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The equivalent reinforced concrete model for simulating the behavior of walls under dynamic shear loading

P. Kotronis ^a, J. Mazars ^b, L. Davenne ^{a,*}

^a *Laboratoire de Mécanique et Technologie, ENS de Cachan, 94235 Cachan cedex, France*

^b *RNVO, Laboratoire Sols, Solides, Structures, INP Grenoble, BP 53, 38041 Grenoble cedex 9, France*

In order to have a simplified model useful for dynamic shear loading analysis the equivalent reinforced concrete model (ERC) has been proposed. The ERC model is derived from the framework method and uses lattice meshes for concrete and reinforcement bars, and uniaxial constitutive laws based on continuum damage mechanics and plasticity. After describing the basis of the model, results showing its capacity to analyze shear walls tested in the framework of the SAFE program are presented. The use of the ERC model should reduce the computational time and allow parametrical studies. In this respect the performance of such a model is demonstrated. However it is shown that for low reinforcement ratios results are sensitive to the angle formed by the diagonals of the concrete lattice and the horizontal bars. Specific attention must be paid in this case.

Keywords: Equivalent reinforced concrete model; Framework method; Continuum damage mechanics; Dynamic shear loading

1. Introduction

The simulation of the non-linear behavior of reinforced concrete structures submitted to severe dynamic shear loading is an important problem for the engineering community. Recent earthquakes in Asia (Kobe, Chi-Chi), Europe (Izmit, Athens) or America (Northridge, Santa-Tecla) have proven once more the need to study thoroughly the shear mechanism and to develop simplified tools for everyday practice. Reinforced concrete squat columns and shear walls suffering from severe dynamic shear loading collapsed suddenly and lead to catastrophic failures.

The earthquake research group in the “Laboratoire de Mécanique et Technologie” has already proposed a simplified modeling strategy for bearing walls dominated by flexure. It consists in using multi-layered 2D Bernoulli beam elements coupled with damage mechanics [1,2]. However, when the effects of shear deformation are prevailing, as is the case of shear walls (bearing walls that have small slenderness), the problem is more complex. On this specific topic, an experimental program performed at JRC Ispra by the

* Corresponding author. Tel.: +33-1-4740-2243; fax: +33-1-4740-2240.

E-mail address: luc.davenne@lmt.ens-cachan.fr (L. Davenne).

french firms EDF and Cogema has given valuable information on the seismic behavior of such structures [3,4].

A simplified modeling strategy used to reproduce the non-linear behavior of shear walls is presented in this paper. Examples of the applications of the model are also discussed. This model is called equivalent reinforced concrete (ERC).

ERC model, inspired by the Truss models developed in the early 1900s [5,6], is based on the framework method [7] and uses a lattice type mesh coupled with modern constitutive laws. In this work lattice models, also adopted to describe fracture performances of concrete [8], are used for the description at the macroscopic level of structures as composite materials.

The verification of the reliability of the model is carried out through the comparisons with the results of the SAFE experimental program. Different type of walls are tested and for low percentage of reinforcement a sensitivity to the angle formed by the diagonals of the concrete lattice and the horizontal bars is observed. Discussion is conducted on this specific point.

2. Numerical tools and concepts for a simplified modeling

2.1. Finite element code

In order to perform non-linear dynamic calculations the code EFICOS was developed. The code uses 1D bar elements and 2D Bernoulli multi-layered beam elements. In both cases only uniaxial constitutive laws are required for the materials. 2D Timoshenko multi-layered beam elements are also used but specific kinematics conditions must be added in order to see some limited shear effects [9,10] and the ERC model has been introduced to compensate for the limitation of such a description, as shown in [11].

The initial secant stiffness matrix algorithm is implemented where the non-linear behavior (internal forces) is rejected in the second member of the equilibrium equation with the external load. For stability and precision reasons, a classical Newmark algorithm is used to solve the equation of motion. For dynamic calculations, masses are concentrated at specific nodes and loading is applied as an input motion at the base of the structure.

2.2. Constitutive models

Seismic loading, which includes cyclic aspects, produces micro-cracking in concrete. The major phenomena—a material stiffness decrease as the micro-cracks open, a stiffness recovery as the cracks close (unilateral behavior of concrete) and inelastic strains concomitant to damage—have to be taken into account. The constitutive law used for concrete is based on the principles of continuum damage mechanics [12,13]. The law, known as the “Unilateral damage law”, elaborated for the description of the micro-cracks, involves two damage scalar variables, one in tension and one in compression, and the description of isotropic inelastic strains. The introduction of two damage scalar variables allows the separation of the mechanical effect of micro-cracking depending on the sign of the stress. The model is able to simulate the unilateral behavior of concrete via a recovery stiffness procedure at re-closure. The total strain in the 3D formulation of the law is given by

