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# Discrete linear functional observer for the thermal estimation in power modules

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**Abstract**—High integrated power electronic modules are designed with the emergence of new semi-conductor technologies. The increase of reliability of power modules induces the precise knowledge of the local temperature, even if it can not be measured at any location. In this paper, the application of an observer in the discrete time framework is proposed. It allows to estimate the temperature at any location using measurements provided from thermal sensors located at a few precise points. The aim is to design a reduced size observer that could be implemented on a real-time embedded target such as Digital Signal Processor. Consequently, this work pays attention on the design procedure and the computation complexity of the resulting observer.

**Index Terms**—Thermal modeling, Sample and hold state space representation, Discrete linear functional observer.

## I. INTRODUCTION

The joint emergence of Wide Band Gap materials (SiC, GaN, C) and new generation hybrid integration techniques significantly enhance performances of power electronic modules. Such modules should operate in severe environment and constraints: high temperature and high power density, fast switching, etc. Consequently of high temperature, new constraints appear and become critical for power electronics assemblies. Several studies aim at identifying failure modes or critical interfaces [1], [2]. Thus, estimation of local temperatures becomes a real challenge in new generation of power modules to increase their lifetime. Indeed, it has been shown in [3], [4] that the evolution of local constraints in a power electronic module, which can be thermal or thermo-mechanical, have a negative effect on the lifetime of the module. These constraints increase the occurrence of potentially critical defects and failures on the module. Consequently, it becomes necessary to have a precise knowledge of the temperatures at specific locations in the module, such as the temperature of semi-conductor chips or wire bondings. However, due to the size of sensors and possible electromagnetic field disturbances close to measurement points, the use of thermal sensors may be difficult at some locations inside of the power module. For these reasons the objective of the following work is to estimate this physical variable in a specific non measured location, using measured data by few sensors.

As a case study, a simple two-dimensions (2D) thermal system is considered in this paper and then modeled using an analogy between thermal and electrical domains. Then,

equations of thermal evolution of the system with respect to time and space can be rewritten using a linear state-space representation. Using this representation, the temperature can be estimated at any location with a linear functional observer or a partial state observer. In order to be implemented on a digital target, observers have to be designed in the discrete time domain. The first way of designing such observer is to consider the sample and hold model of the system and design the corresponding discrete observer.

Section II deals with the construction of a thermal model and its state space representation. In this work, the thermal behavior of a  $(30 \times 30mm)$  2D heated plate, which may represent a section of a power electronics module is considered as a test benchmark for our technics. The continuous and discrete matrix representations of the previous model is established and its purpose is to design a reduced size observer. We propose in section III a way to design a discrete linear functional observer, based on the use of successive derivatives of the measured outputs. The interest of the observer lies in the possibility to estimate the temperature at any location in the system. Thus, our main objectives are to give a straightforward procedure to find the minimal functional observer for the discrete time system, followed by a simulation results from the application on the 2D heated plate in section IV. Finally, a discussion about the performances of the observer is presented.

## II. 2D THERMAL MODEL

### A. Thermal model

The thermal evolution of a 2D heated plate is given by the heat equation (1), [5], [6] with:

- $T$  the local temperature in  $K$ ,
- $t$  the time in  $s$ ,
- $\rho$  the mass density of the material in  $kg.m^{-3}$ ,
- $C_p$  the thermal capacity in  $J.kg^{-1}.K^{-1}$ ,
- $\lambda$  the thermal conductivity in  $W.m^{-1}.K^{-1}$ ,
- $S$  the heat source in the system in  $W.m^{-3}$

$$\rho C_p \frac{\partial T(x, y, t)}{\partial t} = -\lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S \quad (1)$$

The heat equation (1) reflects linear transfer phenomena such as conductive and convective transfers induced by the presence of the temperature gradient represented by (2), [7], [8], [9]:

$$\vec{\varphi} = -\lambda \cdot \overrightarrow{\text{grad}} T \quad (2)$$

where  $\vec{\varphi}$  stands for the heat flux density.

Radiative transfers are non considered in this equation. However, in this work, this kind of transfers are neglected. As (2) is similar to Ohm's law in electrical domain, a thermo-electrical analogy between the different domains can be defined and summarized in Tab. I, [10].

