

A theoretical comparison of different proton imaging set-ups

N. Krah¹

F. Khellaf¹, JM. Létang², S. Rit¹, I. Rinaldi^{3,4}

¹CNRS - Creatis, Lyon, France

²INSA Lyon - Creatis, Lyon, France

³Maastric Clinic, Maastricht, Netherlands

⁴CNRS - IPNL, Lyon, France

4th LLU Algorithm Workshop

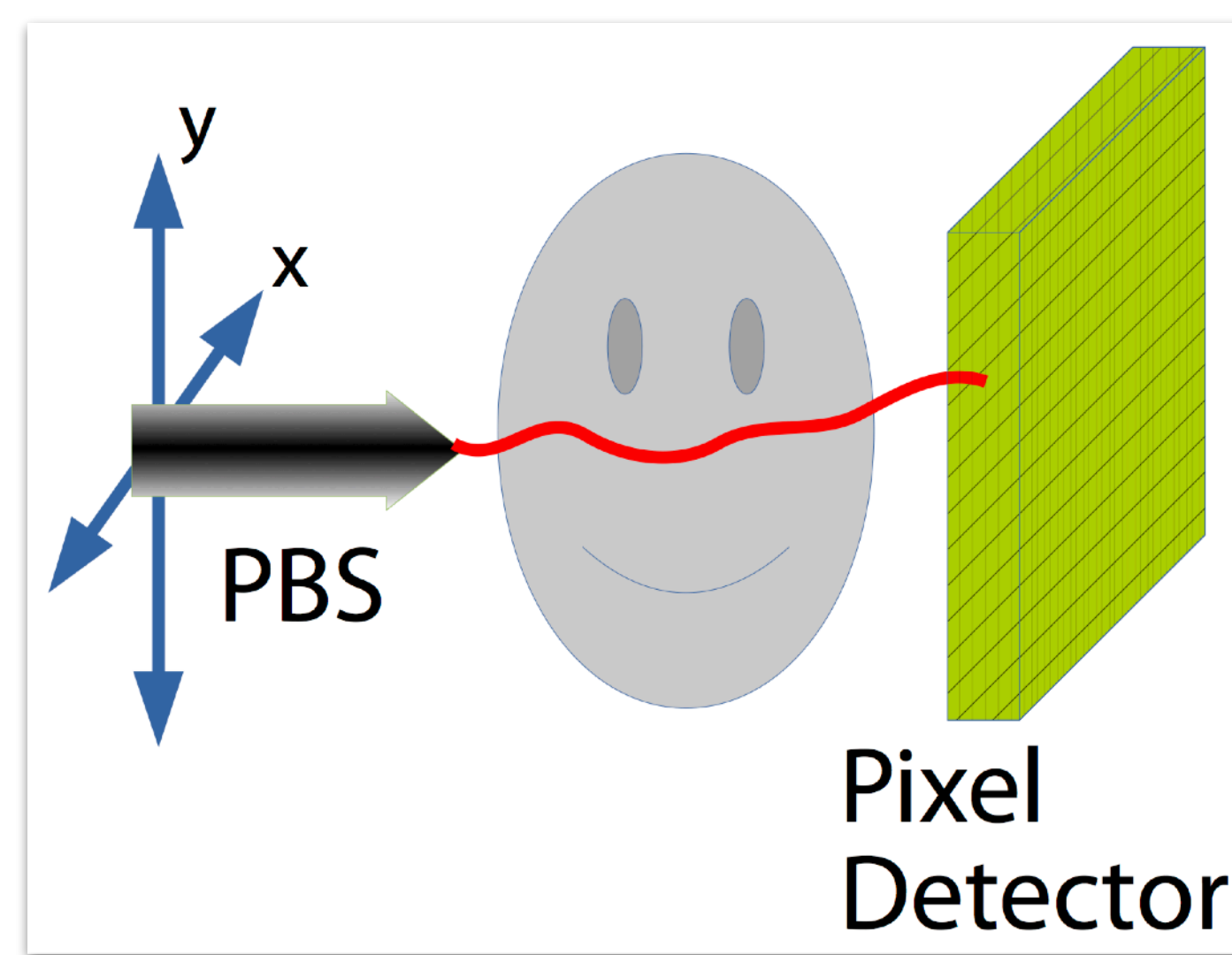
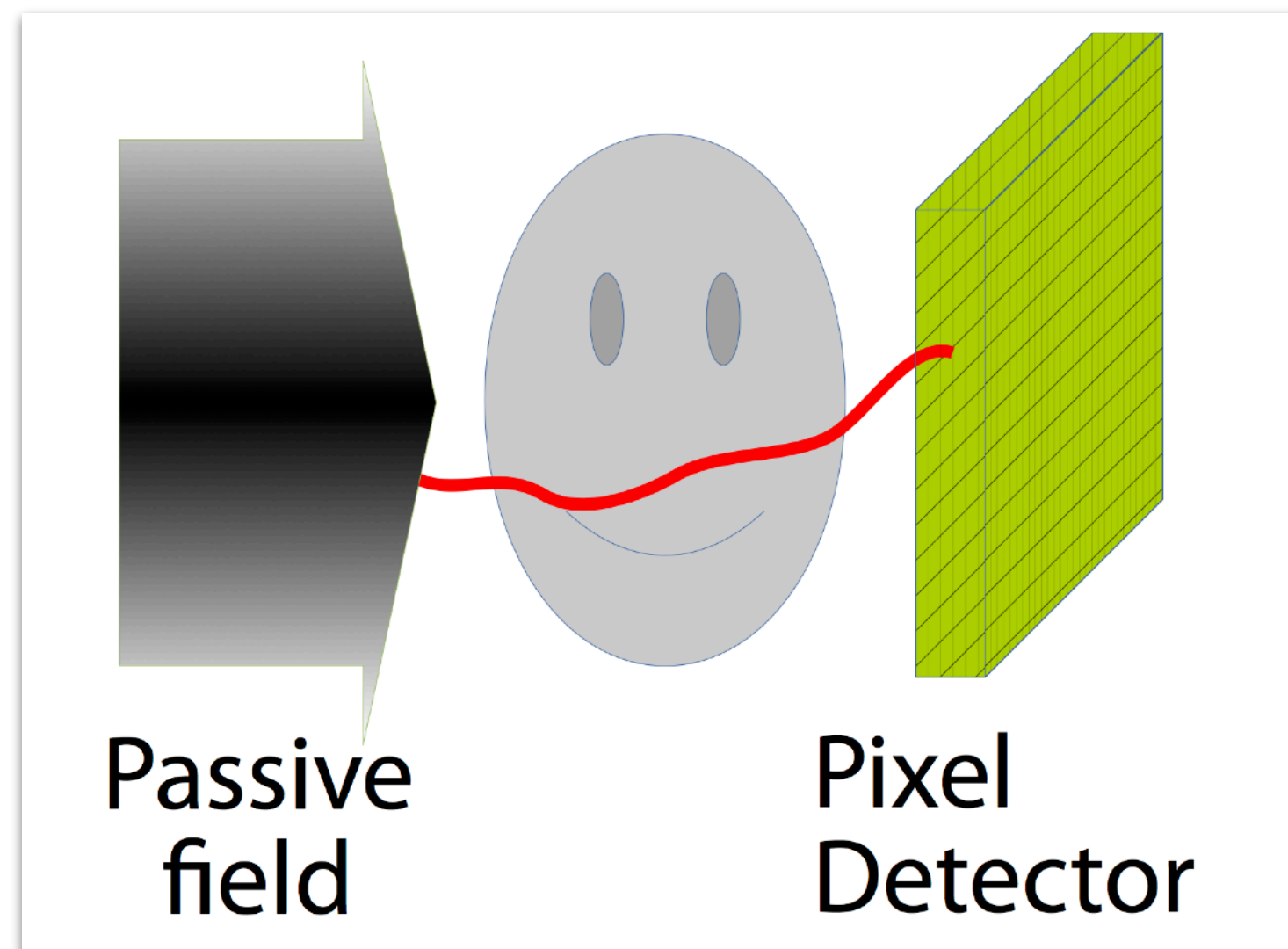
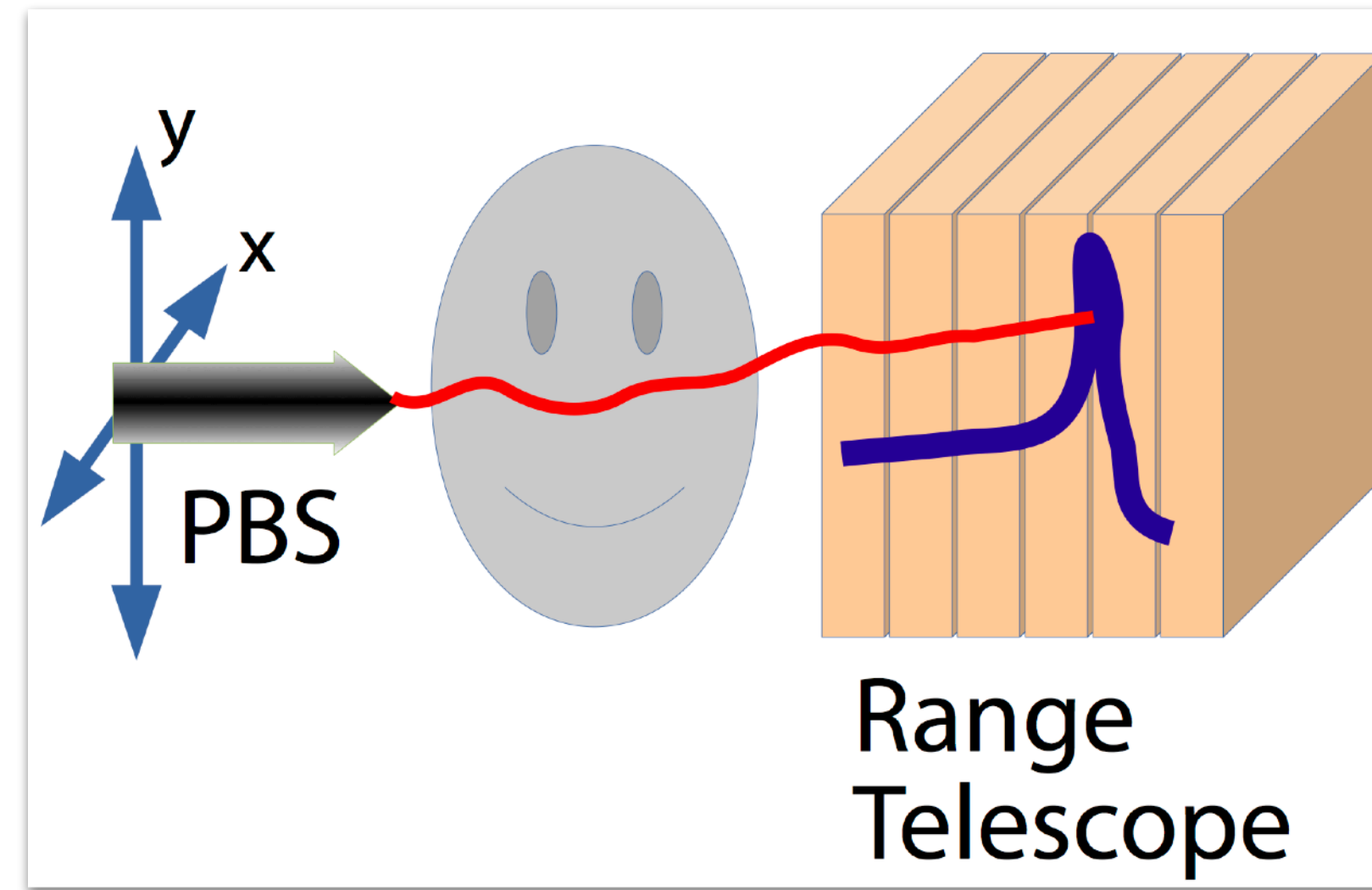
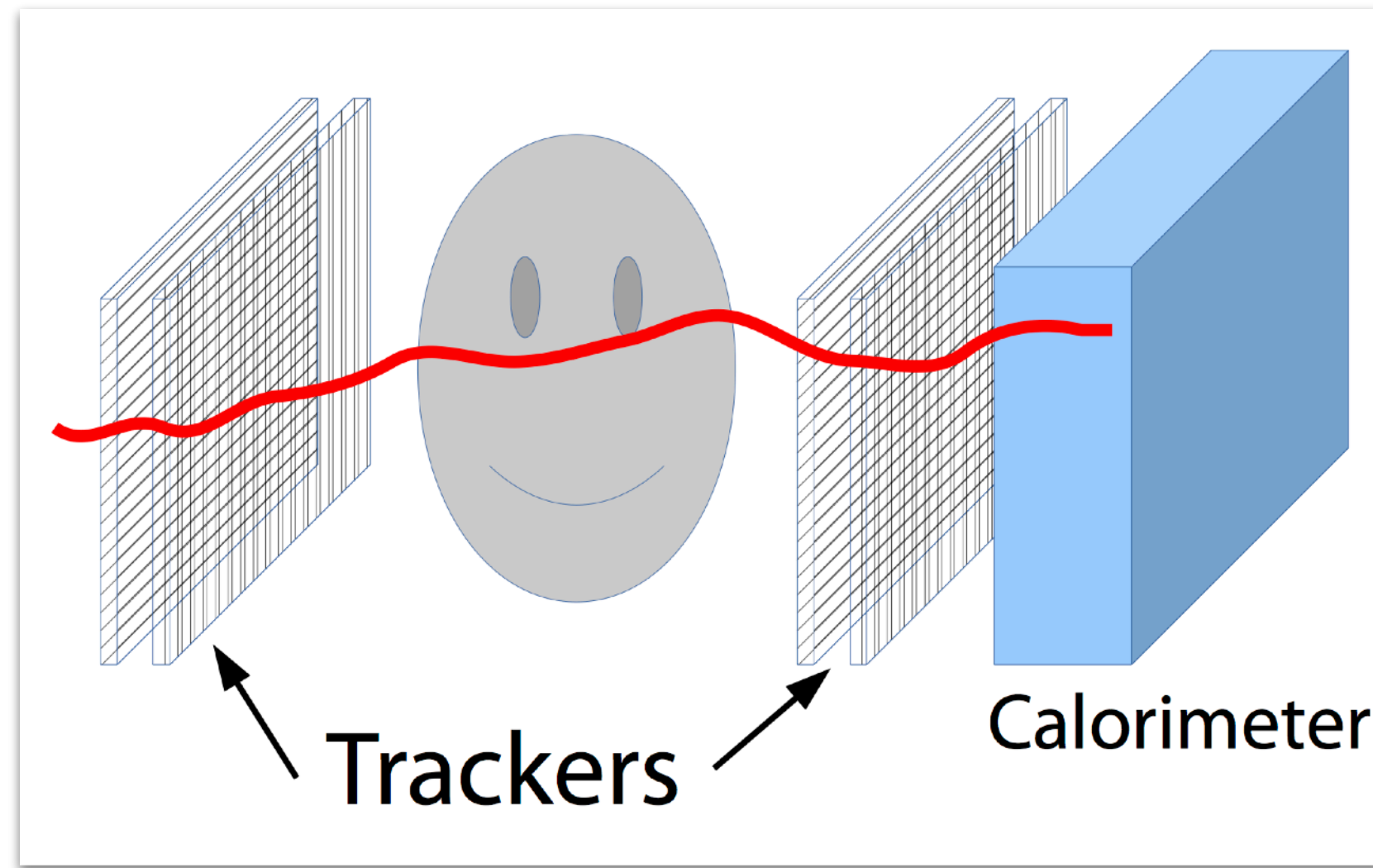


INSA

CREATIS



Selection of proton imaging set-ups



Also:

- **Use Helium or carbon instead of protons (-> Lenny)**
- **Pencil beam plus down stream tracker (-> Helge)**

How to choose the set-up?

