What would be Belgium without probability and probability without Belgium ? Paul Mansion and the scientific approach of randomness

Laurent Mazliak

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Abstract

The present paper studies how the Belgian mathematician Paul Mansion became interested in probability theory. The Belgian mathematical environment, in which probability was present more than in many other countries at the same time appears to have been favorable but also the fact that Mansion, a declared and militant Catholic, found in probability a source of reflection about determinism and randomness in the context of the "modernist crisis" in the Church. Mansion's activity developed on the background of the scholar wars and the foundation of Catholic institutions such as the Institute for philosophy in Louvain, of the consolidation of mathematical education in Belgium as well as of a new interest for probabilistic results in science. We expose how these aspects intersected at the turn of the 20th century

Introduction :

In 1902, the Belgian mathematician Paul Mansion (1845-1919) gave a conference at Brussels Royal Academy of Science about the objective significance of calculus of probability, subsequently published as the paper (Mansion, 1903). The first sentence of the article stated that *there are few countries where the calculus of probability holds so important a place as in Belgium*. And Mansion illustrated this fact by the presence of a course of probability in the syllabus of the *doctorat en sciences physiques et mathématiques*, corresponding more or less to a Master degree, since 1835 and since 1838 in the syllabus of the *hautes écoles techniques* (high technical schools). Let us recall that Belgium had obtained its independence from Netherlands only in 1831. It seems therefore that the rulers of this young country, at least those in charge of education, had paid a particular attention to the presence of the mathematics of randomness in the scientific education of the youth.

The contrast is striking when compared to the situation in France at the same time. A significant presence of probability in the syllabus of higher mathematical education is observed only after the First World War or even later. The difference between Belgium and France as emphasized by Mansion in his 1902 conference was already commented on in (Schneider, 1987) (see in particular p.193). In Paris, at the turn of the 20th century, the mathematics of randomness were the topic of a few lectures at the *Ecole Polytechnique* and the *Collège de France* taught in the 1880s by Joseph Bertrand (1822-1900), who did not hide his lack of consideration towards a topic he regarded above all as a source of recreation (though he admitted that it had some practical utility, for instance through the so-called law of errors)[2]. It was also the subject of a course by Henri Poincaré (1854-1912) at the Sorbonne in the 1890s. Poincaré was the first important scientist in France since Laplace, Poisson and Cournot, to consider probability theory with an acute mathematical look, because new physical theories,

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[2] On Bertrand's lectures, see for instance (Bru, 2006).

especially the kinetic theory of gases, required its use[3]. But in fact, France had to wait for Emile Borel and the 1920s to see the beginning of a large-scale transformation of the situation of probability in scientific higher education.

Though Belgium was, as today, a multilingual country, economical and high-culture elites - and therefore rulers – of that time belonged above all to the francophone community. The latter had been the leader of the 1831 revolution. Moreover these years were not so distant from the French Napoleonic administration time so that the south neighbor of Belgium was the great francophone power and an unavoidable model to take into account. Some institutions in Belgium were created under a clear French influence such as the justice organization which kept the Napoleonic code as its basis. Belgian students came to France for their higher studies in prestigious French institutions as the *Ecole Polytechnique* in Paris and most Belgian academics had studied for a time in France of were even of French origin. Very soon after the independence was proclaimed, however, Belgian rulers expressed their desire for developing a local educational system that enabled to produce the elites needed by the country without calling on their know-it-all southern neighbor in the first place, but also from their a little bit too self-interested neighbor on the other side of the English Channel. On the contrary, Germany, the young emerging European power on the East was considered with an increasing sympathy. An interesting account of this early evolution can be seen in the review of (Arendt, 1837), a book expressing much sympathy towards the young independent country written by W.A.Arendt, a German scholar in Liège. The review was anonymously published in the *Revue de Bruxelles* in 1837 (p.149-167). The reviewer insisted on the fact that French influence in Belgium was generally considerably overestimated abroad. For the reviewer, Arendt's book was to celebrate because it was an example of the German genuine interest for Belgium, while French and British visitors were often only concerned with the profit they could withdraw from the new and fragile country. Gradually, and mostly after 1870 when the Bismarckian victory over France resulted in the emergence of the German Empire, Belgium's Eastern neighbor became the main attractor and a new source of inspiration. Many students came to Germany for the continuation of their education.

However, and more significantly, there were also many aspects in Belgian society that depended only on absolutely local decisions or on the particularities of the local situation. This was due in particular to the presence in the country of some influential actors who proposed genuine original orientations and favored the emergence of their country as a leader in some specific domain. Belgium had a champion for the mathematics of randomness in the person of Adolphe Quetelet (1796-1874). During almost 40 years, Quetelet had been at the same time the most influential statistician in the world, a powerful man of action, a formidable academic politician, and the true personification of the Brussels Academy. Quetelet's influence on the limited Belgian mathematical stage was so high that it was quite natural for probability to become a central topic in higher education.

This kind of situation can be observed in various places and times: being a scientist (but the observation is certainly also true for a writer, a philosopher and so on) in a small and young country is at the same time a misfortune, because of isolation, and a great bit of luck for a man of action because it offers a virgin land on which it is possible to build. Such an example, in a completely different environment and time is also provided by the Czech mathematician Bohuslav Hostinsky who obtained his first position in the newly created Masaryk university in Brno in the newly created Czechoslovakia in 1919 and exploited this a priori unfavorable

[3] On Poincaré and probability, see for instance (Mazliak, 2015) and numerous references included.

situation to the full so that at the eve of WWII, Brno had become a major school in mathematical physics in Europe under his impulsion (Mazliak and Šišma, 2015). This is what Quetelet produced in Belgium with probability. As its geographical position made Belgium a transition place between the German and the French cultural zones, Quetelet's task was to expose the German probabilistic techniques (law of errors, least squares and so on) along with the French rigor in analysis. (Jozeau, 1997) has studied in details the situation of geodetic and the shift of leadership from France to Germany, seen from the Belgian point of view. She showed how after Quetelet, Belgian mathematicians as Guillaume-Adolphe Nerenburger (1804-1869), Anton Meyer (1801-1857) or Jean-Baptiste Liagre (1815-1891) dealing with geodetic, particularly in relation with their teaching at the school for artillery, considered Laplace too complicated and Cournot too philosophical and elementary for application, and preferred to use Gauss, Struve and Bessel's approach.

Mansion, born in 1844, belonged therefore to a generation entirely educated inside the Belgian system so that he developed his ideas rather freely from any external influence. It is not by chance that Mansion's first publication concerned probability, though probability theory remained only a minor part of his activity in the sequel. But during his whole life he remained interested by the topic and succeeded in maintaining a small independent network of probabilists around him, based on a rather original personal agenda that partly explains why Mansion remained isolated from other expanding circles of development of probabilistic thinking of the time. The present paper aims at giving some information on this highly original and today relatively forgotten character and at explaining Mansion's vision of probability, strongly articulated with some scientific debates of the time and his deep personal catholic engagement. The first part of the paper is devoted to general biographical information on Mansion. The second part studies Mansion's interest for probability. Finally, in the third part we examine what Mansion's 1902 conference at Brussels Academy reveals of his vision of probability as well as of his understanding of the importance of probability for dealing with some urgent scientific questions, especially for approaching the theory of evolution after Darwin's seminal works.

Mansion was one of the most active Belgian mathematicians during more than forty years, especially on the institutional stage. He was therefore at the center of a large network of scientists with whom he exchanged ideas, letters and publications, and historians are lucky as his personal papers were largely preserved in several archives such as the Royal Library in Brussels, the archive of the University of Ghent, the Royal Academy in Brussels. The extent of Paul Mansion's correspondence is impressive, and, happily, the archive of several of his important correspondents are also accessible: an instance is Pierre Duhem's papers in Paris Academy of Science containing some 50 letters from the Belgian mathematician. Mansion is therefore a scientist about whom a lot of information is accessible, and we were able to make an extensive use of these sources to feed our study.

1- Paul Mansion : A Belgian and Catholic mathematician

a- Mansion, a young mathematician of an emerging Belgium

Mansion's disciple Alphonse Demoulin, who became in 1893 his colleague in Ghent and was himself a reputed specialist in differential geometry, collected many details about Paul Mansion's childhood. Paul Mansion was born in 1844 in Marchin-lez-Huy in the province of Liège. He was the ninth child of a family of ten, and his father Paul-Joseph Mansion was a communal receiver. Paul-Joseph was fifty-three years old at Paul's birth and died soon after.

The family remained nevertheless relatively well off as Paul-Joseph had acquired several properties and lands. Paul's mother died in turn when he was only seventeen and his older brothers took care of him.

In 1844, Belgium was still quite a newly independent country. If formal independence had been acquired from Netherlands in 1830 after a short revolution and the access of Leopold I to the throne, it was only in 1839 that the king of Holland accepted to sign the treaty of the 24 articles through which he accepted the borders of Belgium and the neutrality of the new state, under the guarantee of the Great Powers: the violation of this treaty by Germany in August 1914 would force the United Kingdom to enter the war. The school question appeared early in the political life of the new state, that is, more precisely, the necessity to fix the role played by the Catholic church in public education. We shall return at length to the religious question in education later in this paper. Let us just mention here that the liberal Prime-Minister Jean-Baptiste Nothomb (1805-1881) obtained in 1842 a compromise with the Catholic party. Each commune was due to have a first-degree school for children but this school may either be maintained by the commune (*subsidiée*) or *adopted*, meaning that an existing private school was officially chosen by the commune which provided funding for this service, and, if necessary, a building. The law stipulated that teachers must be graduated and that diplomas had to be issued only by state-approved schools. Religion was part of the syllabus and had to be taught by Catholic priests, so that the Church kept a right of inspection in the schools (Deprez, 1970).

Mansion entered the communal school of Marchin when this regulation was in force. His master, Jean-Joseph Blaise, belonged to the category of state graduated teachers. A brilliant pupil, Mansion pursued his studies in the secondary school in Huy. When he was eighteen, in 1862, he decided to become himself a teacher of sciences and was admitted to the *École Normale* in Ghent to follow the training courses for teaching in secondary schools. After having taken care of the primary schools, the Belgian government decided to reorganize secondary schools. In 1847, special training courses (*cours normaux*) were created in the state universities of Liège (for the humanities) and Ghent (for the sciences) to prepare students to a cooptation (*agrégation*) as teacher[4]. But this proposition was rejected: other faculties, such as the Law faculty of Ghent and the Medicine faculty in Liège feared a possible contamination resulting in a drastic decrease of the number of students. The law promulgated in 1850 listed the requirements to become habilitated to teach in public secondary schools: the student had to obtain a cooptation from a normal school or a *doctorat* in sciences or in humanities. The mentioned doctorate was in fact conceived as a final diploma, in the spirit of the medieval universities, and not in the modern sense of a genuine research work. The research doctorate, the equivalent of a PhD, was instituted some years later as the special doctorate (*doctorat spécial*).

