Hybrid adaptive control for the half-bridge inverter

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***Abstract*— In this work, we propose a hybrid adaptive control law for the half-bridge inverter subjected to the common problem of unknown load. This controller ensures the system robustness with respect to the output, taking into account the real nature of the signals, which means the continuous-time variables, (voltage and current signals) and the discrete-time variables, (switching signals). The adaptation is accomplished using a state observer and assuming that all states are measurable. Then, stability properties can be ensured using hybrid dynamical system theory and singular perturbation analysis. Finally, the proposed hybrid adaptive controller is validated in simulation.**

***Index Terms*— Half-bridge inverter, hybrid dynamical system, adaptive control, singular perturbation analysis.**

I. Introduction

Nowadays, power converter control is a subject of more and more research topics. This is mainly due to the fact that power converters have a very wide range of applications, for example, the half and full bridge inverters, which are used in Adjustable Speed Drives (ASDs), Uninterruptible Power Supplies (UPSs), static var compensator, active filters, Flexible AC Transmission Systems (FACTSs) and voltage compensator [1]. Frequently, their connected loads can change or suffer perturbations. This problem is usually solved by using adaptation mechanisms, guarantying a robust output signal with respect to these load changes. Hence, in [2], a load adaptive control algorithm is designed to cover a variety of loads largely inductive using square wave, asymmetrical duty cycle, and pulse density modulation. Likewise, in [3] a load adaptive tuned frequency tracking control strategy using PLL (Phase Locked Loop) is used to control a half bridge series resonant inverter for induction heating operations. And in [4], a multiloop nonlinear adaptive control is designed by using a backstepping technique. It is important to emphasize that all these control laws are based on averaged models, losing the properties of the discrete signals.

In this work, we aim to use a more accurate model of half-bridge inverters, considering the real nature of the signals, which means, continuous (voltages and currents) and discrete (on-off state of the switches) signals. To this end, we extend the results presented in [5], where a hybrid control is designed for the half-bridge inverter considering a constant and known load.

The novelty of this work lies in the design of a hybrid adaptive control for a half-bridge inverter regarding an unknown or perturbed load. A similar problem is considered in [6], where the authors stabilize the output of a DC-DC converter to a desired reference, considering a regulation problem and an unknown load. However, here the problem is different, because we are dealing with a tracking problem. To this end, we transform the problem in an output regulation problem [7][8] and we propose an indirect adaptation mechanism in the hybrid dynamical scheme, in order to adapt the load variations. More precisely, we consider an adaptive law fed by a state observer by assuming that all states are measurable. This method is simpler than the one presented in [6]. Then, uniformly locally asymptotically stability is ensured by applying the scale-time separation and by using a singular perturbation analysis.

The paper is organized as follows: in Section II, a model is defined under some assumptions and the problem is stated in Section III. Section IV proposes a hybrid adaptive control that achieves the objectives of our problem. Section V deals with the proof of our main result. Some simulations are performed in Section VI. Section VII draws the conclusion and future works.

Notation: Throughout the paper, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all real $m \times n$ matrices. The set $\mathcal{S}^n$ denotes the set of symmetric positive definite matrices of matrices $\mathbb{R}^{n\times n}$. $\mathfrak{Re}(\varsigma)$ is the real part of a complex number ς.

II. system modeling

A. Inverter model

We consider a half-bridge inverter presented in Fig. 1, which is fed by a DC input, $2V_{in}$, and generates an AC output. This inverter is composed by a load filter, L, C_0, a purely resistive load, R_0. The parasite resistance R_{LS} models not only the switching energy dissipated, but also the resistive component of the inductance. The dynamic behavior of this system can then be described by the following model:

$$\frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_{LS}}{L} & -\frac{1}{L} \\ \frac{1}{C_0} & -\frac{1}{R_0 C_0} \end{bmatrix}\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{V_{in}}{L} \\ 0 \end{bmatrix} u, \quad (1)$$

where i_L is the inductance current, v_C is the capacitor voltage and these two states are considered as the

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continuous-time state variables. $u{:=}\, U_1 - U_2$ is the control input and it is assumed to be a discrete variable:

$$\begin{cases} u = -1, & \text{if } U_1 = \text{ OFF and } U_2 = \text{ON}. \\ u = 1, & \text{if } U_1 = \text{ ON and } U_2 = \text{OFF}. \end{cases}$$

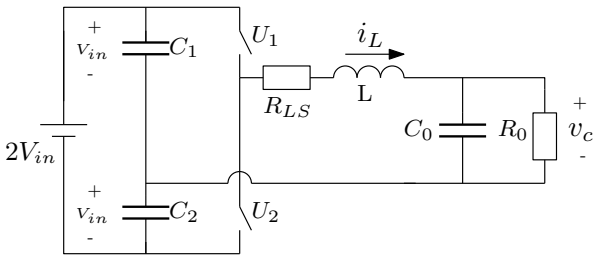

Fig. 1. Half-bridge inverter.