Cost

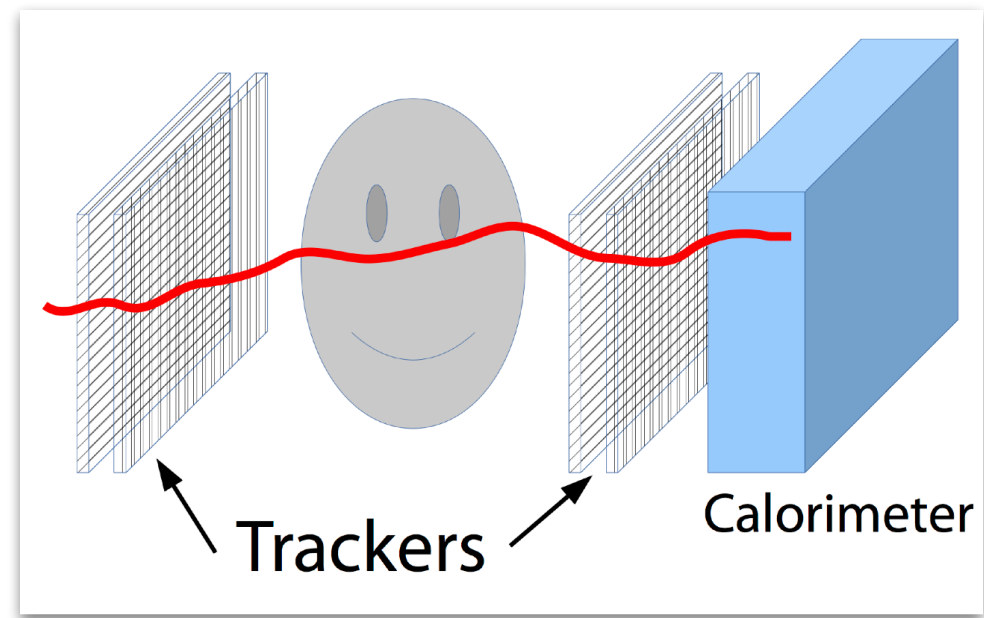
**Integration into
clinical reality**

RSP/WET accuracy

Dose to the patient

Spatial resolution

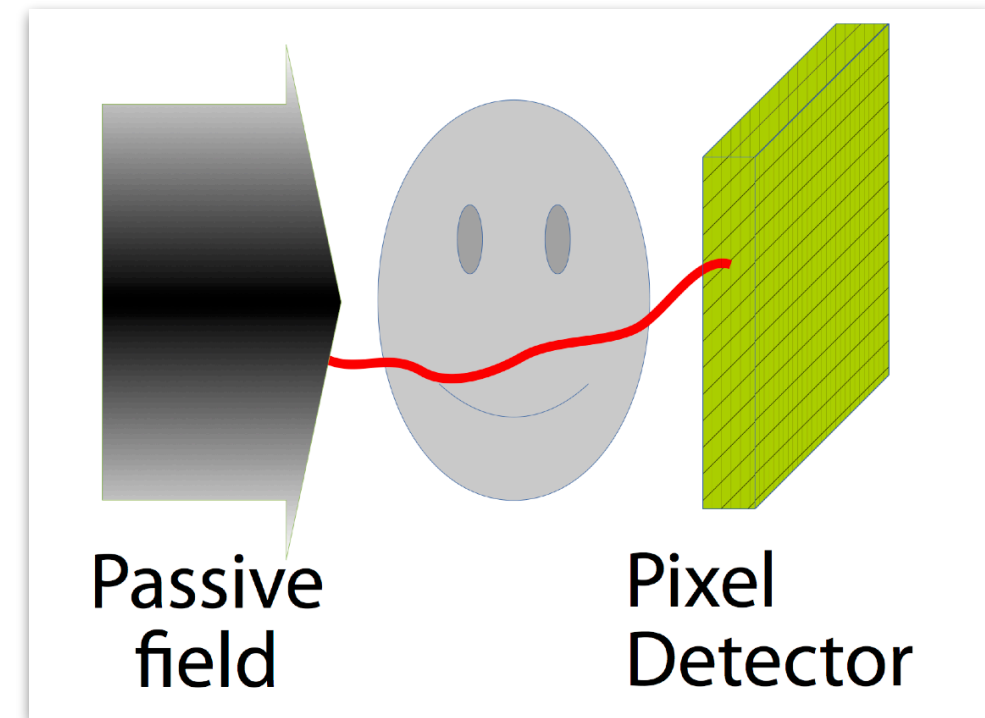
Cost



You need to construct:

- 8 tracker devices
- 1 calorimeter
- 1 fast electronics

Integration

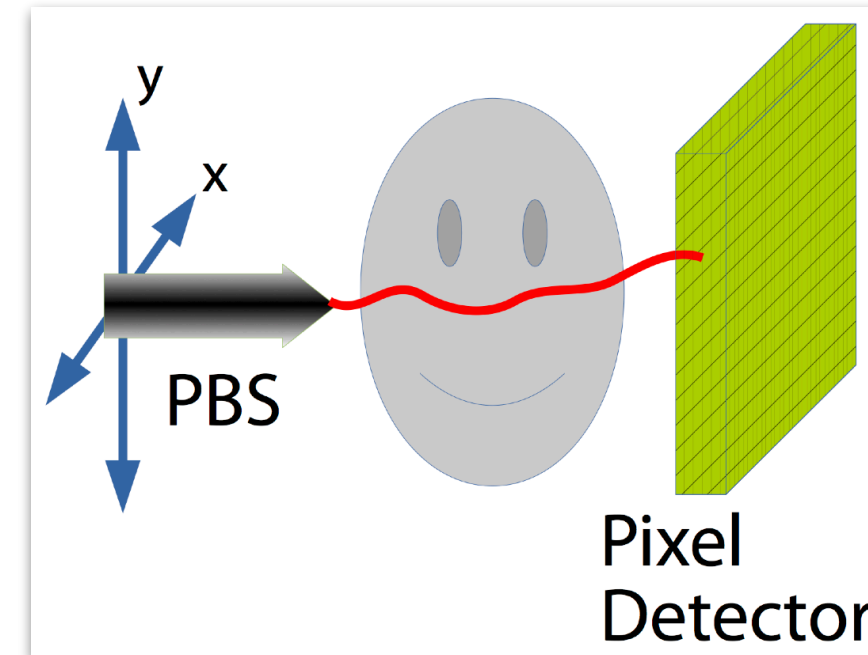


You need to construct:

- 1 Range modulator wheel

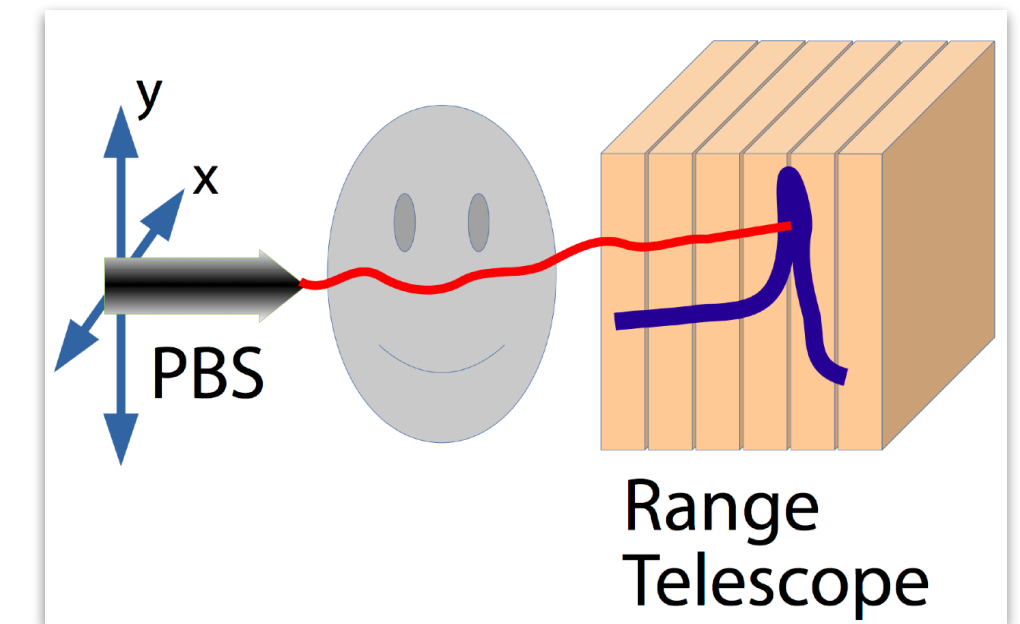
You can re-use:

- 1 X-ray flat panel detector



You can re-use:

- 1 X-ray flat panel detector



You can re-use:

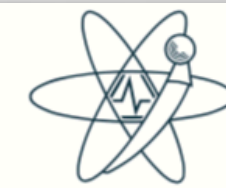
- 1 Range telescope

Simple integration into treatment room

Integration into workflow?

Spatial resolution

Physics in Medicine & Biology



IPEM Institute of Physics and
Engineering in Medicine

PAPER

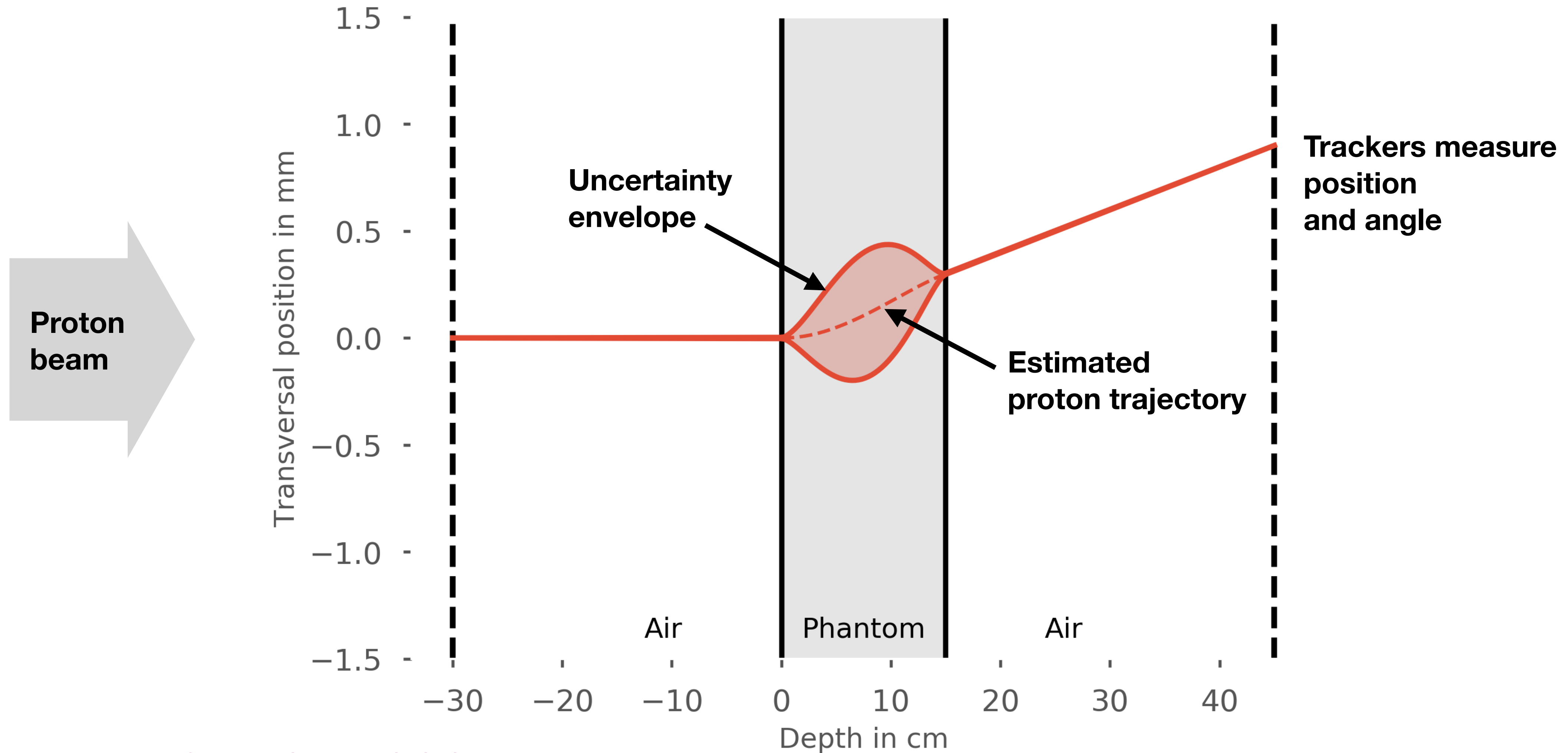
A comprehensive theoretical comparison of proton imaging set-ups in terms of spatial resolution

N Krah^{1,2}, F Khellaf¹, J M Létang¹, S Rit¹ and I Rinaldi² 

¹ Lyon University, INSA-Lyon, University Lyon1, UJM-Saint Etienne, CNRS, Inserm, CREATIS UMR5220, U1206, France

² CNRS/IN2P3 and Lyon 1 University, UMR 5822, Villeurbanne, France

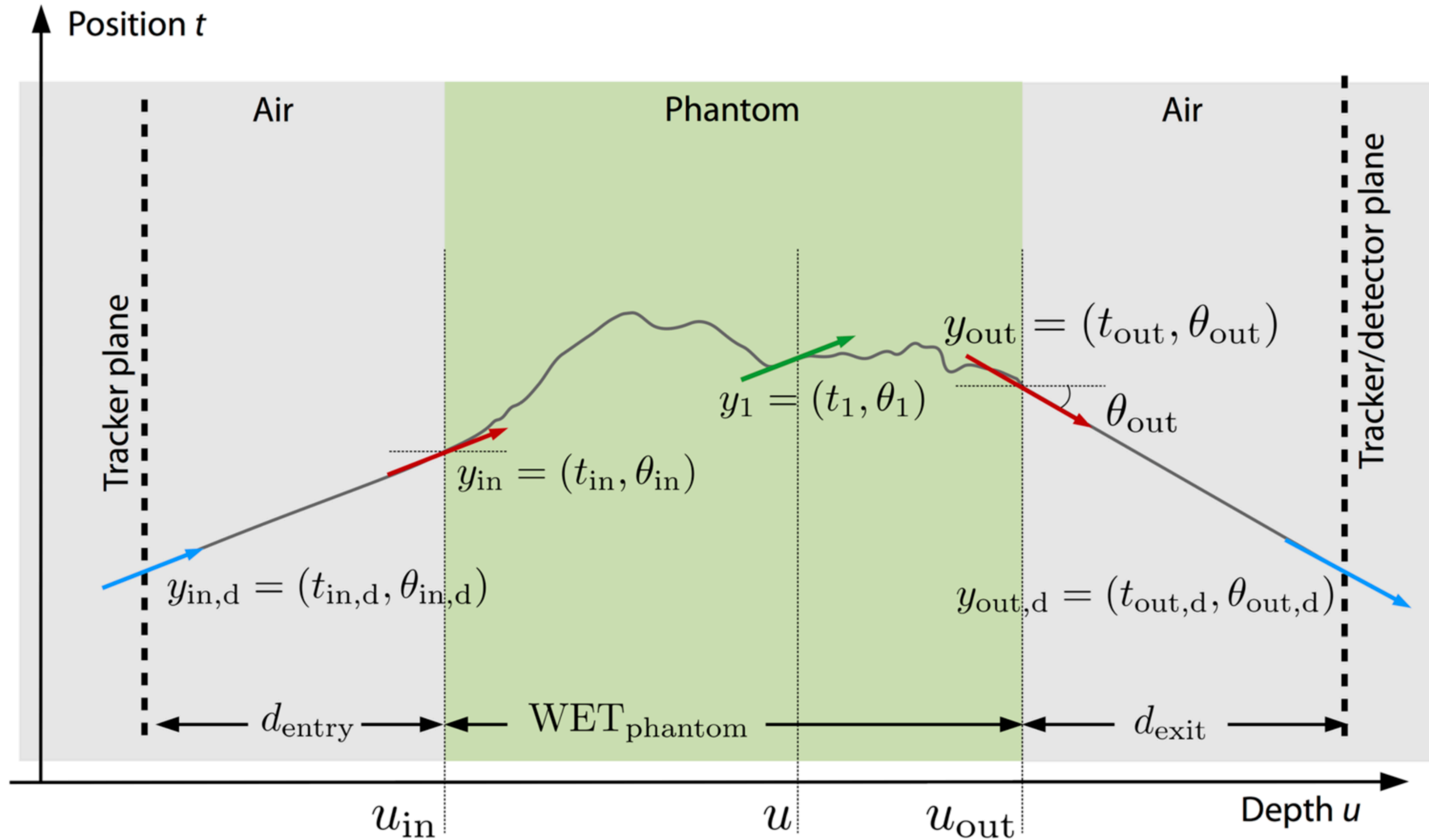
Single tracking set-ups



Williams 2004, PMB, DOI: 10.1088/0031-9155/49/13/010

Schulte 2008, Med. Phys., DOI: 10.1118/1.2986139

Collins-Fekete 2017, PMB, DOI: 10.1088/1361-6560/aa58ce



Likelihood: $L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \rightarrow y_1) \times L_{\text{scat}}(y_1 \rightarrow y_2 = y_{\text{out}} | y_{\text{in}})$