In fact, the government had a clear preference for selecting the future teachers of public secondary schools -- in particular of the most reputed Athénées -- among the *agrégés* of the *École Normale*. In 1852 indeed, the training course for future teachers in the normal school was organized as a genuine special section dedicated to pedagogy but also to a better disciplinary mastering under the noticeable influence of the French model of the *École Normale Supérieure* in Paris. This policy resulted in an almost total institutional independence of the normal schools from the university: the duration of studies gradually increased from two years to four years

4 Joseph Roulez, the rector of Ghent, had even proposed a more radical evolution in order to rationalize higher education in the country: suppressing altogether the humanities section in Ghent and the scientific section in Liège.

and the students presented the final exam before a specific jury and not a commission appointed by the local faculty[5]. Mansion had two first-rate teachers during his schooling on Ghent, the mathematicians Félix Dauge (1829-1889) and Mathias Schaar (1817-1867). About Dauge, Mansion declared later that he had never forgotten his lectures about mathematical methodology, a topic on which Dauge had published a celebrated book in 1883[6]. Schaar, an original polymath, taught arithmetic and analytical mechanics. In 1865, Mansion graduated from the *École Normale* as an *agrégé* and, in October 1865, he began teaching at the preparatory school for civil engineering in Ghent where Schaar was inspector. Probably as Mansion, at this stage of his career, had the desire to get a university degree (which the *agrégation* was not), he also defended a doctorate in physics and mathematics on 13 August 1867. He may have been encouraged to do so (maybe by Schaar himself), as Schaar, seriously ill, could not teach anymore since January 1867. Schaar died very soon (in April) so that his position had become vacant. Actually Paul Mansion was appointed to the chair of Differential and Integral Calculus and Higher Analysis of the University of Ghent on 3 October 1867 at the quite young age of 23 years. This appointment seems to result from a bet made by Dauge and Schaar to make sure that their brilliant *protégé* stays in Ghent.

b- A mathematician in Ghent

Mansion's appointment in Ghent appears to have consolidated a local mathematical tradition well implanted since the beginning of the century. The founding figure of this development of Ghent mathematics had been the French mathematician Jean-Guillaume Garnier (1766-1840) who arrived in 1817 following a rather original trajectory. Born in France in 1766, Garnier received all his higher mathematical education in Paris and was particularly close to Laplace. In 1795, he was chosen as an examiner for the aspirants to the *École Polytechnique* and the *École Normale* meaning that he was supposed to travel across France to uncover promising young men who could become the first recruits of the newly created institutions. His peregrinations led him to Auxerre where he met the young Fourier and convinced him to go to Paris. Fourier and Garnier always remained close friends. In 1800, however, Garnier made a diplomatic mistake, refusing to help the touchy, and already almighty, Laplace to edit his monumental treaty of celestial mechanics. Laplace asked his second favorite disciple, Poisson, to do the work and Garnier lost Laplace's support. At the time, this meant that Garnier was set aside from forefront mathematical life in Paris[7]. In the 1810s, he published several textbooks in geometry and mechanics that had been quoted by mathematicians as Gergonne in France or Grünert in Germany in the next decades. Garnier's complicated relationship with Laplace at least partly explains why he accepted the Dutch king's offer to get the chair of mathematics in Ghent when the university was founded in 1817.

In a notice devoted to Garnier, Mansion wrote the following comments, that he probably obtained first-hand from his older colleagues who had known Garnier well.

> *[Garnier's] teaching was diffuse and boring; on the other hand, he had with his students of personal relationships and his rewarding conversation, clear and demonstrative, was eminently exciting and full*

5 On the evolution of the Belgian *Écoles Normales*, see (Dhondt, 2011), in particular, p.202-204.
6 On Dauge and Mansion, see (Voelke, 2005), p.371.
7 On Laplace's domination over the French mathematical stage of the time, see for instance the comments by Humboldt in (Knobloch, 2012).

> *of accurate historical information on recent progress of science. Also, it can be said of him that he was the main renovator of the study of high mathematics in Belgium, by his works and especially by his students. Among these will be Quetelet, Timmermans, Verhulst, Lemaire, Ed Lannoi, L. Casterman, A. Leschevin, Mareska, Ch. Morren, E. Manderlier, Fr. Duprez, A. Goethals. Garnier was pretty much the only professor in the Faculty of Science from Ghent who did not do his lessons in Latin. He was one of the founders and collaborators of the two scholarly collections of the time, the Annals of Belgium and Mathematical and Physical Correspondence.*

As wrote Mansion, the outstanding importance of Garnier's presence in Ghent can be seen through the list of his students who formed the first generation of mathematicians of the independent Belgium, with Quetelet in the first rank among them. Garnier supervised Quetelet's thesis on conical sections, defended on July 1819 in Ghent. In 1825 the two men founded the first Belgian journal specialized in mathematics, entitled *Correspondance mathématique et physique*[8].

As already mentioned in the introduction of the present paper, Quetelet played a central role to impose the mathematics of randomness as a central topic in Belgium. This point will be developed later but let us right now observe the significant fact that Mansion's first publication in 1868[9] was devoted to a probabilistic question. The paper *On the problem of points* (Mansion, 1870) was submitted as a memoir to the Royal Academy of Sciences in Brussels and published in 1870 by the Academy. It is not clear why Mansion had chosen this topic that may have been suggested by his colleague Emmanuel-Joseph Boudin (1820-1893) who taught an original course of probability at the time in Ghent (see the next part). Anyway, the paper (Mansion, 1870) was an occasion for Mansion to get acquainted with the papers of another mathematician who would play an important role in his scientific life, a professor in Liège, the French born Eugène Catalan (1814-1894). Apart from a mention of Poisson's research on Pascal's problem of points, a paper proving some results of combinatorics, published by Catalan in the *Journal de Liouville* in 1842[10], was indeed the only reference in Mansion's paper. Using Catalan's results, Mansion proved that it was possible to obtain the solution to the general problem of points only with combinatorics, an alternative to the use of integral calculus as Laplace had done or of a more probabilistic direct approach as Poisson's one.

As it had been the case for Garnier, but for quite different reasons, Catalan's trajectory had been original and eventful[11]. Catalan was born in Belgium (then a part of the French Empire) in Bruges in 1814: his mother was Belgian but was not the legal wife of his father, a Parisian jeweler who traveled in 1813 in the Netherlands probably for professional reasons. However, they eventually married in 1821, and Catalan came to Paris at the age of 11. He studied science and entered the *École Polytechnique* in 1833. He met Liouville there, and both always remained in friendly terms. Already during his schooling at the *École Polytechnique*, he began to have some troubles with the police due to some political Republican activities. With Liouville's help Catalan obtained a position of teacher of descriptive geometry at the *École*

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8 On Garnier and Quetelet, see (Droesbeke, 2015).
9 (Mansion, 1870)
10 (Catalan, 1842)
11 For all the details about Catalan, we refer to (Jongmans, 1996).

Polytechnique and was also a teacher at the lycée Charlemagne in Paris. Catalan's mathematical research mainly focused on combinatorics with noticeable connections with other fields such as continuous fractions and theory of numbers (he introduced precisely the Catalan numbers to solve a combinatorics problem).

Catalan's enthusiastic adhesion to the 1848 revolution and his deep republican feelings, resulted in a complicated situation for him after Louis-Napoleon seized the power in France in December 1851. In May 1852, he refused to take the oath of fidelity to the Prince-President and was immediately dismissed from his teaching positions[12]. Though he had enough contacts in the scientific world to be able to make a living from various expedients, his mathematical career was clearly in a stalemate. Fortunately, Catalan had several Belgian mathematicians in his network. He was related to Quetelet. They got acquainted when, in 1840, the Brussels Royal Academy of science crowned Catalan's memoir on change of variables in multiple integrals. Quetelet was then the secretary of the Academy and expressed his satisfaction that a prize was offered to a Belgian compatriot[13]. Another member of Catalan's network was the physicist Michel Gloesener (1794-1876) from Liège. Both men met when Gloesener attended the seances of the Société philomatique when he was in Paris[14]. In 1864, the Liège mathematician Mathias Schaar (1817-1867) had to resign because of health problem, and Gloesener thought that this would be a good opportunity for Catalan. He asked a colleague from Ghent, Ernest Lamarle (1806-1875), to support Catalan's application with his friend the Minister Vandenpeereboom[15]. On January 1865, Lamarle wrote to Catalan to warn him that the chair of mathematics was vacant in Liège. Catalan immediately accepted the proposal and began to teach algebra, calculus and probability in Liège.
Though probability was not a major topic in Catalan's publications, he had continuously devoted some interest to the discipline, probably in connection with his focus on combinatorics. Moreover, as a student in the *École Polytechnique* during the 1830s, Catalan had followed the almost unique high-level teaching in probability at the time in France, though few students seemed to have been seduced by it: among them, more or less, Bravais and Catalan were the only future mathematicians. (Jongmans and Seneta, 1994) for instance has investigated how Catalan in 1841 had proposed a martingale model of successive drawings of balls from an urn.

In 1869, Mansion contacted Catalan, and probed him about the possibility of embarking on a PhD (*doctorat spécial*) about elliptic functions. In his detailed answer on 20 October 1869, Catalan explained that the Belgian system for the PhDs remained still quite obscure to him, but he was encouraging and proposed to Mansion to come and visit him in Liège. This was the beginning of a close friendship between the two men, despite their difference in age (thirty years) and political views. In 1870, with the help of Catalan, Mansion completed a PhD on the theory of multiplication of elliptic functions, subsequently defended in Liège. Despite of the vibrations resulting from the Franco-Prussian war (but the fact that Catalan was in Belgium probably considerably simplified the matter), the network-man Catalan managed also to have Mansion invited in Göttingen for a research stay with Alfred Clebsh (1833-1872) and Ernst Schering (1833-1872). Clebsh was Riemann's successor in Göttingen and he was considered as one of the best expert on elliptic functions of his time. A small collection of letters from Clebsh

12 (Jongmans, 1996), p.60

13 (Jongmans, 1996), p.37

14 (Jongmans, 1996), p.66

15 (Jongmans, 1996), pp.89-90.

to Mansion is kept in Mansion's archive and would certainly deserve a detailed study. But their relationship brutally ceased, as Clebsh died very young, two years after Mansion's visit to Göttingen. Mansion's journey to Germany is also significant of the already mentioned gradual change in the Belgian mathematical community. The looks of Belgian mathematicians turned more and more towards Germany.

In 1871, Mansion married Cécile Belpaire, the daughter of the engineer Alphonse Belpaire (1817-1854), who played an important technical role in the newly independent state. The Belpaire family was wealthy and well introduced in the leading and intellectual circles of the country and this wedding certainly facilitated Mansion's access to these circles and his role of an academic authority in Ghent despite his young age.

In 1874, Catalan decided to resume the publication of Garnier and Quetelet's journal *Correspondance mathématique* under the transparent new title *Nouvelle correspondance mathématique* and asked Mansion to second him. Though the new journal was supposed to deal with mathematical topics taught in the upper classes of secondary school and in sections for engineering schools, Catalan, who was from 1876 at 1880 its main inspirer, gradually raised the level, so that the journal, being less and less adapted to the needs of education in Belgium, was a failure. In 1881, it was replaced by a new publication, *Mathesis*, founded by Mansion and Joseph Neuberg (1840-1926). The program was the same as the primitive program of the *Nouvelle correspondance* but Mansion and Neuberg *had the wisdom to keep Mathesis at the level they had assigned to it, so that it could live and develop*[16]. They produced the journal without interruption for thirty-five years until the end of 1915 when the war interrupted the publication.

