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Poster: Formal Controller Synthesis from Hybrid Programs

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ABSTRACT

We consider a new way of describing complex control problems for dynamic systems called *hybrid programs*. Hybrid program is a finite state automaton whose states describe elementary tasks of reachability and safety defined on a transition system ([3, 5]). The proposed approach to complex control problems description could be seen as an alternative to linear temporal logic (see e.g. [1]). We provide an example to illustrate the approach.

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1 HYBRID TASKS

In our work we consider the dynamics described by transition systems.

DEFINITION 1. A transition system S is a tuple (X, U, Y, Δ, X^0) where X is a set of states; U is a set of inputs; Y is a set of outputs; Δ is a transition relation: $\Delta \subseteq X \times U \times X \times Y$; $X^0 \subseteq X$ is a set of initial states.

Here we assume that $y \in Y$ is a function from $C([0, \theta_y], \mathbb{R}^n)$, $\theta_y > 0$. An input $u \in U$ is *enabled* at state x ($u \in \text{enab}(x)$) if $\Delta(x, u) \neq \emptyset$. The control goal is formally described by a *hybrid program*, the notion of which is introduced in the next section. Hybrid programs consist of elementary hybrid tasks.

DEFINITION 2. An elementary hybrid task is a tuple $T = (Q, E, I, G, Q^0, Q^1, s)$ consisting of a finite set of states Q ; a transition relation $E \subseteq Q \times Q$; a set of invariants $I = \{I_q \subseteq \mathbb{R}^n, q \in Q\}$; a set of guards $G = \{G_e \subseteq \mathbb{R}^n, e \in E\}$; a set of initial states $Q^0 \subseteq Q$; a set of terminal states $Q^1 \subseteq Q$; a task semantic $s \in \{\text{"reachability"}, \text{"safety"}\}$.

We use elementary hybrid tasks to compose transition systems $S||T$ which are derived from the definition of hybrid automata [4] and satisfy the corresponding safety and reachability requirements.

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DEFINITION 3. The composition of $S = (X, U, Y, \Delta, X^0)$ with $T = (Q, E, I, G, Q^0, Q^1, s)$ is the transition system $S||T = (X_T, U_T, Y, \Delta_T, X_T^0, X_T^1, s)$ where the set of states is $X_T = Q \times X$; the set of inputs is $U_T = Q^{X \times Y} \times U$; the set of outputs is Y ; the set of initial states is $X_T^0 = Q^0 \times X^0$; the set of terminal states is X_T^1 . A control $v = (p, u) \in U_T$ is enabled in the state z ($v \in \text{enab}(z)$) if and only if $u \in \text{enab}(x)$ and for all $(x', y) \in \Delta(x, u)$ one of the following conditions holds:

- $q = p(x', y)$ and $\forall t \in \mathcal{T}_y, y(t) \in I_q$;
- there exist $t_i \in \mathcal{T}_y, i = 0, \dots, N + 1$ with $0 = t_0 \leq t_1 \leq \dots \leq t_{N+1} = \theta_y$ (or $t_{N+1} = N_y$ in the discrete case) and $e_i = (q_i, q_{i+1}) \in E, i = 0, \dots, N - 1$ with $q_0 = q, q_N = p(x', y)$ and

$$\begin{cases} \forall t \in [t_i, t_{i+1}) \cap \mathcal{T}_y, y(t) \in I_{q_i}, & i = 0, \dots, N; \\ y(t_{i+1}) \in G_{e_i}, & i = 0, \dots, N - 1. \end{cases}$$

The transition relation is given for $z = (q, x), z' = (q', x') \in X_T, v = (p, u) \in U_T, y \in Y$ by $(z', y) \in \Delta_T(z, v)$ if and only if $v \in \text{enab}(z), (x', y) \in \Delta(x, u), q' = p(x', y)$.