$$\varepsilon = \varepsilon^e + \varepsilon^{\text{in}} \quad (1)$$

$$\varepsilon^e = \frac{\langle \sigma \rangle_+}{E(1 - D_1)} + \frac{\langle \sigma \rangle_-}{E(1 - D_2)} + \frac{\nu}{E} (\sigma - \text{Tr}(\sigma)\mathbf{I}) \quad (2)$$

$$\varepsilon^{\text{in}} = \frac{\beta_1 D_1}{E(1-D_1)} \frac{\partial f(\sigma)}{\partial \sigma} + \frac{\beta_2 D_2}{E(1-D_2)} \mathbf{I} \quad (3)$$

where ε^e is the elastic strain tensor; ε^{in} , the inelastic strain tensor; \mathbf{I} , the unit tensor; and $\text{Tr}(\sigma) = \sigma_{ii}$; $\langle \sigma \rangle_+$ and $\langle \sigma \rangle_-$ denote the positive and the negative parts of the stress tensor σ respectively; E , the initial Young's modulus; ν , the Poisson's ratio; D_1 and D_2 are damage variables for traction and compression respectively. Damage criteria are expressed as

$$f_i = Y_i - Z_i \quad (4)$$

where Y_i are the forces associated to the damage and Z_i is the threshold at a given value of D_i

$$Y_1 = \frac{\langle \sigma \rangle_+ : \langle \sigma \rangle_+ + 2\beta_1 f(\sigma)}{2E(1-D_1)^2} \quad \text{and} \quad Y_2 = \frac{\langle \sigma \rangle_- : \langle \sigma \rangle_- + 2\beta_2 \text{Tr}(\sigma)}{2E(1-D_2)^2} \quad (5)$$

The evolution laws for the damage variables D_i and the closure function $f(\sigma)$ are written as

$$D_i = 1 - \frac{1}{1 + [A_i(Y_i - Y_{0i})]^{B_i}} \quad (6)$$

$$\begin{cases} \text{Tr}(\sigma) \in [0, +\infty] & \rightarrow f(\sigma) = \text{Tr}(\sigma) & \frac{\partial f(\sigma)}{\partial \sigma} = \mathbf{I} \\ \text{Tr}(\sigma) \in [-\sigma_f, 0] & \rightarrow f(\sigma) = \left(1 - \frac{\text{Tr}(\sigma)}{\sigma_f}\right) \text{Tr}(\sigma) & \frac{\partial f(\sigma)}{\partial \sigma} = \left(1 - \frac{\text{Tr}(\sigma)}{\sigma_f}\right) \mathbf{I} \\ \text{Tr}(\sigma) \in [-\infty, -\sigma_f] & \rightarrow f(\sigma) = \frac{-\sigma_f}{2} \text{Tr}(\sigma) & \frac{\partial f(\sigma)}{\partial \sigma} = 0 \cdot \mathbf{I} \end{cases} \quad (7)$$

where σ_f represents the crack closure stress; Y_{0i} is the initial elastic threshold ($Y_{0i} = Z_i(D_i = 0)$); A_i , B_i are damage evolution parameters, β_i are inelasticity parameters. The material parameters can be determined by fitting the uniaxial stress–strain responses (E , ν , Y_{01} , A_1 , B_1 , and β_1 from a tension test), σ_f is usually of the same order of the strength in tension (3–4 MPa). Fig. 1 gives the uniaxial cyclic response, from tension to compression, of this model.

A plasticity model with kinematic hardening is used for steel. Hardening can be either linear or non-linear depending on the information provided from the steel tensile strength tests. The stress–strain relation is given in Fig. 2.

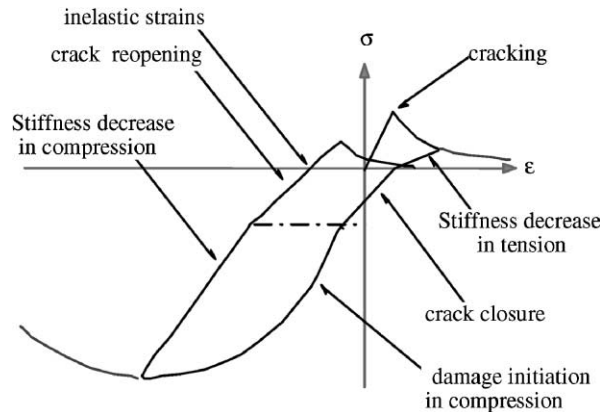


Fig. 1. 1D cyclic response of the “Unilateral damage model” for concrete.