**from entrance to
some depth:**

$$L_{\text{scat}}(y_{\text{in}} \rightarrow y_1) \propto \exp \left[-\frac{1}{2} (y_1^T - y_{\text{in}}^T R_0^T) \Sigma_1^{-1} (y_1 - R_0 y_{\text{in}}) \right]$$

**from some depth
to exit:**

$$L_{\text{scat}}(y_1 \rightarrow y_2 = y_{\text{out}}) \propto \exp \left[-\frac{1}{2} (y_{\text{out}}^T - y_1^T R_1^T) \Sigma_2^{-1} (y_{\text{out}} - R_1 y_1) \right],$$

with: $\Sigma_1 = \begin{pmatrix} \sigma_{t_1}^2 & \sigma_{t_1 \theta_1}^2 \\ \sigma_{t_1 \theta_1}^2 & \sigma_{\theta_1}^2 \end{pmatrix}, \Sigma_2 = \begin{pmatrix} \sigma_{t_2}^2 & \sigma_{t_2 \theta_2}^2 \\ \sigma_{t_2 \theta_2}^2 & \sigma_{\theta_2}^2 \end{pmatrix}, R_0 = \begin{pmatrix} 1 & u - u_{\text{in}} \\ 0 & 1 \end{pmatrix}, R_1 = \begin{pmatrix} 1 & u_{\text{out}} - u \\ 0 & 1 \end{pmatrix}$

$$\sigma_{t_1}^2 = E_0^2 \left(1 + 0.038 \ln \frac{u - u_{\text{in}}}{X_0} \right)^2 \times \int_{u_{\text{in}}}^u \frac{(u - u_{\text{in}})^2}{\beta^2 p^2} \frac{du}{X_0}$$

Likelihood: $L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \rightarrow y_1) \times L_{\text{scat}}(y_1 \rightarrow y_2 = y_{\text{out}} | y_{\text{in}})$

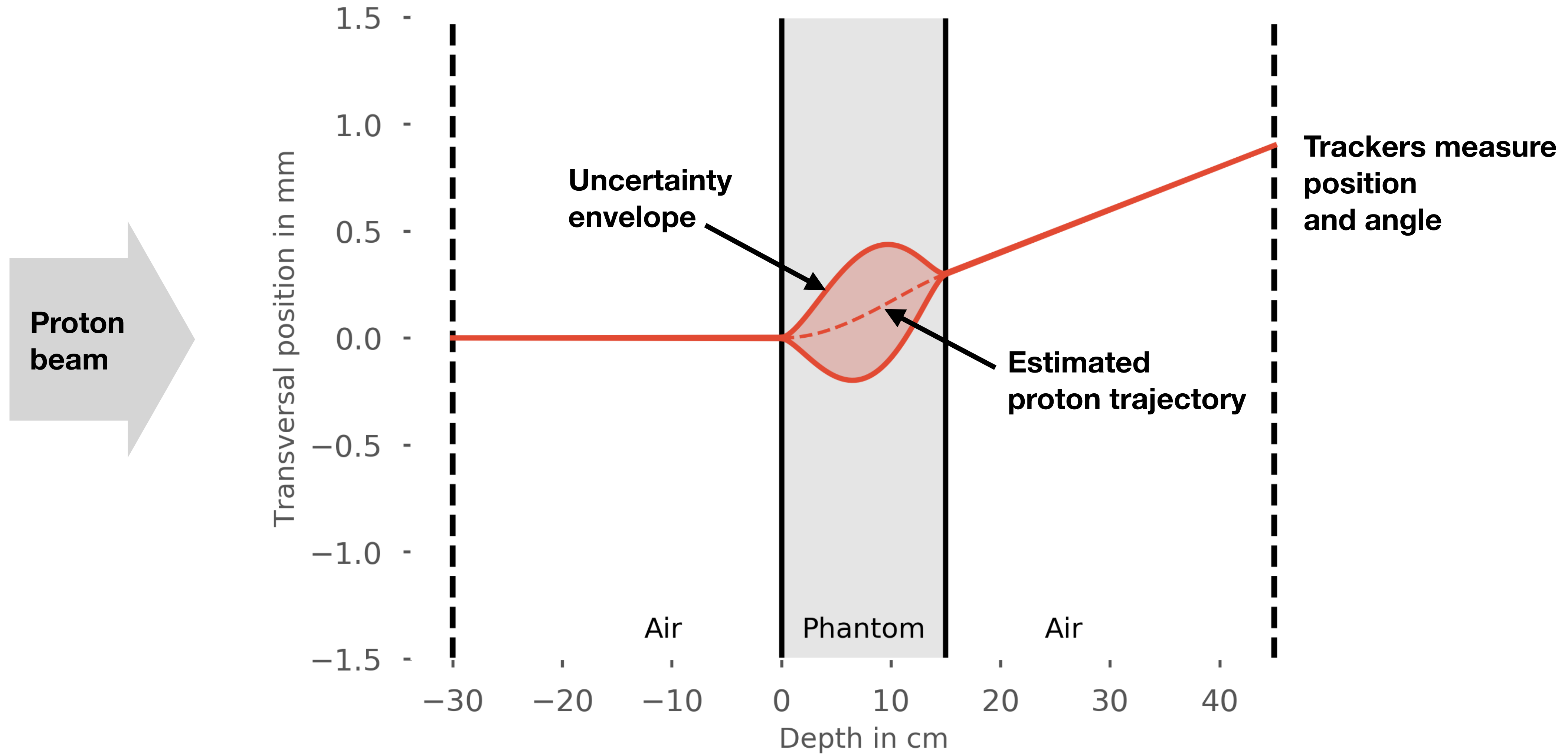
Most likely path (MLP):

$$\begin{aligned} y_{\text{MLP}}(u) &= (\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1)^{-1} \cdot (\Sigma_1^{-1} R_0 y_{\text{in}} + R_1^T \Sigma_2^{-1} y_{\text{out}}) \\ &= R_1^{-1} \Sigma_2 (R_1^{-1} \Sigma_2 + \Sigma_1 R_1^T)^{-1} \cdot R_0 y_{\text{in}} + \Sigma_1 (R_1 \Sigma_1 + \Sigma_2 (R_1^{-1})^T)^{-1} \cdot y_{\text{out}} \end{aligned}$$

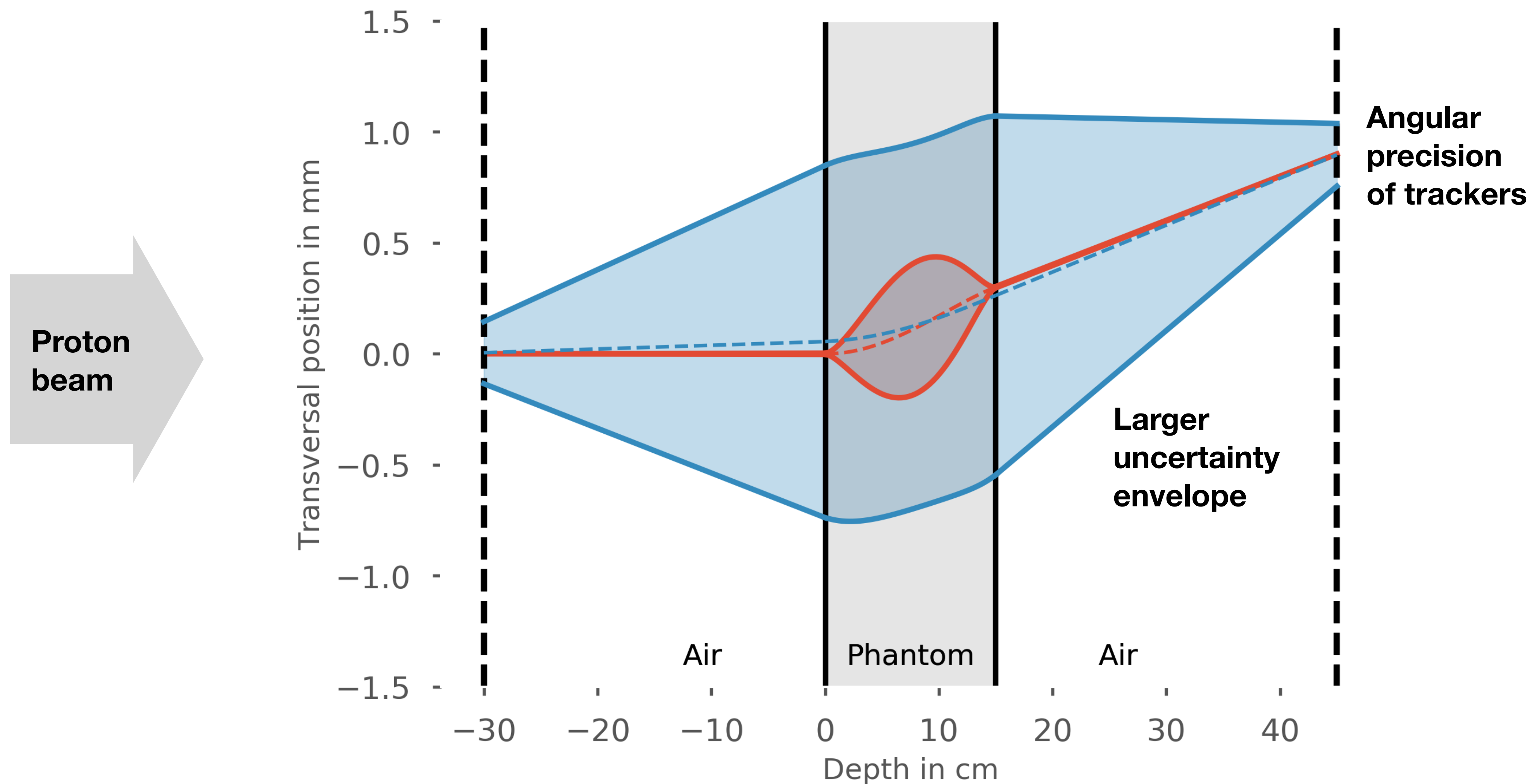
Uncertainty envelope around MLP:

$$\Sigma_{\text{MLP}}(u) = (\Sigma_1^{-1} + R_1^T \Sigma_2^{-1} R_1)^{-1} = \Sigma_1 (\Sigma_2 (R_1^{-1})^T + R_1 \Sigma_1)^{-1} \Sigma_2 (R_1^{-1})^T$$

Single tracking set-ups



Tracker uncertainties



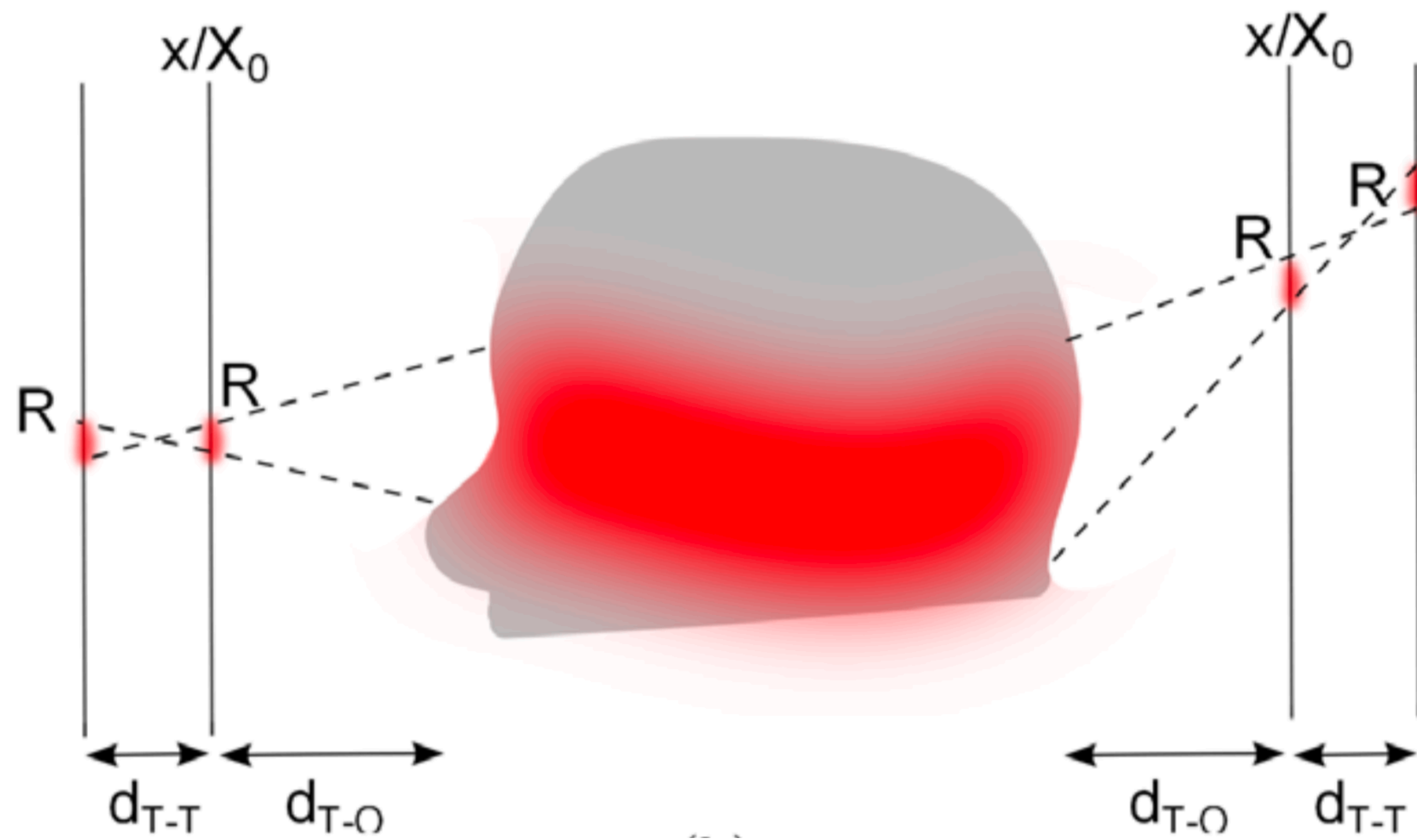


Figure from Bopp et al. 2014, PMB, DOI: 10.1088/0031-9155/59/23/N197

Likelihood of measurement process:

$$\Rightarrow L_{\text{meas}}(\tilde{y}_{\text{in}}, y_{\text{in}}) \propto \exp \left[-\frac{1}{2} (\tilde{y}_{\text{in}} - y_{\text{in}})^T (\mathcal{S}_{\text{in}} \Sigma_{\text{in}} \mathcal{S}_{\text{in}}^T)^{-1} (\tilde{y}_{\text{in}} - y_{\text{in}}) \right]$$

$$\Rightarrow L_{\text{meas}}(\tilde{y}_{\text{out}}, y_{\text{out}}) \propto \exp \left[-\frac{1}{2} (\tilde{y}_{\text{out}} - y_{\text{out}})^T \mathcal{S}_{\text{out}}^T \Sigma_{\text{out}}^{-1} \mathcal{S}_{\text{out}} (\tilde{y}_{\text{out}} - y_{\text{out}}) \right]$$

$$\Sigma_{\text{in}} = \sigma_p^2 T_{\text{in}} \cdot T_{\text{in}}^T + \Sigma_{\text{sc}} \quad \text{and} \quad \Sigma_{\text{out}} = \sigma_p^2 T_{\text{out}} \cdot T_{\text{out}}^T + \Sigma_{\text{sc}}$$

$$\Sigma_{\text{sc}} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{\text{sc}}^2 \end{pmatrix} \quad \text{with} \quad \sigma_{\text{sc}} = \frac{13.6 \text{ MeV}}{\beta(E) p(E)} \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln \left(\frac{x}{X_0} \right) \right]$$

$$T_{\text{out}} = \begin{pmatrix} 1 & 0 \\ -1/d_T & 1/d_T \end{pmatrix} \quad T_{\text{in}} = \begin{pmatrix} 0 & 1 \\ -1/d_T & 1/d_T \end{pmatrix}$$

Ideal trackers

$$L(y_1, y_2 = y_{\text{out}} | y_{\text{in}}) = L_{\text{scat}}(y_{\text{in}} \rightarrow y_1) \times L_{\text{scat}}(y_1 \rightarrow y_2 = y_{\text{out}} | y_{\text{in}})$$

Trackers with uncertainties

$$L(y_1, y_2 = \tilde{y}_{\text{out}} | \tilde{y}_{\text{in}}) = \int L_{\text{meas}}(\tilde{y}_{\text{in}}, y_{\text{in}}) L_{\text{scat}}(y_{\text{in}} \rightarrow y_1) \mathrm{d}y_{\text{in}} \times \int L_{\text{scat}}(y_1 \rightarrow y_2 = y_{\text{out}}) L_{\text{meas}}(\tilde{y}_{\text{out}}, y_{\text{out}}) \mathrm{d}y_{\text{out}}$$

