing R-curve response was observed. This
521 means that the extent of the process zone spans over most of the crack surface.
522 The rather long fracture process zone is related to a low tensile strength (*i.e.*,
523 easy crack initiation) and high fracture energy. It is believed that most of the
524 energy is dissipated through friction by aggregate bridging in the present case,
525 since they are weakly bonded to the matrix after low temperature firing.

526 **Acknowledgments**

527 JAR thanks CNPq for the productivity scholarship, grant #307127/2013-3.
528 RV's stay at LMT was supported through an RIA scholarship, grant #2017/20911-
529 9, São Paulo Research Foundation (FAPESP).

530 **References**

- 531 [1] Lee WE, Vieira W, Zhang S, Ghanbari Ahari K, Sarpoolaky H, Parr
532 C. Castable refractory concretes. *International Materials Reviews*
533 2001;46(3):145–67.
- 534 [2] Wachtman J. *Materials and Equipment - Whitewares - Refractory Ceramics*
535 - Basic Science: Ceramic Engineering and Science Proceedings, Volume 16.
536 No. 1 in *Ceramic Engineering and Science Proceedings*; Wiley; 2009. ISBN
537 9780470316306.
- 538 [3] Luz AP, Braulio MAL, Pandolfelli VC. *Refractory Castable Engineering*;
539 vol. 1. 1 ed.; São Carlos, SP: Göller Verlag; 2015.
- 540 [4] Lee WE, Moore RE. Evolution of in situ refractories in the 20th century.
541 *Journal of the American Ceramic Society* 1998;81(6):1385–410.
- 542 [5] Steinbrech RW. R-curve behavior of ceramics. In: Bradt R, Hasselman D,
543 Munz D, Sakai M, Shevchenko V, editors. *Fracture Mechanics of Ceramics*;
544 vol. 9 of Fracture Mechanics of Ceramics. Springer US. ISBN 978-1-4613-
545 6477-1; 1992, p. 187–208.
- 546 [6] Launey ME, Ritchie RO. On the fracture toughness of advanced materials.
547 *Advanced Materials* 2009;21(20):2103–10.
- 548 [7] Tschegg E. Prüfeinrichtung zur Ermittlung von bruchmechanischen Ken-
549 nwerten sowie hierfür geeignete, Prüfkörper, Austrian Pat. AT 390328B,
550 registered. 1986.
- 551 [8] Brühwiler E, Wittmann FH. The wedge splitting test, a new method of
552 performing stable fracture mechanics tests. *Engineering Fracture Mechanics*
553 1990;35(1-3):117–25.
- 554 [9] Ribeiro S, Rodrigues JA. The influence of microstructure on the maximum
555 load and fracture energy of refractory castables. *Ceramics International*
556 2010;36(1):263–74.

- 557 [10] Miyaji DY, Tonnesen T, Rodrigues JA. Fracture energy and thermal shock
558 damage resistance of refractory castables containing eutectic aggregates.
559 *Ceramics International* 2014;40(9, Part B):15227–39.
- 560 [11] Sutton MA, McNeill SR, Helm JD, Chao YJ. Advances in two-dimensional
561 and three-dimensional computer vision. In: *Photomechanics*; vol. 77 of
562 Topics in Applied Physics. 2000, p. 323–72.
- 563 [12] Sutton MA, Orteu JJ, Schreier H. Image correlation for shape, motion
564 and deformation measurements: basic concepts, theory and applications.
565 Springer Science & Business Media; 2009.
- 566 [13] Sutton MA. Computer vision-based, noncontacting deformation measure-
567 ments in mechanics: A generational transformation. *Applied Mechanics*
568 *Reviews* 2013;65(AMR-13-1009, 050802).
- 569 [14] Belrhiti Y, Pop O, Germaneau A, Doumalin P, Dupré JC, Harmuth H,
570 et al. Investigation of the impact of micro-cracks on fracture behavior of
571 magnesia products using wedge splitting test and digital image correlation.
572 *Journal of the European Ceramic Society* 2015;35(2):823–9.
- 573 [15] Vargas R, Neggers J, Canto RB, Rodrigues JA, Hild F. Analysis of wedge
574 splitting test on refractory castable via integrated DIC. *Journal of the*
575 *European Ceramic Society* 2016;36(16):4309–17.
- 576 [16] Dai Y, Gruber D, Harmuth H. Observation and quantification of the frac-
577 ture process zone for two magnesia refractories with different brittleness.
578 *Journal of the European Ceramic Society* 2017;37(6):2521–9.
- 579 [17] Dai Y, Gruber D, Harmuth H. Determination of the fracture behaviour of
580 MgO-refractories using multi-cycle wedge splitting test and digital image
581 correlation. *Journal of the European Ceramic Society* 2017;37(15):5035–43.
- 582 [18] Dupré JC, Doumalin P, Belrhiti Y, Khelifi I, Pop O, Huger M. Detection
583 of cracks in refractory materials by an enhanced digital image correlation
584 technique. *Journal of Materials Science* 2018;53(2):977–93.