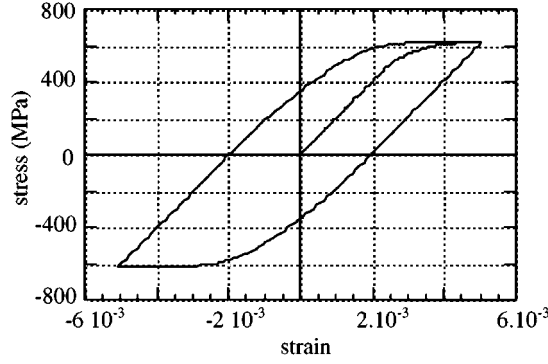


Fig. 2. 1D response of the elasto-plastic constitutive law for reinforcement.

As presented in the applications below, reinforcement is introduced with special bars or layers [1]. Damping is introduced in the analysis through viscous forces generated by a global damping matrix taken as a linear combination of the global stiffness matrix and the mass matrix (Rayleigh damping). This damping matrix remains constant during the calculation.

3. Equivalent reinforced concrete model

3.1. Background

The ERC model uses lattice meshes to predict the non-linear behavior of shear RC walls and is based on the framework method proposed by Hrennikoff [7]. The basic idea of the framework method consists in replacing the continuum material of the elastic body under investigation by a framework of bars, arranged according to a definite pattern, whose elements have suitable elastic properties. Set up only for linear elastic behavior, the criterion of suitability is associated, at a given deformation, to an energy equivalence between the framework pattern and the continuum material. If the unit size of the pattern of such a framework is made infinitesimal, then the latter will be representative of a complete mechanical model of the solid prototype, with identical displacements, strains and unit stresses. Some of the patterns proposed by Hrennikoff for plane stress elastic problems and homogeneous material are shown in Fig. 3. The first pattern implies a Poisson's ratio, ν equal to $1/3$. For the second pattern it can be calibrated.

$$\text{For pattern 1: } A_v = \frac{3}{8} \frac{3k^2 - 1}{k} at, \quad A_h = \frac{3}{8} (3 - k^2) at, \quad A_d = \frac{3}{16} \frac{(1 + k^2)^{3/2}}{k} at \quad (8)$$

$$\text{For pattern 2: } A = \frac{at}{1 + \nu}, \quad A_1 = \frac{at}{(1 + \nu)\sqrt{2}}, \quad A_2 = \frac{3\nu - 1}{2(1 + \nu)(1 - 2\nu)} at \quad (9)$$

3.2. Proposed lattice model

The idea is to use the patterns proposed by Hrennikoff in a non-linear context and for a heterogeneous material. The new model is called the ERC and its principles are summarized in Fig. 4.

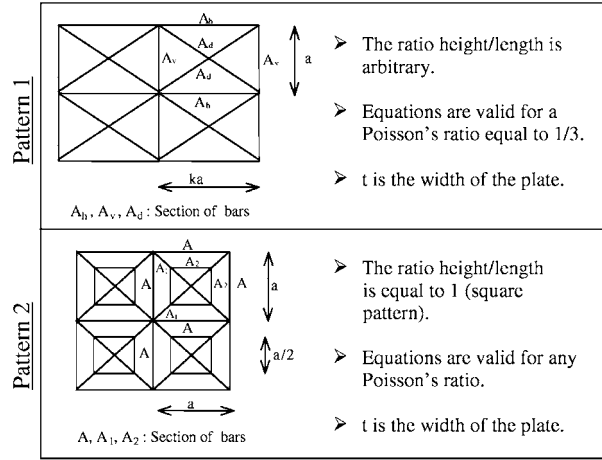


Fig. 3. Two different patterns of the framework method for plane stress problems.

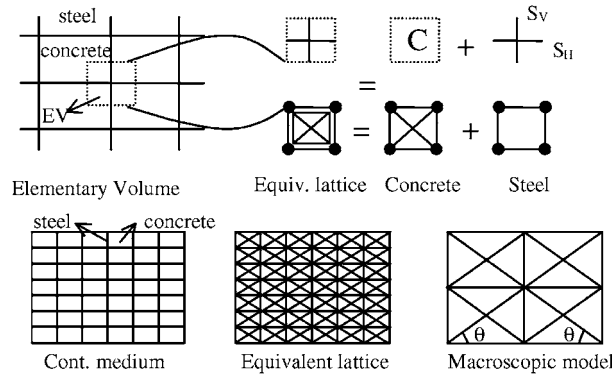


Fig. 4. Principles of the meshing for the ERC model.