Assumption 1: We assume that,

- current and voltage are measurable.
- All the components are ideal.
- The load R_0 is an unknown constant and can suffer disturbances in the interval $[R_0^m, R_0^M]$.

let define $\beta := \frac{1}{R_0} \in [\beta_m, \beta_M]$, which belongs to the following polytope

$$\Omega := \lambda_{\beta,m}\beta_m + \lambda_{\beta,M}\beta_M, \quad \forall\ \lambda_{\beta,m}, \lambda_{\beta,M} \in [0,1],$$

with $\quad \lambda_{\beta,m} + \lambda_{\beta,M} = 1.$

Using definition of β, the system (1) is rewritten as follows

$$\begin{aligned} \dot{x} &= A(\beta)x + Bu, \\ y &= x, \end{aligned} \tag{2}$$

with $x = [x_1\ x_2]^T = [i_L\ v_C]^T$, and $A(\beta)$ and B are easily deduced from (1). Note that $A(\beta)$ is Hurwitz for all $\beta > 0$. Moreover, we assume that the following property is verified.

Property 1: Consider matrix $A(\beta)$ in (2), with $\beta \in [\beta_m, \beta_M]$, and a chosen matrix $Q \in \mathcal{S}^2$. Then there exists a matrix $P \in \mathcal{S}^2$ satisfying the two following Lyapunov inequalities:

$$\begin{aligned} A^T(\beta_m)P + PA(\beta_m) + Q < 0, \\ A^T(\beta_M)P + PA(\beta_M) + Q < 0. \end{aligned}$$

This property imposes finally a common Lyapunov function for the polytopic model (2). In the next subsection, we propose a desired tracking trajectory, described by a linear time-invariant model.

B. A reference model

The desired trajectory for the voltage and the current can be modeled by the following equations:

$$\begin{aligned} x_{2e} &:= V_{C_d}(t) = V_{max}\sin(wt), \\ x_{1e} &:= i_{L_d}(t) = C_0 w V_{max}\cos(wt) + \beta V_{max}\sin(wt), \end{aligned} \tag{3}$$

where V_{max} and ω are the desired amplitude and frequency of the voltage, respectively, applied to the load R_0. In order to impose such behavior, let us define the exogenous input $z \in \mathbb{R}^2$ produced by this time-invariant exosystem:

$$\dot{z} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix} z = \Theta z, \qquad z(0) = \begin{bmatrix} 0 \\ V_{max} \end{bmatrix}. \tag{4}$$

From (3), and considering that $\hat{\beta}$ is the estimated value of β, we have

$$\begin{bmatrix} V_{C_d}(t) \\ i_{L_d}(t) \end{bmatrix} = \Pi(\hat{\beta})z, \quad \text{with } \Pi(\hat{\beta}) = \begin{bmatrix} wC_0 & \hat{\beta} \\ 0 & 1 \end{bmatrix}. \tag{5}$$

Remark 1: Note that from (4), it is simple to see that

$$z_1^2(t) + z_2^2(t) = V_{max}^2.$$

Consequently, let define the following compact set

$$\Phi = \{(z_1, z_2) \in \mathbb{R}^2, \quad z_1^2(t) + z_2^2(t) = V_{max}^2\}.$$

Firstly, define $\Gamma(\hat{\beta})$, such that, the following algebraic equation is verified:

$$A(\beta)\Pi(\hat{\beta}) + B\Gamma(\hat{\beta}) = \Pi(\hat{\beta})\Theta. \tag{6}$$

A simple calculation shows that, $\Gamma(\hat{\beta})$ does not depend on β and can be written as:

$$\Gamma(\hat{\beta}) = \left[-\tfrac{wL\hat{\beta}}{V_{in}} - \tfrac{wR_{LS}C_0}{V_{in}} \quad \left(\tfrac{1}{L} - C_0 w^2 + \tfrac{R_{LS}\hat{\beta}}{L}\right)\tfrac{L}{V_{in}} \right].$$

The dynamics of the overall system is defined by

$$\begin{aligned} \dot{x} &= A(\beta)x + Bu, \\ \dot{z} &= \Theta z, \\ e &= Cx + Dz = x - \Pi(\hat{\beta})z, \end{aligned} \tag{7}$$

where $e = [e_1\ e_2]^T \in \mathbb{R}^2$ is the tracking error, $C \in \mathbb{R}^2$ is the identity matrix and $D = -\Pi(\hat{\beta})$.

Note that, equation (5) and (6) are the well known "regulator equations" [7][8].

Then, by using (6)–(7), the tracking error dynamic is given by

$$\dot{e} = \dot{x} - \Pi(\hat{\beta})\dot{z} - \dot{\hat{\beta}}\Pi'(\hat{\beta})z = A(\beta)e + Bd(\dot{\hat{\beta}}, \hat{\beta}, z),$$

with

$$d(\dot{\hat{\beta}}, \hat{\beta}, z) := \left(v(\hat{\beta}, z) - \frac{L}{V_{in}}\dot{\hat{\beta}}z_2 \right), \tag{8}$$

$$v := u - \Gamma(\hat{\beta})z, \tag{9}$$

where $\Pi' := \frac{d\Pi}{d\hat{\beta}}$.