- 585 [19] Vargas R, Neggers J, Canto RB, Rodrigues JA, Hild F. Comparison of two
586 full-field identification methods for the wedge splitting test on a refractory.
587 Journal of the European Ceramic Society 2018;38(16):5569–79.
- 588 [20] Hillerborg A, Modéer M, Petersson PE. Analysis of crack formation and
589 crack growth in concrete by means of fracture mechanics and finite ele-
590 ments. Cement and Concrete Research 1976;6(6):773–82.
- 591 [21] Elices M, Guinea G, Gómez J, Planas J. The cohesive zone model:
592 advantages, limitations and challenges. Engineering Fracture Mechanics
593 2002;69:137–63.
- 594 [22] Dugdale DS. Yielding of steel sheets containing slits. Journal of the Me-
595 chanics and Physics of Solids 1960;8(2):100–4.
- 596 [23] Song SH, Paulino GH, Buttlar WG. Simulation of crack propagation in
597 asphalt concrete using an intrinsic cohesive zone model. Journal of Engi-
598 neering Mechanics 2006;132(11):1215–23.
- 599 [24] Aure T, Ioannides A. Simulation of crack propagation in concrete beams
600 with cohesive elements in abaqus. Transportation Research Record: Jour-
601 nal of the Transportation Research Board 2010;(2154):12–21.
- 602 [25] Su X, Yang Z, Liu G. Finite Element Modelling of Complex 3D Static and
603 Dynamic Crack Propagation by Embedding Cohesive Elements in Abaqus.
604 Acta Mechanica Solida Sinica 2010;23(3):271–82.
- 605 [26] Evangelista F, Roesler JR, Proença SP. Three-dimensional cohesive zone
606 model for fracture of cementitious materials based on the thermodynamics
607 of irreversible processes. Engineering Fracture Mechanics 2013;97:261–80.
- 608 [27] Shen B, Paulino GH. Identification of cohesive zone model and elastic
609 parameters of fiber-reinforced cementitious composites using digital image
610 correlation and a hybrid inverse technique. Cement and Concrete Compos-
611 ites 2011;33(5):572–85.

- 612 [28] Shen B, Paulino GH. Direct extraction of cohesive fracture properties
613 from digital image correlation: a hybrid inverse technique. *Experimental*
614 *Mechanics* 2011;51(2):143–63.
- 615 [29] Fedele R, Sessa S, Valoroso N. Image correlation-based identification
616 of fracture parameters for structural adhesives. *Technische Mechanik*
617 2012;32(2):195–204.
- 618 [30] Alfano M, Lubineau G, Paulino GH. Global sensitivity analysis in the iden-
619 tification of cohesive models using full-field kinematic data. *International*
620 *Journal of Solids and Structures* 2015;55:66–78.
- 621 [31] Blaysat B, Hoefnagels JPM, Lubineau G, Alfano M, Geers MGD. Interface
622 debonding characterization by image correlation integrated with double
623 cantilever beam kinematics. *International Journal of Solids and Structures*
624 2015;55:79–91.
- 625 [32] Affagard JS, Mathieu F, Guimard JM, Hild F. Identification method for
626 the mixed mode interlaminar behavior of a thermoset composite using dis-
627 placement field measurements and load data. *Composites Part A: Applied*
628 *Science and Manufacturing* 2016;91:238–49.
- 629 [33] Ruybalid AP, Hoefnagels JPM, van der Sluis O, van Maris MPFHL, Geers
630 MGD. Mixed-mode cohesive zone parameters from integrated digital im-
631 age correlation on micrographs only. *International Journal of Solids and*
632 *Structures* 2018;.
- 633 [34] Abanto-Bueno J, Lambros J. Experimental determination of cohesive fail-
634 ure properties of a photodegradable copolymer. *Experimental Mechanics*
635 2005;45(2):144–52.
- 636 [35] Ferreira M, Venturini W, Hild F. On the analysis of notched concrete
637 beams: From measurement with digital image correlation to identification
638 with boundary element method of a cohesive model. *Engineering Fracture*
639 *Mechanics* 2011;78(1):71 – 84.

