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# INTERNAL MODEL BOUNDARY CONTROL OF HYPERBOLIC SYSTEM : APPLICATION TO THE REGULATION OF CHANNELS

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**Abstract:** This paper deals with the regulation problem of irrigation channels using a particular form of control by internal model (IMC). The control problem is stated as a boundary control of hyperbolic Saint-Venant Partial Differential Equations (pde). Regulation is done around an equilibrium state and spatial dependency of the operator parameters is taken into account in the linearized model. The Internal Model Boundary Control (IMBC) used in a direct approach allows to make a control parameters synthesis by semigroup conservation properties. In this paper previous stability results are generalized using perturbation theory in infinite dimensional Hilbert space, including more general hyperbolic systems and sufficient conditions for the closed loop stability are given explicitly by the spectrum calculation e.g.. Simulation and experimental results from Valence experimental micro-channel show that this approach should be suitable for more realistic situations.

**Keywords:** Shallow water equations, infinite dimensional perturbation theory, stabilization, multivariable internal model boundary control, hyperbolic PDE.

## 1. INTRODUCTION

Open surface hydraulic systems were studied by different approaches (Georges and Litrico, 2002; Malaterre, 2003) in modelling or control for mono and multireaches. The usual model is the Saint-Venant equations with regard to the control. In this area, two approaches are currently used: indirect approach in finite dimension (the pde's are approximated) and the direct one in infinite dimension (methods and tools directly relate to pde's). This paper belongs to the second approach, using directly partial differential equations for control synthesis (Pohjolainen, 1982; Pohjolainen, 1985; Touré and Rudolph, 2002). The

internal model boundary control is investigated for control synthesis for multireach regulation. The spatial dependency of variables is taken into account. Conservation properties of semigroup stability give the control synthesis, using some previous perturbations theory results (Kato, 1966; Pohjolainen, 1982; Pohjolainen, 1985).

In the first section, the non linear model for a rectangular channel is given in order to define a linear regulation model around an equilibrium state. The equations include lateral flow perturbations. The regulation problem is then defined for a channel composed of reaches in cascade. In the third section, the boundary control model is well posed to set up the essential properties of the open loop system to be conserved. Previous stability results are developed in order to consider a more general class of hyperbolic operators. In the fourth part, the closed loop system,

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considered as a structural perturbation of the open loop one, is associated to a particular form of the internal model control structure (Touré and Rudolph, 2002). The internal multivariable control law chosen is a proportional integral feedback. Then, synthesis parameters obtained by a direct application of some previous results (Kato, 1966; Pohjolainen, 1982) are recalled and the analytical expression of the resolvent allows to get best estimations of those parameters. In the last part, simulations and experimentations are given in mono and multireaches case for water level control.

## 2. THE CANNAL REGULATION PROBLEM: A BOUNDARY CONTROL SYSTEM

### 2.1 Non Linear Multireach Model

The hydraulic system considered in this paper is a cascade of  $p$  reaches separated by underflow gates and ended with an overflow as represented in Fig. 1. Considering a reach, e.g.  $i^{th}$  one, the following notations are used:

- $L_i$  is the reach length,
- $Q_i(x, t)$  denotes the water-flow,  $x \in ]0_i, L_i[$ ,  $t > 0$ ,  $Q_i \in L^2$ ,
- $Z_i(x, t)$  is the water level,  $x \in ]0_i, L_i[$ ,  $t > 0$ ,  $Z_i \in L^2$ ,
- $U_i(t)$  is the opening of the  $(i + 1)^{th}$  gate,  $U_0$  is the first one.

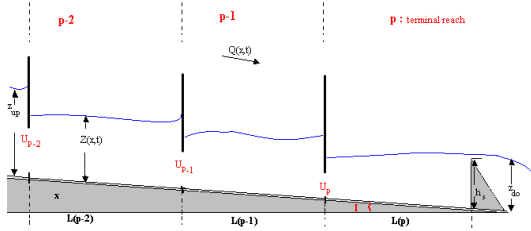


Fig. 1. Channel scheme: multireaches in cascade

The shallow water non linear pde for a rectangular channel can be written as follows for a given reach (Georges and Litrico, 2002; Malaterre, 2003):