Mansion's central research topic was analysis. He was a prolific author and wrote more than 400 papers, a lot of them being published in one of the journal he edited (*Nouvelle correspondance, Mathesis*) or in the transactions of Brussels academy of sciences. Mansion was also a regular contributor to the British journal *Messenger of mathematics*. In 1873, he won a prize attributed by the Belgian Academy of Sciences with a memoir on the theory of partial differential equations of the first two orders that was subsequently published in Paris as a book in 1875[17], which was largely diffused in Europe. Mansion also undertook a huge work of referencing Belgian mathematical activity, for instance in the Italian Bulletin of Boncompagni. Since the very beginning of his career, Mansion also published a lot of studies on the history of science and numerous obituaries of Belgian colleagues. In 1882, he was elected a corresponding member of the class of science of the Royal academy of Belgium, and then a full member in 1887. Only in 1892 he was officially put in charge of the course of probability in Ghent, when he replaced Emmanuel-Joseph Boudin (1820-1893) to the chair of calculus of probability at the university. However, we shall see later that Mansion never left his interest for the mathematics of randomness after his 1868 work we have commented on.

c- A Catholic scientist in the Modern crisis

An original aspect of Mansion needs to be taken in consideration to understand his scientific engagement. During his whole life, he was a convinced catholic, and a deeply committed militant for his faith. At the time of his activity and in the country he belonged to, this aspect is by no means secondary. The Catholic Church was extremely present in the political processes

16 (Demoulin, 1929), p.107
17 (Mansion, 1875)

of the new independent Belgium, in particular in the discussions about the control of higher education. In 1834, a catholic university was founded in Mechelen under the impulsion of the assembly of bishops, provoking an immediate reaction from anti-clerical liberals who founded Brussels free university (among the founders was Quetelet who belonged to freemasonry). The Church asked then the Government to decide to close the state university in Louvain and to allow the transfer of the new small and weak university of Mechelen to Louvain: this was done in 1835. The only remaining state universities in the country were Liège and Ghent. In Ghent, as already mentioned, Mansion obtained his first position in 1867. During the almost fifty years of presence in this not-religious institution, Mansion openly displayed his catholic involvement and probably saw himself as a defender of the faith.

The relationship between the Belgian state and the Catholic Church had been a subject of recurrent tension since the 1840s. On 1 June 1850, the promulgation of a new law about the *enseignement moyen*, in which the state created new public institutions for the education of the youth (the "Athénées" and the *écoles moyennes*), had generated harsh protestations from the assembly of Belgian bishops[18]. In Ghent, Mansion observed the Belgian first school war, that raged in the country between 1879 and 1884 after the new liberal minister, Pierre Van Humbeeck (1829-1890) decided to propose a new law on the public educational system which considerably restricted the influence of the Church, which in Belgium still played an even more important role than in France. In France, indeed, anti-clericalism had acquired momentum for decades, and the recent fall of the clerical Second empire and the new Republican majority in Parliament gave a strong impulse to the laws on education of the 1880s (recall the picturesque event of the removal of crucifixes in the public schools in 1881). Both parties in Belgium probably scrutinized what was happening in France with major interest and this generated violent reactions[19]. The extreme tension in France at the beginning of the 20th century, in particular when in 1901 the French government decided the expulsion of a great number of teaching congregations, resulted in a complicated situation in her neighbors, and especially in Belgium. Between 1901 and 1903, the country experienced the arrival of thousands of religious, men and women, from the south. If some of them chose a contemplative life and the integration in a Belgian convent was relatively easy, it was more complicated for those who chose to continue their teaching mission, in particular because they were concentrated on a relatively small number of places: most of the French teaching institutions logically chose to settle down in great cities or close to the border, in order to facilitate the access for French pupils. This implied local and political tensions. The anticlerical party (the liberals and the socialists) tried to exploit the situation in order to attack the catholic government in Brussels, alleging that the French institutions and religious created an unfair competition -- a classical comment about exiles -- with their Belgian counterpart[20].

Mansion had in fact numerous activities connected to his catholic devotion. He was a member of the congregation of Our Lady of the Seven Sorrows, a member of Saint Vincent de Paul society in Ghent, and remained close to the Jesuits during his whole life. But, more important for the present paper's topic, very early in his scientific life, Mansion had been concerned with the possibility of conciliate his faith and his professional life. An occasion was given in 1870

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18 (Fassbender, 1969)

19 The instance of the region of Tournai, close to the French border, had been studied in (Soete, 1980): at the end of the 1870s, the town was at the same time an anti-clerical liberal stronghold as well as the see of one among the most ultramontanist bishops of Belgium, Edmond Dumont (1828-1892). Dumont's intransigence would besides lead the pope Leo XIII to dismiss him.

20 On this point, see (Leplae, 2005).

when he created the *Cercles Cauchy* with his friend the engineer Charles Lagasse de Locht (1845-1837)[21]. The idea was to institute regular meetings of several of the brightest students in Ghent to counter atheistic propaganda based on alleged scientific arguments. Lagasse had thought at the beginning that the *Cercles* would be called after Leibnitz, but Mansion insisted to replace the protestant philosopher by the catholic mathematician Cauchy. Due to Mansion and Lagasse's activity, other *Cercles Cauchy* appeared in Belgium. The Belgian Jesuit and mathematician, a teacher at Saint-Michel College in Brussels, Ignace Carbonnelle (1829-1889) instituted a *Cercle Cauchy* in Brussels, and some years later, in 1875, founded Brussels Scientific Society. The explicit motto of the society was to oppose anti-religious positivism. Its aim was to help catholic scholars to promote the advancement and diffusion of science in order to fight rationalist and atheist doctrines and to establish a dyke against emerging materialist and anti-religious movements. The Scientific Society of Brussels was very active and published the proceedings of its meetings under the title *Annales de la Société Scientifique de Bruxelles.* Mansion had been one of the founding members of the society. In 1877, a scientific journal was launched and published in Louvain and in Paris, under the title *Revue des Questions Scientifiques* – still existing today -- to which Mansion gave a lot of contributions. He presided Brussels Scientific Society during the year 1889-1890. (Nye, 1976) provides a deep study of the first years of Brussels society, and emphasizes how questions about determinism were a central concern of its members in those years. It is noticeable that among the French members of the society, at the side of Hermite or Pasteur, one finds a Catholic mathematical physicist who had a profound reflection about randomness and probability such as Joseph Boussinesq (1842-1929), then a professor in Lille and the future successor (in 1896) of Poincaré at the Sorbonne to the chair of Probability calculus and mathematical physics.

Mansion was close to another important prelate, and future cardinal, Désiré Mercier (1851-1926). Following Mercier's biograph Laveille, while he was a student of theology, Mercier's interest for science prompted him to attend various lectures from Mansion in Ghent[22]. Certainly, their shared Catholic involvement was important in their relationship from the beginning. A letter Mansion sent to Mercier some years later (in 1882) gives an interesting insight into Mansion's feelings about the Belgian academic situation with regard to Catholicism. At that time, a cousin of Mercier was a student in Ghent. Mansion wrote to Mercier :

> *I will do my best so that your cousin Gustave [..] does not lose the faith in the dangerous environment in which he finds himself. He has already come to see me twice, this augurs well and he spoke to me quite frankly about his ideas. I think I have hired him to become a member of the congregation; I will engage him again at the next opportunity and I will also recommend him to be part of the small Catholic society of the students of Ghent University*[23]*.*

Mercier was probably the most famous Belgian clerk in the 19th and 20th centuries. Of course,

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21 On the creation of the Cercles Cauchy by Lagasse and Mansion, see (Lagasse, 1920) and (Hilbert, 2000), p.53.
22 (Laveille, 1928), p.64.
23 Je ferai mon possible pour que votre cousin Gustave [..] ne perde pas la foi dans le milieu dangereux où il se trouve. Il est déjà venu me voir deux fois, ce qui est de bon augure et il m'a parlé très franchement de ses convictions. Je crois l'avoir engagé à se faire de la congrégation; je l'y engagerai encore à la prochaine occasion et je lui recommanderai aussi de faire partie de la petite société catholique des étudiants de l'université de Ghent". (Mansion to Mercier, 12 November 1882)

it is for his activity during WW1 that he is mostly remembered today, but the role he played in the Belgian intellectual life began much earlier. A study of this important personality is provided by (De Maeyer and Kenis, 2017). Deeply interested in philosophy, natural sciences and mathematics, Mercier was ordinated priest in 1874 and defended a doctorate in theology at Louvain in 1877 at the precise moment when the reign of Pope Pie IX was finished and his successor Leo XIII was elected (in February 1878) to the papacy, with a strong desire of providing the Catholic Church with tools to face the modern world. The intellectual crisis created new scientific theories - Darwin's theory of evolution in the first place (the Origin of Species was published in 1859) - provoked deep shocks among the Catholic elites and the extremely rigid dogmatic answer given by the old pope (in particular the 1870 council of Vatican) was seen with worry by many who thought that would increase the gap between the Church and academics. This was an opinion Mercier shared: he was convinced that the rigidity of old scholastic caused a real counterproductive glaciation, due primarily to the fact that clerks and theologians mostly were scientific illiterates. Mercier, who had a strong personal relationship with the new pope, took this opportunity to implement his intellectual institutional agenda, whose aim was the emergence in the Church of a renewal of scholastic philosophy (the so-called neo-Thomism) along with a resolutely positive attitude towards modernity, especially when it was based on a scientific approach. Mercier was indeed the leader of the progressive current of neo-Thomism. He was an opponent of much more conservative prelates such as the cardinal Tommaso Zigliara (1833-1893) who essentially saw neo-Thomism as a shield against modernity. On the contrary, Mercier defended the idea that scientific activity must be guaranteed a relative autonomy and freedom. Mercier's major realization was the creation in Louvain of the Higher institute for philosophy (*Institut supérieur de philosophie*) in 1889. In the pre-project presented for Roman approbation in 1887, Mercier suggested that the organization of the institute would provide every day lectures on philosophy in the morning, and scientific conferences in the afternoon[24].

The institute would become soon one of the main research centers on neo-Thomism and spiritual reflection on science. The institution was therefore particularly concerned by the modernist crisis in the Catholic Church between 1890 and 1910, with the rather sharp braking given by Leo XIII's rigid successor, Pie X, in 1907 in his encyclical *Pascendi dominici gregis*. However, Mercier's influence prevented major threats for his Louvain institute. He was both a man of great intellectual perspicacity and a skillful tactician (with some taste for authoritarianism), Mercier succeeded in creating in Belgium a catholic intelligentsia gravitating more or less closely to Louvain[25].

Mansion remained anyway an active member of Mercier's circle. (De Raeymaeker, 1951) mentions that Mansion was Mercier's correspondent for mathematics in Louvain. Mansion read a series of conferences on the fundamental principles of mathematics during the first year of the institute 1890-91 and on the fundamental principles of mechanics during the second one 1891-92[26]. He wrote several papers for Mercier's journal *Revue néo-scolastique* founded in 1894 as the official publication of the institute[27]. The correspondence between Mansion and Mercier kept in Louvain contains only four letters, though it is reliable that many more letters were exchanged between the two. What remains is however significant and eloquently proves

24 See (De Raeymaeker, 1951), p. 534.
25 See De Maeyer-Kenis and (De Raeymaeker, 1951), p.536.
26 (De Raeymaeker, 1951), p.538.
27 In particular, (Mansion, 1896a), (Mansion, 1896b), (Mansion, 1908), (Mansion, 1920a).

that both men were deeply concerned by the role the church must play in the education of youth. In a letter dated 27 November 1906, Mansion wrote:

> *I find that the Catholic press does not attack neutral teaching well. It should constantly say and repeat that it is anti-pedagogical because it does not develop the religious faculties of the child in a harmonious way with others faculties; as a result, it is an inferior education from a literary and scientific point of view*[28].