TABLE I  
ELECTRO-THERMAL ANALOGY

Thermal domain	Electricity domain
Parameter (unit), Notation	Parameter (unit), Notation
Temperature (K), $T$	Electric potential (V), $V$
Thermal flux (W), $Q$	Current (A), $I$
Thermal resistance ( $m^2 K/W$ ), $R_{th}$	Resistance ( $\Omega$ ), $R$
Heat capacity (J/K), $C_{th}$	Capacity (F), $C$

This analogy leads to obtain an equivalent electrical model which represents the thermal behavior of a system [10]. In order to design a continuous-time observer, the first step is to discretize the obtained model with respect to the two dimensions of space (see Fig. 1).

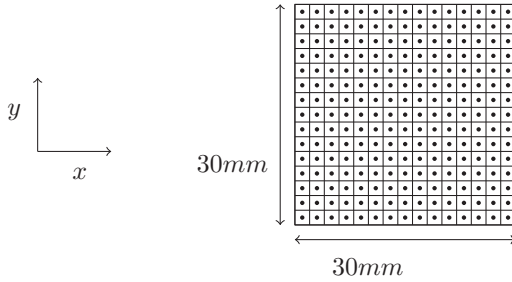


Fig. 1. Surface discretization of the heated plate into elementary surfaces (illustrative example)

Using a spatial finite difference discretization of the heat equation and the previously defined thermo-electrical analogy, thermal behavior of the 2D plate is modeled as a network composed of resistors for spatial thermal conductivity and convection, capacitors for heat storage, voltage sources for temperature sources and current sources for heat sources [11]. On the one hand, the conductive transfer (resp. convective) is characterized by a conduction resistance  $R_{cd}$  (resp. convection resistance  $R_{cv}$ ) defined by :

$$R_{cd} = \frac{e}{\lambda S} \quad (\text{resp. } R_{cv} = \frac{1}{hS}) \quad (3)$$

where  $e$  is the plate thickness,  $S$  is the exchange surface between elementary surfaces and  $h$  is the convection coefficient [12].

On the other hand, the storage of thermal energy in an elementary surface is modeled by a thermal capacity  $C_{th}$

connected between the center and the mass (thermal reference) and given by:

$$C_{th} = \rho C_p V \quad (4)$$

where  $V$  is the volume of the material in  $m^3$ .

Finally, heat sources  $P_{th}$  may be inserted into some elements to induce the dynamic thermal response of the system.

### B. Continuous state space representation of the heated plate

From the spatial discretization, the temperature  $T$  is defined on the centers of the elementary surfaces (denoted nodes in the following) of the plate. It depends on the temperatures of its neighbors and thermal impedances connected to the considered element. Depending on the position of nodes on the plate, two kinds of impedances connected to the nodes must be considered as shown in Fig. 2. Thus, the heated plate is represented by a network of impedances that translates the conduction between center and edges of this surface.

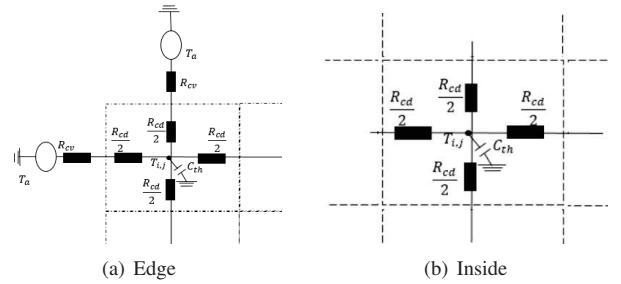


Fig. 2. Representation of an impedance network of an elementary surface at the edge and inside of the plate [13]

These elementary schemes must be combined to build the complete network described in Fig. 3.