$$\partial_t Z_i = -\partial_x \frac{Q_i}{b} + q_{l,i}(t) \quad (1)$$

$$\partial_t Q_i = -\partial_x \left( \frac{Q_i^2}{bZ_i} + \frac{1}{2}gbZ_i^2 \right) + f_i(x, t) \quad (2)$$

$$Z_i(x, 0) = Z_{0,i}(x), \quad Q_i(x, 0) = Q_{0,i}(x), \quad (3)$$

where  $b$  is the channel width,  $g$  the gravity constant. The function

$$f_i(x, t) = gbZ_i(x, t)(I_i - J_i(x, t)) + kq_{l,i}(x, t) \frac{Q_i(x, t)}{bZ_i(x, t)}$$

stands for friction perturbations, where  $I_i$  is the bottom slope,  $J_i$  the slope's friction expressed with the Manning-Strickler expression and  $R_i$  the hydraulic radius:

$$J_i = \frac{n^2 Q_i^2}{(bZ_i)^2 R_i^{4/3}}, \quad R_i = \frac{bZ_i}{b + 2Z_i}. \quad (4)$$

The function  $q_{l,i}(t)$  represents a lateral flow by unit length ( $m^2 \cdot s^{-1}$ ),  $q_{l,i} > 0$  ( $k = 0$ ) for supply (rain) and negative for loss (evaporation) ( $k = 1$ ).

Each underflow gates imposes a boundary condition of the form:

$$Q(0_i, t) = U_{i-1}(t)\Psi_{1,i}(Z(0_i, t)), \quad (5)$$

with  $\Psi_{1,i}(Z(x, t)) = K_{i-1}\sqrt{2g(z_{up} - Z(x, t))}$ ,  $Z < z_{up}$  and  $z_{up}$  is the water level before the upstream gate.  $K_i$  is the product of  $(i)^{th}$  gate (or overflow) width and water-flow coefficient of the gate. In addition for the last reach, the downstream boundary condition is:

$$Z(L_p, t) = \Psi_{2,p}(Q(L_p, t)), \quad (6)$$

with  $\Psi_{2,p}(Q(x, t)) = \left(\frac{Q(x, t)^2}{2gK_p^2}\right)^{1/3} + h_s$ ,  $h_s$  is the overflow height.

The control problem is the stabilization of the height and/or the water-flow, around an equilibrium behavior for each considered reach. The output to be controlled in this paper is the water level at each downstream.

### 2.2 A Regulation Model

Let  $(z_e(x), q_e(x))$  be an equilibrium state for a given reach. A linearized model with variable coefficients can be involved to describe the variations around this equilibrium behavior.

This equilibrium state of the system satisfies the following equations:

$$\left. \begin{aligned} \partial_x z_e &= \frac{a_4 - a_5 - 2q_l \frac{q_e}{z_e}}{gbz_e - q_e^2 / bz_e^2} \\ \partial_x q_e &= bq_l \end{aligned} \right\} \quad (7)$$

$$\text{with } a_5 = 2gbJ_e \frac{z_e}{q_e} - \frac{kq_l}{bz_e} \quad (8)$$

$$a_4 = gbz_e \left( I + J_e + \frac{4}{3} J_e \frac{1}{1 + 2z_e/b} \right) - kq_l \frac{q_e}{bz_e^2} \quad (9)$$

Considering one equilibrium state for the  $i^{th}$  reach, the linearized system around an equilibrium state  $(z_{e,i}(x), q_{e,i})$  is,  $\xi_i = (z_i \ q_i)^t \in X_i = L^2(0_i, L_i) \times L^2(0_i, L_i)$ :

$$\begin{aligned} \partial_t \xi_i(t) &= (\partial_t z_i(t) \ \partial_t q_i(t))^t \\ &= A_{1,i}(x) \partial_x \xi_i(x) + A_{2,i}(x) \xi_i(x) \end{aligned} \quad (10)$$

$$\xi_i(x, 0) = \xi_{0,i}(x) \quad (11)$$

The boundary conditions for an upstream gate (UG) and a downstream overflow (DO) are:

$$\begin{aligned} (UG) \quad q_i(0_i, t) - u_{i-1,e} \partial_z \Psi_1(z_{e,i}(0_i)) z_i(0_i, t) \\ = u_{i-1}(t) \Psi_1(z_{e,i}(0_i)) \end{aligned} \quad (12)$$

$$(DO) \quad z_i(L_i, t) - \partial_q \Psi_2(q_{e,i}) q_i(L_i, t) = 0 \quad (13)$$

