

# Dynamic programming approaches for the temporal knapsack problem

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## Appendices

This appendix reports the detailed results that are summarized in the paper. Table 1 recalls how each configuration of our algorithm we tested is parameterized.

Table 1: Parameters of the tested methods.

Configuration	Strategy	Criterion $\psi_i$
MCF*	Cardinality constrained	$\psi_i^2 (1 - \psi_i^3)$
Stable	Weighted stable set	$\frac{-\psi_i^2 + \max_{j \in \mathcal{J} \neq i} \psi_j^2}{\max_{j \in \mathcal{J} \neq i} \psi_j^2} \psi_i^3$
NbLabels	Cardinality constrained	$\psi_i^2$
Hybrid	Cardinality constrained	First $\psi_i^3$ , then $\psi_i^2$
LagMult	Cardinality constrained	$\psi_i^1$
KColor	K-Color	$\frac{-\psi_i^2 + \max_{j \in \mathcal{J} \neq i} \psi_j^2}{\max_{j \in \mathcal{J} \neq i} \psi_j^2} \psi_i^3$

Table 2: Computing time (in seconds) for each instance and for the best parameters of our algorithm (MCF\*), and degraded parameters. "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	Without partial enum.	Without feas. checks	Without Lag. mult.
U1	3,16	1,88	75,82	3,16
U2	2,02	4,43	52,27	2,08
U3	8,05	3,33	135,56	8,70
U4	3,26	2,43	64,07	3,29
U5	2,31	2,11	74,32	2,51
U6	2,29	2,14	73,55	2,27
U7	2,06	1,91	111,33	2,09
U8	2,06	1,98	89,95	2,13
U9	2,47	2,02	112,47	2,37
U10	2,31	3,11	50,70	2,31
U11	29,17	15,85	118,75	33,82
U12	45,11	23,62	202,95	38,23
U13	22,99	11,52	183,53	28,01
U14	39,67	23,36	217,70	54,21
U15	32,47	20,57	172,65	35,85
U16	33,54	21,98	226,57	43,06
U17	31,11	19,95	200,51	38,75
U18	30,24	14,31	188,12	38,84
U19	27,27	15,13	194,53	35,27
U20	41,77	21,71	204,11	41,53

Table 2: Computing time (in seconds) for each instance and for the best parameters of our algorithm (MCF\*), and degraded parameters. "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	Without Partial Enum	Without Feasibility	Without LagMult
U21	402,02	356,02	956,81	499,05
U22	200,90	120,87	387,18	3344,32
U23	199,28	157,89	460,09	1761,36
U24	228,90	200,88	927,27	548,04
U25	233,67	167,96	493,92	4539,24
U26	314,22	232,09	890,20	1147,02
U27	568,08	373,74	1090,35	956,36
U28	309,73	187,01	614,03	883,73
U29	476,89	412,86	1587,73	1519,72
U30	330,63	290,47	933,56	775,05
U31	1819,88	1304,23	3111,40	3924,08
U32	1878,91	1665,70	2717,74	1707,24
U33	1927,90	1221,23	6723,10	-
U34	4255,41	2741,86	-	-
U35	3404,56	2256,44	7557,14	3595,27
U36	4296,61	2180,70	-	6820,29
U37	1548,95	829,21	5491,99	1925,77
U38	2559,99	1774,11	4044,71	-
U39	7129,55	6395,53	-	-
U40	2463,76	1545,94	7915,86	2050,72
U41	6978,75	7886,43	-	-
U42	7866,61	-	-	-
U43	6229,77	4122,03	9722,41	-
U44	4699,10	5804,35	7079,77	-
U45	5451,66	9003,36	7578,04	-
U46	6493,66	5572,32	9203,96	-
U47	3300,61	3983,97	4196,38	-
U48	9696,93	-	-	-
U49	5152,23	7669,67	8068,14	-
U50	1224,82	2434,46	1493,59	-
U51	3776,10	7723,64	6048,46	-
U52	6504,37	-	8632,62	-
U53	2684,34	5676,62	3342,95	-
U54	4767,18	9622,51	6050,09	-
U55	7790,36	-	8505,44	-
U56	8249,76	-	-	-
U57	4308,46	8215,55	5152,76	-
U58	5144,68	8166,08	4402,04	-
U59	7587,86	-	8298,94	-
U60	6243,75	-	9241,08	-
U61	7967,24	-	9381,08	-
U62	5000,41	-	5033,60	-
U63	9289,11	-	-	-
U64	7408,70	-	8338,44	-
U65	4412,67	7951,08	6630,94	-
U66	2835,29	5076,61	2798,14	-
U67	5595,89	-	6882,07	-
U68	3494,36	7807,31	4234,24	-
U69	-	-	-	-
U70	2688,13	5532,26	2867,12	-
U71	-	-	-	-