The following assumptions are made [11]:

1. An elementary volume of reinforced concrete can be separated into a concrete element and an horizontal and a vertical reinforcement bars (S_H and S_V , respectively). Concrete and steel are then modeled separately by using two different lattices.
2. The sections of the bars simulating concrete have been derived directly from the framework method. The first pattern of the framework method is used because of its simplicity and the smaller number of required elements. This pattern is accurate for a Poisson's ratio equal to 1/3, which is obviously not the case for reinforced concrete in the elastic regime. This choice is however justified by the fact that the problems we are dealing with are highly non-linear (collapse of the specimen) and therefore the apparent Poisson's ratio is significantly changing (from 0.2 to 0.4 or even more).
3. A crucial parameter for the success of the non-linear simulation is the angle θ that the diagonals of the concrete lattice form with the horizontal bars. This angle depends on the reinforcement ratios in the horizontal and vertical direction, the loading (normal compressive stress at the base of the specimens and shear stress) and the boundary conditions. It is related to the direction of the cracks in the structure

- (the bars are supposed to reproduce the Ritter–Mörsch scheme) in both directions for cyclic loading. It lies between 30° and 60° to avoid negative values for the sections of ‘concrete’ bars calculated according to the first pattern of the framework method. The value of the angle needs a calibration. For the calculations presented in this paper, the experimental curve has been used to reproduce the elastic response and the ultimate strength. Simple pre-calculations can be also used, as discussed at the end of the paper.
4. The ‘Unilateral damage model’ in its 1D formulation is used to simulate the non-linear behavior of concrete. Tests on reinforced concrete elements demonstrated that even after extensive cracking, tensile stresses still exist in the cracked concrete and significantly increase its ability to resist shear stresses [14]. Adjusting the post-peak behavior (A_1 and B_1 , Eq. (6)) of the concrete damage model enables us to simulate this phenomenon known as ‘tension stiffening’ [15]. For compression, the reduction of the compressive capacity due to tension perpendicular to the struts (as suggested by Vecchio and Collins [16] for over reinforced shear walls which is not the case here) has not been modeled. However, if necessary, this could be made possible by calibrating the Y_{02} , A_2 , B_2 parameters of the damage model.
 5. A lattice composed of horizontal and vertical bars coupled with a uniaxial plasticity model simulates steel. The section and position of the bars coincide with the actual section and position of the reinforcement. To simplify the mesh, a method of distribution is used where the bar sections are proportionally increased in order to obtain in a given zone the same global surface area of reinforcement. The mesh is thus independent of the geometry of the specimen.
 6. Perfect bonding is assumed between concrete and steel.
 7. For at least the type of structures tested here, where the stress field is rather homogeneous, the number of elements that simulate concrete or steel does not have a great influence on the result. Two meshes with the same angle θ and different number of bars will give more or less the same results (same global response, same average damage in the trusses) [15]. A ‘macro’ model can be used instead of the ‘equivalent lattice’ (Fig. 4). ERC can be used for part of structure where the stress field is homogeneous and the angle of the cracks does not change significantly during the loading. Otherwise, re-meshing strategies or complete 2D models should be used.

The main interest of the ERC model is that it is based on uniaxial behavior, which makes the calculation easy and robust. This is not always the case for 2D or 3D damage or smeared crack approaches, particularly under severe shear where localization phenomena can compromise the robustness.

4. Modeling of the SAFE specimens

Within the SAFE research project, organized by COGEMA and EDF, 13 squat reinforced concrete walls (with a small slenderness) have been tested at the Joint Research Center of the European Commission at ISPRA in Italy [3,4]. The specimens are very squat (slenderness 0.46). The rotation of the upper part is restrained during the test to ensure a very high level of shear in the mock-ups. A set of four synthetic accelerograms of increasing amplitude is applied pseudodynamically (PSD test method). The following paragraphs describe the modeling of specimens T5 and T12.

4.1. Test characteristics

T5 and T12 mock-ups have the same geometric characteristics. The differences are in the reinforcement ratio and the normal stress at the base of the specimens. T12 has a much more important normal stress and a smaller reinforcement ratio. Characteristics of the walls, the flanges, the top and lower slabs are shown in Fig. 5 and in Table 1. The specimens were excited in the x -direction. The loading levels were determined to get responses ranging from elastic to elasto-plastic regimes.

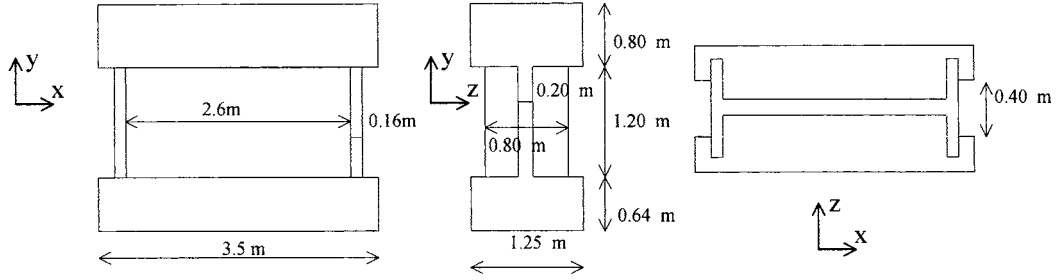


Fig. 5. SAFE tests: geometry of the T5 and T12 specimens.

