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# Perspective-12-Quadrics: An analytical solution to the camera pose estimation problem from conics - quadrics correspondences.

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**Goal:** To estimate the pose of a perspective camera given a set of  $n$  3D quadrics in the world and their corresponding 2D projections (conics) in the image.

## 1 Problem statement.

- **2D:** Homogeneous quadratic form of a conic equation:

$$u^\top C_i u = 0,$$

where  $u \in \mathbb{R}^3$  is the homogeneous vector of a generic 2D point belonging to the conic defined by the symmetric matrix  $C_i \in \mathbb{R}^{3 \times 3}$ . The conic has 5 degrees of freedom given by the 6 elements of the lower triangular part of the symmetric matrix  $C_i$ , except one for the scale. The dual conic is defined by  $C_i^* = \text{adj}(C_i)$ .

- **3D:** Homogeneous quadratic form of a quadric equation:

$$x^\top Q_i x = 0,$$

where  $x \in \mathbb{R}^4$  is the homogeneous vector of a generic 3D point belonging to the quadric defined by the symmetric matrix  $Q_i \in \mathbb{R}^{4 \times 4}$ . The quadric has 9 degrees of freedom given by the 10 elements of the lower triangular part of the symmetric matrix  $Q_i$ , except one for the scale. The dual quadric is defined by  $Q_i^* = \text{adj}(Q_i)$ .

- **Projection equation:** Let's consider the projection matrix

$$P = K[R|t] = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$$

The projection equation [1] is:

$$\beta_i C_i^* = P Q_i^* P^\top \tag{1}$$

## 2 Solution.

In order to recover  $P$  in closed form from the set of conic-quadric pairs, we have to re-arrange Eq. 1 into a linear system [2, 3]. Let us define the operator  $vec()$  that serialises all the elements of a generic matrix, and the operator  $vech()$  that serialises the elements of the lower triangular part of a symmetric matrix. Then let us define  $v_i^* = vech(Q_i^*)$  and  $c_i^* = vech(C_i^*)$ . Then let us arrange the product of the elements of  $P$  and  $P^\top$  in a single matrix  $G \in \mathbb{R}^{6 \times 10}$  as follows:

$$G = D(P \otimes P)E$$

where  $\otimes$  is the Kronecker product and matrices  $D \in \mathbb{R}^{6 \times 9}$  and  $E \in \mathbb{R}^{16 \times 10}$ , are two matrices such that  $vech(X) = Dvec(X)$  and  $vec(Y) = Evech(y)$  respectively, where  $X \in \mathbb{R}^{9 \times 9}$  and  $Y \in \mathbb{R}^{16 \times 16}$  are two symmetric matrices.

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} p_{11}^2 & 2p_{11}p_{12} & p_{12}^2 & 2p_{11}p_{13} & 2p_{12}p_{13} & p_{13}^2 & 2p_{11}p_{14} & 2p_{12}p_{14} & 2p_{13}p_{14} & p_{14}^2 \\ p_{11}p_{21} & p_{11}p_{22} + p_{12}p_{21} & p_{12}p_{22} & p_{11}p_{23} + p_{13}p_{21} & p_{12}p_{23} + p_{13}p_{22} & p_{13}p_{23} & p_{11}p_{24} + p_{14}p_{21} & p_{12}p_{24} + p_{14}p_{22} & p_{13}p_{24} + p_{14}p_{23} & p_{14}p_{24} \\ p_{21}^2 & 2p_{21}p_{22} & p_{22}^2 & 2p_{21}p_{23} & 2p_{22}p_{23} & p_{23}^2 & 2p_{21}p_{24} & 2p_{22}p_{24} & 2p_{23}p_{24} & p_{24}^2 \\ p_{11}p_{31} & p_{11}p_{32} + p_{12}p_{31} & p_{12}p_{32} & p_{11}p_{33} + p_{13}p_{31} & p_{12}p_{33} + p_{13}p_{32} & p_{13}p_{33} & p_{11}p_{34} + p_{14}p_{31} & p_{12}p_{34} + p_{14}p_{32} & p_{13}p_{34} + p_{14}p_{33} & p_{14}p_{34} \\ p_{21}p_{31} & p_{21}p_{32} + p_{22}p_{31} & p_{22}p_{32} & p_{21}p_{33} + p_{23}p_{31} & p_{22}p_{33} + p_{23}p_{32} & p_{23}p_{33} & p_{21}p_{34} + p_{24}p_{31} & p_{22}p_{34} + p_{24}p_{32} & p_{23}p_{34} + p_{24}p_{33} & p_{24}p_{34} \\ p_{31}^2 & 2p_{31}p_{32} & p_{32}^2 & 2p_{31}p_{33} & 2p_{32}p_{33} & p_{33}^2 & 2p_{31}p_{34} & 2p_{32}p_{34} & 2p_{33}p_{34} & p_{34}^2 \end{bmatrix}$$

Given  $G$  we can rewrite Eq. 1 as a linear equation:

$$\beta_i c_i^* = G v_i^* \quad (2)$$

Eq. 2 can thus be solved using DLT algorithm, *e.g.* can be written:

$$0 = Bg,$$

where  $g = \text{vec}(G) = [g_{11} \cdots g_{610}]^\top \in \mathbb{R}^{60}$ , and  $B \in \mathbb{R}^{15n \times 60}$  is the concatenation of matrices  $b_i \in \mathbb{R}^{15 \times 60}$ :

$$b_i = \begin{bmatrix} c_{2i} \mathbf{v}_i^\top & -c_{1i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & c_{3i} \mathbf{v}_i^\top & -c_{2i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & c_{4i} \mathbf{v}_i^\top & -c_{3i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & c_{5i} \mathbf{v}_i^\top & -c_{4i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & c_{6i} \mathbf{v}_i^\top & -c_{5i} \mathbf{v}_i^\top \\ -c_{3i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & c_{1i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & -c_{4i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & c_{2i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & -c_{5i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & c_{3i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & -c_{6i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & c_{4i} \mathbf{v}_i^\top \\ c_{4i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & -c_{1i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & c_{5i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & -c_{2i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & c_{6i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & -c_{4i} \mathbf{v}_i^\top \\ -c_{5i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & c_{1i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top \\ \mathbf{0}_{10}^\top & -c_{6i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & c_{2i} \mathbf{v}_i^\top \\ c_{6i} \mathbf{v}_i^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & \mathbf{0}_{10}^\top & -c_{1i} \mathbf{v}_i^\top \end{bmatrix},$$

with:

$$\begin{aligned} \mathbf{c}_i^\top &= [c_{1i} \ c_{2i} \ c_{3i} \ c_{4i} \ c_{5i} \ c_{6i}], \\ \mathbf{v}_i^\top &= [v_{1i} \ v_{2i} \ v_{3i} \ v_{4i} \ v_{5i} \ v_{6i} \ v_{7i} \ v_{8i} \ v_{9i} \ v_{10i}], \\ \mathbf{0}_{10}^\top &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \end{aligned}$$

Each pair of conic-quadric provides 15 homogeneous linear equations in the unknown elements of  $G$ . However, only 5 are linearly independent. Thus 12 pairs of 3D-2D correspondences are required to solve the system. A total least squares solution can then be found by choosing  $g$  as a right singular vector corresponding to the smallest singular value of  $B$ . Finally,  $P$  can be inferred from  $G$ .

## References

- [1] R.I. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, Cambridge University Press, second edition, 2004.
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