- 640 [36] Richefeu V, Chrysochoos A, Huon V, Monerie Y, Peyroux R, Wattrisse
641 B. Toward local identification of cohesive zone models using digital image
642 correlation. *European Journal of Mechanics-A/Solids* 2012;34:38–51.
- 643 [37] Rethore J, Estevez R. Identification of a cohesive zone model from digital
644 images at the micron-scale. *Journal of the Mechanics and Physics of Solids*
645 2013;61(6):1407–20.
- 646 [38] Abaqus 6.14 Documentation. Dassault Systèmes Simulia Corp.; Provi-
647 dence, RI, USA.; 2014.
- 648 [39] Diógenes HJF, Cossolino LC, Pereira AHA, El Debs MK, El Debs ALHC.
649 Determination of modulus of elasticity of concrete from the acoustic re-
650 sponse. *Revista IBRACON de Estruturas e Materiais* 2011;4(5):803–13.
- 651 [40] Hild F, Bouterf A, Chamoin L, Leclerc H, Mathieu F, Neggiers J, et al.
652 Toward 4D mechanical correlation. *Advanced Modeling and Simulation in*
653 *Engineering Sciences* 2016;3(1):17.
- 654 [41] Besnard G, Hild F, Roux S. “Finite-Element” displacement fields analysis
655 from digital images: Application to Portevin-Le Chatelier bands. *Experi-
656 mental Mechanics* 2006;46(6):789–803.
- 657 [42] Leclerc H, Périé J, Roux S, Hild F. Integrated digital image correlation
658 for the identification of mechanical properties; vol. LNCS 5496. Berlin
659 (Germany): Springer; 2009, p. 161–71.
- 660 [43] Geuzaine C, Remacle JF. Gmsh: A 3-D finite element mesh generator
661 with built-in pre-and post-processing facilities. *International Journal for*
662 *Numerical Methods in Engineering* 2009;79(11):1309–31.
- 663 [44] Hild F, Roux S. Comparison of local and global approaches to digital image
664 correlation. *Experimental Mechanics* 2012;52(9):1503–19.
- 665 [45] Pagnacco E, Caro-Bretelle AS, Ienny P. Parameter Identification from Me-
666 chanical Field Measurements using Finite Element Model Updating Strate-
667 gies; chap. 9. Wiley-Blackwell; 2012, p. 247–74.

- 668 [46] Maginador RV. Analysis of crack propagation in the Wedge Splitting Test
669 via Digital Image Correlation and Finite Element Analyses. Master's thesis;
670 Federal University of São Carlos (UFSCar) - Brazil; 2019.
- 671 [47] Park K, Paulino GH, Roesler JR. A unified potential-based cohesive model
672 of mixed-mode fracture. *Journal of the Mechanics and Physics of Solids*
673 2009;57(6):891–908.
- 674 [48] Park K, Paulino GH. Computational implementation of the PPR potential-
675 based cohesive model in ABAQUS: educational perspective. *Engineering*
676 *Fracture Mechanics* 2012;93:239–62.
- 677 [49] Park K, Choi H, Paulino GH. Assessment of cohesive traction-separation
678 relationships in ABAQUS: A comparative study. *Mechanics Research Com-*
679 *munications* 2016;78:71–8.
- 680 [50] Fedele R, Raka B, Hild F, Roux S. Identification of adhesive properties in
681 GLARE assemblies using digital image correlation. *Journal of the Mechan-*
682 *ics and Physics of Solids* 2009;57(7):1003–16.
- 683 [51] Falk ML, Needleman A, Rice JR. A critical evaluation of cohesive zone
684 models of dynamic fracture. In: *Proceedings of the 5th European mechanics*
685 *of materials conference on scale transitions from atomistics to continuum*
686 *plasticity*. 2001, p. 43–50.
- 687 [52] Davila C, Camanho P, de Moura M. Mixed-mode decohesion elements for
688 analyses of progressive delamination. In: *19th AIAA Applied Aerodynam-*
689 *ics Conference*. 2001, p. 1486.
- 690 [53] Moës N, Belytschko T. Extended finite element method for cohesive crack
691 growth. *Engineering Fracture Mechanics* 2002;69(7):813–33.
- 692 [54] Moslemi M, Khoshnavan M. Cohesive zone parameters selection for mode-I
693 prediction of interfacial delamination. *Strojnicki Vestnik/Journal of Me-*
694 *chanical Engineering* 2015;61(9):507–16.

- 695 [55] Saucedo L, Rena CY, Ruiz G. Fully-developed FPZ length in quasi-brittle
696 materials. *International Journal of Fracture* 2012;178(1-2):97–112.
- 697 [56] Dong W, Zhou X, Wu Z. On fracture process zone and crack extension
698 resistance of concrete based on initial fracture toughness. *Construction*
699 *and Building Materials* 2013;49:352–63.
- 700 [57] Pereira AHA, Miyaji DY, Cabrelon MD, Medeiros J, Rodrigues JA. A
701 study about the contibution of the α - β phase transition of quartz to ther-
702 mal cycle damage of a refractory used in fluidized catalytic cracking units.
703 *Cerâmica* 2014;60:449–56.