MLP considering tracker uncertainties

$$y_{\text{MLP}}(\mathbf{u}) = C_2 (C_1 + C_2)^{-1} R_0 S_{\text{in}} \cdot \tilde{y}_{\text{in,d}} \\ + C_1 (C_1 + C_2)^{-1} R_1^{-1} S_{\text{out}}^{-1} \cdot \tilde{y}_{\text{out,d}}$$

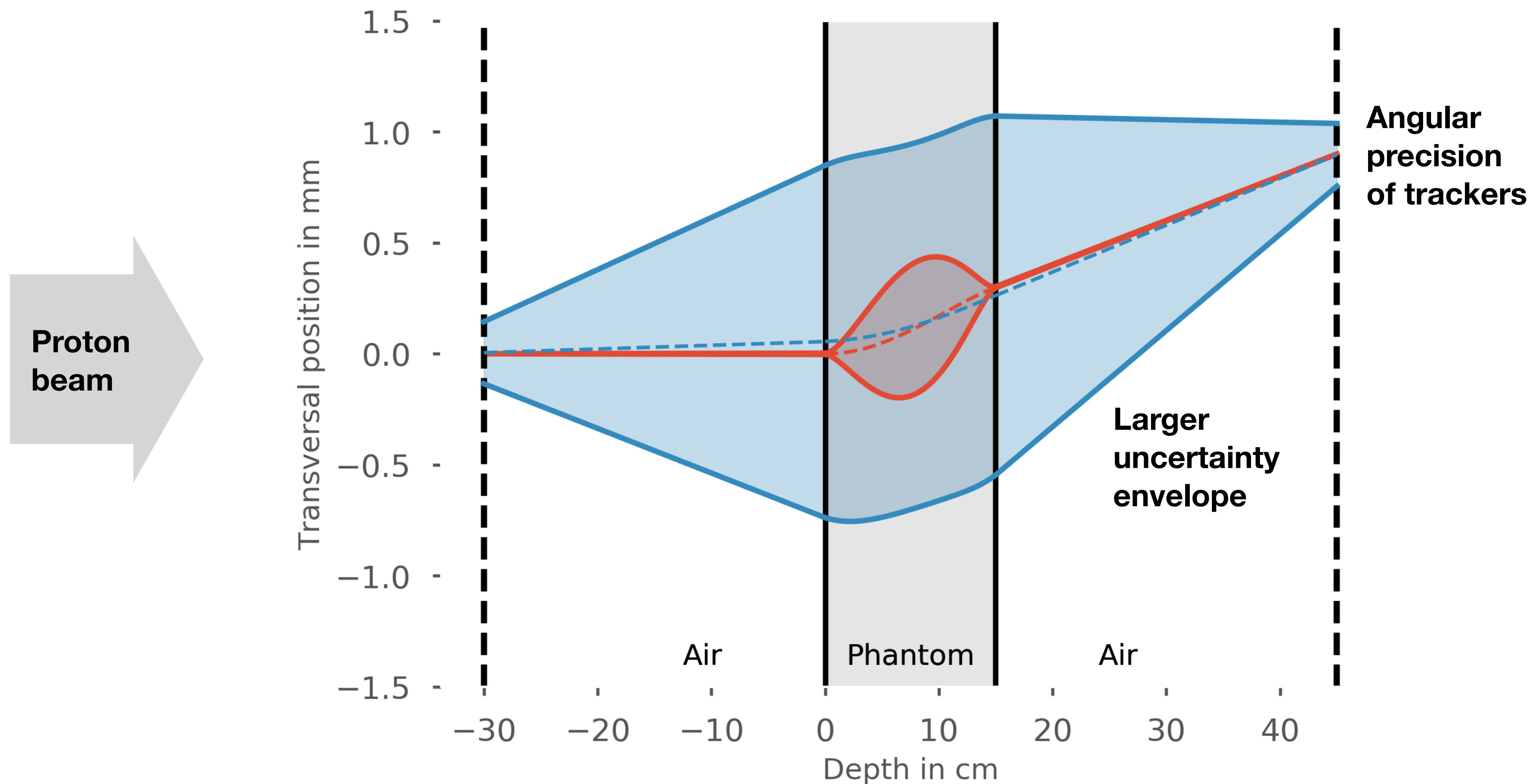
$$\Sigma_{\text{MLP}}(\mathbf{u}) = C_1 (C_1 + C_2)^{-1} C_2$$

$$\sigma_{\text{MLP}}(\mathbf{u}) = (\Sigma_{\text{MLP}}(\mathbf{u}))_{1,1}$$

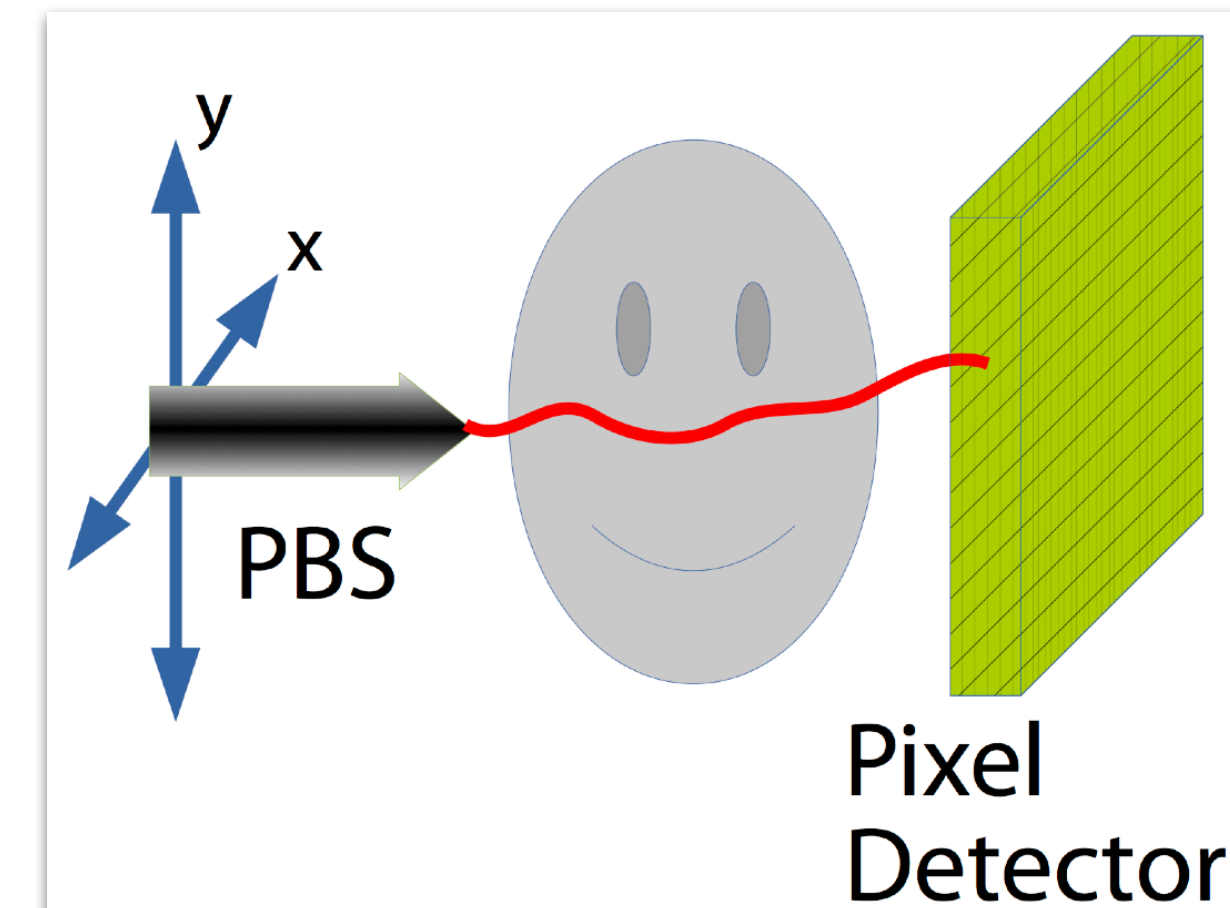
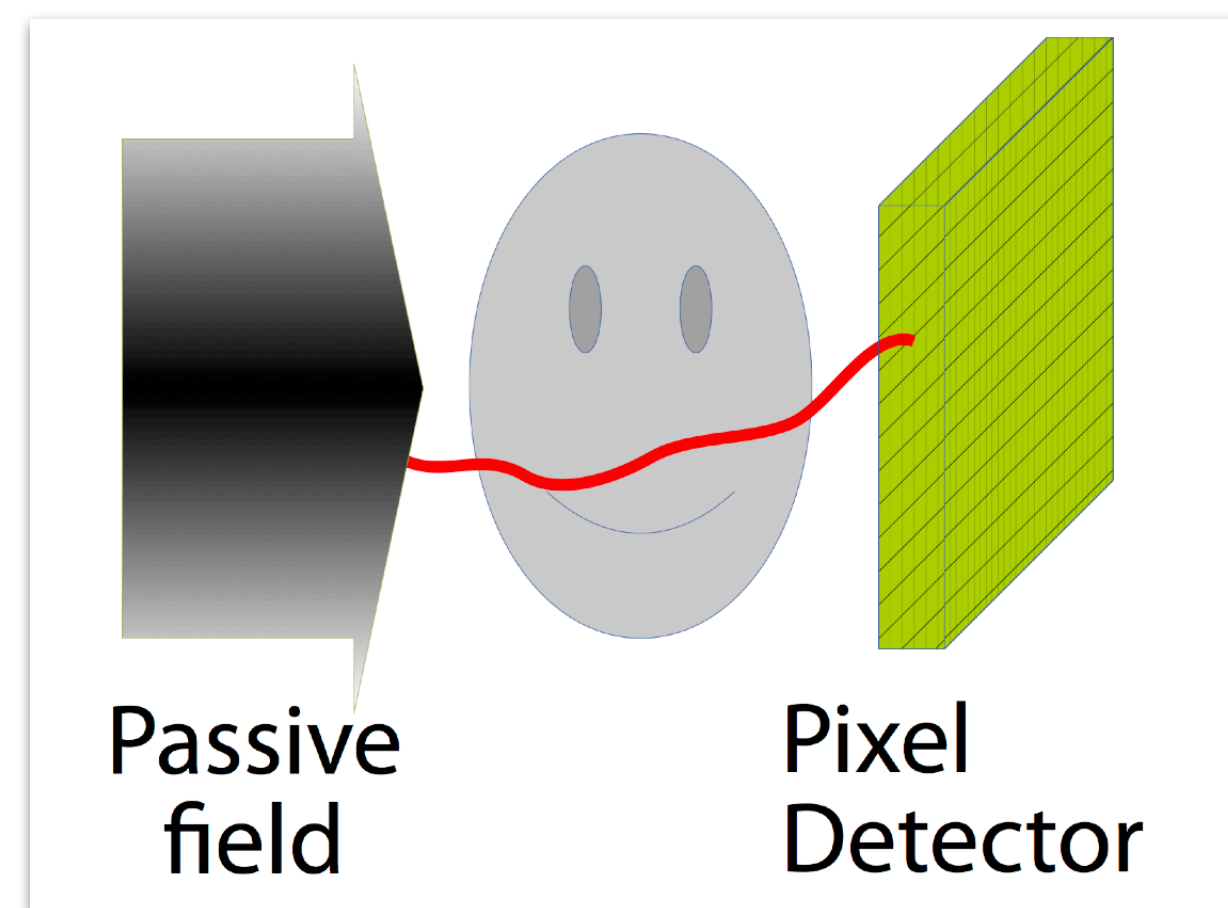
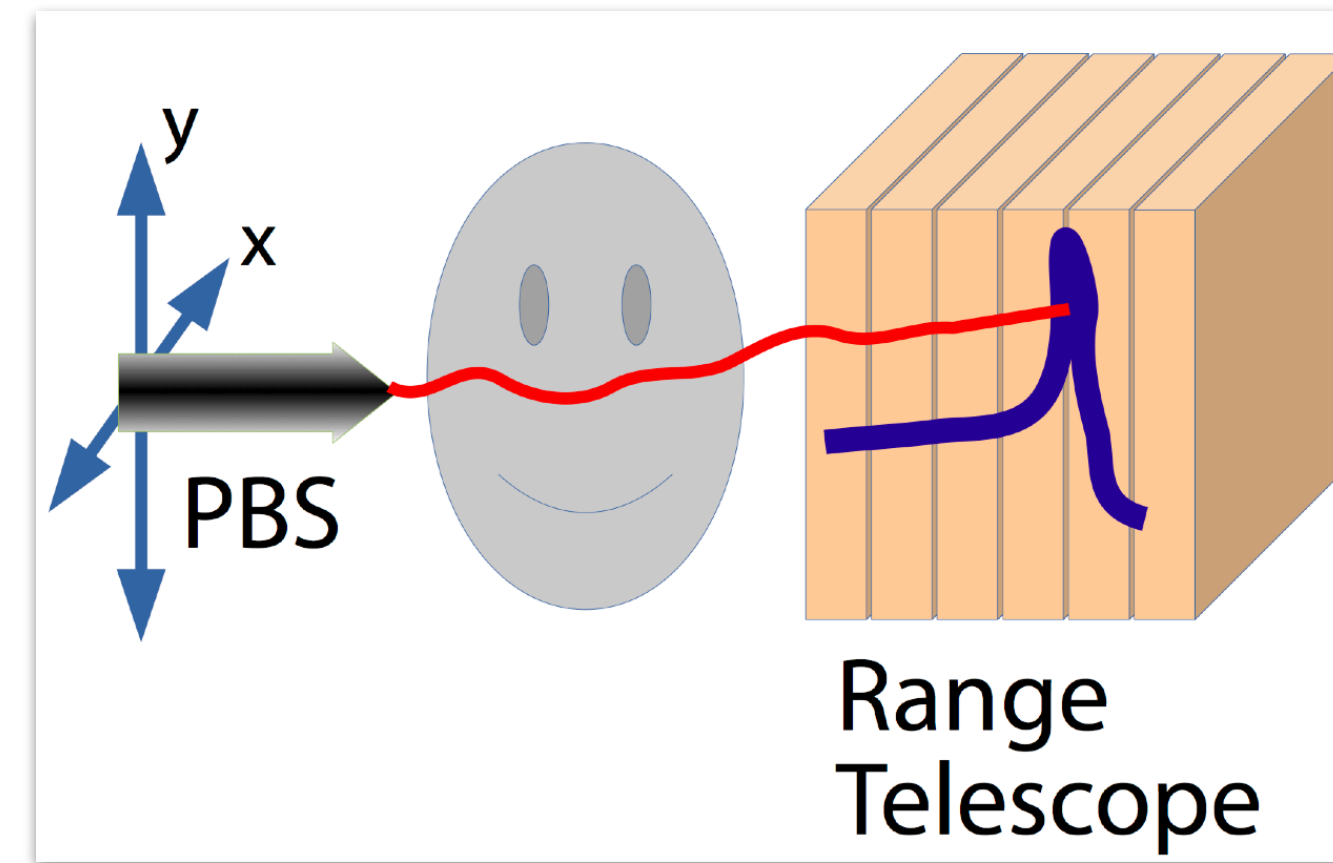
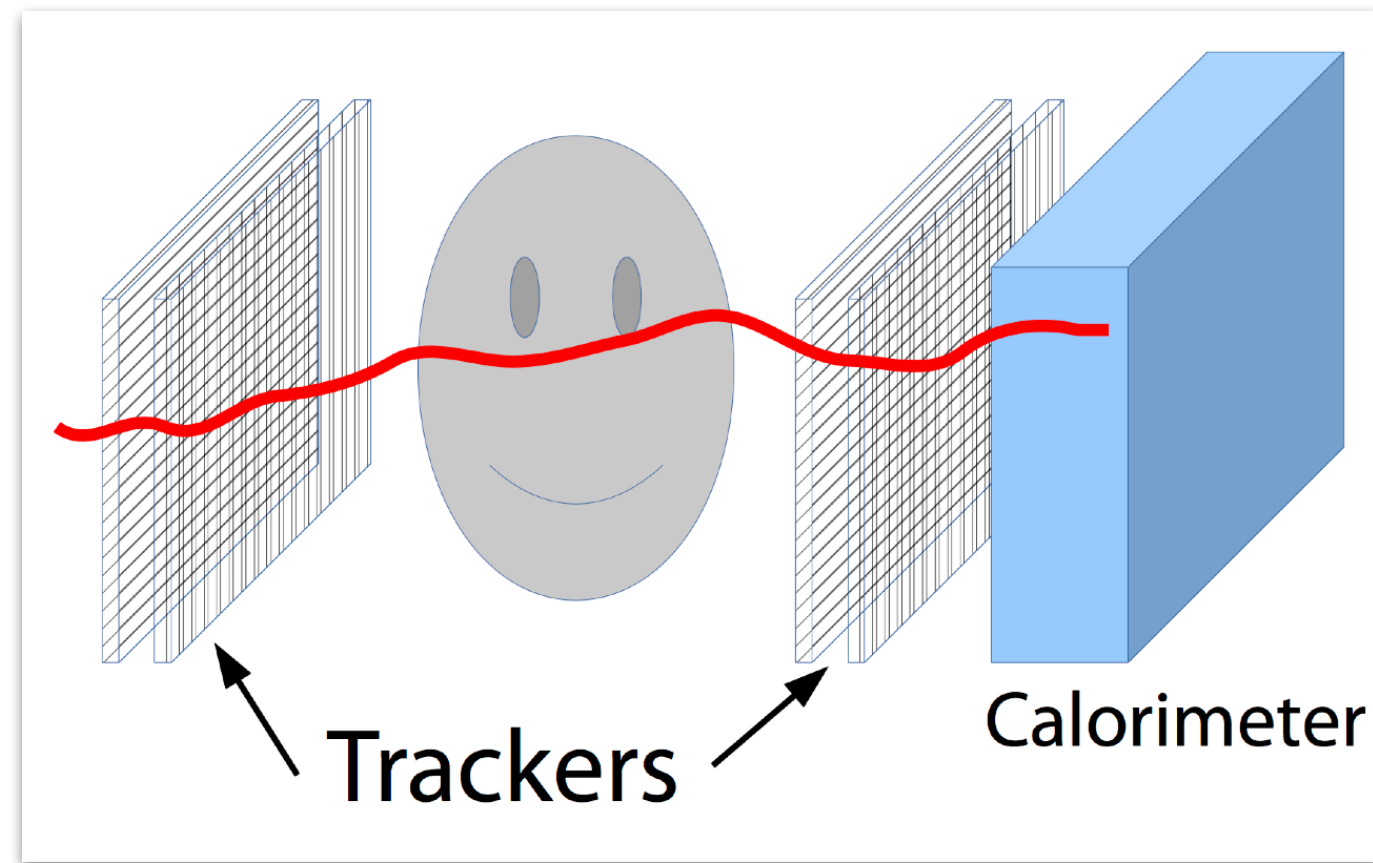
with $C_1 = R_0 S_{\text{in}} \Sigma_{\text{in}} S_{\text{in}}^T R_0^T + \Sigma_1$