where  $u_{i,e}$  is the  $i^{th}$  gate equilibrium state opening and  $u_i$  is the opening variations of this gate. Moreover

$$A_{1,i}(x) = - \begin{pmatrix} 0 & a_{1,i}(x) \\ a_{2,i}(x) & a_{3,i}(x) \end{pmatrix}, \quad (14)$$

$$A_{2,i}(x) = \begin{pmatrix} 0 & 0 \\ a_{4,i}(x) & -a_{5,i}(x) \end{pmatrix}, \quad (15)$$

with  $a_{1,i}(x) = \frac{1}{b}$ ,  $a_{2,i}(x) = gbz_{e,i}(x) - \frac{q_{e,i}^2}{bz_{e,i}^2(x)}$ ,  $a_{3,i}(x) = \frac{2q_{e,i}}{bz_{e,i}(x)}$ . Coefficients  $a_{4,i}(x)$  and  $a_{5,i}(x)$  are given by the relations (8) and (9).

The overall linearized system around an equilibrium state is then written as:

$$\partial_t \xi(t) = A_e(x) \partial_x \xi(x) + B_e(x) \xi(x) \quad (16)$$

$$\xi(x, 0) = \xi_0(x) \quad (17)$$

$$F(\xi, u_e) = G(u(t)), \quad (18)$$

where  $\xi = (z_1 \ q_1 \ z_2 \ q_2 \ \dots \ z_p \ q_p)^t \in X$  where  $X = \left[ \prod_{i=1}^p L^2(0_i, L_i) \times L^2(0_i, L_i) \right]$ . Equation (18) represents the boundary conditions (12)-(13).

Operators  $A_e(x)$  and  $B_e(x)$  are the generalization of operators  $A_{1,i}(x)$  and  $A_{2,i}(x)$  respectively:

$A_e = \text{diag}(A_{1,i})_{1 \leq i \leq p}$ ,  $B_e = \text{diag}(A_{2,i})_{1 \leq i \leq p}$ . Output variable  $y$  is the water levels variation around the equilibrium behaviour at each  $x_j = L_j$ ,  $1 \leq j \leq p$ ,

$$y(t) = C\xi(t) \in Y = \mathbb{R}^p, \quad t \geq 0$$

where  $C$  is a bounded operator (representation of the measurement):

$$C\xi = (\text{diag}(C_i))_{1 \leq i \leq p} \xi dx, \quad \mu > 0,$$

and  $C_i \xi = \left( \frac{1}{2\mu} \int_{x_i-\mu}^{x_i+\mu} \mathbf{1}_{x_i \pm \mu} \quad 0 \right) \xi dx$ ,  $\mu > 0$ , with  $\mathbf{1}_{x_i \pm \mu}(x) = \mathbf{1}_{[x_i-\mu, x_i+\mu]}(x)$  the function that equals 1 if  $x \in [x_i - \mu, x_i + \mu]$ , else 0, and  $\mu > 0$ .

The control is given by  $u(t) \in U = \mathbb{R}^p$ ,  $u \in C^\alpha([0, \infty], U)$  (Regularity coefficient is generally taken as  $\alpha = 2$ ). The control problem is to find the variations of the control action  $u(t)$  such that the water levels at each downstream reach  $x = L_i$  (i.e. the output variables) track reference signals  $r_i(t)$ , different for each reach.

The reference signal  $r_i(t)$  is chosen, for all cases, constant or no persistent.

### 3. OPEN LOOP CHARACTERIZATION

The system is first written as a classical boundary control system. Associated to the internal model structure, the closed loop system is described as an open loop perturbation. The control problem can be expressed as a stabilization problem around an equilibrium state, defined e.g. as  $\partial_t \xi = 0$ . The linearized boundary control model can be formulated as follows:

$$\partial_t \xi(t) = A_d(x) \xi(t), \quad x \in \Omega, \quad t > 0 \quad (19)$$

$$F_b \xi(t) = B_b u(t), \quad \text{on } \Gamma = \partial\Omega, \quad t > 0 \quad (20)$$

$$\xi(x, 0) = \xi_0(x) \quad (21)$$

where  $A_d(x) = A_e(x) \partial_x + B_e(x)$  is an hyperbolic operator, and  $F_b(\xi) = F_0 \xi(0, t) + F_L \xi(L, t)$ .