704 **Appendix A**

705 The space-time history of the cohesive tractions for the back face is shown
 706 for the two analyzed cases in Figure 17. When $\alpha = 7$, the damaged zone is
 707 smaller and develops later on. At the end of the test, the extent of the damaged
 708 zones is similar in both cases.

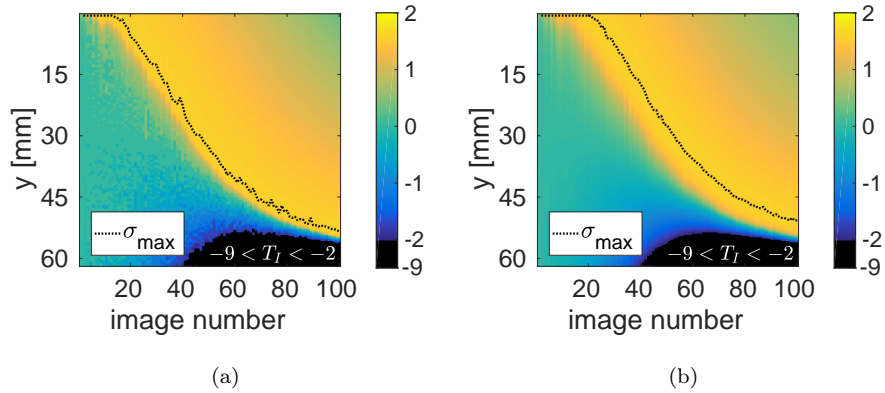


Figure 17: Normal traction (expressed in MPa) history for the back face, with $\alpha = 2$ (a) and $\alpha = 7$ (b). The black dotted line shows the location for which the cohesive strength σ_{\max} was reached.

709 The damage history for the back face is shown in Figure 18. Although the
 710 most damaged element reaches a level less than 0.9, most of the specimen is
 711 damaged at the end of the test. This observation applies in both cases.

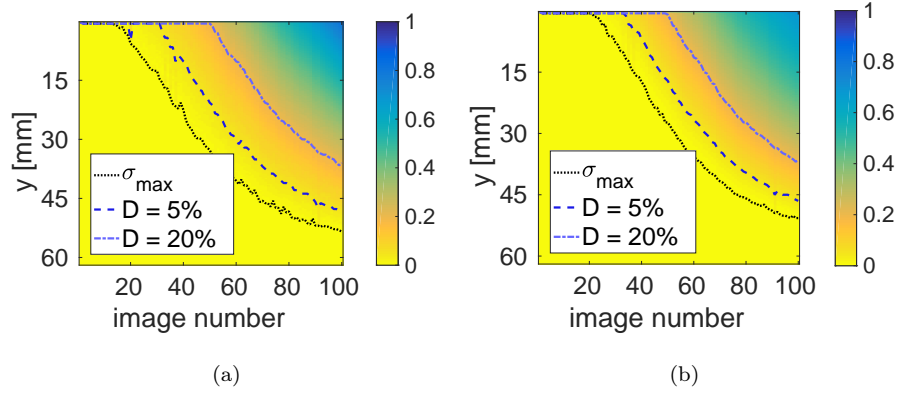


Figure 18: Damage history for the back face with $\alpha = 2$ (a) and $\alpha = 7$ (b). The black dotted line shows the location for which the cohesive strength σ_{\max} was reached.

712 The spacetime history of elementary dissipated energy for the back face is
 713 shown in Figure 19. The maximum level that can be reached is equal to J_c , but
 714 no element has achieved such dissipation in both cases.

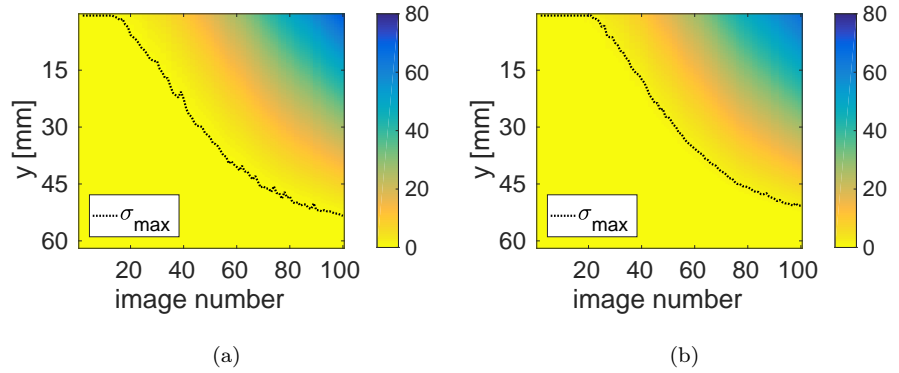


Figure 19: Elementary dissipated energy (in J/m^2) history for the back face when $\alpha = 2$ (a) and $\alpha = 7$ (b). The black dotted line shows the location for which the cohesive strength σ_{\max} was reached.