In order to present the control law, consider the following model:

$$\begin{aligned} \dot{e} &= A(\beta)e + Bd(\dot{\hat{\beta}}, \hat{\beta}, z), \\ \dot{z} &= \Theta z, \end{aligned} \tag{10}$$

where v is the available input given in (9) and it is composed of a continuous-time signal $\Gamma(\hat{\beta})z$ and a switching signal u. The following assumption must be hold to ensure the existence of signal u, making $e = 0$.

Assumption 2: There exist two functions of time $\lambda_1, \lambda_{-1} \in [0,1]$ satisfying $\lambda_1 + \lambda_{-1} = 1$, such that the following equation holds:

$$\lambda_1 - \lambda_{-1} - \Gamma(\hat{\beta})z = 0. \tag{11}$$

Note that this assumption provides the existence of $e = 0$ with $\hat{\beta} = \beta$, allowing to be $v = 0$ through a convex combination of the two operating modes of u as follows

$$v = \lambda_1 - \lambda_{-1} - \Gamma(\hat{\beta})z.$$

Remark 2: It is important to understand that in Assumption 2, the solution $e = 0$ with $\hat{\beta} = \beta$, is obtained in sense of Filoppov solution. Indeed, the signal $u = \lambda_1 - \lambda_{-1}$, is a periodic sequence of arbitrarily small period T, spending a time equal to $\lambda_1 T$ in mode $u = 1$, and $\lambda_{-1} T$ in mode $u = -1$.

III. Problem statement

This paper focuses on the design of a switching signal $u \in \{-1, 1\}$ and an adaptive law $\hat{\beta}$, which guarantee the two following properties:

- a suitable trajectory tracking properties of the voltage $x_2(t)$ to a desired trajectory $x_{2_e}(t)$, ensuring that, the error $e = x - x_e$ with $x_e = [x_{e_1} x_{e_2}]^T$, converges to the equilibrium $e = 0$ in the Filippov sense.
- The convergence of the estimation of the load $\hat{\beta}$ to its real value of β.

Inspired by [6], we extend the work presented in [5], for model (2) with an unknown parameter β.

Problem 1: The idea represented by Fig. 2 is to design a hybrid adaptive controller that considers the continuous-time dynamics, x_1, x_2, and the discrete-time dynamic, u, estimating β in continuous-time, at the same time that x_1 and x_2 converge to a sinusoidal references given by (3).

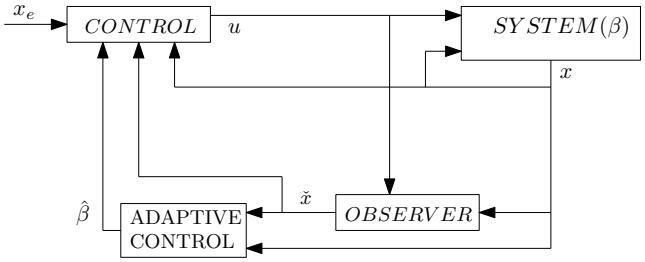

Fig. 2. Hybrid adaptive control scheme.

In order to solve Problem 1, we propose firstly an adaptation control law with the aim of estimating the unknown constant β. Then, we design a hybrid adaptive controller that considers both, the continuous and the discrete dynamics.

IV. Hybrid model and proposed control law

Consider that $\hat{e}_2$ and $\hat{\beta}$ are the estimated states of e_2 and β, respectively. Then, the system (10) and the estimated variables mentioned before, can be modeled as a hybrid dynamical system, following the paradigm given in [9], wherein

- the continuous-time behavior encompasses the evolution of e, z, $\hat{e}_2$ and $\hat{\beta}$ and,
- the discrete-time behavior captures the evolution of the input signal u, (through v in (9)).

We characterize the overall dynamics by the following hybrid model:

$$\mathcal{H}: \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{z} \\ \dot{v} \\ \dot{\hat{e}}_2 \\ \dot{\hat{\beta}} \end{bmatrix} = f(\xi), & \xi \in \mathcal{C}, \\ \begin{bmatrix} e^+ \\ z^+ \\ v^+ \\ \hat{e}_2^+ \\ \hat{\beta}^+ \end{bmatrix} \in G(\xi), & \xi \in \mathcal{D}, \end{cases} \tag{12}$$

where $\xi = [e\ z\ v\ \hat{e}_2\ \hat{\beta}]$, f is the flow map and and G is the jump map containing the evolution of the state after jumps

$$f(\xi) := \begin{bmatrix} A(\beta)e + Bd(\xi), \\ \Theta z \\ -\Gamma(\hat{\beta})\Theta z \\ \frac{1}{C_0}(e_1 - \hat{\beta}e_2) + \alpha(e_2 - \hat{e}_2) \\ -\frac{\gamma e_2(e_2 - \hat{e}_2)}{C_0} \end{bmatrix},$$