Results from (Fattorini, 1968; Don-Washburn, 1979) works, show that the abstract boundary control system (19)-(21) has a solution that exists and belongs to  $D(A_d)$  if  $A_d$  is a closed, densely defined operator, and generates a  $C_0$ -semigroup. Conditions have been yet given in order to get a well-posed system (Dos-Santos and Touré, 2005; Dos-Santos *et al.*, 2004), when  $q_i = 0$  e.g.. An extension to a larger class of operators is proposed here.

#### 3.1 Well defined operator

*Proposition 1.* The operator  $A_d(x) = A_e(x) \partial_x + B_e(x)$  of the system (19)-(21) is a closed and densely defined operator, if:

- $B_e(x)$  is  $A_e(x) \partial_x$ -bounded with  $b < 1$  on a Hilbert space ( $b < 1/2$  for a Banach),
- $-A_e(0)F_0 - A_e(L)F_L$  is invertible,
- $B_e(x)$  is densely defined,
- $A_e$  is invertible, densely defined and  $A_e^{-1}$  is bounded.

*Proposition 2.* Open loop system is well posed, i.e. generator of a  $C_0$ -semigroup if  $A_e(x)$  and  $B_e(x)$  are bounded and  $A_e(x)$  invertible, densely defined and  $A_e^{-1}$  is bounded  $\forall x \in \Omega$ .

Those properties established, the stability can be studied.

#### 3.2 Open Loop Stability

The idea is to consider  $A_d(x)$  as a perturbation of the operator  $A_e(x) \partial_x$  by an operator  $B_e(x)$  which is  $A_e(x) \partial_x$ -bounded. Recall that the open loop system, without control is:

$$\dot{\varphi}(t) = A\varphi(t) \quad t > 0, \quad x \in \Omega$$

$$\varphi(0) = \varphi_0 \in D(A(x))$$

and  $\varphi(t) = T_A(t) \varphi_0$  is the open loop state, where  $T_A(t)$  is the  $C_0$ -semigroup generated by  $A(x) = A_e(x) \partial_x + B_e(x)$ , and  $D(A) = D(A_d) \cap \text{Ker}(F_b)$ .

*Proposition 3.* Let suppose that  $\Re e(\sigma(A_e(x) \partial_x)) < 0$ ,  $\forall x \in \Omega$ . Then,  $A_e(x) \partial_x$  generates a  $C_0$ -semigroup exponentially stable. Moreover,  $\langle A_e(x) \partial_x \varphi, \varphi \rangle \leq 0$ ,  $\forall \varphi \in D(A_e(x) \partial_x)$ .

*Proof:* The idea of the proof is to use the resolvent compactness of  $A_e(x) \partial_x$  (Kato, 1966), and the spectral growth property (Triggiani, 1975), then results from (Curtain and Zwart, 1995) allow to conclude.

*Proposition 4.* Let consider  $A(x) = A_e(x)\partial_x + B_e(x)$ ,  $x \in \Omega$  such that  $A_e(x)\partial_x$  verifies  $\Re e(\sigma(A_e(x)\partial_x)) < 0$  and  $B_e(x)$  is  $A_e(x)\partial_x$ -bounded with  $b < 1$ ,  $\forall x \in \Omega$ . Suppose that:

- i)  $B_e(x)$  is semi-definite negative,
- ii)  $0 \in \rho(A(x)) = \rho(A_e(x)\partial_x + B_e(x))$ .

Then,  $A(x)$  is generator of a  $C_0$ -semigroup exponentially stable.

*Proof:* The idea is similar as for the previous proposition, indeed i) implies that:

$$\langle A(x)\varphi, \varphi \rangle < 0, \quad \forall \varphi \in X \text{ et } \forall x \in \Omega,$$

and if  $0 \in \rho(A(x))$  then  $\Re e(\sigma(A(x))) < 0$ . Moreover  $A(x)$  has a compact resolvent too.

The channel operator  $A(x) = A_e(x)\partial_x + B_e(x)$ , (16)-(18), generates a  $C_0$ -semigroup exponentially stable, as it verifies propositions 3 and 4 (with  $b = 0$ ) in the fluvial case, with  $q_l = 0$  or not (Dos-Santos *et al.*, 2005; Dos-Santos and Touré, 2005).