$$C_2 = R_1^{-1} S_{\text{out}}^{-1} \Sigma_{\text{out}} (S_{\text{out}}^{-1})^T (R_1^{-1})^T + R_1^{-1} \Sigma_2 (R_1^{-1})^T$$

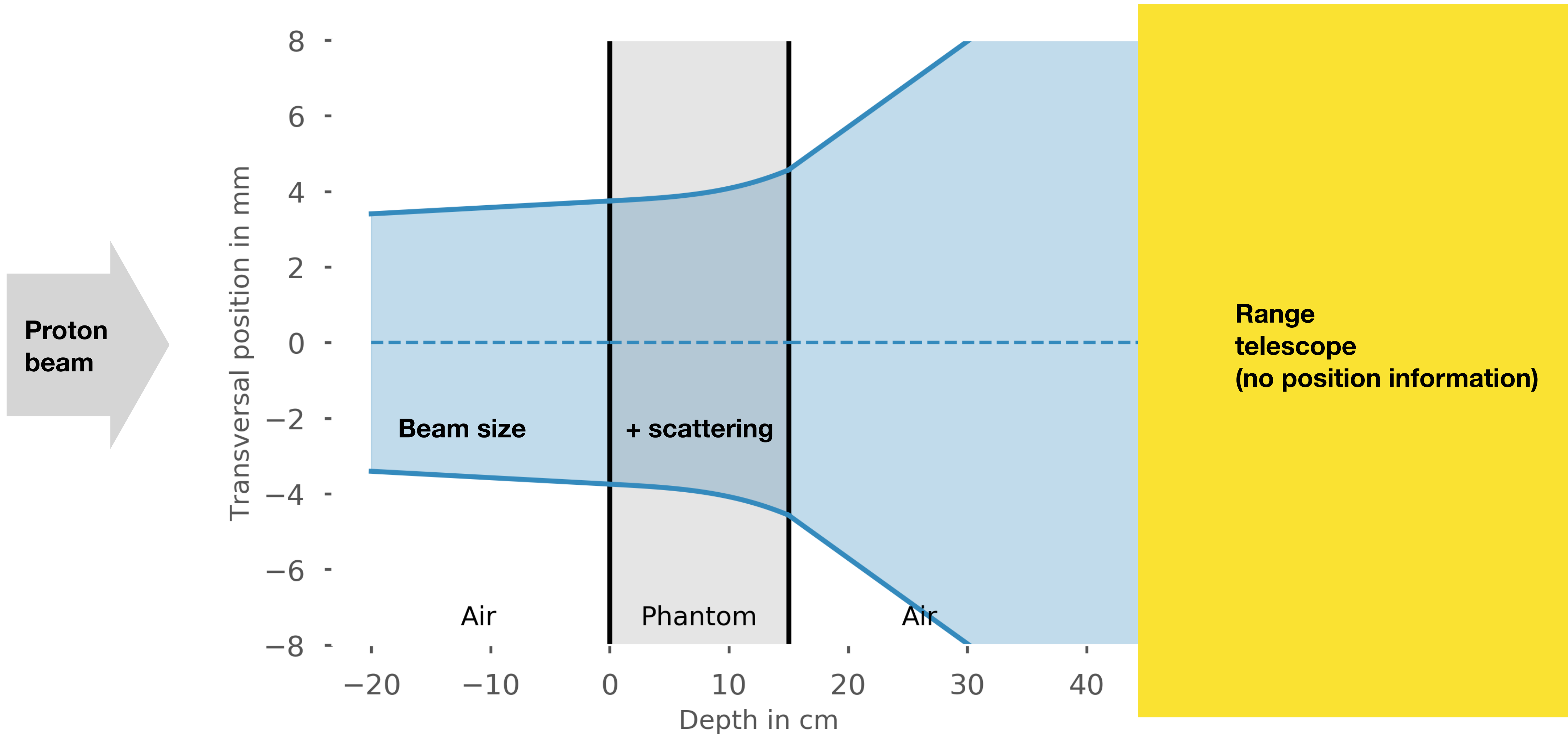
Tracker uncertainties



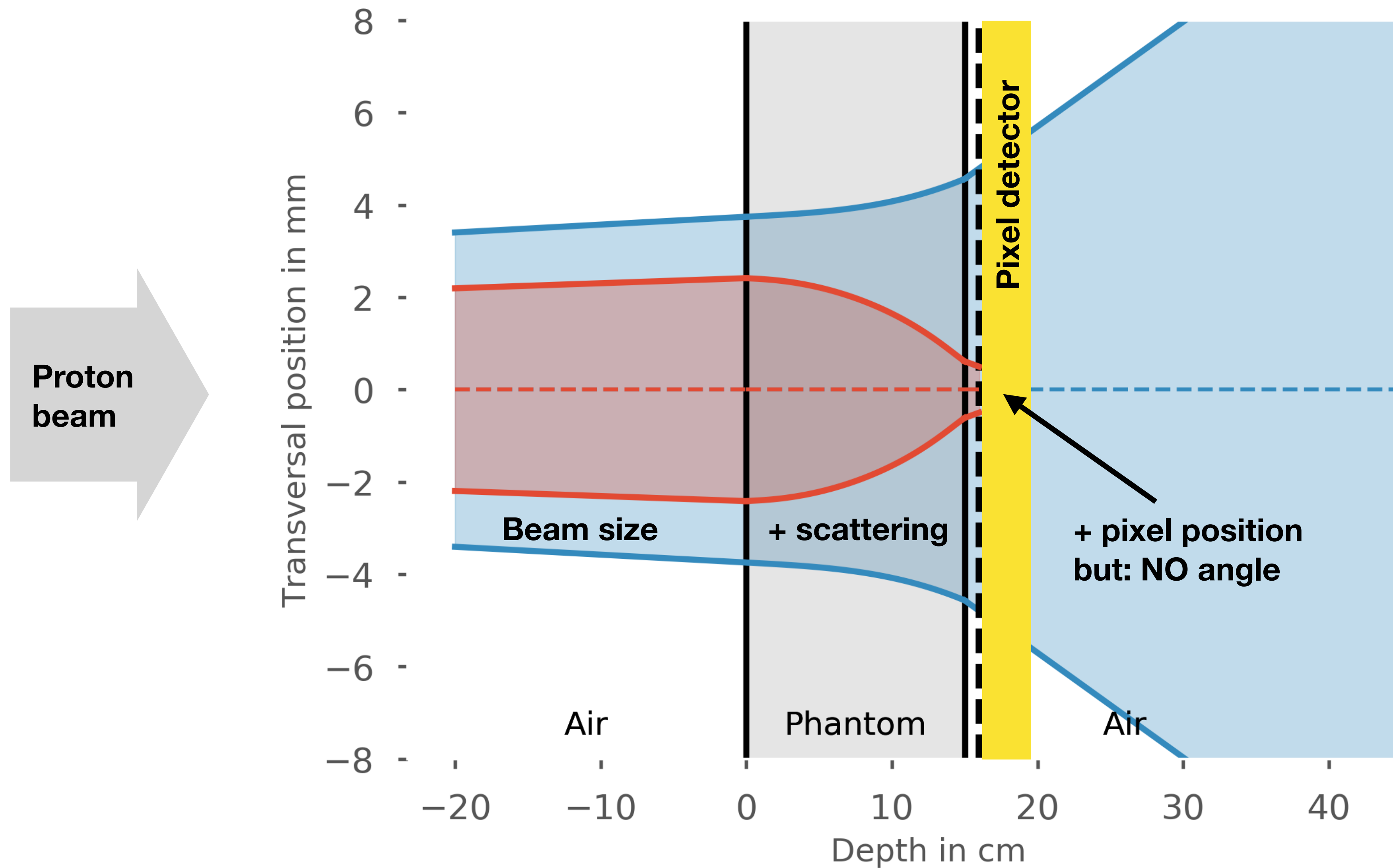
Apply formalism to integral mode set-ups