Mercier and Mansion had a good opportunity for exposing their views to a broader audience at the occasion of the third international congress of catholic scientists held in Brussels between 3 and 8 September 1894. These congresses had been launched in 1886 under the impulsion of another clerk, Maurice Le Sage d'Hauteroche d'Hulst (1841-1896). D'Hulst was, as Mercier, convinced of a necessary modernization of the relationship between the society and the Church, and he was especially interested into improving mutual dialogue between academics and Catholics. The situation for d'Hulst, who was French, was in fact much more difficult than for Mercier, in the context of a strong anti-clerical climate of the Third Republic of the 1880s and of the process of laicization of education. The Minister for public instruction Jules Ferry for instance, asked the parliament, which had now a clear Republican majority, to vote a law on March 1880 that severely limited the viability of catholic universities. D'Hulst had therefore to fight on two fronts[29]: against inflexible anti-clericals such as the successor of Ferry as Minister for public instruction, Paul Bert (1833-1886), for which the major virtue of science had been to free intellectual work from clericalism and Christian dogma[30]. Some rather excited declarations were besides exchanged in the French parliament to which d'Hulst belonged as a deputy of the department of Finistère. But he had also to face against inflexible (*intransigeants*) catholic who permanently attacked him for his dangerous liberalism[31]. In 1886, d'Hulst had launched the organization of an international congress of Catholic scientists that was to be held in Paris at Easter 1887. In his circular letter of 1 February 1886 published in the *Annales de Philosophie Chrétiennes*[32], d'Hulst mentions that the aim is to gather *professors and writers who were known for adding a real scientific value at the service of Christian convictions*[33] and to invite all the Catholics who are interested in the development of science for the defense of the faith[34]. Mansion and Mercier were both members of the organizing committee. The proceedings of the congress testify of Mansion's continuous involvement in the preparation phase. He was a member of the preparatory commission (in fact a vice-president) and, as

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28 Je trouve que la presse catholique n'attaque pas bien l'enseignement neutre. Elle devrait sans cesse dire et répéter qu'il est antipédagogique parce qu'il ne développe pas les facultés religieuses de l'enfant d'une manière harmonieuse avec les autres; par suite, c'est un enseignement <u>inférieur</u> au point de vue littéraire et scientifique.

29 On d'Hulst's engagement, see (Beretta, 1996). D'Hulst had a lot of exchanges at the end of the 19th century with Georg Cantor (1845-1918) during the latter's evolution towards catholic mysticism. Cantor's correspondence reveals how French scholars played an important role in this evolution. On this question, see the beautiful study provided in (Décaillot, 2008). Chapter 3 (pp.59 and seq.) provides also many general considerations about Catholic intellectual and scientific stage during the last two decades of the century.

30 (Beretta, 1996), p.69

31 (Beretta, 1996) , pp.46-47

32 Volume 111 of the *Annales de Philosophie Chrétiennes* on pp.401-404 (1885-1886). Reproduced in (Beretta, 1996), pp.267-272.

33 des professeurs et des écrivains qui se sont fait connaître en mettant une valeur scientifique réelle au service des convictions chrétiennes

34 (Beretta, 1996), p.269

reported in the historical exposition[35], probably the most active who *attracted almost all his catholic colleagues of the university of Ghent (28 out of 30) and a considerable number of members from Ghent, Anvers and Western Flanders*[36]. The seventh section of the proceedings, devoted to *mathematical and natural sciences,* includes Mansion's presentation to the congress[37]. It contains also a contribution by Charles Hermite[38] who attended the Congress and was certainly considered as most valuable scientific endorsement of the meeting.

2- A diffuse interest for probability

a- Belgium as an improbable mold for probability

We already mentioned how Quételet played a central role within the Belgian academic life and how, under his influence, probability became a central topic of study. Quételet was probably the first to perceive that statistics must fundamentally rely on the calculus of probability[39]. In (Droesbeke, 2005), Droesbeke had studied how Quetelet continuously acted in order to promote mathematical education in Belgium, with a special accent put on popularization. Besides, Quételet seems to have had a real gift for teaching and he was a highly appreciated pedagogue during his whole life, celebrated for his clear and simple style of exposition. Three important books published by Quételet in three consecutive years, at the same time of his campaign for the creation of an Observatory in Brussels, are significant of his desire to disseminate scientific culture in his country. Quételet's source for (Quetelet, 1828) was the *Traité élémentaire du calcul des probabilités* by Sylvestre-François Lacroix published for the first time in Paris in 1816 and again in 1822. In order to facilitate the access of a large audience to this new and important scientific tool, Quételet aimed at building a kind of arithmetic of probability based only on fractions and proportions. The expression "*arithmetic of probability*'" was proposed by Garnier after receiving Quételet's book with enthusiasm. Garnier commented that he ``could not understand how it had not come before''[40]. Though Quételet's frenetic activity began before the Belgian revolution, the newly acquired independence of the new state obviously gave a decisive impulse to it. (Droesbeke, 2005) exposes how Quételet participated, and often directed the elaboration of the Belgian education structure. In particular, he supported the creation of polytechnical schools, whose syllabus had to include theoretical, as well as practical, scientific lectures, and he was also involved in the organization of the Royal military school. Above all, Quételet was one of the promoters of the *Université Libre de Bruxelles* founded in 1834 as an answer to the foundation in Malines of the Catholic institution of higher education that was soon transferred to Louvain, as was already mentioned.

There were therefore two state universities in Belgium: Ghent and Liège. Garnier taught probability theory in Ghent. His student Jean-François Lemaire (1797-1852) was appointed Chair of mathematical analysis in Liège but it was Lemaire's successor in 1849, Anton Meyer who promoted probability in Liège.

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35 (CSIC, 1895)
36 (CSIC, 1895), p.9.
37 (Mansion, 1895)
38 (Hermite, 1895)
39 (Droesbeke et Tassi, 1997; p.44),
40 (Droesbeke, 2005), p.15.

This highly original personality had been a valuable mathematician as well as a highly appreciated professor in various Belgian institutions, but he is mostly known today as one of the major authors in Luxembourgish literature. Born in Luxembourg City, the son of a shoemaker, Meyer studied mathematics in Liège where he paid for his tuition by giving private lessons to his fellow students and helping out in the university library. After receiving his doctorate, Meyer spent the year 1825 in Paris, following lectures at the *Collège de France* and at the *Sorbonne.* (Jozeau, 1997) observes (p.98) that the same year, Quetelet and Nerenburger were also in Paris, and they may have got acquainted at the time. When he came back to his native country in 1826, he was professor of mathematics in various institutions such as the college of Echternach or military schools in Breda, Louvain and Bruxelles. In 1835, Quetelet, Nerenburger and Meyer were all three professors at Brussels military school. During the 1830s, Meyer participated to several geodetic campaigns for the determination of the borderline between Belgium and Netherlands and in the 1840s he taught several courses on geodetic. (Jozeau, 1997) (p.99 *and seq.*) studies how Meyer began to learn at this occasion the German techniques, and would even stay in Germany for several weeks in 1846 in order to study new probabilistic methods for geodetic In 1838, probably with Quételet's support, Meyer had been offered the chair of higher mathematics in the *Université Libre de Bruxelles,* and later in Liège in 1849. (Jozeau, 1997) observes (p.106) that Meyer's arrival in Liège coincided with a new syllabus for the doctorate course of analysis, probably inspired by Quetelet: it now contained two explicitly separated parts: superior analysis, elliptic functions and calculus of variations on one hand, and probability and social arithmetic on the other hand. Meyer implemented this division upon his arrival and between 1849 and 1857, date of his sudden death, Meyer taught a course of probability in Liège that may have been the most structured course proposed in a European university at the time, realizing Quételet's program of connecting the theoretical approach of probability of the French mathematicians Laplace, Lacroix and Poisson and the German practical developments about the law of errors.

In 1856, Anton Meyer submitted a proof of the Central limit theorem in the special case of two-valued random variables to the Academy in Brussels. Meyer's proof was not based on the usual procedure which can be traced back to de Moivre, and which had also been elaborated in (Laplace, 1812). Meyer used instead an extension of Laplace's generating functions. Meyer's article seems unfortunately lost but we know its contents through a brief report (Brasseur, 1856) by Jean-Baptiste Brasseur. The latter hoped that Meyer's method would lead to a more exact discussion of how it was possible to neglect the "terms of higher order of smallness." Meyer's paper was accepted for publication under the condition that a better examination of the "convergence of the series" should be made. The publication failed, as Meyer died the following year[41].

At the time he disappeared, Meyer was completing the manuscript of a textbook on the calculus of probability. The manuscript remained unexploited for more than 15 years, but was

41 (Fischer, 2010) pp.24-25. For sake of completeness, let us briefly mention that a few months before his death, Meyer had been involved in a polemic with Liagre, Liagre was a captain of the military engineering and had studied with Quételet and taught probability in military academies mostly from a practical point of view. He had presented in 1855 a note to Brussels Royal Academy in which he introduced the first elements for the so-called turning-point test. The aim of this test is to check the randomness of a sequence of data. This test was rediscovered, apparently independently, some years later by Bienaymé -- see (Heyde and Seneta, 1977), pp.124-128. Meyer published his reaction in 1857 as a note in which he criticized Liagre for his alleged sloppiness without much tact. A controversy resulted at the Royal Academy (Liagre being an important member of the institution while Meyer was only a corresponding member) illustrating some difficulty for maintaining the balance between a theoretical and a practical point of view in probabilistic questions. On this, see (Breny, Jongmans et Seneta,1992).

eventually published in 1874 by the *Société royale des sciences de Liège* under the supervision of François Folie (1833-1905). Folie was a Belgian astronomer and between 1872 and 1884 the administrator-inspector of Liège University, under whom the faculties of science and medicine in Liège were entirely modernized in particular with new buildings, as a new observatory in the suburb of Cointe[42]. A systematic use of Laplace's method for the asymptotic development of integrals is used in Meyer's book for the presentation of the law of errors. The method itself is exposed in a very clear mathematical appendix (Meyer, 1874; pp.421 et seq.). Meyer's course of probability was clearly conceived for students with a quite extended mathematical education. The book offers thus a rather remarkable solution of continuity with (Laplace, 1812) of which Meyer had probably been an enthusiast reader when he was in Paris.

Mansion, in (Mansion, 1903) wrote that Meyer's textbook was the only genuine treatise on probability theory written in a European language between Poisson's 1837 and Bertrand's 1889 books. This judgment was possibly somewhat exaggerated: (Hald, 1998), for instance, seems to consider that the value of Meyer's book is more connected to its clarity than to its originality. Hald (p.598) emphasizes however that Meyer's textbook – as well as (Liagre, 1852) – is a hint of the remarkably high level of the teaching of probability in Belgium at the time. Anyway, a remarkable fact concerning Meyer's book is its translation into German in 1879 by the Prague mathematician Emanuel Czuber (1851-1925). We have no evidence of how Czuber was aware of the existence of Meyer's treatise, but his specialization in geodesy offers a reliable clue. In 1876, Czuber had indeed defended a habilitation in geodesy with the title *Theorie und Praxis der Ausgleichsrechnung*. At the same time, he may also have begun to lecture on probability at the *Deutsche Technische Hochshule* as a privatdozent from 1876 as claims (Hyksova, 2011; p.112). One of Czuber's first publications (Czuber, 1878) clearly shows that in 1878 in addition to geodesy, he was concerned with actuarial sciences. Czuber was probably attracted by the mathematical soundness of Meyer's approach in his textbook for basing the law of errors on a precise mathematical apparatus that would enable a more clear exploitation for application. Czuber added a preface to the translation of Meyer's treatise, containing the complete translation of Folie's preface for the original edition, and some comments by Czuber about the contents of the book. Czuber explained that due to their importance for application to actuarial science, he had felt the need of rewriting and completing the book. He in particular profoundly reshaped Chapter 8 (for him, the most essential) devoted to compensation of errors. The aim was to make such a chapter useful for approaching standard situations but also to include the most recent theoretic developments of the question. Czuber had also completely revised chapter 9 devoted to insurance problems, of major importance for his actuarial interests. He wrote:

> *We thought we should take into account the progress made in recent years in this field, at least as far as can be allowed by the volume and purpose of the present work. In the introduction the fundamentals were explained according to the Knapp and Zeuner's approach, and we followed the method of the latter for the determination of the average lifespan. We have tried to develop ourselves a method to establish the average duration of marriages and widowhoods. We have kept, mainly for historical reasons, the theory of Euler on the renewal of populations;*

42 In the 1880s, Folie was embarked in a scientific controversy with the astronomer German Moravian astronomer Norbert Herz (1858-1927) about questions of stellar positions. The mathematicians Gustav Mittag-Leffler and Vito Volterra took part in this controversy in 1899 after Folie's submission of a paper in Acta Mathematica. See (Jaeck, Mazliak and Sallent del Colombo, 2019), Letters 117-119.