The control objective can be now achieved by a simple control law employed in the IMBC control structure.

#### 4. THE IMBC STRUCTURE: CLOSED LOOP

The Internal Model Boundary Control (IMBC) structure is an extension of the classical IMC structure with an additional internal feedback on the model (Fig. 2). The tracking model  $M_r$  and

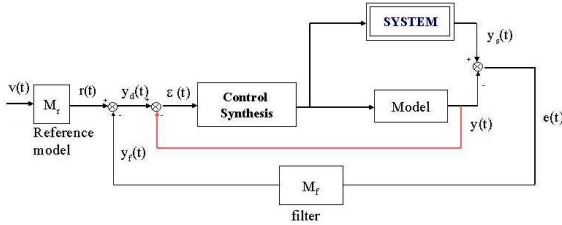


Fig. 2. IMBC structure

the low pass filter model  $M_f$  are stable systems of finite dimension (states  $x_r(t)$  and  $x_f(t)$  are associated to matrices  $A_r$ ,  $A_f$  resp.).

A multivariable proportional-integral feedback control is chosen for the control law:

$$\begin{aligned} u(t) &= \alpha_i \kappa_i \int \varepsilon(s) ds + \alpha_p \kappa_p \varepsilon(t) \\ &= \alpha_i \kappa_i \zeta(t) + \alpha_p \kappa_p \dot{\zeta}(t), \end{aligned}$$

with  $\dot{\zeta}(t) = \varepsilon(t)$ . Moreover,  $\varepsilon(t) = y_d(t) - y(t)$  acts like an integrator compared to the "real" measured output, indeed:  $\varepsilon(t) = r(t) - y(t) - y_f(t)$ . The exogeneous signals  $r(t)$  and  $e(t)$  is supposed to be no persistent, i.e.:  $\forall \epsilon > 0, \exists t_0 > 0 : \|r(t) - r(t_0)\| < \epsilon, \forall t > t_0$ , idem for  $e(t)$ .

#### 4.1 Closed Loop State Space

Let  $x_a(t) = (\varphi(t) \quad \zeta(t))^t$  the new state space then,

$$\begin{cases} \dot{x}_a(t) = \mathcal{A}(\alpha)x_a(t) + Bv(t) \\ x_a(0) = x_{a0} \end{cases} \quad (22)$$

As the extended IMBC state space  $X_a(t) = (x_r(t) \quad x_f(t) \quad x_a(t))^t$  does not improved the comprehension and has yet been discussed (Dos-Santos and Touré, 2005), we only focus on (22).

$\mathcal{A}$  can be viewed as a bounded perturbation of  $A$ :

$$\mathcal{A}(\alpha) = \mathcal{A}_e(\alpha) + \alpha_i \mathcal{A}_e^{(1)}(\alpha) + \alpha_p^2 \mathcal{A}_e^{(2)}(\alpha), \quad (23)$$

$$\text{and where } \mathcal{A}_e(\alpha) = \begin{pmatrix} (I + D\tilde{\kappa}_p C)A & 0 \\ -(I - CDW\tilde{\kappa}_p)C & 0 \end{pmatrix}$$

contains the open loop operator  $A$ .

$W$  is the left pseudo inverse of  $(I + \alpha_p \kappa_p CD)$ , such that  $W(I + \alpha_p \kappa_p CD) = I$  and  $\tilde{\kappa}_p = \alpha_p \kappa_p$ ,  $\tilde{\kappa}_i = \alpha_i \kappa_i$ ,  $\alpha = (\alpha_i, \alpha_p)$ .  $\mathcal{A}_e^{(1)}$  and  $\mathcal{A}_e^{(2)}$  are bounded operators as  $C$ ,  $D$  and  $CD$ .

Following the stability of both tracking and filter models ( $M_r$  and  $M_f$ ), matrices  $A_r$  and  $A_f$  can be chosen as stable Hurwitz ones. So the stability of the global system depends on the stability study of  $\mathcal{A}(\alpha)$  in (23).

#### 4.2 Closed Loop Stability Results

Now the perturbation theory, from Kato's works (Kato, 1966), for control problem of infinite dimensional system (Pohjolainen, 1982; Pohjolainen, 1985) can be used.