$$G(\xi) := \begin{bmatrix} x \\ z \\ \left(\underset{\substack{v=u-\Gamma(\beta)z \\ u\in[-1,1]}}{\operatorname{argmin}} \tilde{e}^T P(A(\hat{\beta})e + Bd(\xi)) \right) - \Gamma(\hat{\beta})z \\ \hat{e}_2 \\ \hat{\beta} \end{bmatrix}, \tag{13}$$

being

$$d(\xi) = \left(v(\hat{\beta}, z) + \frac{\gamma L e_2(e_2 - \hat{e}_2)}{V_{in} C_0} z_2 \right), \tag{14}$$

and with α, $\gamma > 0$ are design parameters. The error $\tilde{e} = [\tilde{e}_1\ \tilde{e}_2]$ is defined as follows

$$\begin{cases} \tilde{e}_1 = e_1 \\ \tilde{e}_2 = e_2 + (e_2 - \hat{e}_2). \end{cases} \tag{15}$$

Inspired by [10] and as proposed by [6], we select the so-called flow and jump sets

$$\mathcal{C} := \{\xi : \tilde{e}^T P(A(\hat{\beta})e + Bd(\xi)) \leq -\eta \tilde{e}^T Q \tilde{e}\}, \tag{16}$$

$$\mathcal{D} := \{\xi : \tilde{e}^T P(A(\hat{\beta})e + Bd(\xi)) \geq -\eta \tilde{e}^T Q \tilde{e}\}, \tag{17}$$

with $\eta \in (0, 1)$ is a tunned parameter.

Remark 3: To achieve the control objectives, i.e. reach the desired equilibrium, $e = 0$, the proposed control generates arbitrary a fast switching in the steady-state, due to the fact that this desired equilibrium is achieved in Filippov sense. But from a practical point of view, the switching frequency must present a minimal dwell-time between consecutive switches. Note that the switching frequency is reduced if $\eta \to 0$ and, it is increased if $\eta \to 1$.

Proposition 1: The hybrid system $(12)-(17)$ satisfies the basic hybrid conditions given in [9, Assumption 6.5]:

- sets $\mathcal{C}$ and $\mathcal{D}$ given in (16) and (17) respectively, are closed.
- f is a continuous function, thus is therefore outer semicontinuous and locally bounded. Moreover, f is convex for all $\xi \in \mathcal{C}$.
- G is closed, therefore it is outer semicontinuous [9, Lemma 5.1] and locally bounded.

Then, we can conclude that the hybrid system $(12)-(17)$ is well-posed.

Next, we invoke Lemma in [10, Lemma 1], for setting up stability properties.

Lemma 1: Consider matrices $P, Q \in \mathcal{S}^2$ satisfying Property 1. Then for each $e \in \mathbb{R}^2$,

$$\min_{\substack{v=u-\Gamma(\hat{\beta})z \\ u\in[-1,1]}} \tilde{e}^T P(A(\beta)e + Bd(\xi)) \leq -\tilde{e}^T Q\tilde{e}.$$

Remark 4: We note here that for $\eta \in (0,1)$ and if $\tilde{e} \neq 0$, then

$$-\tilde{e}^T Q\tilde{e} < -\eta\tilde{e}^T Q\tilde{e}.$$

Thereafter, using the Hybrid Dynamical System (HDS) theory, we will establish stability properties of the given compact set

$$\mathcal{A} := \{\xi : e = 0, z \in \Phi, u \in \{-1, 1\}, \hat{e}_2 = e_2, \hat{\beta} = \beta\}. \quad (18)$$

Theorem 1: Consider Assumption 1, 2 and matrices $P, Q \in \mathcal{S}^2$ satisfying Property 1. Then attractor (18) is Uniformly Locally Asymptotically Stable (ULAS) for hybrid system (12)–(17).

The proof of this Theorem is given in the next Section. Here we will comment the choice of matrices P and Q. These matrices are selected following some optimization criteria given in (34) for a hybrid system (12)–(17). Specifically, we use [5, Theorem 2], where some LQ performance level is guaranteed, then the following bound holds along any solution of our hybrid system

$$J \leq \eta^{-1}\tilde{e}^T P\tilde{e} \quad (19)$$

defined in a compact hybrid time domain [9, Definition 2.3].

On the other hand, we find that the suboptimal-level corresponds to a high frequency switches, which can increase the dissipated energy. Therefore, the choice of η decides a trade-off between any LQ performance level and switching frequency as noted in Remark (3).

This next section is devoted to prove the stability of hybrid system (12)–(17).