The set X_T^1 is defined as follows

$$X_T^1 = \{z = (q, x) \mid q \in Q^1 \text{ and } \text{enab}(x) \neq \emptyset, \\ \forall u \in \text{enab}(x), \forall (x', y) \in \Delta(x, u) \Rightarrow y(0) \in I_q\}.$$

A controller C for the transition system $S||T$ is defined as a subset of $X_T \times U_T$ such that $C(z) \subseteq \text{enab}(z)$. We denote the domain of the controller C as $\text{dom}(C) = \{z \in X_T \mid C(z) \neq \emptyset\}$. The controlled system is given by the transition system $S||T/C = (X_T, U_T, Y, \Delta_{T/C}, X_{T/C}^0, X_{T/C}^1, s)$ where the transition relation is given for $z \in X_T, v \in U_T$ by $(z', y) \in \Delta_{T/C}(z, v)$ if and only if $v \in C(z)$ and $(z', y) \in \Delta_T(z, v)$; the set of initial states is $X_{T/C}^0 = X_T^0 \cap \text{dom}(C)$.

Next, for each type of tasks we give the definition of the corresponding controller.

DEFINITION 4. For $S||T = (X_T, U_T, Y, \Delta_T, X_T^0, X_T^1, s)$ with safety semantics s the corresponding controller is a controller such that all maximal trajectories of $S||T/C$ are either infinite or have the form $\{(z_k, v_k, y_k)\}_{k=0}^N$ with $z_N \in X_T^1$.

DEFINITION 5. For $S||T = (X_T, U_T, Y, \Delta_T, X_T^0, X_T^1, s)$ with reachability semantics s the corresponding controller is a controller such that all maximal trajectories of $S||T/C$ have the form $\{(z_k, v_k, y_k)\}_{k=0}^N$ with $z_N \in X_T^1$.

Now we are ready to define hybrid programs and controllers for them.

2 HYBRID PROGRAMS

DEFINITION 6. A hybrid program \mathbb{T} is a tuple $(\mathcal{T}, \mathcal{E}, \mathcal{R}, \mathcal{T}^0)$ where \mathcal{T} is a finite set of tasks; $\mathcal{T}^0 \subseteq \mathcal{T}$ is a set of initial tasks; $\mathcal{E} \subseteq \mathcal{T} \times \mathcal{T}$ is a set of transitions; $\mathcal{R} = \{R_e, e \in \mathcal{E}\}$ is a set of transition relations

$$R_e \subseteq Q_T^1 \times Q_{T'}^0, \text{ for } e = (T, T') \in \mathcal{E}.$$

We also consider the definition of the composition $S||\mathbb{T}$ of the symbolic transition system S and the hybrid program \mathbb{T} . Then we use the sets $J_e = \{q \mid R_e(q) \neq \emptyset\}$, $e \in \mathcal{E}$ and $J_T = \bigcup_{T'} \{J_e \mid e = (T, T')\}$ to define a controller for the hybrid program \mathbb{T} .

DEFINITION 7. A controller C for the hybrid program \mathbb{T} is a set of controllers $\{C_T, T \in \mathcal{T}\}$ such that

- C_T is a controller which solves the task T ;
- For all $T \in \mathcal{T}$ for all states $z = (q, x) \in X_T^1$ reachable with C_T the following property holds: if $q \in J_T$ then there exists $q' \in Q_{T'}^0$, such that $q' \in R_e(q)$, $(q', x) \in X_{T'}^0$, and $C_{T'}(q', x) \neq \emptyset$ for some $e = (T, T')$, $q \in J_e$.

To compute the hybrid program controllers we utilize an algorithm which iteratively updates the individual task controllers ([2, 5]). Algorithm 1 terminates when a fixed point is reached or, in other words, when the maximal safety set of the hybrid program is found.