Table 1

SAFE tests: main characteristics of the T5 and T12 specimens

Specimens	T5	T12
Type of test	Pseudo-dynamic test	Pseudo-dynamic test
Boundary conditions	Rotation at the top not allowed	Rotation at the top not allowed
Height/length	0.46	0.46
Section of web wall, m ²	0.52	0.52
Section of flanges, m ²	0.128	0.128
Horizontal reinforcement, %	0.8	0.11
Vertical reinforcement, %	0.8	0.11
Compression strength of concrete, MPa	34.7	34.7
Tensile strength of concrete, MPa	3.0	3.0
Young 's modulus of concrete, MPa	30 000	30 000
Yield strength of steel, MPa	500	400
Young 's modulus of steel, MPa	200 000	200 000
Normal stress at the base, MPa	0.34	1.0
Mass (top slab + extra mass), kg	$(29 + 0) \times 10^3 = 29000$	$(29 + 56.06) \times 10^3 = 85065$

4.2. Mesh characteristics

In order to reproduce the elastic behavior and ultimate strength, the angle θ for the concrete mesh has been calibrated from experimental results ($\theta = 41.6^\circ$ for T5 and $\theta = 30.1^\circ$ for T12—Fig. 7a and b). As allowed by the use of EFICOS, each flange is described by multi-layered Bernoulli beam elements to account for bending. The width of these beam elements equals the actual length of the flange (0.80 m). Four stiff beam elements, the rotation of which is not allowed, simulate the top slab. Vertical displacement is free and the walls are fixed at the base. Horizontal and vertical reinforcement is simulated with horizontal and vertical truss elements (Fig. 6c and d). Their section has been found by using the distribution method. Reinforcement in flanges is introduced through special layers in the beam elements. Specific values used for the materials are the ones already reported in Table 1. Since the experimental data gave us for concrete only strength in tension and compression was available for concrete from experimental data, we have chosen a set of parameters in order to obtain these strengths and to reproduce “classical” post-peak behavior. Some numerical tests have been performed to check the sensitivity of the results to the non-fitted parameters. The only important parameter resulted to be the residual stress in tension (the so-called “tension stiffening” effect) which has been adjusted in order to give better results at the structural level.

Finally the parameters used are: $E = 30 \times 10^9$ Pa; $A_1 = 6 \times 10^{-3}$ Pa⁻¹; $B_1 = 1.2$; $Y_{01} = 250$ Pa; $\beta_1 = 10^6$ Pa; $A_2 = 5 \times 10^{-6}$ Pa⁻¹; $B_2 = 1.2$; $Y_{02} = 1.5 \times 10^4$ Pa; $\beta_2 = -40 \times 10^6$ Pa; $\sigma_f = 3.5 \times 10^6$ Pa.

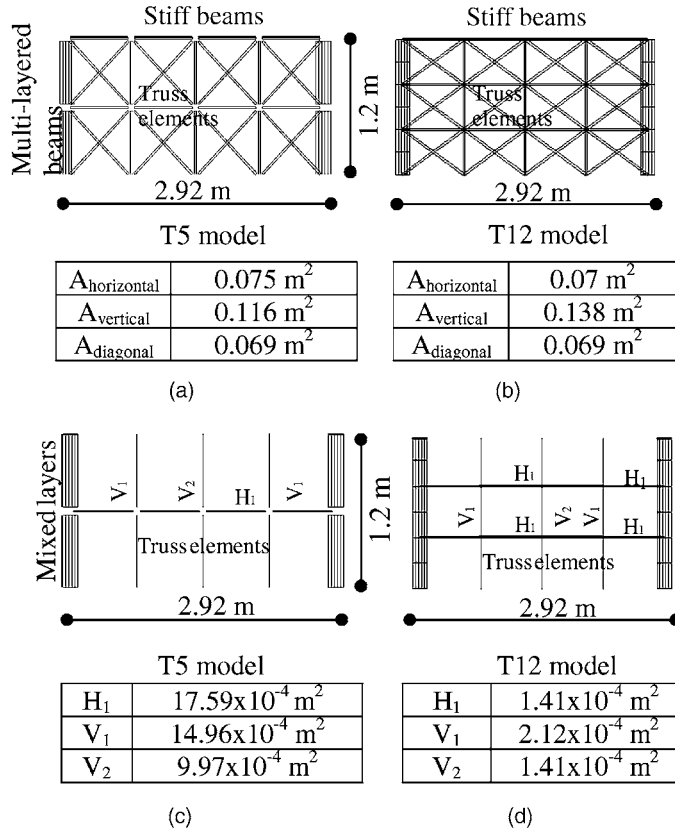


Fig. 6. SAFE tests: concrete mesh and section of truss elements for (a) T5 specimen and (b) T12 specimen—steel mesh and section of truss elements for (c) T5 specimen and (d) T12 specimen.