V. Proof of theorem 1

Let us now remind the observer and adaptation dynamics given as

$$\dot{\hat{e}}_2 = \frac{1}{C_0}(e_1 - \hat{\beta}e_2) + \alpha(e_2 - \hat{e}_2), \quad (20)$$

$$\dot{\hat{\beta}} = -\frac{\gamma e_2(e_2 - \hat{e}_2)}{C_0}. \quad (21)$$

Lemma 2: Considering Assumption 1, then $\hat{e}_2 = e_2 = 0$, $e_1 = 0$ and $\beta \neq \hat{\beta}$ is not an equilibrium of the differential equation (1), (20) and (21).

Proof: Consider that β is unknown and constant. Note that (5) is not accomplished, i.e. if $\beta \neq \hat{\beta}$, then the reference $\Pi(\hat{\beta})z$ is different to the desired trajectory (3), which directly depends on the real value of β. Therefore, $\dot{e} \neq 0$, $\dot{\hat{e}}_2 \neq 0$ and $\dot{\hat{\beta}} \neq 0$ ∎

Lemma 3: Regard system (10), Lemma 2 and Assumption 1 and assume that its solutions are bounded. The extended observer (20)–(21) has the following properties:

i) The estimated states $\hat{e}_2$, $\hat{\beta}$ are bounded.
ii) $\lim_{t\to\infty} \hat{e}_2(t) = e_2(t)$.
iii) $\lim_{t\to\infty} \hat{\beta}(t) = \beta$.

Proof: We can guarantee that the solutions of (10) are bounded from [5, Theorem 1]. Let define the following error variables:

$$e_2 := e_2 - \hat{e}_2, \quad \tilde{\beta} := \beta - \hat{\beta}. \quad (22)$$

and from Assumption 1, we have $\dot{\tilde{\beta}} = -\dot{\hat{\beta}}$. Next, from (II-B), (20) and (21) the errors dynamics are given by

$$\dot{\tilde{e}}_2 = -\frac{\tilde{\beta}}{C_0}e_2 - \alpha e_2 \quad (23)$$

$$\dot{\tilde{\beta}} = \frac{\gamma e_2 e_2}{C_0}. \quad (24)$$

To ensure that the errors converge to 0, we propose to design a suitable candidate Lyapunov function, given by

$$W = \frac{1}{2}\left({e_2}^2 + \frac{\tilde{\beta}^2}{\gamma}\right). \quad (25)$$

The derivative of W along the trajectories (23)–(24) gives:

$$< \nabla W(e_2, \tilde{\beta}), f(e_2, e_2, \tilde{\beta}) > = -\alpha {e_2}^2,$$

and from standard Lyapunov arguments, it follows that the error variables e_2 and $\tilde{\beta}$ are bounded. In addition, by LaSalle invariant principle, we easily conclude that $e_2 \to 0$, which implies from (24) that $\dot{\tilde{\beta}} \to 0$. Likewise, from (23), and concluding from $e_2 \to 0$ and Lemma 2, we get $\tilde{\beta} \to 0$. ∎

To prove the stability of a hybrid system (12)–(17), let us use the fact that we have two distinct behaviors. In one hand, there are slow time-continuous variables $\xi_r := (e, z, v)$, in other hand, $\xi_f := (\hat{e}_2, \hat{\beta})$ represents the fast time-continuous variables. Using this last assumption and by considering these variables change, $\nu := \frac{1}{\alpha}$,

$\bar{\gamma} := \frac{\gamma}{C_0}$, we will rewrite the complete system in a singular perturbation form given as follows:

$$\mathcal{H}_p : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{z} \\ \dot{v} \\ \nu\dot{\hat{e}}_2 \\ \nu\dot{\hat{\beta}} \end{bmatrix} = f_p(\xi), & \xi \in \mathcal{C}, \\ \begin{bmatrix} e^+ \\ z^+ \\ v^+ \\ \hat{e}_2^+ \\ \hat{\beta}^+ \end{bmatrix} \in G_p(\xi), & \xi \in \mathcal{D}, \end{cases} \tag{26}$$

$$f_p(\xi) := \begin{bmatrix} A(\beta)e + Bd(\xi) \\ \Theta z \\ -\Gamma(\hat{\beta})\Theta z \\ \frac{\nu}{C_0}(e_1 - \hat{\beta}e_2) + (e_2 - \hat{e}_2) \\ -\nu\bar{\gamma}e_2(e_2 - \hat{e}_2) \end{bmatrix},$$

$$G_p(\xi) := \begin{bmatrix} x \\ z \\ \left(\underset{\substack{v=u-\Gamma(\hat{\beta})z \\ u\in[-1,1]}}{\operatorname{argmin}} \ \tilde{e}^T P(A(\hat{\beta})e + Bd(\xi)) \right) - \Gamma(\hat{\beta})z \\ \hat{e}_2 \\ \hat{\beta} \end{bmatrix} \tag{27}$$

where $\xi = [e \ z \ v \ \hat{e}_2 \ \hat{\beta}]$.