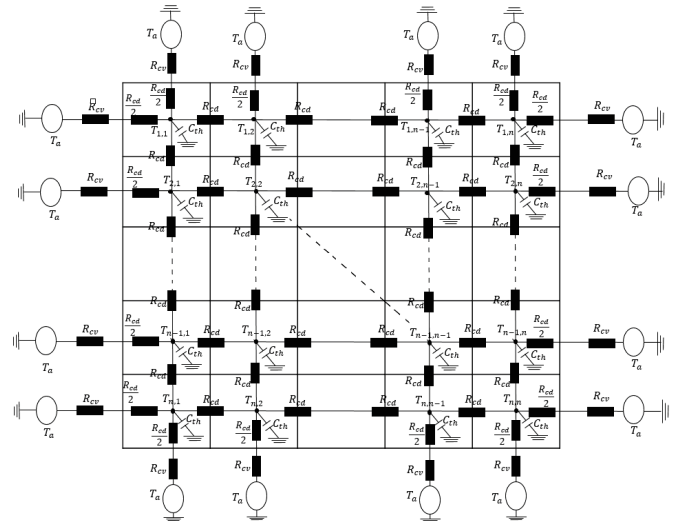


Fig. 3. Electro-thermal nodal model of the heated plate [13]

Millman's theorem [14] allows to express the temperature for each node with a first order differential equation. By

combining all the node's equations, a state space representation is obtained (5).

$$\begin{cases} C_{th}\dot{\mathbf{T}}(t) = A\mathbf{T}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) = C\mathbf{T}(t) \end{cases} \quad (5)$$

where  $\mathbf{T}(t)$  is the vector of local temperatures,  $C_{th}$  is the diagonal matrix of thermal capacities,  $A$  is the thermal resistances matrix,  $\mathbf{y}(t)$  is the measurements vector,  $C$  reflects the sensors position,  $\mathbf{u}(t)$  is the vector of boundary conditions for temperatures and heat sources and  $B$  reflects their influence on the plate.

Simulation of local temperatures is then obtained through the state space model (5). As a contrary to experiments, simulation of (5) allows the knowledge of all temperatures. Consequently, the estimation of the temperature in a specific non measured location is necessary. In this paper, we propose to achieve this objective through linear functional observers in discrete time.

### C. Sample and hold state space representation

For digital target applications, a discretization of the state space system is necessary. Assume that the continuous time system of the heat equation is given by the following state space representation:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) = C\mathbf{x}(t) \end{cases} \quad (6)$$

Equation of the sampled system is given by:

$$\begin{cases} \mathbf{x}(k+1) = A_d\mathbf{x}(k) + B_d\mathbf{u}(k) \\ \mathbf{y}(k) = C_d\mathbf{x}(k) \end{cases} \quad (7)$$

with:

$$\begin{cases} A_d = e^{AT_s} \\ B_d = \int_0^{T_s} e^{A\theta} B d\theta \\ C_d = C \end{cases} \quad (8)$$

where  $T_s$  is the sampling period. For every  $t$  in  $\mathbb{R}^+$ ,  $\mathbf{x}(t)$  is a  $n$ -dimensional state vector,  $\mathbf{u}(t)$  is a  $p$ -dimensional control vector supposed to be known,  $\mathbf{y}(t)$  is a  $m$ -dimensional measured vector and,  $A_d(n \times n)$ ,  $B_d(n \times p)$  and  $C_d(m \times n)$  are constant matrices.

**Notation:** in the whole paper, all the discrete signals  $s(k)$  may be written in one of three ways  $s(kT) \equiv s(k) \equiv s_k$ , where  $k \in \mathbb{Z}$ .

The model in (7) is therefore called a zero-order hold of the system in (6) [15], [16].

## III. ESTIMATION OF NON-MEASURED VARIABLES

### A. Discrete functional linear observers

Let us consider a system described by the sample and hold state space equations in (7).

The aim of a functional observer is to estimate state variables, at least asymptotically, from the measurements on the system. Estimated state variables are defined by :

$$\mathbf{v}(k) = L\mathbf{x}(k) \quad (9)$$

where  $L$  is a constant full row rank ( $l \times n$ ) matrix selecting estimated components.