Table 2: Computing time (in seconds) for each instance and for the best parameters of our algorithm (MCF\*), and degraded parameters. "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	Without Partial Enum	Without Feasibility	Without LagMult
U72	5107,50	-	5937,80	-
U73	2974,47	9870,35	3714,81	-
U74	1227,69	2460,48	1215,57	-
U75	9498,18	-	-	-
U76	4656,98	10603,35	5275,76	-
U77	3440,62	6735,61	4321,70	-
U78	-	-	-	-
U79	3726,25	7409,43	4517,62	-
U80	-	-	-	-
U81	5872,25	-	6523,62	-
U82	1822,24	3774,01	1822,33	-
U83	1506,61	2351,75	1409,55	-
U84	1014,26	1886,80	1049,82	-
U85	2066,76	3800,80	1993,27	-
U86	5443,40	-	5217,95	-
U87	3121,01	6376,74	3680,24	-
U88	2229,73	4477,31	2431,04	-
U89	2856,67	5607,44	3446,39	-
U90	-	-	-	-
U91	7565,47	-	7770,56	-
U92	-	-	9862,26	-
U93	2582,98	6512,43	3087,42	-
U94	607,07	1019,77	573,63	-
U95	2980,81	5785,34	2604,21	-
U96	1562,09	3429,97	1658,19	-
U97	968,16	1770,61	810,62	-
U98	2328,19	5141,22	2317,87	-
U99	2726,39	6527,84	2778,63	-
U100	698,49	977,78	500,68	-

Table 3: Computing time (in seconds) for each instance and for the best parameters of our algorithm (MCF\*), a basic version of SSDP applied on TKP without the improvements we propose (MCF), the algorithm of [GI14] (GI) and the ILP solver IBM Ilog Cplex on model (4)-(6) (CPLEX). "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	GI	CPLEX	MCF
U1	3,16	0,54	0,02	1123,87
U2	2,02	2,48	0,02	51,14
U3	8,05	0,31	0,02	9714,50
U4	3,26	0,25	0,02	-
U5	2,31	1,04	0,02	-
U6	2,29	0,25	0,02	-
U7	2,06	0,27	0,21	-
U8	2,06	0,3	0,02	-
U9	2,47	0,25	0,02	-
U10	2,31	0,34	0,02	-
U11	29,17	1,54	0,09	-
U12	45,11	6,99	0,29	-
U13	22,99	1,34	0,37	61,92
U14	39,67	11,43	0,32	-
U15	32,47	4,19	0,24	-

Table 3: Computing time (in seconds) for each instance and for the best parameters of our algorithm (MCF\*), a basic version of SSDP applied on TKP without the improvements we propose (MCF), the algorithm of [GI14] (GI) and the ILP solver IBM Ilog Cplex on model (4)-(6) (CPLEX). ”-” means that the algorithm could not solve the instance within three hours.