4.3. Pseudo-dynamic analysis

Due to its nature, a PSD test can be simulated by static calculations by using the displacements applied to the structure by the servo-controlled hydraulic actuators. The displacements applied to the mock-ups during the experiments are used as input data for the cyclic analysis. Displacement is applied at the top of the specimens and the total horizontal reaction is monitored. Global results correctly capture the history of the horizontal reaction for both specimens (Fig. 7). The deformed shape of the T5 specimen for the equivalent lattice mesh (with respect to the rebar positions) and the macroscopic model (steel reinforcement is distributed on a lower number of trusses) is shown in Fig. 8. The deformed shapes in Fig. 8 show the rising of the top slab in which the rotation is prevented. This rising is more important in the simulation than in the test. This difference is partly due to the value used for the Poisson's ratio (fixed to 1/3 for the chosen pattern). Based on damage concepts, the model gives damage evolutions into the concrete lattice. Tensile damage is representative of cracks more or less perpendicular to the corresponding damaged bar. Compressive damage is representative of struts effects, but, due to the coarse mesh used, the results are not really local. However, the analysis for the 2 specimens shows that the non-linearity is firstly due to cracking and finally the plateau is reached when reinforcement yields. For the T5 specimen damage in compression is limited to 0.3 which means that no crushing is observed.

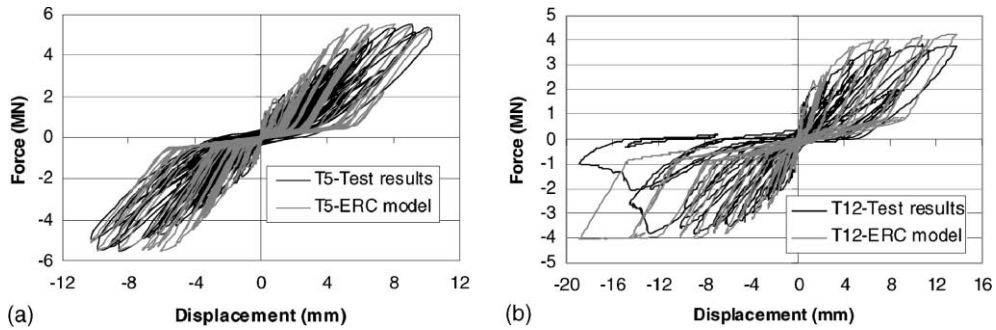


Fig. 7. SAFE tests: results of the cyclic calculations for the T5 and T12 specimen.

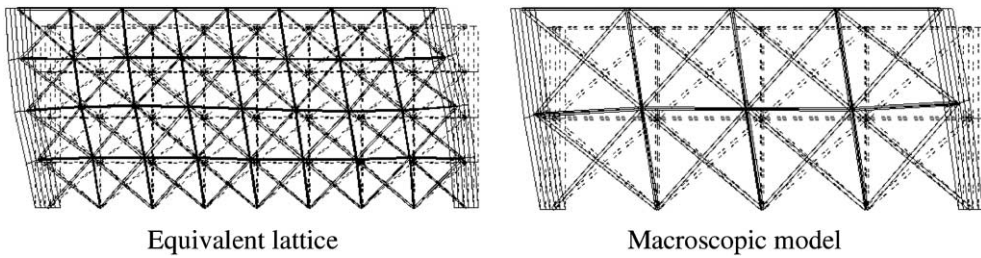


Fig. 8. Deformed shape of T5 specimen for two different meshes.

4.4. Modal and transient dynamic analysis

To verify our modeling strategy a dynamic calculations for the T5 mock-up is also performed by using the artificially generated earthquake ground acceleration histogram used as input in the simulation. Attention has to be paid to the fictitious mass used during the pseudo-dynamic test. The latter only influences the translation mode of the specimen and does not change its normal stress at the base. It is simulated as an extra mass linked to the shear wall via a rigid bar.

The modal analysis adequately predicted the fundamental frequency of the mock-up (6.7 Hz for the test and 6.8 Hz for the numerical model). The Rayleigh damping coefficients have been adjusted to ensure a 1% damping on the first two modes. This small value is justified by the nature of the pseudo-dynamic test, which is carried out quasi-statically. Nevertheless, a sensitivity analysis of the damping ratio has shown that special attention must be paid to these coefficients [2]. The maximum shear forces are the same for each damping ratio because in any case the plateau is reached. Fig. 9 shows the results of the dynamic calculations for displacements and shear forces with a focus on the third level of loading. One can observe a shift in frequencies but the maximum values of displacements and forces are close to the experimental ones.