The observation of  $\mathbf{v}(k)$  can be carried out by the designed linear functional observer or Luenberger observer [17], [18], which is described by the state equations:

$$\begin{cases} \mathbf{z}(k+1) = F_d\mathbf{z}(k) + G_d\mathbf{u}(k) + H_d\mathbf{y}(k) \\ \hat{\mathbf{v}}(k) = P_d\mathbf{z}(k) + V_d\mathbf{y}(k) \end{cases} \quad (10)$$

where  $\mathbf{z}(k)$  is a  $q$ -dimensional state vector and  $\hat{\mathbf{v}}(k)$  is a  $l$ -dimensional vector. The constant matrices  $F_d, G_d, H_d, P_d, V_d$  and the order  $q$  are determined such that  $\lim_{k \rightarrow +\infty} (\mathbf{v}(k) - \hat{\mathbf{v}}(k)) = 0$ . Moreover, it must be kept in mind that we look for a minimal order observer. The asymptotic tracking is ensured if  $F_d$  is a Schur matrix, i.e. all the eigenvalues of  $F_d$  are inside a open unit disk.

Following [19], the linear functional observer (10) exists if and only if there exists a  $(q \times n)$  matrix  $M$  such that:

$$MA_d - F_dM = H_dC_d \quad (11)$$

$$L = P_dM + V_dC_d \quad (12)$$

$$G_d = MB_d \quad (13)$$

### B. Design of a discrete Luenberger observer

This section deals with the search for a minimum order of a functional observer. This point is achieved in order to obtain a fast and implementable observer.

Let  $q$  be the smallest integer such that,

$$\text{rank } \Sigma_q = \text{rank} \begin{pmatrix} \Sigma_q \\ LA_d^q \end{pmatrix} \quad (14)$$

with:

$$\Sigma_q = \begin{pmatrix} C_d \\ L \\ C_dA_d \\ LA_d \\ \vdots \\ C_dA_d^{q-1} \\ LA_d^{q-1} \\ C_dA_d^q \end{pmatrix} \quad (15)$$

1) *First step:* The design of the observer uses the successive derivations of  $\mathbf{v}(k)$ . After  $q$  derivations of  $\mathbf{v}(k) = L\mathbf{x}(k)$ , we obtain:

$$\mathbf{v}(k+q) = LA_d^q\mathbf{x}(k) + \sum_{i=0}^{q-1} LA_d^{q-1-i}B_d\mathbf{u}(k-i) \quad (16)$$

From (14), two matrices can be defined,  $\Gamma_i, i \in \llbracket 0 ; q \rrbracket$  and  $\Lambda_i, i \in \llbracket 0 ; q-1 \rrbracket$  such that:

$$LA_d^q = \sum_{i=0}^q \Gamma_i C_d A_d^i + \sum_{i=0}^{q-1} \Lambda_i LA_d^i \quad (17)$$

Using (17), (16) can be written as:

$$\mathbf{v}(k+q) = \sum_{i=0}^q \Gamma_i C_d A_d^i \mathbf{x}(k) + \sum_{i=0}^{q-1} \Lambda_i L A_d^i \mathbf{x}(k) + \sum_{i=0}^{q-1} L A_d^{q-1-i} B_d \mathbf{u}(k-i) \quad (18)$$

2) *Second step:* The second step is to eliminate the state  $\mathbf{x}(k)$  from the (18) so that  $\mathbf{v}(k+q)$  will be expressed only in terms of  $\mathbf{v}(k)$ ,  $\mathbf{y}(k)$ ,  $\mathbf{u}(k)$  and their successive derivatives. To do so, the state equation (7) is used after each derivation of  $\mathbf{v}(k) = L\mathbf{x}(k)$  and  $\mathbf{y}(k) = C_d\mathbf{x}(k)$ , [20], [21]. Thus, we get:

$$\mathbf{v}(k+q) = \sum_{i=0}^q \Gamma_i \mathbf{y}(k+i) + \sum_{i=0}^{q-1} \Lambda_i \mathbf{v}(k+i) + \sum_{i=0}^{q-1} \Phi_i \mathbf{u}(k+i) \quad (19)$$

where, for  $i \in \llbracket 0 ; q-2 \rrbracket$ :

$$\Phi_i = \left[ L A_d^{q-1-i} - \sum_{j=i+1}^q \Gamma_j C_d A_d^{j-i-1} - \sum_{j=i+1}^{q-1} \Lambda_j L A_d^{j-i-1} \right] B_d \quad (20)$$

and

$$\Phi_{q-1} = [L - \Gamma_q C_d] B_d \quad (21)$$

3) *Third step:* The third step consists in realizing the input-output differential equation (19) [20], [22], as:

$$\left\{ \begin{array}{l} \mathbf{z}(k+1) = \begin{pmatrix} 0 & & \Lambda_0 \\ 1 & \ddots & \Lambda_1 \\ & \ddots & 0 \\ & & 1 & \Lambda_{q-1} \end{pmatrix} \mathbf{z}(k) + \begin{pmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_{q-1} \end{pmatrix} \mathbf{u}(k) \\ \quad + \begin{pmatrix} \Gamma_0 + \Lambda_0 \Gamma_q \\ \Gamma_1 + \Lambda_1 \Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1} \Gamma_q \end{pmatrix} \mathbf{y}(k) \\ \hat{\mathbf{v}}(k) = [0 \ 0 \ \dots \ 0 \ 1] \mathbf{z}(k) + \Gamma_q \mathbf{y}(k) \end{array} \right. \quad (22)$$

When  $F_d$  is a Schur matrix, it is demonstrated that (22) is an asymptotic observer of the functional linear  $L\mathbf{x}(k)$ . Otherwise, it becomes necessary to increase the order  $q$  and to do again the building procedure with a higher order, [23], [24].

#### IV. APPLICATION TO THE 2D HEATED PLATE

##### A. Design of a minimal-order observer

In this section, the discrete functional observer is applied on temperature estimation of a heated plate. In order to closely follow the design of the observer, the plate is spatially discretized into 9 elementary surfaces (Fig. 4) leading to a 9-order state space model. This choice is only achieved to detail the design procedure of the observer and the obtained results. Any partitioning of the heated plate could be chosen. Moreover, the more precise the partitioning is, the larger the matrices are. However, in case of large matrices, the details of calculus cannot be explicitly shown.  $Ca$ ,  $S$  and  $O$  denotes respectively the sensor, the heat source and the estimated temperature locations. Let us remark that from symmetrical reasons, for identical initial conditions, the temperature of cell  $Ca$  shall be equal to the one of cell  $O$ .

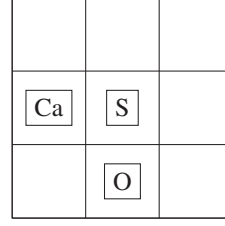


Fig. 4. Discretization of the heated plate

Considering the modeling method in section II, a continuous [25] and the associated sample and hold state space representation are obtained for a sampling period  $T_s = 1s$ :

$$\left\{ \begin{array}{l} A_d = \begin{pmatrix} a & b & c & b & d & e & c & e & f \\ b & g & b & d & h & d & e & i & e \\ c & b & a & e & d & b & f & e & c \\ b & d & e & g & h & i & b & d & e \\ d & h & d & h & j & h & k & h & d \\ e & d & l & i & h & g & e & d & l \\ c & e & f & l & d & e & a & l & c \\ e & i & e & d & h & d & l & g & l \\ f & e & c & e & d & b & c & b & a \end{pmatrix} \\ B_d = \begin{pmatrix} 0,74 & 6,74 \times 10^{-6} \\ 0,50 & 1,907 \times 10^{-3} \\ 0,74 & 6,74 \times 10^{-6} \\ 0,50 & 1,907 \times 10^{-3} \\ 7,683 \times 10^{-3} & 0,68 \\ 0,50 & 1,907 \times 10^{-3} \\ 0,74 & 6,74 \times 10^{-6} \\ 0,50 & 1,907 \times 10^{-3} \\ 0,74 & 6,74 \times 10^{-6} \end{pmatrix} \\ C_d = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ L = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \right. \quad (23)$$

where  $a = 0.24, b = 2.45 \times 10^{-3}, c = 7.55 \times 10^{-6}, d = 2.45 \times 10^{-5}, e = 7.54 \times 10^{-8}, f = 2.32 \times 10^{-10}, g = 0.48, h = 4.88 \times 10^{-3}, i = 1.50 \times 10^{-5}, j = 0.97, k = 2.45 \times 10^{-5}$  and  $l = 2.45 \times 10^{-3}$ .

The design procedure in section III-B is applied and the minimal order such as  $\text{rank}(\Sigma_3) = \text{rank} \begin{pmatrix} \Sigma_3 \\ L A_d^3 \end{pmatrix}$  is  $q = 3$ . Thus, a minimal third-order candidate observer can be envisaged.