Remark 5: In order to ensure a singular perturbation form, we need to establish that the response time of the slow subsystem, $t_R(A(\beta))$, is larger than the fast subsystem time response. In this aim, we select $\gamma = C_0\alpha$ and

$$\alpha >> \max\left\{ \frac{1}{t_R(A(\beta))} \right\} \quad \text{for} \quad \beta \in [\beta_m, \beta_M].$$

Note that, in the hybrid scheme (26)–(27), the fast variables directly impact the stability of the slow variables. However, the jumps do not affect the fast variables, because they do not present any jump ($\hat{e}_2^+ = \hat{e}_2$ and $\hat{\beta}^+ = \hat{\beta}$).

In order to fulfill a singular perturbation analysis, we will check the assumptions given in [11] for the model (26):

A. Regularity of system's data

Regularity of system's data comes directly from Proposition 1.

B. Regularity of "manifold"

The "manifold", which corresponds to the quasi-steady-state equilibrium manifold of classical singular perturbation theory [12], that means when $\nu \to 0^+$ is

$$\begin{aligned} e_2 - \hat{e}_2 &= 0 \\ \beta - \hat{\beta} &= 0. \end{aligned} \tag{28}$$

Note that $\beta - \hat{\beta} = 0$ comes from Lemma 2,3. As (28) is continuous, we can consider that the manifold is empty outside of $\mathcal{C}$, letting take the following set-valued:

$$\mathcal{M}(e_2) := \begin{cases} \begin{bmatrix} e_2 \\ \beta \end{bmatrix} & e_2 \in \mathcal{C} \\ 0 & e_2 \notin \mathcal{C}, \end{cases}$$

for a given β. Note that $\mathcal{M}$ is outer semi-continuous, locally bounded and nonempty.

C. Stability for reduced system

The reduced system is the system (12)–(13) in the manifold $\mathcal{M}$, which is

$$\mathcal{H}_r : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{z} \\ \dot{v} \end{bmatrix} = f_r(\xi_r), & \xi_r \in \mathcal{C}(\mathcal{M}), \\ \begin{bmatrix} e^+ \\ z^+ \\ v^+ \end{bmatrix} \in G_r(\xi_r), & \xi_r \in \mathcal{D}(\mathcal{M}), \end{cases} \tag{29}$$

$$f_r(\xi_r) := \begin{bmatrix} A(\beta)e + Bv \\ \Theta z \\ -\Gamma(\hat{\beta})\Theta z \end{bmatrix},$$

$$G_r(\xi_r) := \begin{bmatrix} e \\ z \\ \left(\underset{\substack{v=u-\Gamma(\hat{\beta})z \\ u\in[-1,1]}}{\operatorname{argmin}} \ \tilde{e}^T P(A(\hat{\beta})e + Bv) \right) - \Gamma(\hat{\beta})z \end{bmatrix}, \tag{30}$$

being $\xi_r = (e, z, v)$. Note that, the reduced system ignores $\hat{e}_2$ and $\hat{\beta}$ when determining jumps. Then, we can guarantee asymptotic stability from [5, Theorem 1]. Indeed the attractor 18 is ULAS for the reduced system.

D. Stability of the boundary layer

The boundary layer, is given by

$$\mathcal{H}_{bl} := \begin{cases} \dot{\xi}_r = 0 \\ \dot{\hat{e}}_2 = \frac{\nu}{C_0}(e_1 - \hat{\beta}e_2) + (e_2 - \hat{e}_2) & \xi \in \mathcal{C} \cap r\mathbb{B} \\ \dot{\hat{\beta}} = -\nu\bar{\gamma}e_2(e_2 - \hat{e}_2) \end{cases}$$

being $r\mathbb{B}$ a closed ball of radius r. Note that the boundary layer system ignores the jumps, and during flows ξ_r remains constant.

In order to evaluate the stability of the boundary layer, let consider the error equations of $\mathcal{H}_{bl}$ and let take as particular solution $\breve{e}_{2_{bl}} \in \mathbb{R}$ equal to a constant value. and re-scale time t to $\tau = (t - t_0)/\nu$, getting

$$\frac{d}{d\tau}\breve{e}_2 = -\frac{\nu}{C_0}\tilde{\beta}\hat{e}_{2_{bl}} - \breve{e}_2 \tag{31}$$

$$\frac{d}{d\tau}\tilde{\beta} = \nu\bar{\gamma}\hat{e}_{2_{bl}}\breve{e}_2. \tag{32}$$

Then, the stability property of (31)–(32) can be established using the Lyapunov function given in (25). Note that, in this case, the derivative of W along the

trajectories in the boundary layer is relative to τ, instead of to ordinary time t.

$$\frac{d}{d\tau}z \quad = \quad Jz,$$

with $z = [\check{e}\ \tilde{\beta}]^T$, and

$$J = \begin{pmatrix} -1 & -\frac{\nu}{C_0}\hat{e}_{2_{bl}} \\ -\frac{\nu}{C_0}\gamma\hat{e}_{2_{bl}} & 0 \end{pmatrix} z.$$

Therefore, we can define the next property:

Property 2: By replacing $\nu := 1/\alpha$, then the real part of the eigenvalues of J, for $\hat{e}_{2_{bl}} \in \{\mathbb{R}\backslash\{0\}\}$ are all strictly negative, i.e.