5. Discussion about the angle θ

As mentioned after the calibration of the different strength values and the tension stiffening effects, the angle θ formed between the diagonal and the horizontal bars of the mesh is a crucial parameter for the success of the simulations. In this paper, its value has been calibrated using the envelope curves of the experimental results. To study its influence, a sensitivity analysis is presented in Fig. 10. For normally

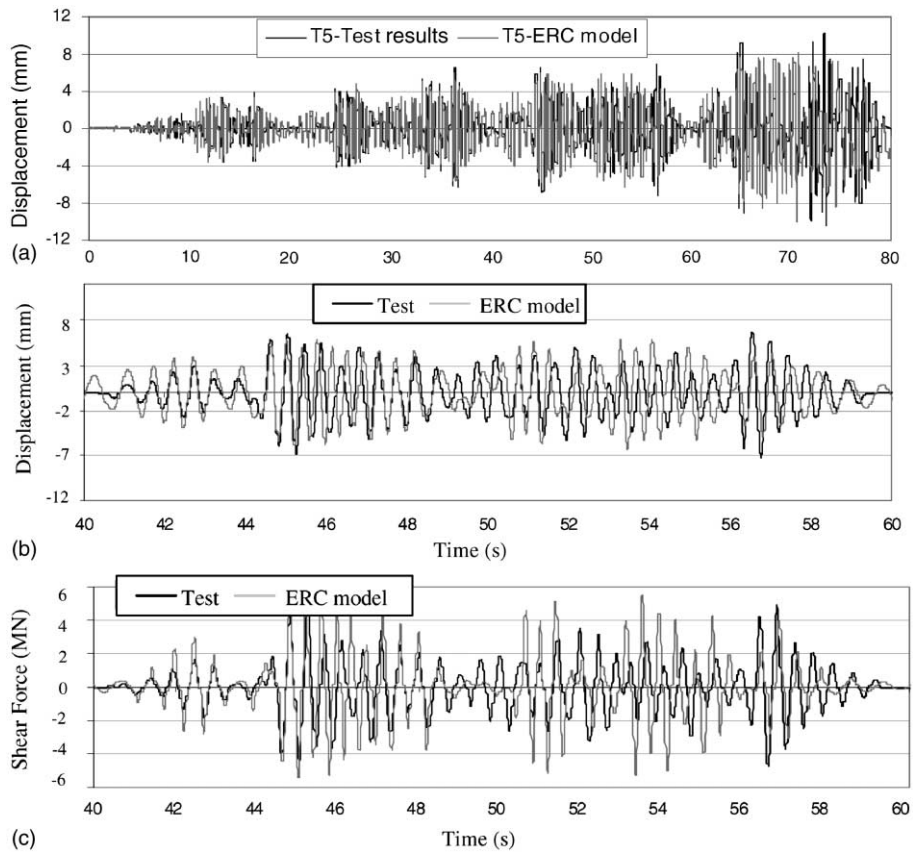


Fig. 9. Transient dynamic analysis of the T5 specimen: (a) top slab displacement for the four successive earthquake levels, (b) zoom of the top slab displacement during the third step of loading, (c) zoom for the shear force at the base of the wall during the third step of loading.

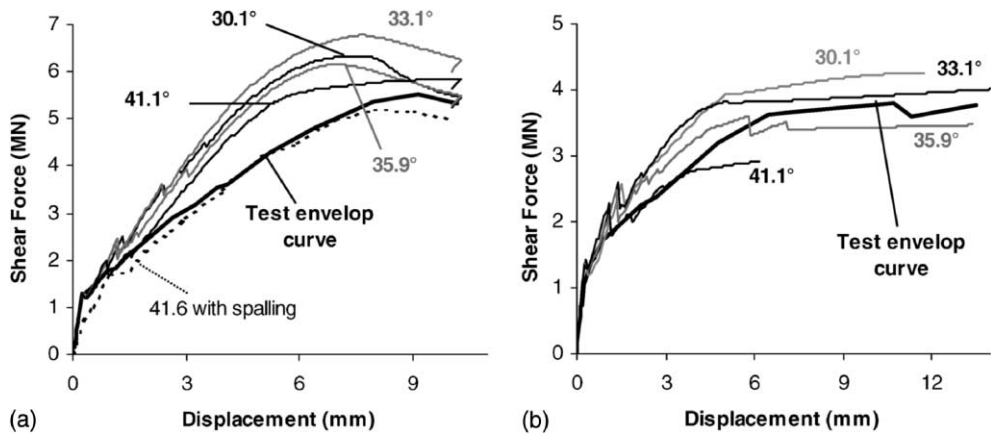


Fig. 10. Monotonic simulations with various angles compared to the test envelope curve: (a) T5 specimen, dash line is a calculation including spalling effects, (b) T12 specimen.

reinforced concrete structures (T5 specimen, Fig. 10a), the results of the simulation do not change significantly with the angle. A value between 35° and 45° correctly reproduces the ultimate behavior of the T5 mock-up (at the end, the concrete is almost completely cracked).