It has to be verified that  $F_d$  is a Schur matrix:

$$L A_d^3 \Sigma_3^{-1} = \begin{pmatrix} -0.05 & 0.05 & 0.47 & -0.47 \\ -1.22 & 1.22 & 1 \end{pmatrix} \quad (24)$$

Then,  $\Lambda_0 = 0.05, \Lambda_1 = -0.47, \Lambda_2 = 1.22$ . It yields to:

$$F_d = \begin{pmatrix} 0 & 0 & 0.05 \\ 1 & 0 & -0.47 \\ 0 & 1 & 1.22 \end{pmatrix} \quad (25)$$

The eigenvalues of  $F_d$  are  $(0.24, 0.48, 0.48)$ .



$F_d$  is then a Schur matrix. Consequently, the observer is asymptotically convergent.

From,  $LA_d^3\Sigma_3^{-1}$ , it gets  $\Gamma_0 = -0.05, \Gamma_1 = 0.47, \Gamma_2 = -1.22$  and  $\Gamma_3 = 1$ .

Using (22), the matrices of the observer are deduced:

$$\begin{cases} G_d = \begin{pmatrix} 1.28 \times 10^{-13} & -3.23 \times 10^{-16} \\ -6.46 \times 10^{-13} & 1.13 \times 10^{-15} \\ 5.41 \times 10^{-13} & 2.05 \times 10^{-15} \end{pmatrix} \\ H_d = \begin{pmatrix} 6.17 \times 10^{-14} \\ -3.80 \times 10^{-13} \\ 5.04 \times 10^{-13} \end{pmatrix} \\ P_d = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}, \text{ and } V_d = 1 \end{cases} \quad (26)$$

Considering orders of magnitude of the coefficients in  $G_d$  and  $H_d$ , they can be approximated by:

$$G_d = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, H_d = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, P_d = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}, V_d = 1 \quad (27)$$

The observer design is ended. However, when the obtained poles (0.24, 0.48, 0.48) are not suitable to ensure a sufficiently fast dynamic of the observer error, the observer order has to be increased.

For an initial condition  $T(0) - \hat{T}(0) = 1$ , the simulation results are given in Fig. 5.

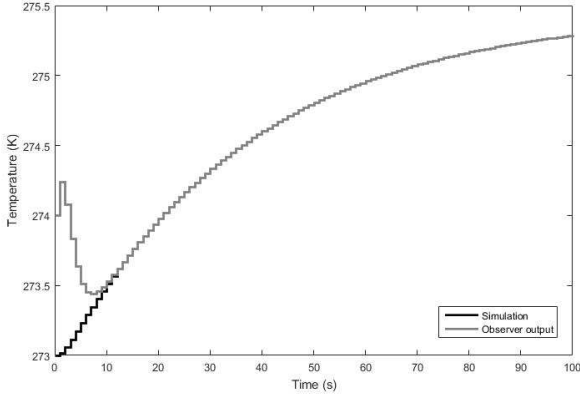


Fig. 5. Simulation result of observer of order  $q = 3$ , for a discrete state model, with non identical initial conditions

It is clear that the convergence between the observer output is asymptotically ensured to the simulation value. Moreover, the observer is designed with an arbitrary asymptotic convergence speed due to the eigenvalues of  $F_d$ . To improve performances and to obtain a faster convergence, the order of the observer has to be increased.

#### B. Convergence time and computation complexity of the observer

In order to evaluate the performances of the observer previously designed, several sampling periods  $T_s$  are tested.

Obviously, the maximum sampling period remains lower than the time constant of the temperature evolution. A convergence criteria  $\epsilon = |T - \hat{T}| < 10^{-3}$  is chosen to measure the convergence time  $T_{cv}$ . Then, convergence time and maximum estimation error are given in Tab. II for an initial condition  $T(0) - \hat{T}(0) = 1$ . Note that in this comparison, the observer order  $q$  stays equal to 3.