$$\lambda_1 = \mathfrak{Re}\left\{\left((-1+\sqrt{1-4\frac{\gamma}{\alpha^2 C_0^2}\hat{e}_{2_{bl}}^2}\right)/2\right\} < 0$$
$$\lambda_2 = \mathfrak{Re}\left\{\left(-1-\sqrt{1-4\frac{\gamma}{\alpha^2 C_0^2}\hat{e}_{2_{bl}}^2}\right)/2\right\} < 0$$

Proof of theorem 1: From the analysis given in V-A, V-B, V-C and V-D for the model (26) as $\nu \to 0^+$, we prove ULAS of attractor (18) by applying [11, Theorem 1].

VI. Simulations on half-bridge inverter

In this section, we perform some simulations in MATLAB/Simulink by exploiting the HyEQ Toolbox [13] to verify the properties of the closed loop (12)–(17).
Let us consider for these simulations, the parameters given in Table I.

TABLE I
Simulation parameters

Parameter	Convention	Value	Units
DC input voltage	V_{in}	96	V
Reference peak voltage	V_{max}	$220\sqrt{2}$	V
Nominal angular frequency	ω	100π	rad/s
Nominal load resistance	R_0	240	Ω
Estimated series resistance	R_{LS}	2	Ω
Inductor	L	50	mH
Output capacitor	C_0	200	μF

We assume that the load can vary in the following set:

$$R_0 = 240\Omega \in [120, 360]\Omega \Leftrightarrow \beta = 0.0042 \in [0.0028, 0.0083], \tag{33}$$

which corresponds to $\pm 50\%$ of variation with respect to the nominal value of R_0.

The desired trajectories are given following

$$x_e = \begin{bmatrix} v_{Ce} \\ i_{Le} \end{bmatrix} = \begin{bmatrix} 220\sqrt{2}\sin(100\pi t) \\ 19.5\sin(100\pi t + 86°) \end{bmatrix}.$$

Considering [10, theorem 2], let take the following cost function to get any LQ performance level.

$$J = \min_u \sum_{k\in \mathrm{dom}_j(\xi)} \int_{t_k}^{t_{k+1}} \frac{\rho}{R_0}(v_c(\tau,k)-v_{Cd})^2 + R(i_L(\tau,k)-i_{Ld})^2 d\tau \tag{34}$$

with $\rho = 1000$, and

$$P = \begin{bmatrix} 21.6862 & 0.1721 \\ 0.1721 & 0.0888 \end{bmatrix},\ Q = \begin{bmatrix} R_{LS} & 0 \\ 0 & \frac{\rho}{R_0} \end{bmatrix}.$$

It satisfies Property 1 and constraint given in (19). Finally, in order to avoid arbitrary fast switching, as mentioned in Remark 3, we introduce practically a sampling time $T_s = 10^{-6}s$. Moreover, from (1), (3) and the convex combination $u = \lambda_1 - \lambda_{-1}$ with $\lambda_1 + \lambda_{-1} = 1$, we get to stabilize e in 0 in Filippov sense, noted in Remark 2, with $\lambda_1 = 0.5 + 0.036\sin(100\pi t) + 0.32\cos(100\pi t)$, satisfying the condition (11). Furthermore, we take $\eta = 0.1$ which corresponds to a sub-optimal value that guarantees a trade-off between performance level and switching frequency, as shown in [10].

We select the convergence speed of the observer state, α, according to Remark 5. Thus, we need to satisfy $\max\{30,\ 40\} << \alpha$. For this issue, we choose $\alpha = 400$ and $\alpha = 4000$. Moreover, $\gamma = C_0\alpha$.

Finally, we force two load changes, in the transient time at $t = 0.001s$ and in the steady state at $t = 0.03$s, changing β of $\pm 50\%$. From $\beta = 42 \cdot 10^{-3}$ ($R_0 = 240\Omega$) to $\beta = 83 \cdot 10^{-3}$ ($R_0 = 360\Omega$) at $t = 0.001s$ and from $\beta = 83 \cdot 10^{-3}$ ($R_0 = 360\Omega$) to $\beta = 28 \cdot 10^{-3}$ ($R_0 = 120\Omega$) at $t = 0.03s$.

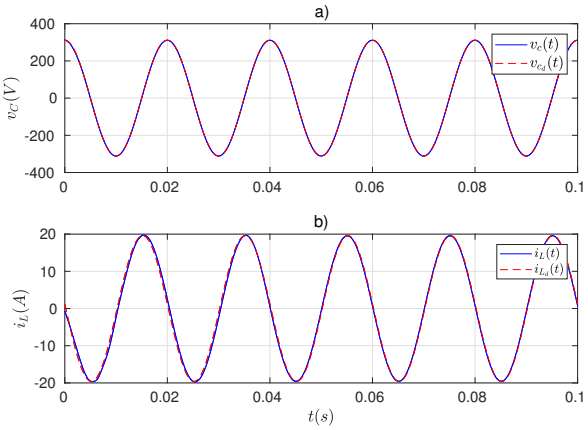

Fig. 3. Evolution of voltage and current for $\alpha = 400$.