However, in the first part of the non-linear regime (between 3 and 9 mm), the simulation overestimates the strength for every angle. During the T5 experiment an important spalling has been observed. The cover concrete was seriously damaged and the reinforcement bars were visible. A simplified, straightforward method to take into account this phenomenon is to change the width of the web wall in the mesh. By changing the width from 0.20 to 0.15 m where 0.025 m corresponds approximately to the width of the cover, and by using this new value for the simulation, numerical results are in better agreement in the non-linear regime as seen with the dash line in Fig. 10a. Nonetheless, the response is not as stiff in the pre-cracking regime. A progressive reduction of the section associated to a spalling criterion should be implemented. It could be a damaging effect linked to the strain state and acting on the cover part of the width of the structure, however some more analysis is necessary before to do this.

For the specific case of the lightly reinforced structure T12 with an important normal stress, the value of the angle significantly influences the results. Only a value between 30° and 33° correctly reproduces the experimental results.

Different methods have been examined to estimate θ . The simplest one would be to consider it equal to the direction ϕ of the principal stress at the end of the linear regime. This gives 42.6° for T5, which is in good agreement with the value calibrated here and 38.5° for T12 that is not as good [17]. The main reason is that the method cannot reproduce the influence of the reinforcement ratio. Other simplified approaches based on the Ritter–Mörsch scheme were tried. These used non-linear equations to estimate the angle with adapted stress–strain relationships for concrete and steel. The approaches used are the compression field theory (CFT) [18] or the rotating angle softened truss model (RA-STM) [19]. However, these relationships are based on experimental results where the rotation of the tested specimens was free. For example, the RA-STM underestimates the strength of the T5 and T12 specimen (the rotation of which was prevented during the tests) (Fig. 11). A comparison between the RA-STM and a simulation with the ERC model, where the top slabs are free to rotate (Fig. 11), shows the great influence of the boundary conditions. It would be interesting to improve the CFT or the RA-STM to take into account these new boundary conditions. This could be the way to determine the best angle θ for the ERC method. Work is in progress in that direction.

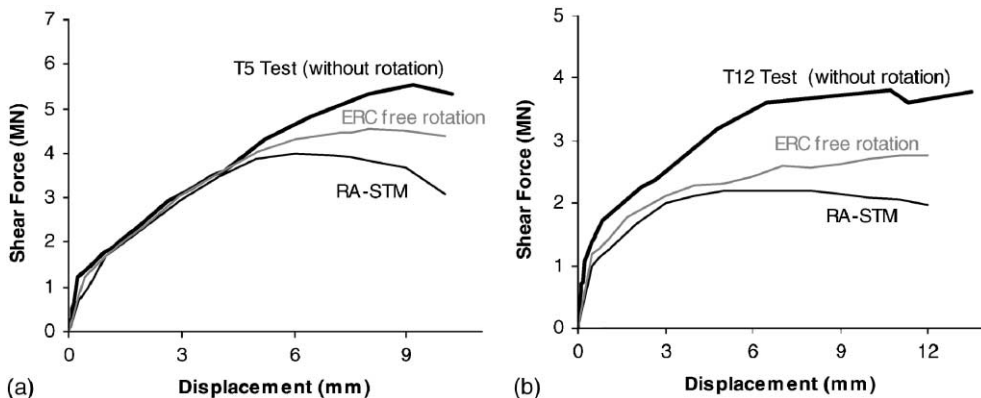


Fig. 11. Simulation with RA-STM and simulation with ERC assuming the rotation of the top slab free.

6. Conclusions

The ERC model is a simplified method to simulate the behavior of walls under dynamic shear loading. It is based upon the framework method and uses a lattice mesh to simulate the walls. It is straightforward and cost effective since it only needs simplified structural elements and 1D constitutive laws for the materials. It allows parametric studies to be performed even under earthquake loading, which is not really possible with classical FEM calculations, due to the size of problems.

In this paper, the results of tests carried out in the SAFE program have been used to show that ERC models are able to reproduce correctly the global behavior of shear walls with different reinforcement ratios in both static and dynamic situations. The comparison with experimental results has shown that the angle θ formed between the diagonal and the horizontal compressive trusses is a very sensitive parameter in this method. The discussion of the results has shown that the calibration of this angle by first performing an elastic calculation to get the orientation of the principal axis when cracking begins is a way to obtain roughly a solution. However, sensitivity increases with low reinforcement ratios then other ways of estimating θ have to be investigated.

A possible, or even necessary, improvement of the model is the modeling of the progressive spalling of the concrete cover in normally or over-reinforced specimens.

The proposed lattice model is very promising and could be extended to the non-linear simulation of parts of structures where the shear field is homogeneous enough. The main interest of this type of model is its simplicity, the possibility to easily implement it in finite element codes and the fact that it leads to trusses and beams analysis, which are familiar in engineering.

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