> *but here again we have sought to present newer approaches. For a fair understanding of the formulas and their practical application, we have provided many examples and we have taken care, in their progress, to eliminate the difficulties.*

In his 1901 entry about probability theory for the *Encyklopädia des Mathematisches Wissenschaft*, Czuber mentioned Meyer's book among the fundamental reference textbooks. Contrary to Mansion's statement in (Mansion, 1903) claiming that Meyer's was the only treatise devoted to probability between Poisson's and Bertrand's books, Czuber mentioned (Cournot, 1843) and (Laurent, 1873). It is plausible that Mansion considered them as textbooks less specifically oriented towards students specialized in mathematics.

As mentioned before, Meyer's treatise contained the lectures on probability given in Liège. In Ghent, the other state university, it was Eugène-Joseph Boudin (1820-1893), Mansion's former teacher and his predecessor to the chair of calculus of probability before 1892, who had taught a course in probability at the University of Ghent and at the Ecole du Génie Civil for more than forty years. In 1913, Mansion wrote in his biographical note on Boudin for the Liber Memorialis of the university of Ghent:

> *As for the course on probability, it is a true masterpiece in terms of the principles and the order of the subjects, deeply influenced by the best ideas of Laplace and superior to the best textbooks on the topic. The theory of errors rests on Hagen's hypothesis[43], of which Boudin, the first and for a long time the only one, had recognized all the philosophical value, shifting away with reason from Laplace on this point. The author of the present notice hopes sometime to pay a debt of gratitude to his former master by publishing a final edition of this beautiful course, with a slightly modernized analytical point of view. Boudin had given him that permission a few years before he passed away[44].*

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43 Gotthilf Hagen (1797-1884), a disciple of Bessel, had rediscovered and developed the hypothesis of elementary errors, already proposed by Daniel Bernoulli in 1778 (but ignored by Hagen) as basis for the law of errors. Hagen in 1837 made the following assumption

> *The error in the result of any measurement is the algebraic sum of an infinitely large number of elementary errors ["elementäre Fehler"], which are all equally large, and of which each single one can be just as positive as negative.*

Understanding the total error as a combination of (an infinite number of small and identically distributed) partial errors was a different perspective from, or rather went beyond Laplace's approximating normal distribution in that it gives a systematic cause for the normality of the total error. As (Fischer, 2010; 81-82) comments, this hypothesis was related to a model of error causation, which was according to a very large number of drawings with replacements from an urn containing black and white balls corresponding to the positive and negative values of elementary errors.

Though, as (Fischer, 2010; p.94) comments, it is not obvious that Quetelet statistical research which made an extensive use of the normality of error in measures, was directly influenced by Quetelet's reading of Hagen's work, one observes at least that Hagen was quoted by Quetelet, for instance in (Quetelet 1846). What was the respective influence of Quetelet and Boudin about Hagen remains to be clarified.

44 Quant au Cours de calcul des probabilités, tout imprégné des idées les meilleures de Laplace, c'est un vrai chef-d'oeuvre sous le rapport des principes et de l'ordre des matières, supérieur aux meilleurs manuels. La théorie des erreurs y repose sur l'hypothèse de Hagen dont Boudin, le premier et longtemps le seul, avait reconnu toute la valeur philosophique, s'écartant avec raison de Laplace sur ce point. L'auteur de cette notice espère quelque jour s'acquitter d'une dette de reconnaissance envers son ancien maître en publiant une édition définitive et un peu

Boudin's treatise waited in fact a long time for being properly published. Between 1865 and 1889, three rather confidential autographic editions were published (Boudin, 1865), (Boudin, 1870) and (Boudin, 1889). In 1869, the young Mansion provided a copy of Boudin's first autographic edition to Eugène Catalan who wrote to him

> *I must begin by thanking you for sending Boudin's Probability lessons: so far, I am delighted with this book, which will make me an excellent base of operations. Not only will it serve me a great deal, but it could also serve others: I believe that M. Boudin would do a good deed by having it printed with modifications, additions, and so on. In the meantime, present him, I beg you, my sincere compliments*[45].

The final edition of Boudin's lessons would be published only in 1916 when Mansion succeeded in having them published in Paris with considerable extensions and complements. In the book, Mansion included in particular his conference (Mansion, 1903) as an annex. Boudin's (and Mansion's) rather unnoticed 1916 publication would certainly deserve an independent study as it seems to offer one of the best survey of the state of the art of probability theory before its resolute turn towards measure theory. It is worth noting that Mansion had the book published in Paris in 1916, while Ghent was under German occupation and at the precise moment when Germans were supporting the creation of a Flemish university in Ghent. The German decree was published on 13 March 1916 followed on March 18th by the arrest and deportation of the historians Paul Fredericq (1850-1920) and Henri Pirenne (1862-1935) to Germany. Fredericq and Pirenne were two professors in Ghent University who protested against the German occupation. Mansion's preface is dated from August 1916 seeming to confirm that the book was not ready before the war. Therefore, the publication was probably seen by Mansion as a patriotic action, and a lot of subtle comments included in the book, emphasizing the role and presence of Belgium in the concert of nations, can be interpreted as such. For illustration sake, let us only quote the final section of Mansion's preface dedicating the book to the memory of his father-in-law (whom he never met, due to his premature death) the hydraulic engineer Alphonse Belpaire, who had played an important role in the new independent Belgium and who had been Boudin's mentor:

> *In all his works, Alphonse Belpaire has pointed out, concerning Belgium, the fundamental importance of the economic history of nations, without forgetting the influence of the other factors of civilization; for, he says, the moral world obeys eternal invariable laws, like the physical world*[46].

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rajeunie au point de vue analytique de ce beau cours. Boudin lui en avait donné l'autorisation, quelques années avant sa mort.

45 Je dois commencer par vous remercier de votre envoi du Cours de Probabilités: jusqu'à présent, je suis enchanté de cet ouvrage, qui va me faire une excellente base d'opérations. Non seulement il me servira beaucoup, mais encore il pourrait servir à d'autres: je crois que M.Boudin commettrait une bonne action en le faisant imprimer avec modifications, additions, etc. En attendant, présentez-lui, je vous prie, mes compliments sincères. (Eugène Catalan to Paul Mansion, 1 December 1869).

46 Dans tous ses ouvrages, Alphonse Belpaire a fait ressortir, à propos de la Belgique, l'importance capitale de l'histoire économique des nations, sans oublier d'ailleurs l'influence des autres facteurs de la civilisation ; car, dit-il, le monde moral obéit comme le monde physique à des lois éternelles, invariables.

b- Some probabilistic steps in Mansion's career

As was already mentioned, if Mansion's papers dealing with probability do not represent a large proportion of his production (there are less than 15), it is noticeable that their publication was spread out during the 45 years of Mansion's activity, proving his recurrent interest for the discipline. Probability was in particular the topic of his first published study in 1868. Another fact is that Mansion seems to have been interested at the same time in the technical aspects of probability (mostly limit theorems: law of large numbers and central limit theorem) as well as in the epistemological interpretation of probability's role in contemporary science. There is an obvious link between both. As it was the case for Poincaré in the last third of the 19th century, Mansion questioned objective part contained in a probabilistic approach of the world. Is this approach only a mathematical trick, or does it reflect a reality of the world? Even for Poincaré, conventionalism was not the ultimate answer to the question: the so-called method of arbitrary functions he invented in order to attribute probabilities to events was seen as a hint of an objective contents in the probability approach[47].

A first appearance of the question in Mansion's works was a small note included by the philosopher Paul Janet in 1882 in the second edition of his essay on final causes (Janet, 1882). Janet commented in the preface of this edition that he had to modify his original text after having received many reactions to the first edition in 1876. As the book had been conceived as a polemical text, aimed at provoking comments on a topic rarely commented in France, this had been quite natural for Janet. He mentioned that in the Anglo-Saxon culture, cross-examination of philosophical questions is a more common feature. (Janet, 1876) questioned if the recent progresses of science imply necessarily a doubt on God's existence and how the classical approaches of the question have to be adjusted to fit this evolution. In the second chapter of the book, Janet analysed the so-called physical-theological proofs.

Among the comments he had received, wrote Janet, one of the most interesting was due to Paul Mansion. Janet decided to reproduce Mansion's letter as an appendix in (Janet, 1882). Mansion's note is entitled "the epicurean argument and the calculus of probability". Mansion contested an assertion by Janet in (Janet, 1876) concerning the Epicurian objection to the final cause proof of existence of God. This proof states that the world is too well organized not to reflect the intelligent design of a creator. Epicurians object that if the world is an assembly of elementary particles randomly associated, one must necessarily conclude that at some moment, it assumes any kind of distribution, including the present state. Janet asserted that the calculus of probability may be used to study the validity or invalidity of this argument: in particular because if the number of particles is infinite, the probability of assuming the present state of the world is zero so that the Epicurian argument is dismissed. Mansion contested this use of probability, though, as he himself wrote, it seems highly tempting at first glance. First, writes Mansion, infinity is a mathematical abstraction, so that it cannot be used in a proof about the real world. Second, even if the number of particles is finite, the conclusion that the world must assume all the possible states is valid only if one assumes that some internal forces guarantee that the atoms have a behaviour allowing to apply the rules of the calculus of probability (in modern terms that their repartition is uniformly distributed over the various possibilities). This is an assumption we have no specific reason to make. Therefore, writes Mansion probability cannot be useful to approach the Epicurian objection. He would include later a section along the same lines in (Mansion, 1903) (section XI, p.67).

It is probably because applying the limit theorems was the unique way to attribute an objective

47 See (Mazliak, 2015).

value to a probabilistic result that Mansion's probabilistic technical research focused on them. At the end of the 19th century, especially after Cournot's studies, Laplace and Poisson's works were revisited and Mansion found in them several hints allowing an improvement of the estimation for the rate of convergence in some situations of large numbers. In particular, Mansion was interested in the convergence rate of the probability of

$$P\left(\left|\frac{S_n}{n}-p\right|\leq\epsilon\right)$$

to 1 when n tends to infinity and S_n is a simple random walk with a probability p of success. The estimation of large deviations from the mean is a significant question, but one that did not seem to fascinate many mathematicians at the time. Apart from a small group of probabilists close to Mansion, and some years later Borel, it is only at the end on the 1920s and above all in the 1930s that these questions would attract a lot of attention from a new generation of mathematicians. Another topic present in Mansion's works is a presentation and a deepening of Laplace and Gauss least-square method, extending Quetelet and Catalan's studies. The method was still quite unknown in the francophone cultural sphere[48].