Instance	MCF*	GI	CPLEX	MCF
U16	33,54	35,68	0,28	-
U17	31,11	4,36	0,40	-
U18	30,24	4,88	0,30	-
U19	27,27	3,18	2,96	-
U20	41,77	13,51	0,15	-
U21	402,02	55,9	4,68	-
U22	200,90	18,64	1,10	-
U23	199,28	72,67	0,73	-
U24	228,90	20,34	1,22	3643,25
U25	233,67	54,06	1,71	-
U26	314,22	73,82	7,66	-
U27	568,08	44,07	2,68	-
U28	309,73	17	0,89	-
U29	476,89	242,31	1,80	-
U30	330,63	119,79	0,90	-
U31	1819,88	196,93	5,85	-
U32	1878,91	229,97	4,12	-
U33	1927,90	207,09	1,27	-
U34	4255,41	3726,14	27,14	-
U35	3404,56	575,07	18,04	380,60
U36	4296,61	2000,77	34,30	-
U37	1548,95	71,88	13,12	-
U38	2559,99	886,81	37,05	-
U39	7129,55	2190,42	36,53	-
U40	2463,76	692,63	30,01	-
U41	6978,75	6903,51	57,26	-
U42	7866,61	-	-	-
U43	6229,77	2690,56	19,93	-
U44	4699,10	1577,27	124,41	-
U45	5451,66	6844,36	6561,22	-
U46	6493,66	4445,63	1455,45	397,68
U47	3300,61	1217,71	530,65	-
U48	9696,93	3255,63	558,23	-
U49	5152,23	-	406,57	-
U50	1224,82	9054,96	57,91	-
U51	3776,10	2238,36	610,97	-
U52	6504,37	6540,73	3466,28	-
U53	2684,34	428,32	22,56	-
U54	4767,18	893,85	578,92	-
U55	7790,36	-	736,85	-
U56	8249,76	-	-	-
U57	4308,46	-	564,25	882,27
U58	5144,68	-	-	-
U59	7587,86	-	438,55	-
U60	6243,75	-	-	-
U61	7967,24	-	-	-
U62	5000,41	-	2165,84	-
U63	9289,11	-	-	-
U64	7408,70	-	-	-
U65	4412,67	-	2519,71	-

Table 3: Computing time (in seconds) for each instance and for the best parameters of our algorithm (MCF\*), a basic version of SSDP applied on TKP without the improvements we propose (MCF), the algorithm of [GI14] (GI) and the ILP solver IBM Ilog Cplex on model (4)-(6) (CPLEX). "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	GI	CPLEX	MCF
U66	2835,29	-	5998,32	-
U67	5595,89	9620,61	5313,91	-
U68	3494,36	-	1036,55	1817,05
U69	-	-	1606,55	-
U70	2688,13	-	467,26	-
U71	-	-	-	-
U72	5107,50	-	-	-
U73	2974,47	-	3511,75	-
U74	1227,69	-	4416,41	-
U75	9498,18	-	-	-
U76	4656,98	-	-	-
U77	3440,62	-	-	-
U78	-	-	-	-
U79	3726,25	-	-	193,73
U80	-	-	-	-
U81	5872,25	-	-	-
U82	1822,24	-	-	-
U83	1506,61	-	-	-
U84	1014,26	2257,06	2666,00	-
U85	2066,76	-	-	-
U86	5443,40	-	-	-
U87	3121,01	-	-	-
U88	2229,73	5371,19	-	-
U89	2856,67	-	5306,25	-
U90	-	-	-	1298,43
U91	7565,47	-	-	-
U92	-	-	-	-
U93	2582,98	-	-	-
U94	607,07	-	2356,61	-
U95	2980,81	-	-	6532,08
U96	1562,09	-	-	-
U97	968,16	-	-	-
U98	2328,19	-	-	-
U99	2726,39	-	-	-
U100	698,49	-	958,07	-