Figures 3 and 4 show the evolution of the voltage and current for $\alpha = 400$ and for $\alpha = 4000$, respectively. In both cases, the states converge to their corresponding references. On the other hand, Fig. 5 and Fig. 6 show the evolution of $\hat{\beta}$ and $\check{e}_2$, under the load change of β in the transient time (t=0.001s) and in the steady state (t=0.03s), respectively. The simulations in Fig. 5 a), Fig. 6 a), Fig. 5 b) and Fig. 6 b) are performed for a value of $\alpha = 400$ and Fig. 5 c), Fig. 6 c), Fig. 5 d) and , Fig. 6 d) are done using $\alpha = 4000$. Note as α larger is, the convergence of $\hat{\beta}$ to β faster is. Similar performance is obtained with the error $\check{e}_2$.

Therefore, these figures show Theorem 1 statement.

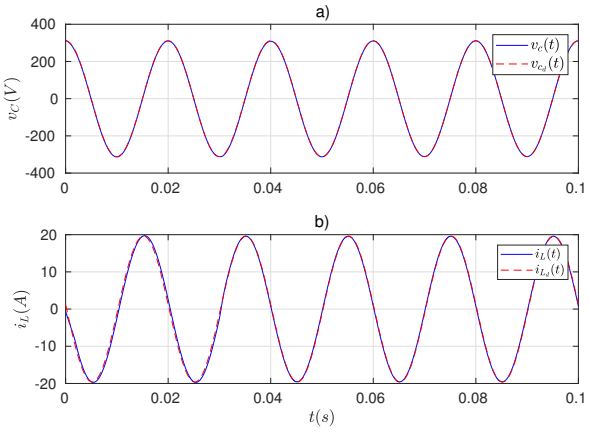

Fig. 4. Evolution of voltage and current for $\alpha = 4000$.

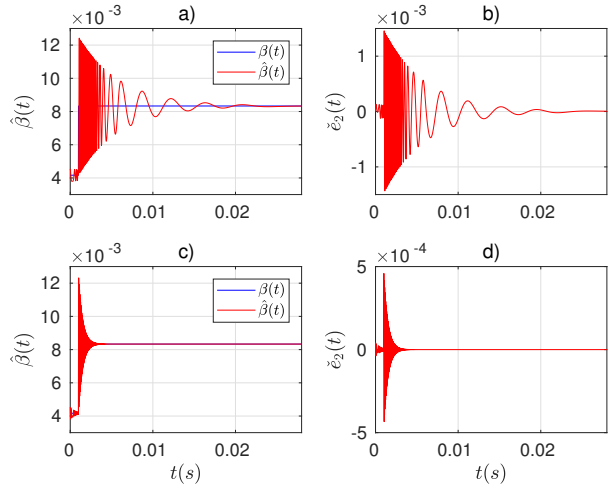

Fig. 5. Evolution of the estimation of β and the error $\check{e}_2$ in the transit time, for $\alpha = 400$ and for $\alpha = 400$.

VII. Conclusions and future works

In this article, we have proposed a hybrid adaptive control for a half-bridge inverter with unknown variable load, which guarantees the robustness of the convergence of the states toward the desired trajectories.

The interest of such approach is the use of a hybrid control scheme that considers the continuous-time dynamics as well as the discrete-time dynamics, avoiding the use an average control signal. An indirect adaptive control is proposed to estimate the unknown resistive load. Finally, ULAS of the full system is proven by using a standard singular perturbation analysis.

A future work is to reduce the switching in the steady-state by considering a time- or space-regulation in order to generate a minimum dwell-time.

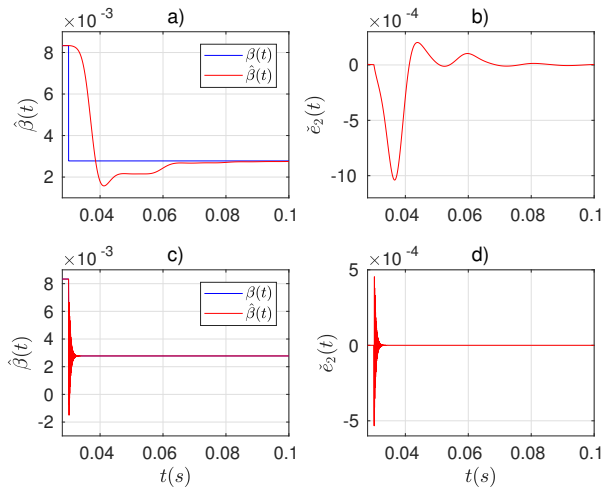

Fig. 6. Evolution of the estimation of β and the error $\check{e}_2$ in the steady state for $\alpha = 400$ and for $\alpha = 400$.

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