As mentioned above, Mansion's probabilistic interests were shared by a network of mathematicians, mostly Belgian or at least having strong links with Belgium, and rather isolated from other scholars who reflected on randomness, in particular French scholars in Paris such as Poincaré or Borel. Let us consider again Mansion's studies about the law of large numbers. A first paper (Mansion, 1892) was published in 1892 in the Annals of the Brussels Scientific Society. In 1895, it was followed by the paper (Goedseels, 1895) by the astronomer and meteorologist Edouard Goedseels (1857-1928) in the same journal. Then, two new papers by Mansion (Mansion, 1902) and (Mansion, 1904) appeared in 1902 and 1904. In 1907, the Louvain mathematician Charles de La Vallée Poussin (1866-1962) extended Goedseels and Mansion's results in two articles on the Bernoulli's theorem, (Vallée Poussin, 1907a) and (Vallée Poussin, 1907b). De la Vallée-Poussin proved in particular that, denoting by N the number of heads obtained in tossing a coin $2n$ times, and by p the probability of obtaining head, one has

$$P(|\frac{N}{2n}-p|>\frac{\lambda_n}{\sqrt{n}})\to 0$$

if and only if λ_n tends to infinity[49].

Mansion and De la Vallée Poussin's results are quoted in (Montessus de Ballore, 1908). Montessus was a professor of mathematics at the Catholic University of Lille and probability had long been a central topic in his exchanges with Mansion. Mansion sent a copy of (Mansion, 1903) to Montessus, and proposed that he visited him in Ghent[50]. Two years later, Montessus suggested that they publish together a book on probability. However, Mansion refused[51] and explained to have had himself for a long time the project of publishing an edition of the lectures notes of his former colleague Boudin. However, he remained interested by Montessus' project, and when the book was published in 1908, Mansion advised his

48 For the contextualization of Mansion's research about limit theorems and the least squares, one can consult the recent two great volumes of (Bru and Bru, 2018) – see in particular n° 167 pp.251 *et seq.*

49 On this, see (Le Ferrand, 2014).

50 Paul Mansion to Robert Montessus, 28 April 1904.

51 Paul Mansion to Robert Montessus, 25 December 1906.

colleague[52] to send his book to the Jesuit father Fernand Willaert (1877-1953) in Louvain, an astronomer who was precisely conducting an investigation about the meaning of probability, so that he may write a review for the Revue des Questions Scientifiques. Willaert indeed wrote a review[53] for the journal, in which he praised the clarity and readability of the book. He also devoted several pages to express his skepticism about Montessus' assertion that Bernoulli theorem provided an objective value to the calculus of probability. Willaert denied in fact a genuine objective value to any mathematical abstraction: such a conclusion would be merely a psychological illusion because mathematics constitutes a powerful theory, and many results are closed to what can be observed in reality. It is therefore somewhat impossible to look for an exact determination of the conditions under which an abstract model coincides with reality. Though he was slightly more positive about this possibility in his book, Montessus seems however to have appreciated the review and wrote to Willaert to express his gratitude. Willaert answered[54] to thank Montessus for his kindness and to insist on the fact that the central problem in the reflection about probability was indeed to estimate the degree of coincidence between the experimental result (for instance when one throws a dice) and the theoretical result. As seen, for Willaert, as for Mansion (but also Poincaré or Borel at the same time) the question of objectivity of the results of the calculus of probability was under scrutiny. It is significant to compare Montessus' book with (Borel, 1909) published at the same moment. While Borel placed himself in the continuity of the mathematical approach proposed by Poincaré, Montessus was clearly more on the side of the validity of practical application, so that he exposed Mansion and de la Vallée-Poussin's inequalities for the theorem of Bernoulli. Montessus' book is also the first in which the results of Bachelier's 1900 thesis on the theory of speculation were quoted. Unfortunately, we did not find information about how Montessus had got acquainted with Bachelier's work. A reasonable hypothesis is that Poincaré was the origin of this information: Montessus defended his PhD (on continuous fractions) in 1905 at the Sorbonne, before a jury presided by Poincaré; the second thesis (as usual, its topic had been chosen by the faculty, probably by Poincaré himself) was precisely the calculus of probability. For this occasion, Montessus needed to prepare a survey of recent works on probability, and necessarily took Bachelier's thesis into account. As mentioned above, Montessus was professor at the Catholic Institute in Lille – not a neutral fact in those years of tension between the Church and the state. Though Montessus de Ballore does not seem to have been as involved in the Catholic Church as Mansion was, this common characteristic of both mathematicians certainly played a role in their relationship. In his letter on 25 December 1906, Mansion congratulated his colleague for the reception of the great prize of Paris Academy of Sciences for his thesis on continuous fractions and wrote that he would communicate the information to the Brussels Scientific Society. He expressed his satisfaction because

> *it has always had a good effect on the public, and young people in particular, to learn that one of the world's leading scientific bodies crowns the work of young Catholic scientists*[55]

Another member of Mansion's "Belgian" probabilistic network was the Italian mathematician Ernesto Cesarò (1859-1906). After some disappointment in Italy during his secondary studies,

52 Paul Mansion to Robert Montessus, 2 April 1908.
53 (Willaert, 1908)
54 Willaert to Montessus, 9 August 1908
55 [c]ela fait toujours bon effet sur le public, et sur les jeunes gens en particulier, d'apprendre que l'un des plus éminents corps scientifiques du monde couronne les travaux de jeunes savants catholiques. (Mansion to Montessus, 25 December 1906).

the young Cesarò was sent to Liège (his brother was already a student there) to study engineering: he entered the mining school in 1874 and obtained his graduation in 1878. However, Cesarò decided to continue his scholarship at Liège University and began to publish mathematical works, receiving many encouragements from Catalan. Cesarò came back to Italy in 1883 and after several complicated years, eventually obtained a chair in Palermo and then in Naples. Though Cesarò is today above all known for his contributions to differential geometry and the theory of series, he was also interested by probability as is proved by several publications, mostly in the *Giornale di Battaglini* or in Mansion's journal *Mathesis*. Most of these publications are devoted to specific problems, such as the "breakage of diamond"[56] a geometrical probability problem in which the question is to decide which division in three pieces would be optimal for the jeweler. However, Cesarò was also interested in broader aspects of probability theory. In his two-part paper (Cesaro, 1891), Cesaro studied how probability can be interpreted in cases where there are an unlimited number of possibilities. Contrary to Bertrand's negative attitude condemning geometrical probability altogether, Cesarò wrote that the value of a probability seen as a limit was perfectly acceptable, and required a part of subjectivity comparable to the attribution of a value to the sum of a non absolutely convergent series. This comparison by Cesarò, a mathematician who devoted much energy to the study of general series displays a remarkable effort to keep a unified conception of mathematics. Mansion, in (Mansion, 1903) (p.59), emphasized the soundness of Cesarò's approach, and regretted that the latter's probabilistic work was not better known, due to his choice for publishing in a hardly accessible Italian journal.

3- An other look on probability

We have seen that though he did not write a lot of papers about probability, Mansion had constantly kept an eye on the subject. What can explain that interest apart from the noticeable presence of probability in the Belgian scientific life he had himself emphasized in (Mansion, 1903)? A closer look at this text reveals that a search for a spiritual interpretation of probability was a basis for Mansion's interest. This last section of the present paper is devoted to explain the religious context in which Mansion's reflection on probability took place and how (Mansion, 1903) let appear his personal approach of the subject.

a- A search for a spiritual interpretation

If it was mainly the changes in Physics, in particular the emergence of statistical mechanics, that forced a reluctant Poincaré to accept the presence of probabilities in science in the 1890s[57], a major scientific crisis related to randomness occurred 30 years earlier after the publication of Charles Darwin's "Origins of species" in 1859. The status of man in the natural world was at stake and became the topic of numerous controversies about various possible interpretations of Darwin. The reception of Darwin's book by Catholic theologians, and more generally by Catholic thinkers, immediately generated discussions and polemics. The end of the rigid Pius IX's pontificate, when the Pontifical states were collapsing under the pressure of Italian unification, was certainly not a proper time for open discussions. The situation began to evolve with his successor Leo XIII, pope in 1878, and his already mentioned wish for an evolution of the dialogue between the Church and modern science. This wish concerned in the first place the questions of evolution. For many Catholic thinkers, it was simply sloppiness to

56 (Cesaro, 1886)
57 See (Mazliak, 2015).

leave those to anti-clerical militants. Therefore, the last decades of the 19th century were a period of intense reflection on evolution by catholic theologians, philosophers and scientists. Belgium seems to have been a particularly fecund zone on that prospect and Mercier's institute in Louvain had been at the forefront of this research. (O'Brien, 1931) mentions (pp.115-116):

> *Of especial significance in symbolizing the rapprochement that had been effected between evolution and the Scriptures was the action of the University of Louvain, one of the greatest, if not indeed the greatest, of all the universities conducted under the auspices of the Catholic Church. (…) In 1909, Canon Dordelot declared: "it is no exaggeration to say that, in showing us a creation more grandiose than we had ever suspected it, Charles Darwin completed the work of Isaac Newton; because for all those whose ears are not incapable of hearing, Darwin was the interpreter of the organic world; just as Newton was the voice from heaven come to tell us of the glory of the Creator, and to proclaim that the universe is a work truly worthy of His hand".*

A renewed approach of Lamarckian transformism was considered as a possible adaptation of Darwin's conceptions, but it was strongly opposed by supporters of an evolution theory strictly based on random natural selection. Transformism was tainted with a teleological bias by anti-clericals who suspected it was an attempt from the Church to infiltrate the idea of evolution by reintroducing the possibility of a divine plan. The Belgian philosopher and psychologist Joseph Delbœuf (1831-1887), who had a strong interest for mathematics during his whole life, tried to provide a mathematical study for supporting transformism. Delbœuf had been a professor at the university of Ghent before his transfer to Liège. He published in 1877 the paper (Delbœuf, 1877) in which he presented how combinatorics demonstrates that evolution systematically favors the predominance of mutations. The French zoologist Alfred Giard (1846-1908), who was a major introducer and exponent of evolution theory in France, was very positive about Delbœuf's approach[58]. However, Giard mentions that

> *other naturalists, on the contrary, more Darwinists than Darwin, refuse to admit any other cause of evolution than natural selection. They excommunicate both Romanes and Delboeuf, and the new supporters of Lamarck's views, rejuvenated and improved by modern science. At the head of these ultra-Darwinists is Weismann, who in many works has attempted to show that others drawn solely from the mechanism of selection could replace Lamarck's explanations. Weismann's essays, partly translated into English, were warmly welcomed by most biologists in Britain and gave powerful help to the eminent Alfred Russel Wallace.*

Delbœuf's interest for evolution theory seems more related to the fact that it was at his time a

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58 See (Giard, 1890). It is worth observing that Giard was seduced by Lamarckism and accepted to write the preface for several books of his disciple, the biologist Félix Le Dantec (1869-1917), one of the most passionate opponents to the presence of randomness in scientific modeling. A fearsome polemicist, the atheist and materialist Le Dantec was, during his whole life, a passionate defender of a strict determinism, and a slayer of probability under which a deist idea was always more or less hidden. The introduction to his book (Le Dantec, 1898) is worth reading to seize Le Dantec's thunderous rage to defend his point of view against philosophers, and, especially, the neo-Thomists. The most profound controversy about probability involving Le Dantec was happened with his friend Emile Borel in the latter's journal Revue du Mois: on this, see the profound study (Bru, Bru and Chung, 2009).

burning scientific question than to its religious implications. But, in the line of O'Brien's previous quotation, evolution and the fight against ultra-Darwinist became a capital theme of reflection in Louvain University and philosophers who passed through Louvain proposed various approaches of the question. The English biologist St George Jackson Mivart (1827-1900) presented one of them. Mirvart had converted to Catholicism when he was 17 and this choice had barred him the access to the university of Oxford. First an ardent supporter of Thomas Henry Huxley's conception of natural selection, Mirvart distanced gradually from it and eventually became a resolute opponent to what seemed to him a diversion from Darwin's original idea. Mivart's intention always remained to reconcile Darwin's theory of evolution with the beliefs of the Catholic Church, and was eventually rejected by Catholic authorities as well as ultra-Darwinists. Mivart contested the role of blind unintentional randomness in evolution. In 1871, he published (Mirvart, 1871) in which he suggested that the idea of transformism by insensible transitional stages was nonsensical. An example was the emergence of wings in birds: would it be relevant to consider that a half-wing could provide any evolutionary advantage? Therefore, the evolution must be based on the existence of significant intermediate stages. In 1876, Mivart later obtained a doctorate in philosophy and made regular visits in Louvain. He defended a PhD in medicine at Louvain in 1884 and the same year he accepted the chair of natural history, but was condemned in 1900 for having published several articles in the journal Nineteenth Century in which he provocatively asserted that it was not always possible to reconcile science and the Bible and that predominance must be given to science in such cases.