Table 4: Computing time (in seconds) for each instance and for the different configurations of our algorithm (see Table 1). "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	Kcolor	Stable	NbLabels	Hybrid	LagMult
U1	3,16	4,30	4,37	3,20	4,27	3,15
U2	2,02	2,18	2,18	2,05	2,22	2,01
U3	8,05	3,26	6,61	25,39	24,27	3,86
U4	3,26	3,49	3,47	3,29	3,43	3,27
U5	2,31	2,56	2,50	2,28	2,52	2,29
U6	2,29	2,46	2,44	2,28	2,45	2,34
U7	2,06	2,25	2,23	2,05	2,23	2,06
U8	2,06	2,27	2,30	2,06	2,29	2,20
U9	2,47	2,53	2,53	2,40	2,54	2,44
U10	2,31	2,45	2,41	2,28	2,40	2,31

Table 4: Computing time (in seconds) for each instance and for the different configurations of our algorithm (see Table 1). "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	Kcolor	Stable	NbLabels	Hybrid	LagMult
U11	29,17	24,86	30,91	29,19	30,85	28,98
U12	45,11	30,79	30,53	30,15	31,11	30,23
U13	22,99	19,85	24,13	23,26	24,32	22,90
U14	39,67	27,00	33,50	33,09	33,70	33,18
U15	32,47	32,54	32,74	43,65	33,06	32,11
U16	33,54	28,71	36,17	33,34	36,85	27,45
U17	31,11	32,65	33,72	45,16	32,59	31,11
U18	30,24	31,96	31,54	30,57	31,63	29,82
U19	27,27	21,19	26,71	28,29	27,59	27,44
U20	41,77	32,84	34,16	30,98	32,44	31,85
U21	402,02	363,62	242,59	371,17	252,06	483,09
U22	200,90	210,70	152,58	207,97	165,65	193,72
U23	199,28	217,74	180,13	259,81	183,96	206,01
U24	228,90	182,98	164,52	266,39	154,74	239,05
U25	233,67	222,90	176,87	248,37	174,03	231,74
U26	314,22	345,90	207,04	375,56	195,12	334,51
U27	568,08	668,54	241,03	503,88	268,25	451,72
U28	309,73	317,42	183,07	341,89	184,23	303,82
U29	476,89	532,33	278,08	558,37	275,36	512,27
U30	330,63	334,87	245,93	382,50	255,15	410,39
U31	1819,88	2765,39	1099,81	2079,20	1301,97	2216,36
U32	1878,91	2577,69	1333,30	2190,76	1681,58	1939,55
U33	1927,90	1669,99	750,74	1557,18	1027,61	2218,95
U34	4255,41	4437,43	1571,61	5284,24	2035,99	3664,80
U35	3404,56	3469,08	1370,17	4734,16	1523,89	4033,79
U36	4296,61	3454,96	1525,90	4829,19	1771,48	3747,05
U37	1548,95	1621,39	703,34	1699,32	864,16	2115,02
U38	2559,99	2666,27	1823,69	2196,95	1255,99	3852,43
U39	7129,55	8430,25	3792,99	9147,75	4578,79	8771,23
U40	2463,76	1591,28	1147,96	2279,09	1182,42	1890,91
U41	6978,75	8551,77	7970,67	7039,08	4630,46	7938,33
U42	7866,61	-	-	-	8895,44	8864,98
U43	6229,77	6729,64	2784,26	6680,04	2912,02	6250,26
U44	4699,10	7538,37	3721,25	4441,14	3790,64	5715,15
U45	5451,66	8057,71	5356,64	5612,56	6396,25	5185,05
U46	6493,66	9125,86	3760,40	5913,34	3823,00	6717,65
U47	3300,61	5469,61	1810,60	3010,22	2996,86	3631,64
U48	9696,93	-	-	9803,14	-	-
U49	5152,23	-	3795,10	5462,49	4544,63	6032,85
U50	1224,82	2418,71	1226,80	1452,91	1229,99	1497,17
U51	3776,10	-	5445,59	4695,00	5341,67	5878,03
U52	6504,37	-	7029,24	5722,99	7694,31	9193,44
U53	2684,34	5374,35	3001,95	3822,32	3424,51	3059,46
U54	4767,18	9632,70	5829,61	5340,91	6434,48	6353,95
U55	7790,36	-	8634,63	7542,14	8986,94	8745,47
U56	8249,76	-	9828,89	-	-	9965,03
U57	4308,46	7509,02	4040,47	4238,83	5080,17	5328,74
U58	5144,68	-	4820,64	5657,23	4894,79	5610,56
U59	7587,86	-	8754,78	8548,50	9499,11	5951,99
U60	6243,75	-	8599,58	7680,75	8059,78	9792,83
U61	7967,24	-	9809,90	8455,66	9132,19	-
U62	5000,41	-	4987,13	4690,07	5311,21	5086,59