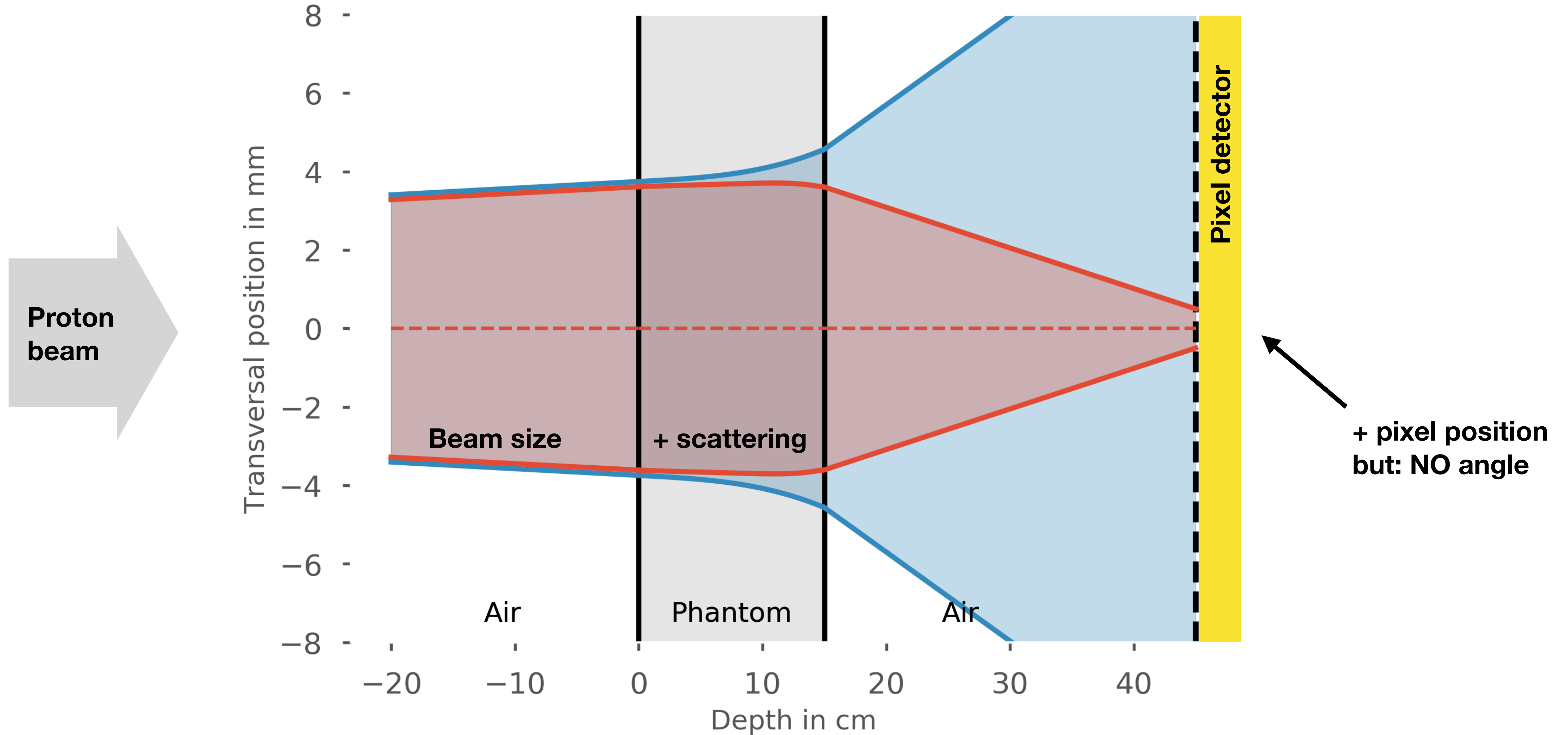
PBS-based set-ups



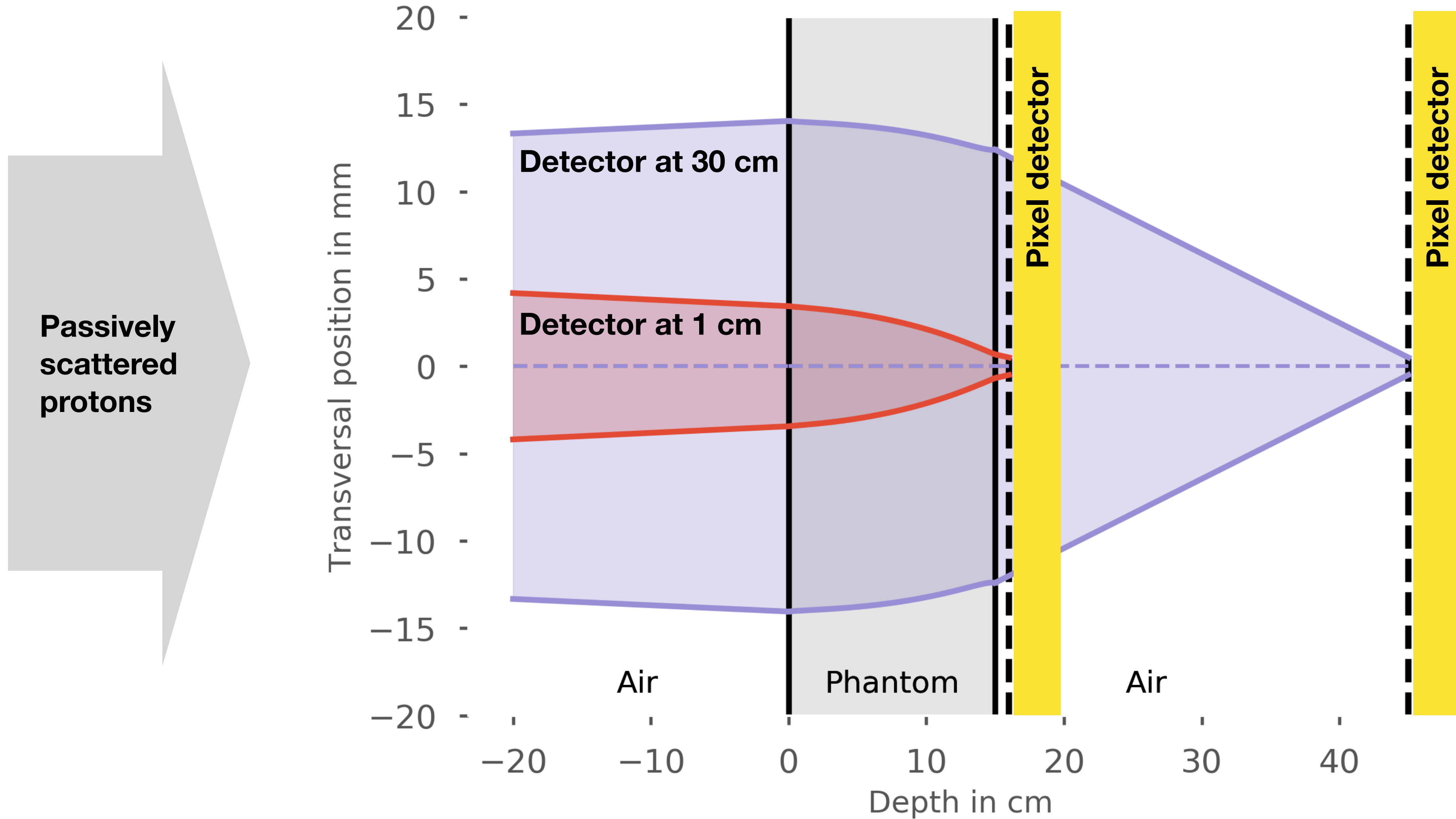
PBS-based set-ups



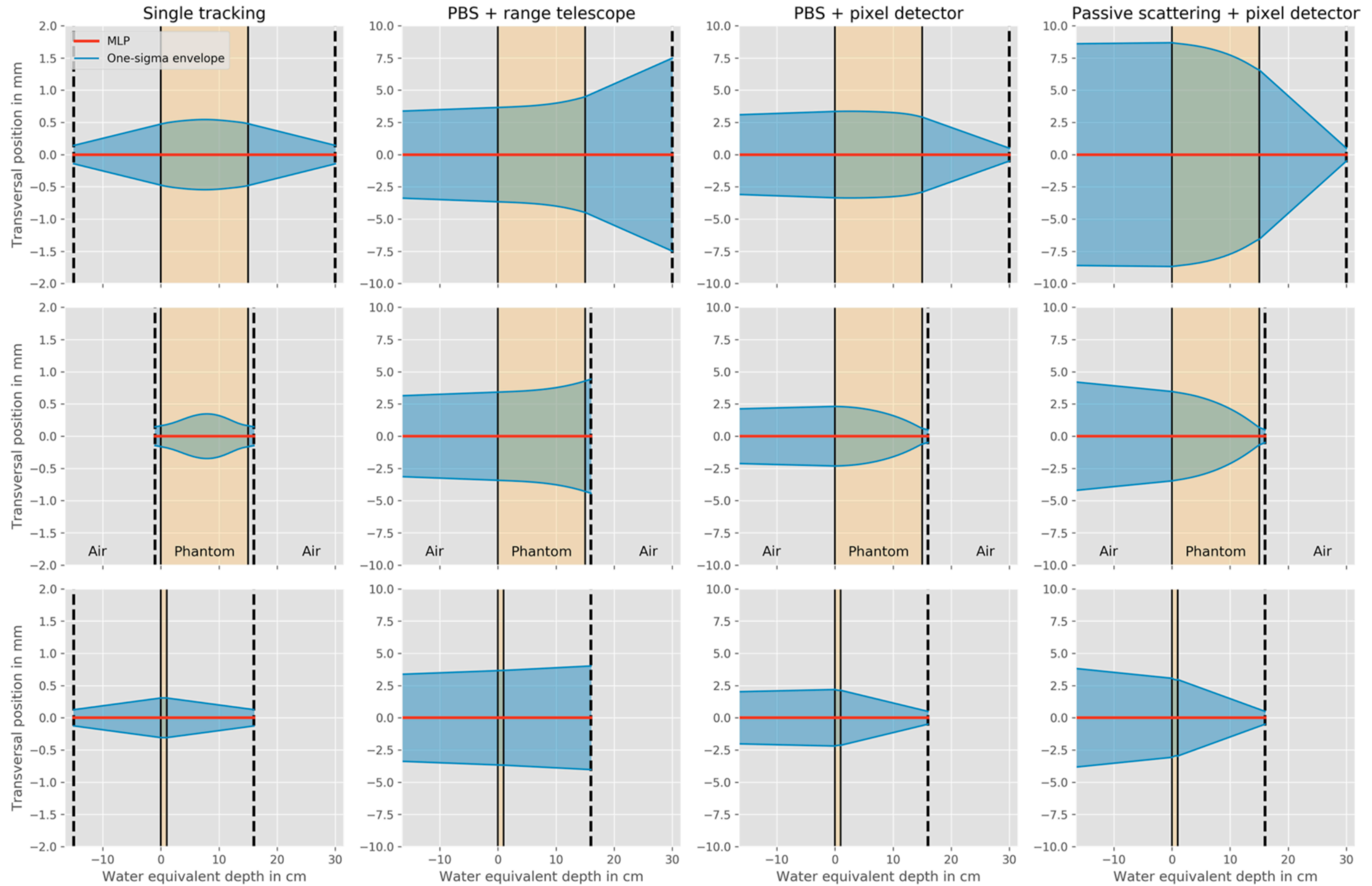
PBS-based set-ups



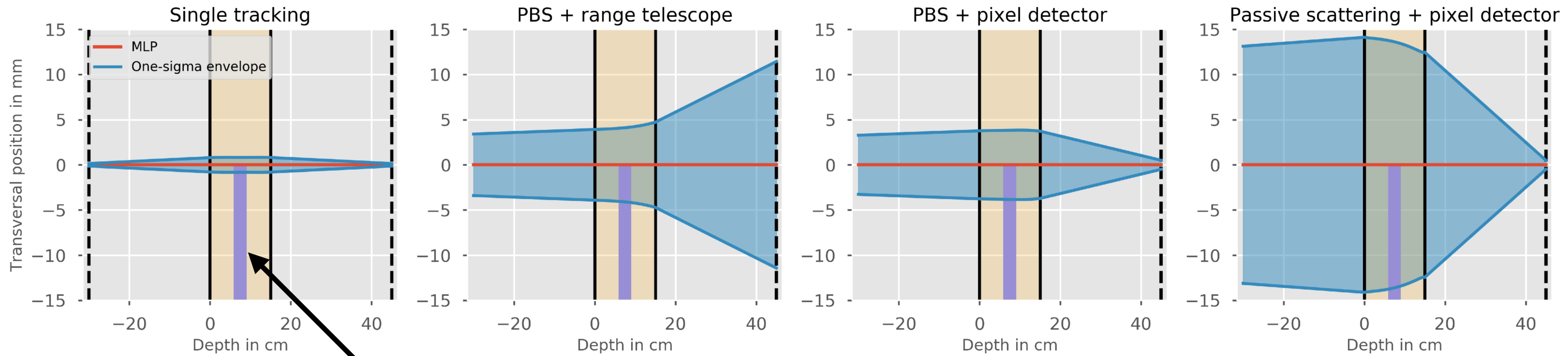
Passive scattering set-ups



Set-up parameter	$\sigma_{t_{\text{in}}}$	$\sigma_{\theta_{\text{in}}}$	$\sigma_{t_{\text{out}}}$	$\sigma_{\theta_{\text{out}}}$
Single tracking idealised	0	0	0	0
PBS + range telescope idealised	0	0	$\rightarrow \infty$	$\rightarrow \infty$
PBS + pixel detector idealised	0	0	0	$\rightarrow \infty$
Passive scattering + pixel detector idealised	$\rightarrow \infty$	0	0	$\rightarrow \infty$
Single tracking (see section 28)	0.15 mm	3 mrad	0.15 mm	3 mrad
Single tracking without angle measurement	0.5 mm	15 mrad	0.5 mm	$\rightarrow \infty$ (45°)
PBS + range telescope	8/2.35 mm	0.1 mrad	20 cm	$\rightarrow \infty$ (45°)
passive scattering + pixel detector	20 cm	15 mrad	0.5 mm	$\rightarrow \infty$ (45°)
PBS + pixel detector	8/2.35 mm	0.1 mrad	0.5 mm	$\rightarrow \infty$ (45°)



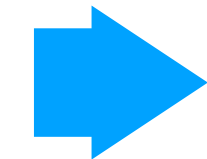
Measure of spatial resolution



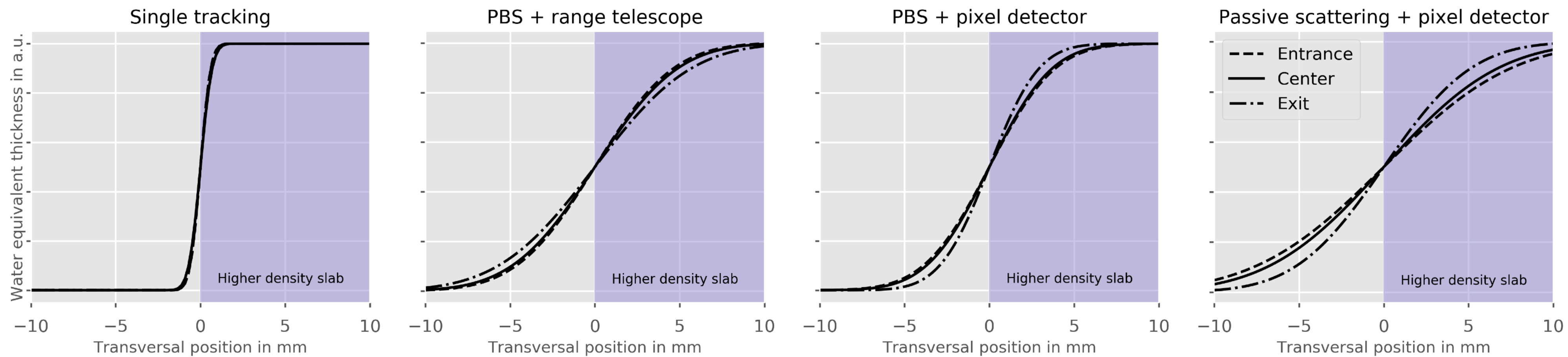
Higher density slab



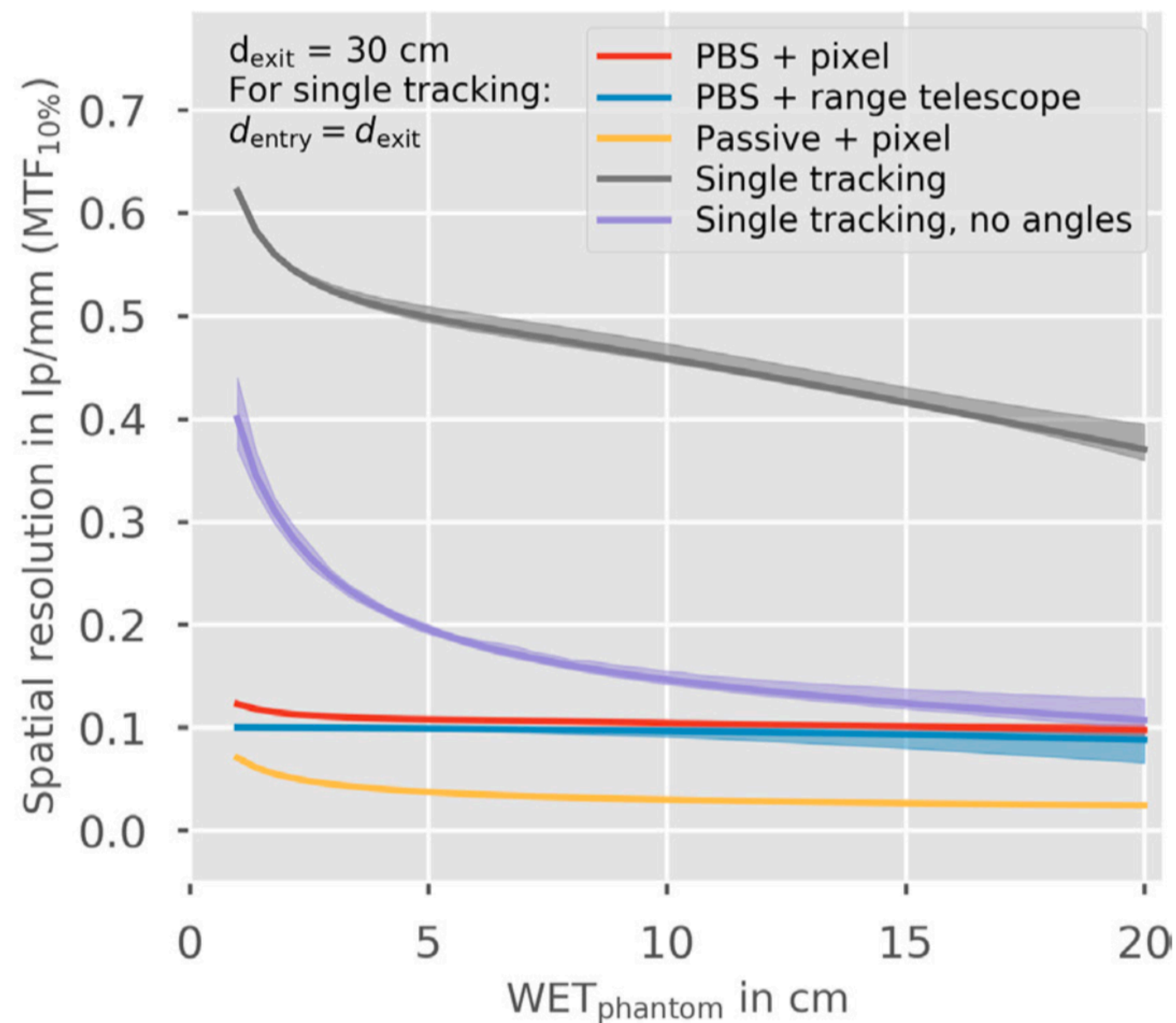
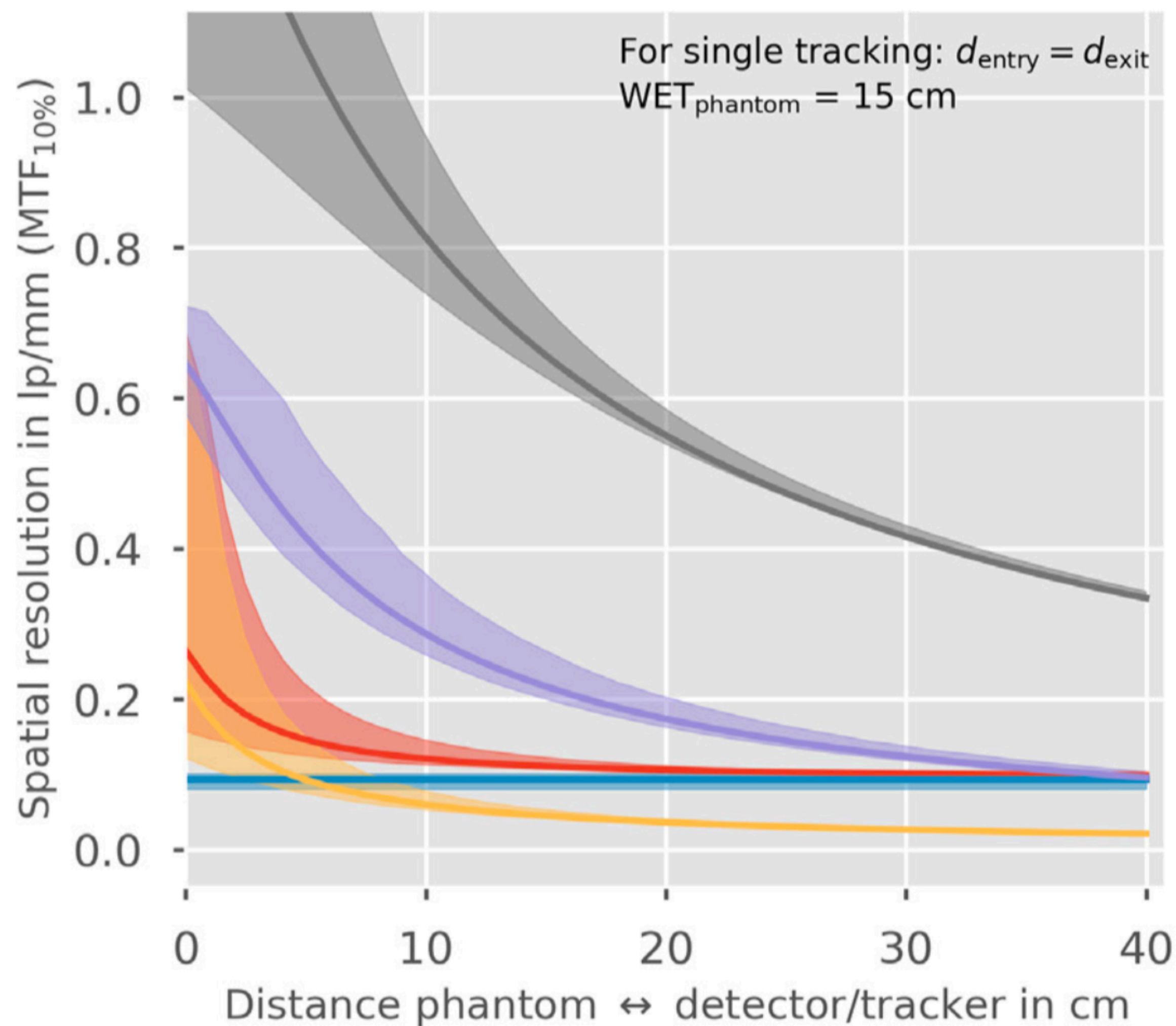
Edge spread distribution