TABLE II  
OBSERVER PERFORMANCES REGARDING  $T_s$

$T_s$	50ms	100ms	1s	2s	4s
$T_{cv}$	22.9s	20.9s	15s	14s	12s
$\epsilon_{max}$	138	37	1.22	1	1

In case of digital implementation of the observer, it is necessary to consider the computation complexity to obtain the estimated temperature for each sample. Considering the structure of  $F_d$ , the worst case such as  $G_d$  and  $H_d$  are full of non-zero values,  $\mathbf{y}$  is a scalar and  $\mathbf{u}$  is a  $d$  dimension vector, the estimated temperature needs  $dq^3 + 1$  multiplications and  $3q + d - 1$  additions. In a digital processors, multiplications are longer to compute than additions. Consequently, it becomes clear that the observer dimension as to be minimal.

Finally, regarding observer performances and computation complexity, it can be said that the sampling period should be as large as possible to increase the performances of the observer in terms of estimation error and convergence time and to avoid high computation complexity.

#### C. Increase of the accuracy of spatial discretization

Let's consider a system model with a more accurate discretization with  $(11 \times 11)$  elementary surfaces leading to a  $121^{th}$  order state space model. The position of the sensor and the estimated point are chosen to ensure non symmetry in the estimation problem. An observer is designed according to the procedure given in section III. Thus, a linear functional observer of order  $q = 14$  is obtained. The asymptotic convergence of the simulated temperature  $T$  and the estimated temperature  $\hat{T}$  is checked. This study points out the advantage of a linear functional observer compared to a reduced observer of order  $n - l = 120$ . The simulation result is given in Fig. 6.

It can be concluded that the observer accurately estimates the temperature of a desired point, whatever the initial conditions.

## V. CONCLUSION

In this paper, a thermal modeling of a heated plate is presented using the finite difference discretization for the 2D heat equation, leading to a state space representation of the system with the thermo-electrical analogy.

From this state space representation expressed in the discrete time framework, a linear functional observer has been designed in order to estimate the temperature at any desired point using few measurements and the knowledge of inputs.

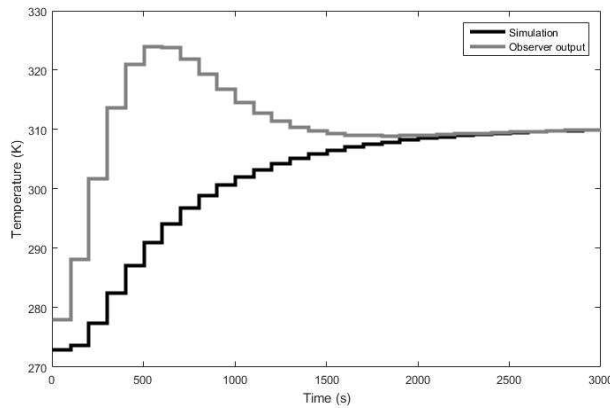


Fig. 6. Simulation result of observer of order  $q = 14$ , for a state model of order 121, with non zero initial conditions

This paper shows that the observer is able to accurately estimate the temperature of the heated plate. Then, performances of the observer in terms of estimation error and convergence time have been studied regarding the sampling period. Computation complexity has also been evaluated. It has been shown that the setting of the sampling period has to be done regarding the physical system and the digital target, especially for large system and observers.

The main objective of this paper was to show the feasibility of our approach to design fast and efficient observers. Our future development will be to establish 3D thermal models for the observation of a complete power electronic module. However, these models will be naturally of large dimension and state space representations will be huge. Thus, it could be supposed that even the linear functional observer will not drastically reduce the order of the problem. In this case, the observer designed from experimentally identified transfers could be studied. Using small order transfers, the associated observer would have a limited dimension. This particular point will be developed in further work.

Two ways of development could also be considered for the presented work. On the one hand, some efforts will be done on the thermal modeling. Anisotropic materials could be on concerned, leading to variations of parameters along spatial dimensions. This point will lead to models of complete power modules. On the other hand, unknown input observers could be studied. This particular point is extremely important in case of unknown or uncontrolled perturbations applied on the system. This will be applied to the observation of real systems where environmental conditions, such as ambient temperature and cooling systems, are not always perfectly stable and controlled. Finally, for stability purposes, the use of sample and hold observers designed from continuous linear functional observers will also be discussed in further work. It will also be compared to numerically integrated continuous observers.

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