Table 4: Computing time (in seconds) for each instance and for the different configurations of our algorithm (see Table 1). "-" means that the algorithm could not solve the instance within three hours.

Instance	MCF*	Kcolor	Stable	NbLabels	Hybrid	LagMult
U63	9289,11	-	-	-	-	-
U64	7408,70	-	9048,81	8083,06	9402,76	9943,09
U65	4412,67	7906,36	3848,64	5496,54	5247,44	4185,19
U66	2835,29	9672,36	3054,83	2985,93	3258,06	3452,69
U67	5595,89	-	5493,93	6062,00	7512,81	6907,83
U68	3494,36	6084,89	3686,61	3874,02	4772,09	3808,80
U69	-	-	-	-	-	-
U70	2688,13	4406,69	2584,61	2754,84	3384,86	2711,68
U71	-	-	-	-	-	-
U72	5107,50	-	5638,88	6083,78	6583,60	5491,18
U73	2974,47	7244,58	3104,87	3693,40	3845,70	3602,71
U74	1227,69	2486,13	1306,68	1460,18	1450,69	1450,25
U75	9498,18	-	-	9351,50	-	-
U76	4656,98	-	5797,05	5350,53	5776,38	7090,83
U77	3440,62	7527,05	4189,03	3818,83	4870,89	4635,45
U78	-	-	-	-	-	-
U79	3726,25	8832,63	4190,03	3950,26	5050,34	4157,69
U80	-	-	-	-	-	-
U81	5872,25	-	8374,61	6970,76	8279,53	9878,74
U82	1822,24	4504,49	1939,14	2011,70	2470,88	2538,53
U83	1506,61	3374,34	1556,77	1697,10	1664,36	1726,74
U84	1014,26	2434,49	1169,75	1104,78	1593,61	1462,48
U85	2066,76	4317,53	2116,15	2615,26	2233,86	2533,06
U86	5443,40	-	7207,21	6263,50	7993,67	7490,21
U87	3121,01	8157,50	4899,37	3751,39	4292,91	4987,38
U88	2229,73	4860,35	3063,91	2793,16	3296,29	3178,46
U89	2856,67	5647,90	3658,24	3225,07	4132,31	4487,62
U90	-	-	-	-	-	-
U91	7565,47	-	8548,51	6353,22	8904,16	-
U92	-	-	-	-	-	-
U93	2582,98	6546,20	2992,37	3453,67	4037,48	3567,62
U94	607,07	1078,72	718,61	630,33	748,85	868,40
U95	2980,81	5921,19	2407,82	2932,52	3972,40	4132,44
U96	1562,09	2793,35	1992,10	1825,48	1901,20	2140,26
U97	968,16	2320,25	1275,93	1056,35	1443,02	1287,42
U98	2328,19	4278,86	2327,92	2500,43	2777,59	2738,50
U99	2726,39	7971,74	3371,99	4097,15	4032,81	3838,40
U100	698,49	1308,97	735,17	700,66	819,76	852,73

## References

- [GI14] T. Gschwind and S. Irnich. Stabilized column generation for the temporal knapsack problem using dual- optimal inequalities. Working Papers 1413, Gutenberg School of Management and Economics, Johannes Gutenberg-Universität Mainz, 2014.