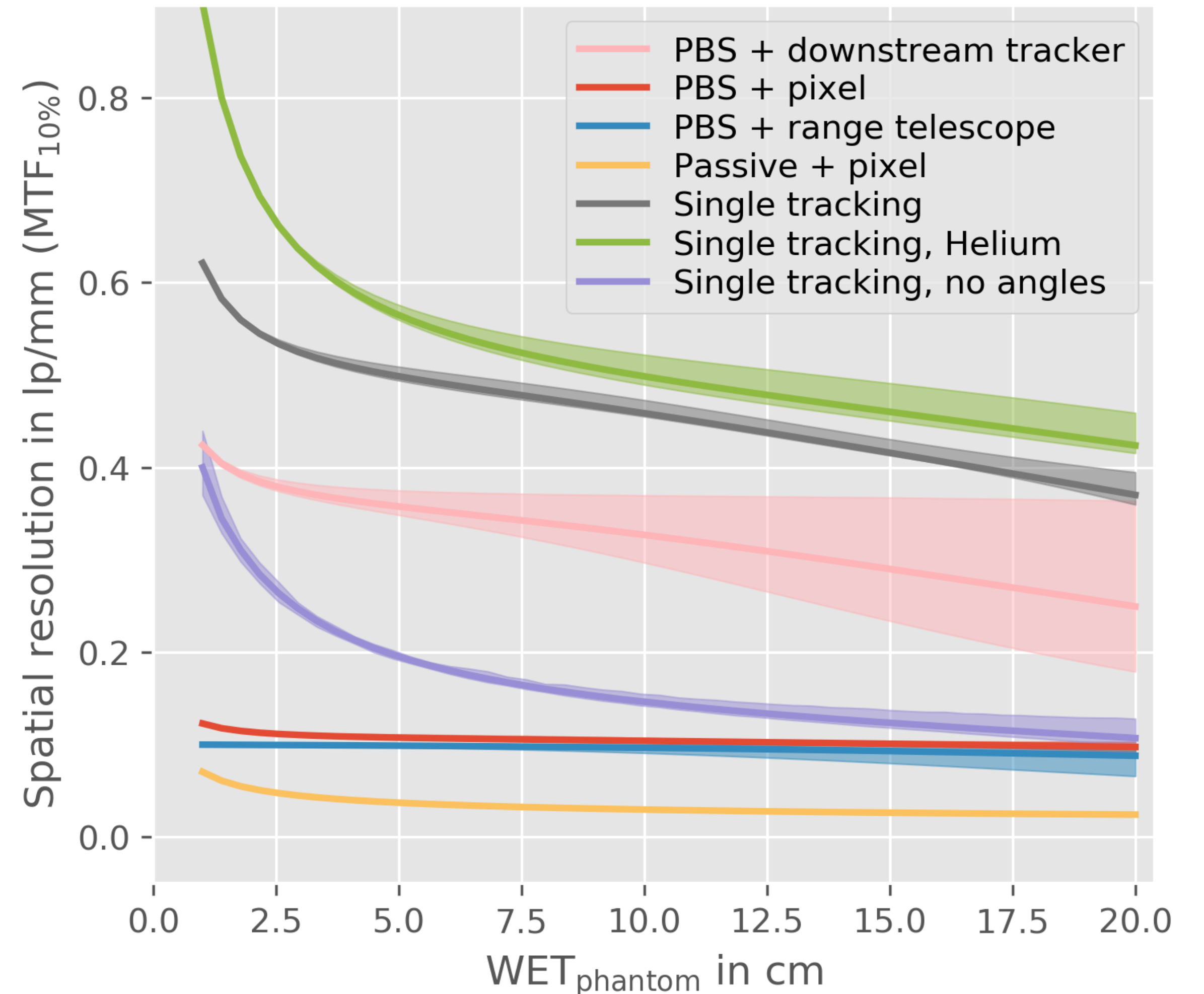
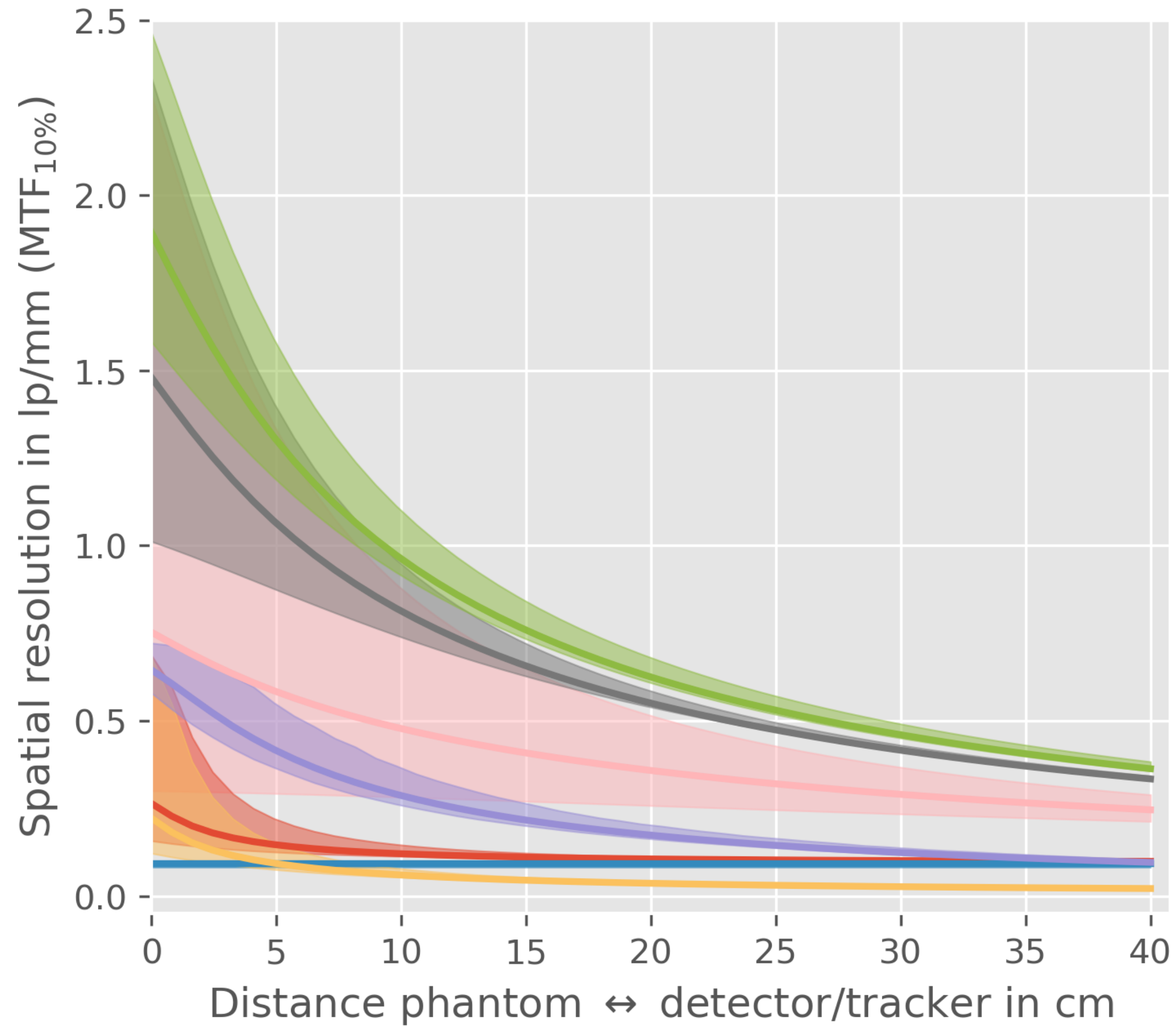
Spatial resolution in I_p/mm



Impact of detector distance on spatial resolution

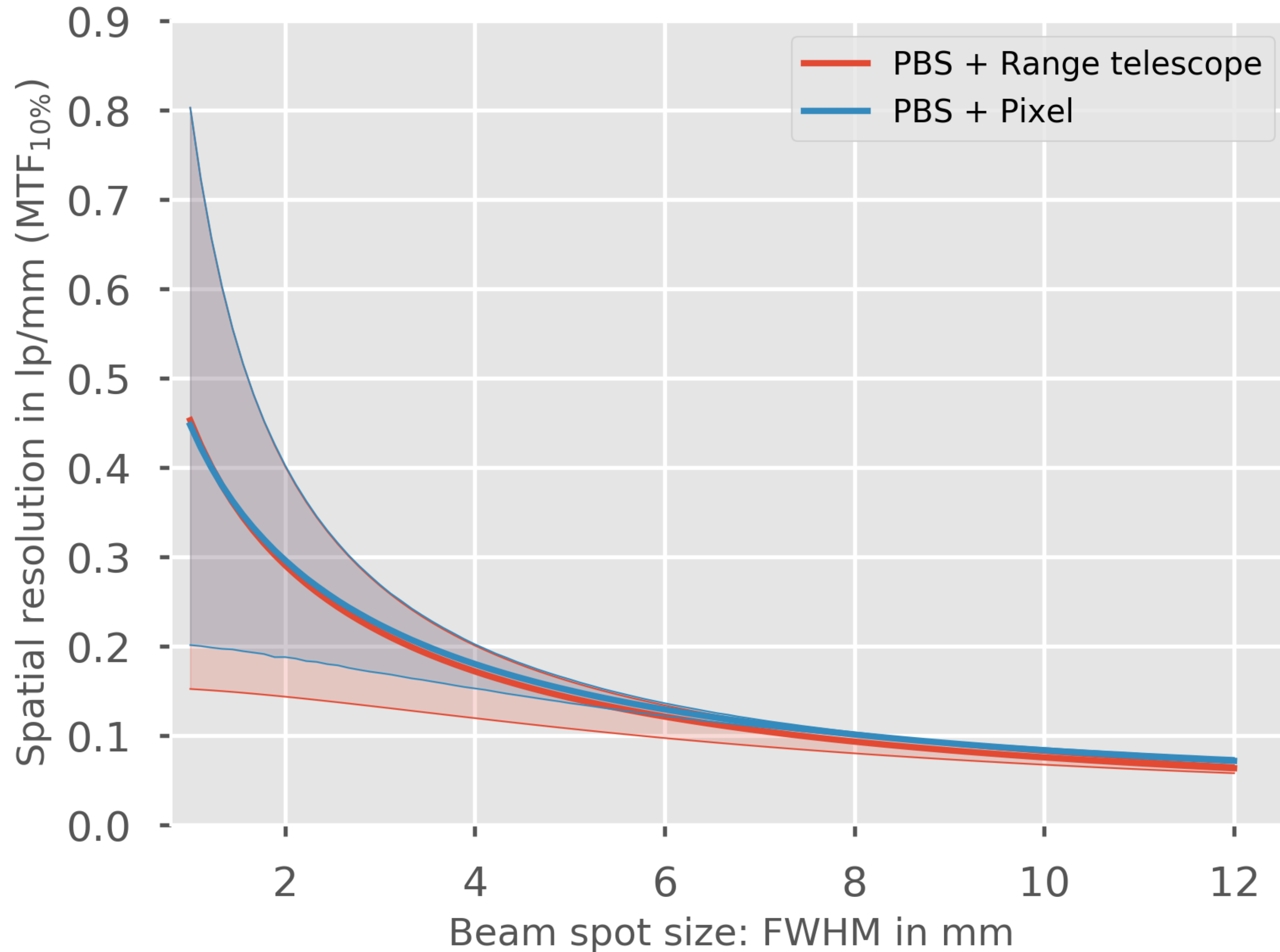


How about Helium and the Bergen set-up?