In relation with the effervescence around the theory of evolution, reflections about the calculus of probability and its presence in scientific modeling were of particular interest in Louvain. John Henry Newman (1801-1890), an English theologian and a former Anglican minister who also converted to Catholicism in 1845. Newman, who was later nominated cardinal by his friend pope Leo XIII was interested in the question of provability, in particular for the problems related to the faith. In 1870, he published his major book "Grammar of Ascent" in which he asserted that in everyday life, it is the accumulation of probabilities that replaces formal logic. The latter is generally inoperative because too rigid to be applied to the moving circumstances of life. I am certain that Britain is an island though I have never checked myself the fact: it is the accumulation of probabilities in favor of the fact that Britain is an island, which leads me to certainty. Many modernist theologians considered Newman's theory as a convenient answer to strict positivism without coming back to a simple apologetic. (Newman, 1870), being mostly a recollection of various homilies, is a slightly confused text, and Newman's ideas were probably better exposed by posterior writers such as Bremond in (Bremond, 1905). Newman's idea was that man has access to an "illative sense" allowing him to form a personal logic built on a cluster of probabilities, and to convert these probabilities into certainty. This "sense" is the central property of mankind that "helps him out" in front of the questions of faith[59].

Mercier himself was engaged in this reflection about randomness. Louvain's archive keeps his notes for lectures on probability to students of the institute in 1891 among his personal papers. The content of the lectures is not particularly original, but two points deserve to be emphasized. First, Mercier's aim was to teach the mathematical bases of probability theory. He presented to his students the principles governing probability (principles of total probabilities and of compound probabilities) and gave many examples of calculation with these principles. The students in Louvain institute were not students in mathematics. This is a hint that Mercier had adopted a rather modern point of view that a philosopher must begin by learning a science

59 See (Bremond, 1905), p.258-259.

before constructing a philosophical approach of it. Mercier had probably the intuition that this was the only proper way to avoid condescension from scientists. Some philosophers shared with Mercier this impression. Léon Brunschwicg, for instance, wrote:

> *The philosophical speculations that relate to the space of geometers without any other specification, whether they make it a reality or a pure idea or a form of intuition, have lost touch with current science*[60].

A second observation is that Mercier does not mention the eventuality of an objective probability but only the concept of subjective probability that mathematical probability is supposed to quantify and make calculable. Though he is not quoted in Mercier's notes, this approach is in line with Newman's use of probability simply as a tool aimed at enlightening free will.

We have already mentioned how Mansion was close to Mercier and to Louvain institute. His Brussels' conference (Mansion, 1903) was therefore composed on that background. It presents as a reflection about the objective contents of the calculus of probability a century after Laplace and Condorcet. As mentioned Mansion, in the second half of the 19th century, several scientists, such as Bertrand or Poincaré claimed that this objectivity was mostly an illusion. In 1905, the German translation of Poincaré's (Poincaré, 1902) was published, and this gave Mansion on occasion to challenge Poincaré's conventionalism, in particular for the use of geometry in physics. To believe in the conventional nature of geometry was to reject the possibility of measuring lengths, which amounted to "denying any possibility of a quantitative knowledge of nature". But the Catholic Mansion doubted that "anybody could go so far[61]. In his 1916 edition of Boudin's lectures, Mansion gave his final judgements about the nature of probability.

> *Our first thesis is as follows: The Calculus of Probability has for object the events which are subjected to a complex law, resulting from a principal law according to which certain numerical relations are constant, and secondary disturbing laws giving birth to weak variations of these ratios. In the study of such events we can consider as legitimate the results deduced from the law of large numbers. We conclude from this principle that the Calculus of Probability can be applied to moral statistics, games of chance, evolutionism, but not to judgments in civil or criminal matters, or to the probability of causes.*
> *Our second thesis is the principle of the accumulation of probabilities independent of Newman. Properly speaking, it cannot be translated into a formula, except in a symbolic form through the intervention of an unknown function. We show that this principle, so far ignored by geometers and logicians, is the perfectly legitimate source of our practical certainties, in all the sciences based ultimately on testimony such as natural sciences or historical sciences.*

60 (Brunschvicg, 1912), p. 444.
61 (Walter, 1997) Scott Walter. La vérité en géométrie: sur le rejet mathématique de la doctrine conventionnaliste. Philosophia Scientiae. 2, 3.. 1997. 103-135

b- *What would be the Jesuits without probability* ?[62]

A rather original trait of Mansion's interest for the calculus of probability is related to his complicated relationship towards its founder, Blaise Pascal. In a short popular biography of Paul Mansion published in a local history journal of the commune of Marchin (in the surroundings of Liège), local historians Paul Grognard and Andrée Hubin wrote that there was just one topic about which the extremely tempered Mansion had real moments of rage. This topic was precisely Blaise Pascal.

> *[Mansion] lost his composure if, occasionally, someone in his presence dared to boast the scientific merits of Blaise Pascal. "Pascal did not invent anything, he would say, he cultivated the paradox without reason, he was not a genius, he was an evil original".*[63]

Another proof of this strong dislike for Pascal is Mansion's last published paper (Mansion, 1920b) written during the Great War but published only posthumously in the Revue des Questions Scientifiques, not without some hesitation because of its polemical character. This paper pretends to be a scientific biography of Pascal and its central thesis is exposed without nuance by the elderely Mansion: *there is no such thing as Pascal's scientific achievement with the noticeable exception of his research on the calculus of probability*. Mansion wants to prove that Pascal had in fact borrowed all his supposed discoveries from various scientists (Torricelli, Descartes, Mersenne, Desargues, Roberval, Otto de Guéricke, Stevin…). French writers or scientists are fully responsible for the fact that a legend of genius was built about Pascal. As Mansion wrote

In France, despite all the documents, the admiration for Pascal is almost idolatry, even among those who know and report most of the deficiencies of his mind and character (...) It is against this half-canonization that we try to react[64].

In a second part of (Mansion, 1920), Mansion wants to prove that Pascal was in fact not the Jansenist until the end of his life that is portrayed by those who want to prove that he was an opponent to the Catholic Church, a position supposedly comforted by his scientific and independent originality. Pascal on his deathbed was repentant, receiving the extreme unction as a submitted son of the Church. The present paper is obviously not the proper place to discuss Mansion's opinion that was judged excessive even by his close friends. What we would like to understand is why Pascal has been such a fixation point during Mansion's life.

The first idea coming to mind is that Mansion was close to many institutions and many people linked to the Jesuits, and most of Pascal's attacks were directed against the Jesuits. In Pascal's view, the Jesuits - who, indeed, during Louis XIV's reign had a direct access to the king's ear and saw the so-called Jansenists as heretics more or less comparable to Protestants - were hypocrites (think of Molière's *Tartuffe*) and their casuistic approach of the faith was a satanic way to seize the direction of conscience of their listeners. Let us recall this famous passage of Pascal's *Pensées*

> What would the Jesuits be without probability and probability without

62 (Pascal, 2004). p.286.
63 (Grognard et Hubin, 1973). p.43.
64 (Mansion, 1920b). p.333-334.

> the Jesuits? Take away probability and you can no longer please the world. Bring in probability we can no longer displease it[65].

However, this influence of the Jesuits on Mansion's mind seems unsufficient to explain his obsession. After all, as (Grognard et Hubin, 1973) recalls (p.43), a Belgian Jesuit, Henri Bosmans, wrote one of the most significant PhD on Pascal mathematical's work at the time and juged Mansion's opinion too violent against Pascal. Nevertheless, he wrote in (Bosmans, 1924) that on the whole it was often true that other scientists had obtained before him many discoveries attributed to Pascal. Bosmans had the desired not to repeat Pascal's legend without criticism as had been done too often, especially in 1923 for the tercentenary of his birth. On the contrary, Bosman's desire was to present Pascal beyond his legend, as a profound and powerful, but *human*, mind. He wrote that he was

> *far from wanting to denigrate Pascal. So I will declare immediately and straightforwardly that I hold him, not only for a good but also for a great geometer. It is impossible, however, to appreciate his true merit, unless one forgets the enthusiastic exaggerations that writers, strangers to the subject, have written about him. The mathematician, who, pen in hand, would read Pascal, dominated by this preconceived idea, would go from disappointment to disappointment. In order to know him with his qualities, but also with his deficiencies, he must be placed in the context of his contemporaries; we must compare his work to theirs; a hard and long work which cannot be the object of a single paper in a journal*[66].

A relevant hypothesis to explain Mansion's hostility is that he had been impressed by the violent discussions about Pascal that took place in France at the beginning of the Third Republic. Anti-clerical Republicans placed Pascal was on a pedestal as a symbol of the fight of freethinking against the clerical hegemony. The paper (Quantin, 2014) studies how the inclusion of Pascal's *Provinciales* by Republican and anti-clerical ministers such as Jules Ferry or Paul Bert in the syllabus for the *baccalauréat* in 1880 and the *Brevet supérieur* in 1884 had been an element of the school war in France. On the one hand, the anti-clericals saw Pascal's accusations against the Jesuits, as a welcome condemnation of the Church desire of domination over the French scholar system. On the other hand, several Catholics were convinced that it was possible and necessary to recover Pascal as a major spiritual and religious thinker. They began to produce new commented editions of the *Provincales* that were diversely received in the catholic world. Though we do not have found a direct proof of Mercier or Mansion's attention to what was happening in France, it is reliable that they could not miss these hot events and, being deeply engaged in the Church, probably looked at them with worry.

Pascal was in fact present on the Belgian intellectual stage during the second half of the 19th century as is proved by several publications. The religious arguments were always important in them though the way they portrayed Pascal was varied. In 1855, Emile Lion, a lawyer from Liège, published the paper (Lion, 1855). The *Moniteur de l'enseignement*, directed by a professor at the Athénée of Tournai was the official journal of teachers. If Lion's paper was clearly a rehabilitation of Pascal, it cannot be said to display a genuine hostility towards the Church though it contains some remarks, which can be interpreted as hostile to the most rigid tendencies of Catholicism. Lion wrote for instance that

65 (Pascal, 2004). p.286.
66 (Bosmans, 1924), p.7-8.