Algorithm 1: Hybrid program controller

Input: Transition system S , hybrid program \mathbb{T}
Output: Controller $C_T = \{C_T, T \in \mathcal{T}\}$
Global variables: Value functions V_T , terminal sets X_T^1
Local variables : *controllerUpdated*, *blockingTerminalState*
begin
 controllerUpdated := true;
 while *controllerUpdated* **do**
 controllerUpdated := false;
 foreach $T \in \mathcal{T}$ **do**
 foreach reachable $z = (q, x) \in X_T^1 \cap (J_T \times X)$ **do**
 blockingTerminalState := true;
 foreach T' s.t. $q \in J_{(T, T')}$ **do**
 if $(R_e(q) \times \{x\}) \cap X_{T'}^0 \cap \text{dom} C_{T'} \neq \emptyset$
 then
 blockingTerminalState := false;
 if *blockingTerminalState* **then**
 $X_T^1 := X_T^1 \setminus \{z\}$;
 $C_T = \text{UpdateController}(S||T, \{z\})$;
 controllerUpdated := true;
 return C ;

We then prove correctness of the corresponding *UpdateController* algorithms (for safety and for reachability) using dynamic programming techniques.

3 ILLUSTRATIVE EXAMPLE

Finally, we apply this theory to formulate and solve a control synthesis example. Let us consider the following two-dimensional

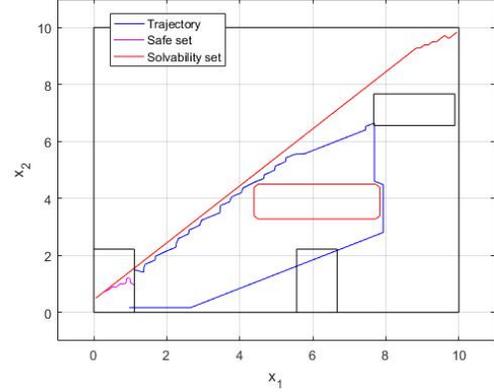


Figure 1: Simulation of the example's system trajectory with the computed controller.

system

$$\begin{cases} \dot{x}_1 &= 2u_1, \\ \dot{x}_2 &= u_2 + 3vh(x_2 - x_1)u_0 \end{cases}$$

where $u \in [-1, 1]^2$ is the control, $v \in [0, 1]$ is the disturbance, $h(x) = \max\{0, \max\{1, x\}\}$. The state space constraint here is $x \in Q = [0, 10]^2 \setminus [4.44, 7.77] \times [3.33, 4.44]$. The control problem is described by a hybrid program presented on Fig. 2. The safety task requires the system to stay in Q_1 or transition to the reachability task which is to visit Q_2, Q_3, Q_1 (in this particular order) and to transition back to safety task. Here $Q_1 = [0, 1.11] \times [0, 2.22]$, $Q_2 = [5.55, 6.66] \times [0, 2.22]$, $Q_3 = [7.77, 10] \times [6.66, 7.77]$. We compute the hybrid program controller using Algorithm 1. The simulated trajectory with this controller is depicted on Fig. 1.

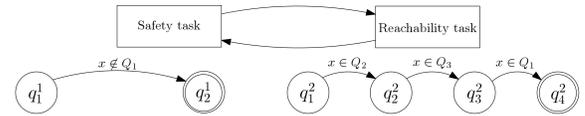


Figure 2: Visual representation of hybrid tasks in the example.

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REFERENCES

- [1] Calin Belta, Boyan Jordanov, and Ebru Aydin Gol. 2017. *Formal Methods for Discrete-Time Dynamical Systems*. Springer.
- [2] Antoine Girard. 2012. Controller synthesis for safety and reachability via approximate bisimulation. *Automatica* 48, 5 (2012), 947–953.
- [3] Antoine Girard, Gregor Gossler, and Sebt Mouelhi. 2016. Safety controller synthesis for incrementally stable switched systems using multiscale symbolic models. *IEEE Trans. Automat. Control* 61, 6 (2016), 1537–1549.
- [4] John Lygeros, Karl Henrik Johansson, Slobodan N. Simić, Jun Zhang, and S. Shankar Sastry. 2003. Dynamical Properties of Hybrid Automata. *IEEE Trans. Automat. Control* 48, 1 (2003), 2–17.
- [5] Paulo Tabuada. 2008. *Verification and Control of Hybrid Systems: a symbolic approach*. Springer.