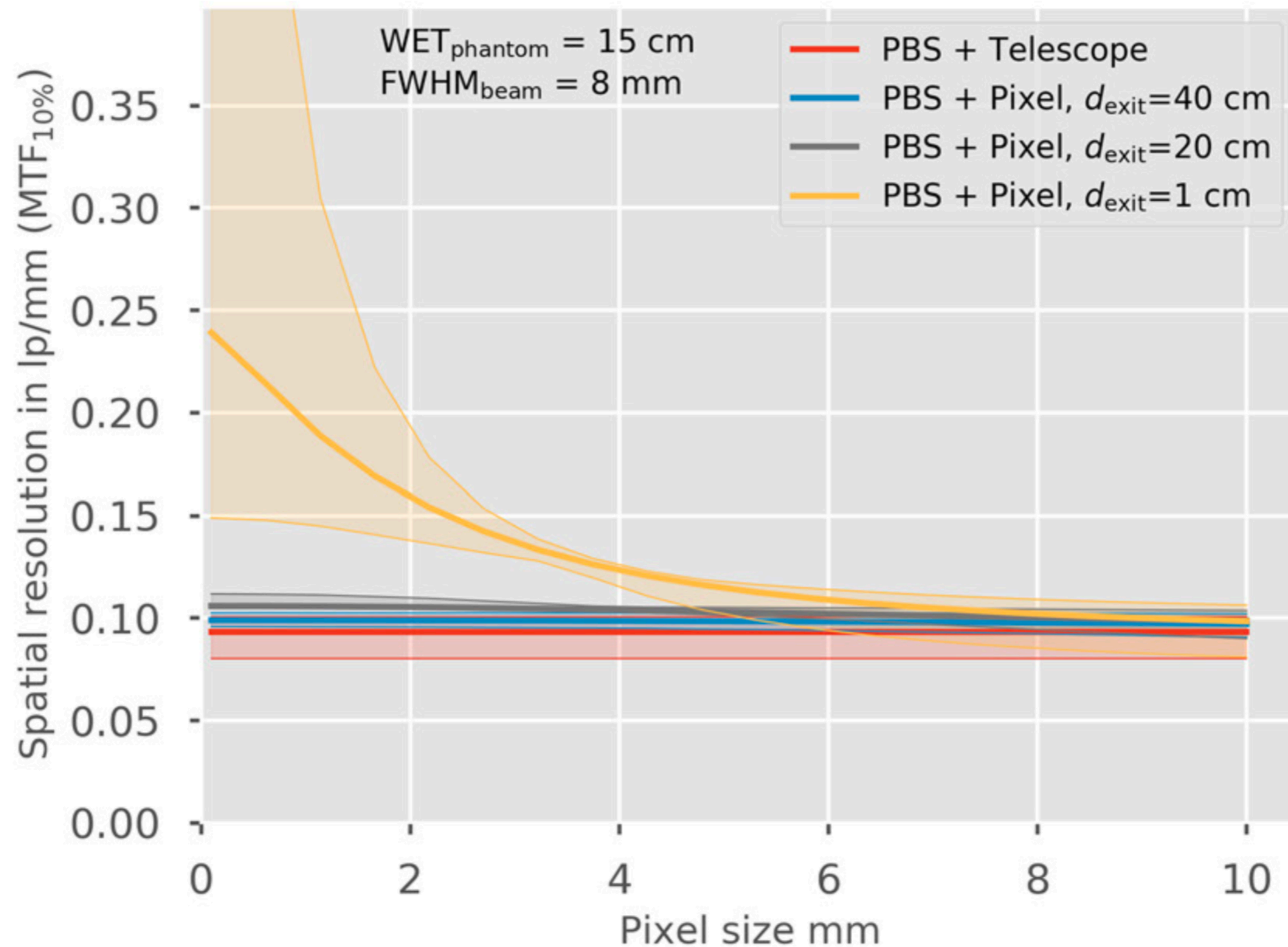
***preliminary**

Impact of beam size in PBS set-ups



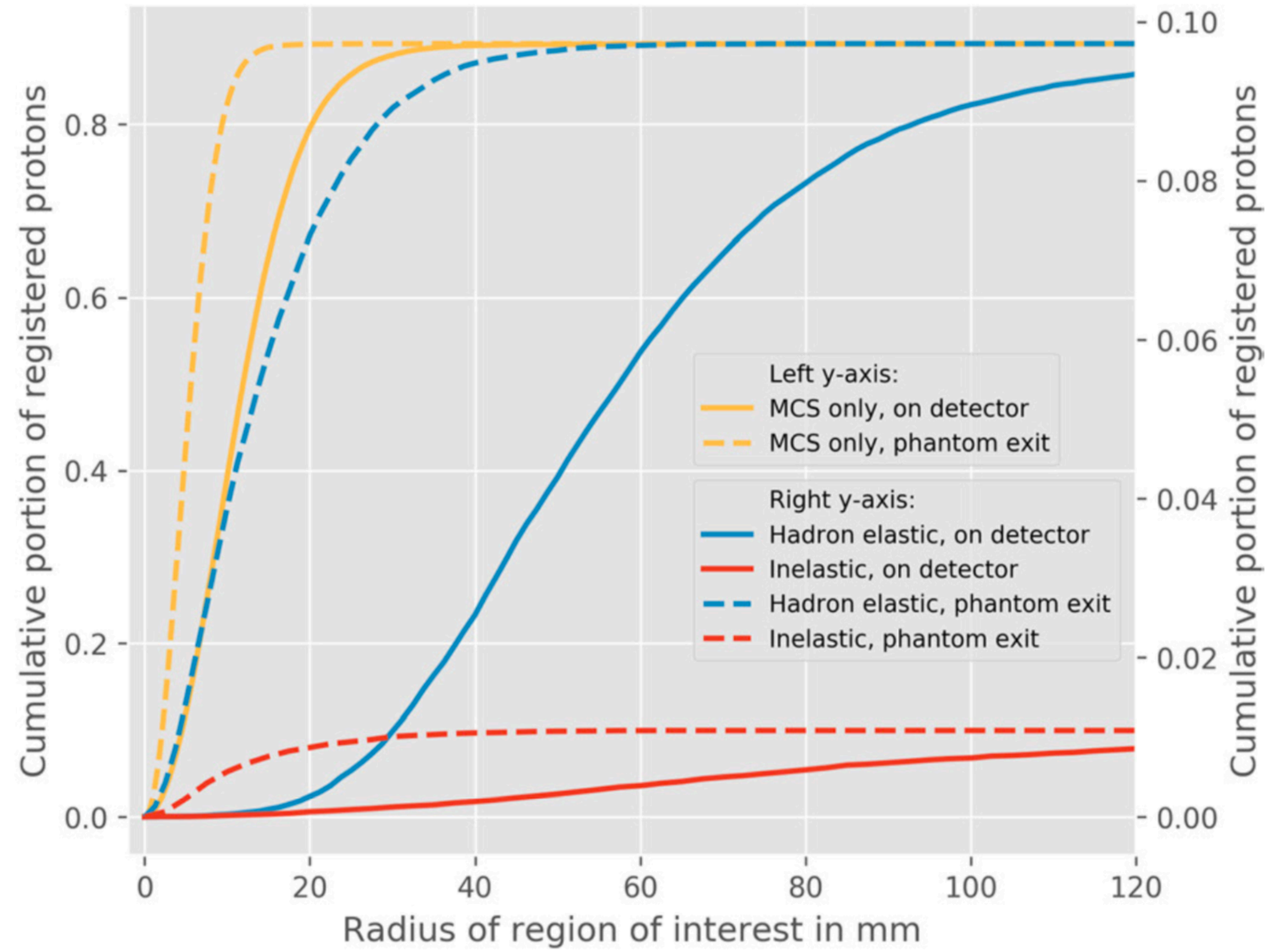
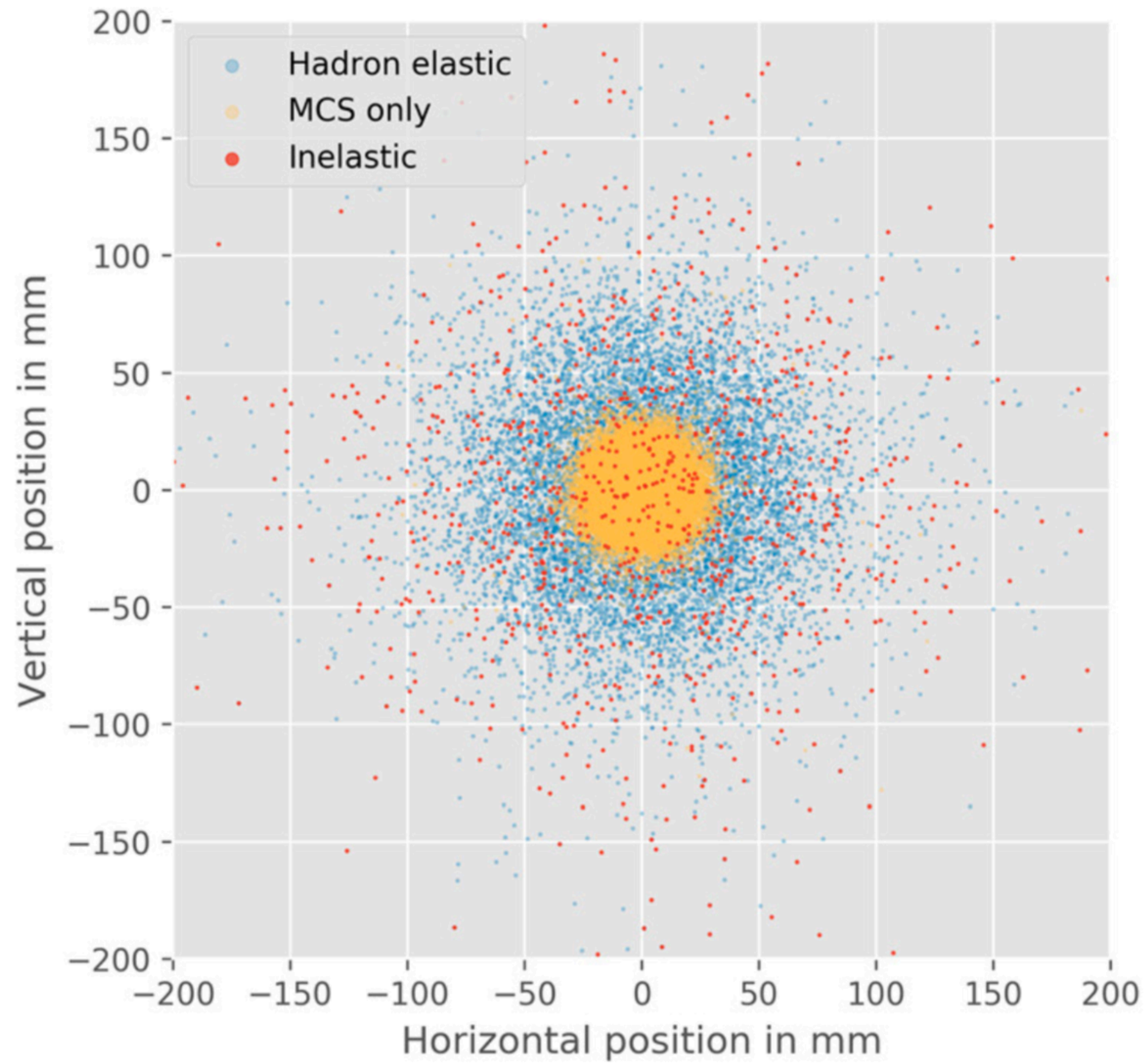
Small beam size greatly improves spatial resolution

Pixel size



... does not really matter unless the detector is placed very close to the phantom

So pixels are useless?

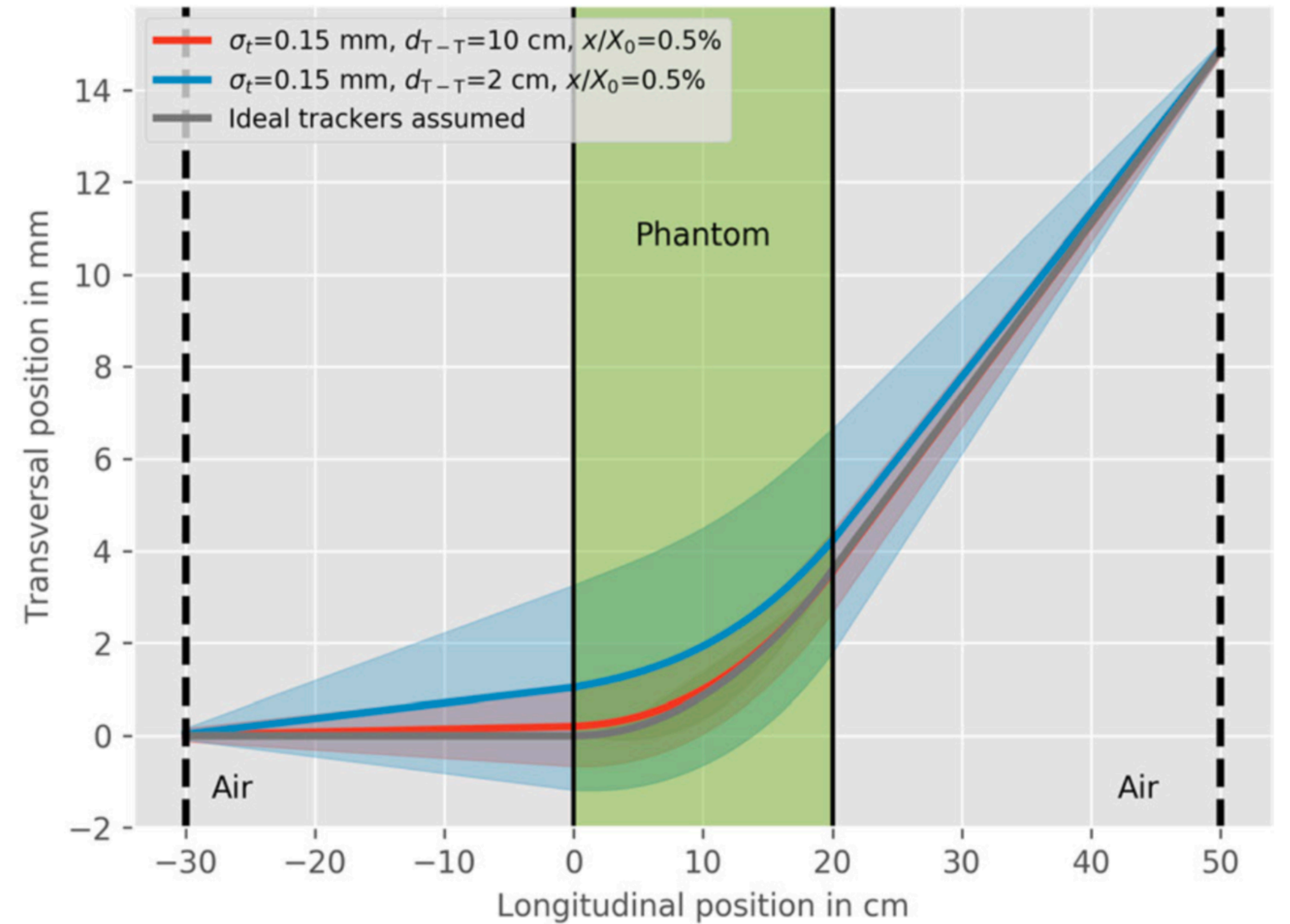
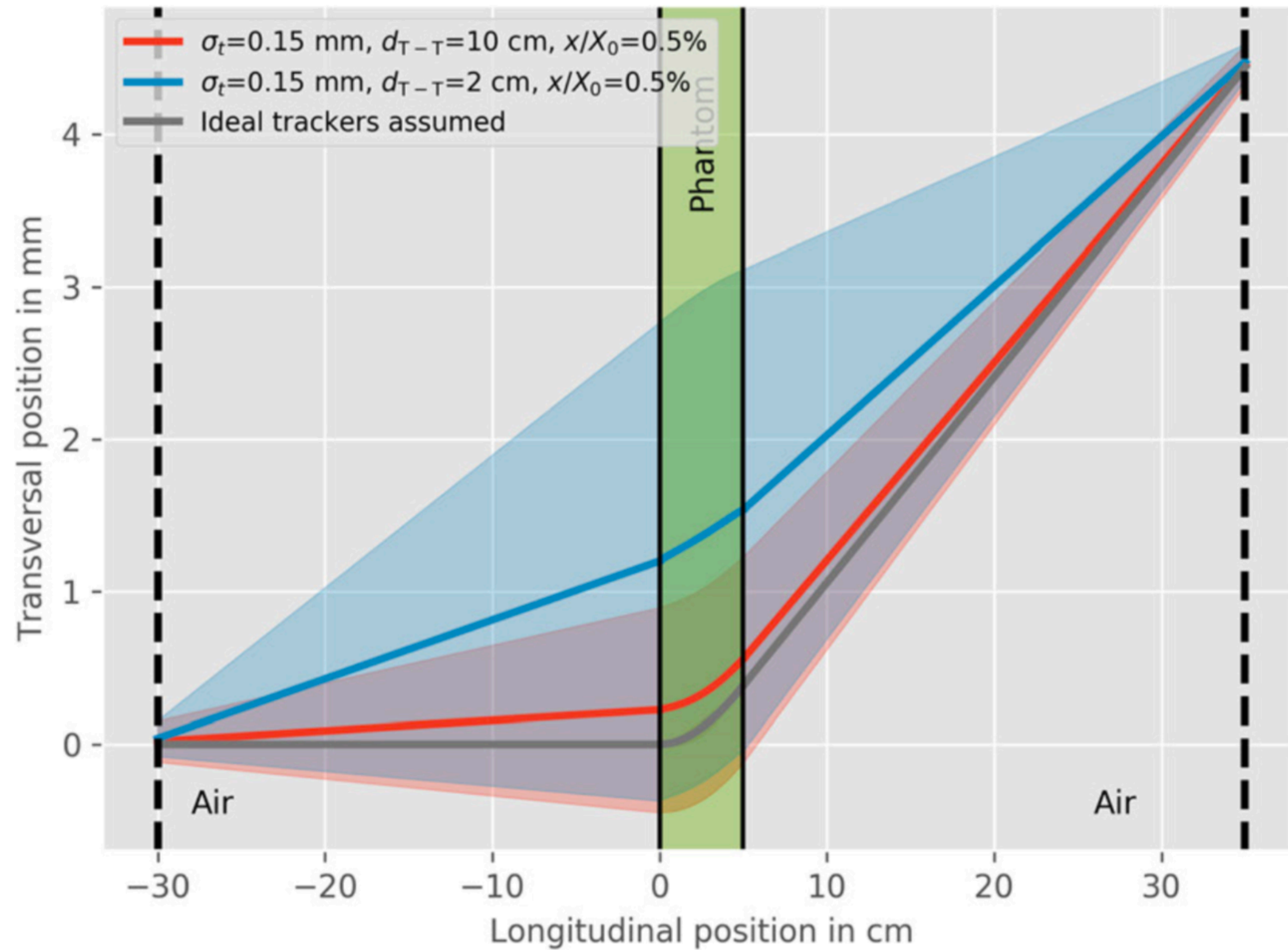


Impact of experimental uncertainties on MLP

$$y_{\text{MLP}}(u) = C_2 (C_1 + C_2)^{-1} R_0 S_{\text{in}} \cdot \tilde{y}_{\text{in,d}} + C_1 (C_1 + C_2)^{-1} R_1^{-1} S_{\text{out}}^{-1} \cdot \tilde{y}_{\text{out,d}}$$

$$C_1 = R_0 S_{\text{in}} \Sigma_{\text{in}} S_{\text{in}}^T R_0^T + \Sigma_1$$

$$C_2 = R_1^{-1} S_{\text{out}}^{-1} \Sigma_{\text{out}} (S_{\text{out}}^{-1})^T (R_1^{-1})^T + R_1^{-1} \Sigma_2 (R_1^{-1})^T$$



Conclusion

- **List mode:**
tracker uncertainty very important
spatial resolution degrades drastically with tracker distance
- **Passive field:** Must put detector close to phantom/patient
- **PBS:** Small beam size improves spatial resolution a lot
- **Pixel size is irrelevant for spatial resolution**
but pixels allows for region of interest filtering
- **Refine MLP estimation?**

Merci !