> *the study of Pascal has shown him related to this order of ideas and feelings that not everybody can fully understand, because not everybody can reach it. In this sense Pascal was an ascetic, not of that gentle and pious asceticism of contemplative souls, which inspired the great Orders of the Middle Ages and produced the Imitation of Jesus Christ; but of that reasoning and militant asceticism, having an original and almost scientific character like all that pertains to this great mind. However, as we know, Pascal was very far from this extreme doctrine, which disregards the personality of man, denies the legitimacy of reason and of our natural forces, and pursuing even in real life the consequences of pantheism, assigns as the sole perfection to be absorbed in ecstasy and to lose oneself in God. The immortal thinker was not one of those exalted spirits who indulge in the great dreams of philosophy, and his geometrical mind would not have endured such mysticism. Sometimes he fell into error, but through the depths of his ideas and convictions he always remained attached to the truth. He was a Jansenist, but did not want to give up Catholicism.*

Thirty years later another book (Laurent, 1884) was published in Tournai about Pascal. This essay was published in the collection "*Musée moral et littéraire de la famille*" and was clearly related to the attempt of the Catholics, and especially of the Jesuits, for promoting an image of Pascal as an obedient son of the Church, though slightly naive and deceived by heretics. For Laurent, Pascal had been subsequently unjustly attacked both by some Catholics who considered him as a Jansenist, and by some anti-clericals who saw him as a fanatic. But, writes the author of (Laurent, 1884) "the day comes when the trick is discovered, and full justice is finally returned to the victims of the Voltairean hatred". The chapter 4 devoted to the *Provinciales* is savourous. Laurent writes (p.43) that "irreligious criticism has exhausted all the formulas of praise in the French language, in favor of the work that we can not pass over in silence "; however he adds, using a notorious casuistic argumentation (p.44)

> *Our deep conviction, based on a long study of the life and works of Pascal, is that God did not judge him so severely as men, because he found in him a Jansenist, a material culprit, but not a formal one, according to the scholastic formulation.*

The underground presence of Pascal in Mansion's scientific life was permanent, and this kind of obsession may have participated to his increased interest for the calculus of probability. In (Mansion, 1920) (p.338), Mansion defended the idea that probability was the only domain for which Pascal may be praised to have been an ``initiator'' and his unique real scientific merit.

It had been mentioned several times that Mansion's first publication was precisely about a very typical Pascalian problem, the problem of points. Another hint is given by an exchange between Mansion, then a member of the board of the journal *Revue des Questions scientifiques* and Pierre Duhem after the latter's submission of a review of Poincaré's textbook on Thermodynamics in which Duhem deeply criticized Poincaré's approach. Duhem concluded his review by one of Pascal's tough judgement against narrow-minded mathematicians. (Stoffel, 2007) explains (p.293) how Duhem preferred to give up this project after Mansion had sent a letter expressing his reservations. Mansion wrote:

> *I have received your appreciation of M. Poincaré's book on Thermodynamics. I must confess to you that this small article, which is full of verve, gives some worry to me: I fear that it will blur both you and me with M. Poincaré. The physicists will enjoy it but we, poor analysts, we are roughly shaken by your criticism and especially by the quote of Pascal in fine. Is there a way to improve things? Could you at least send me another conclusion for your interesting article*[67]*.*

Mansion's hostility towards Pascal was besides not his only manifestation of opposition to a philosopher. In another paper written during the war and published posthumously as (Mansion, 1920a), Mansion wanted to prove that Gauss had considered Kant's conception as totally irrelevant to mathematics. According to Mansion, the history of geometry provides an unequivocal argument against Kant's conception of the space as an innate form of sensibility. It was, for sure, not the first time that Kant's misunderstanding of mathematics was underlined. In particular, the logician Couturat in (Couturat, 1904), published at the occasion of Kant's death centennial, had already harshly condemned what he considered to be a nonsensical anti-analytical approach of mathematics. But Mansion's comment may have been the first one calling out some neo-thomistic arguments. In May and June 1895, Mansion had read a series of lectures in Louvain about non-Euclidian geometries and their anti-Kantian consequences. These questions were subsequently discussed inside Louvain institute. At the beginning of the 20th century, an ecclesiastic, Abbot Charles Sentroul (1876-1933), defended a PhD in which he asserted that contrary to Mansion's opinion non-Euclidian geometries were interpretable in a way reconciling Aristotle and Kant[68]. (Mansion, 1920b) was a response to Sentroul's criticism. Mansion defended the idea that mathematics is necessary to understand philosophy on par of the importance of philosophy for a deep understanding of mathematics. Ironically, Mansion concluded his paper by quoting Pascal:

> *Without philosophy, pure mathematicians are exposed to look at all the truths about moral ideas that depend on the spirit of finesse as unprovable, because, according to Pascal's remark, they cannot be proved* more geometrico.

The difficulty for conciliating Kant's rigid a priori synthetic approach to mathematical concepts with probability is well known[69]. Mansion may have found there another argument for criticizing the German philosopher. Mansion, as Poincaré, had accepted that probability was an essential part of modern science. But, in the wake of Mercier's institute in Louvain, he had always wanted to provide this concept with some spiritual contents that Pascal, because his hostility towards the Jesuits and though he had been an inventor of the mathematical approach to randomness, had always considered with mistrust. This aspect brings Mansion closer to other spiritual interpretation of probability as an alternative approach to positivism at exactly the same time in Russia by Nikolai Vasilievitch Bugaev (1837-1903)[70], Pavel Alexeevich Nekrasov (1853-1924)[71] or Pavel Alexandrovich Florensky (1882-1937). However, it does not

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67 Paul Mansion to Pierre Duhem, 11 February 1892.
68 See in particular (Sentroul, 1907)
69 See for instance (Kamlah, 1987), in particular p.113.
70 On Bugaiev's amazing talk at the international congress of mathematicians in Zurich in 1897, see in particular (Mazliak, 2015b).
71 On Nekrasov's considerations about free-will, and his explosive controversy with Markov, see in particular (Seneta, 2003).

seem that these authors commented on Mansion's papers on probability and in particular on (Mansion, 1903)[72]. In another place, the Czech lands, one may also mention Masaryk and Vorovka's analysis of Hume's skepticism and probability[73] as a tool of learning in "a world that is no longer endowed with the sharp edges of the certainty"[74] as Cavaillès would express it later.

Conclusion

The years following the Great War saw an important evolution of the presence of probability on the mathematical stage in the world. Though this movement was inscribed in the continuity of transformations initiated long before the 1910s, such as the profound modification encountered by Physics, it acquired a considerable momentum during the 1920s and probability gradually became a major topic for research (theoretical as well as applied) in those years. A significant instance of this new situation is given by the rapid development of Markov chains recounted in Bru's remarkable study (Bru, 2003). The transformation was especially spectacular in France where, under the powerful impulse of Borel in particular, probability really began to appear in syllabus or as a dynamic research domain at that moment with the emergence of the interest of first-rate scientists as Maurice Fréchet or Paul Lévy. Therefore, a strong presence of probability that had been a kind of Belgian specificity during the second half of the 19th century as was emphasized in the present paper, gradually became the norm in other countries. As was explained above, some exceptional elements of the Belgian situation allow understanding how Belgium became a leading probabilistic country around 1850. The presence of Quetelet and some other mathematicians of the time, their dynamism on a restricted scientific stage of a small and new country looking for an original scientific development, the curious geographical situation of between two powerful neighbors, France and Germany, as well as the francophone element facilitating the import of French mathematics (and of French mathematicians, such as Garnier or Catalan) during the first half of the 19th century, all these

72 In few occasions Mansion's name and the journal Mathesis appeared in Russian works. The 1875 textbook on differential equations was mentioned, in particular in relation with Korkin's work on the topic (see (Ozhigova, 1968), for instance p.51). Moreover, Mansion's textbooks seem to have been considered as basic litterature in university libraries. For instance, the catalogue of the library of the brand new Tomsk technological institute, founded in 1896, displays in 1906 no less than 5 books by Mansion. Florensky also mentioned Mansion at least at one occasion in his 1922 booklet *Imaginaries in geometry*. However, it seems that the main occasion for Mansion to have contacts with Russia was in the 1890s when Kazan's physical-mathematical society prepared the celebration of Lobatchevsky's centenary. Mansion wrote several papers on Euclid's postulate and non-Euclidian geometries in *Mathesis* and in the journal of Brussels scientific society at various occasions (on this, see (Voelke, 2005), in particular pp.367-376). He had been interested in understanding how the works of the Jesuit mathematician Giovanni Girolamo Saccheri (1667-1733) could have an influence on Lobatchevsky and wrote to the founder of Kazan's physical-mathematical society, Alexandr Vassilievitch Vassiliev (1853-1929), who was one of the main promoters of Lobatchevsky's research to get information. Vassiliev answered only in 1892 (explaining that he had had health problems during the whole year 1891), and mentioned to his Belgian colleague the preparation of brilliant celebrations for the next year centenary, asking him to participate as a member of the committee. Mansion accepted the proposition and was active in circulating a subscription to erect a monument in front of the university. Mansion did not attend the celebration (probably due to his numerous local –including family – obligations) that took place between 22 and 24 October 1893. In a not-dated letter after the ceremonies (probably at the end of 1893 or beginning of 1894), Vassilieff sent a letter to thank Mansion for his support. He wrote the following comment on the Russian mathematical life: « It must be said that most mathematicians in Russia follow M. Tschebycheff's path and are not interested in geometry or philosophy of mathematics. »

73 On this point, see (Mazliak, 2015) and the Czech references inside. See also (Novotny, 2011).

74 (Cavaillès, 1940), p. 154.

aspects merged and participated to the complex picture. Paul Mansion appears therefore as a good representative of the Belgian mathematical community of his time. He had however some personal originality, and it seems that the main one is related to his constant desire of keeping together his scientific work and his religious faith. A striking example of this desire is given in a public letter, quoted in a footnote of Mercier's textbook (Mercier, 1922) (p.38). After the liberal newspaper *La Flandre Libérale* had complained about Mansion's religious commitment that seemed incompatible with his position of scientist in Ghent University, Mansion sent the a scathing answer to the journal. Mansion wrote in particular that

> *It is unscientific to speak incessantly of the antagonism of science and of Catholicism in general, without ever going down to the detail. If La Flandre Libérale and the other Belgian anti-Catholic journals are assured of this antagonism, what prevents them from putting the so-called antinomies of science and faith before us, Catholics, with precision, in a table with two parallel columns? In the first would be the scientific truths borrowed from physics, chemistry, astronomy, mineralogy, geology, botany, zoology, anthropology, biology, and so on. In the second, opposite, if we know it, the contrary decisions of councils and popes, as we find them, for example, in the Enchividion of Denzinger. But we dare to predict that the second column will remain empty, if we insert only authorized interpretations of the Bible and the Catholic Tradition, and if we do not put in the first, under the pretext of science, unprovable assertions*[75].

Because Mansion was so active in publishing and popularizing mathematics, notably with his journal *Mathesis*, he infused some original aspects in Belgian probabilities of the time, through his reflection on determinism and interpretation of probability, on the background of the scholar quarrels of the young country as well as the stirs in the Church due to the contact with scientific modernity. If it is true that Mansion's reflection on the meaning of probability gradually passed to the background in the 20th century due to the empowerment of probability as a mathematical domain, it bears witness of the ambiguous status of a discipline whose concepts always hold something of the original desire of defining randomness and chance.

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75 Il n'est pas scientifique de parler sans cesse de l'antagonisme de la science et du catholicisme, en général, sans jamais descendre au détail. Si *la Flandre libérale* et les autres journaux anticatholiques belges sont assurés de cet antagonisme, qui les empêche de nous mettre sous les yeux, à nous catholiques, les prétendues antinomies de la science et de la foi, avec précision, dans un tableau à deux colonnes parallèles ? Dans la première seraient les vérités scientifiques empruntées à la physique, à la chimie, à l'astronomie, à la minéralogie, à la géologie, à la bota- nique, à la zoologie, à l'anthropologie, à la biologie, etc. ; dans la seconde, en face, si l'on en connaît, les décisions contraires des conciles et des papes, telles qu'on les trouve, par exemple, dans l'Enchividion de Denzinger. Mais nous osons prédire que la seconde colonne restera vide, si l'on n'y insère que des interprétations autorisées de la Bible et de la Tradition catholique, et si l'on ne met pas dans la première, sous prétexte de science, des assertions indéfendables.

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