For the multireach operator, assumptions

- needed to preserve the open loop stability for the closed loop one are (Dos-Santos *et al.*, 2005):
- $\text{rank}(CD) = p$ ,  $\text{rank}(CDW) = p$ ,
- $\kappa_p = [CD]^\ddagger$  ( $\ddagger$  is the right pseudo inverse),
- $\kappa_i = -\theta[CD]^\ddagger$ ,  $0 < \theta < 1$ ,  $\Re e(\sigma(CDW\kappa_i)) < 0$ ,
- $0 \leq \alpha_i < \alpha_{i,max}$ , (for  $\Gamma$  of Fig.3)

$$\alpha_{i,max} = \min_{\lambda \in \Gamma} (a \|R(\lambda; \mathcal{A}_e)\| + 1)^{-1} \quad (24)$$

- $(I + \alpha_p \kappa_p CD)$  is invertible and its inverse is  $W = k(I - \alpha_p \kappa_p CD)$ , with  $k = (1 - \alpha_p^2)^{-1}$  and  $a = \|D\kappa_p C\|$ , such that:

$$0 \leq \alpha_p < \alpha_{p,max} = (\sup_{\lambda \in \Gamma} a \|R(\lambda; A)\|)^{-1}. \quad (25)$$

One of the difficulties is to get correct estimations of the control parameters  $\alpha_{i,max}$  and  $\alpha_{p,max}$ . Values obtained by simulations or experimentations are not optimum. Thus, their expression ((24)-(25)) must be developed before the simulations by the explicit calculus of the resolvent.

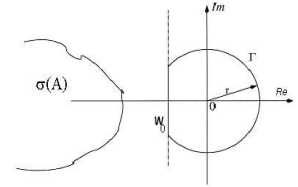


Fig. 3. Spectrum

### 4.3 Analytical Expression of the Synthesis Parameters

For an hyperbolic operator, its resolvent can be given explicitly as its spectrum. Indeed, let consider  $A(x) = A_e(x)\partial_x + B_e(x)$ , it is supposed that it is well posed, it generates a  $C_0$ -semigroup exponentially stable. Let  $\mu(x) = A_e^{-1}(x)(\lambda(x)Id - B_e(x))$ , and the boundary conditions are as follows  $F_0\xi(0) + F_L\xi(L) = 0$ , then  $R(\lambda, A) = (\lambda Id - A)^{-1}$  equals:

$$R(\lambda, A)v = \frac{e^{\mu(0)x} e^{\int_0^x \mu(s) ds} \mathcal{R}(\lambda, A)v}{A_e(0)F_0 + A_e(L)F_L e^{\mu(0)L} e^{\int_0^L \mu(s) ds}} \quad (26)$$

$$\mathcal{R}(\lambda(x), A(x))v(x) = -A_e(0)F_0 \int_0^x f(y) dy + A_e(L)F_L e^{\mu(0)L} e^{\int_0^L \mu(s) ds} \int_x^L f(y) dy$$

$$\text{with } f(y) = e^{-\mu(0)y} e^{-\int_0^y \mu(s) ds} A_e^{-1}(y)v(y)$$

Relation (26) is not defined when

$$A_e(0)F_0 + A_e(L)F_L e^{\mu(0)L} e^{\int_0^L \mu(s) ds} = 0,$$

it gives the spectrum. For example, for the operator  $A_1(x)\partial_x$ , its spectrum is when an overflow and an underflow gates are considered:

$$\sigma(A_1(x)\partial_x) = \{\lambda_n : \lambda_n(x) = \lambda(x) + \frac{2in\pi}{2L}\theta(x)\}$$

with  $\lambda : \Omega \rightarrow \mathbb{R}^- \setminus \{0\}$ ,

$$\lambda(x) = \frac{-\ln(\alpha_L)}{2L}\theta(x) = \frac{-\ln(\alpha_L)}{2L} \frac{a_1 a_2(x)}{a_3(x) + a_2(x)}.$$

Moreover, one get:

$$\|R(\lambda, A)\|_{L^2(\Omega)} = \|(\lambda Id - B_e)^{-1}\|,$$

for the semigroup stability ( $T_A(t)$ ), so the synthesis parameters evaluation depends on the open loop operator;  $\lambda$  expression allows to define  $\Gamma$  (Fig (3)), coupled together with the previous resolvent expression,  $\alpha_{p,max}$  in (25) can be analytically evaluated (and in the same way for  $\alpha_{i,max}$ ).

## 5. RESULTS

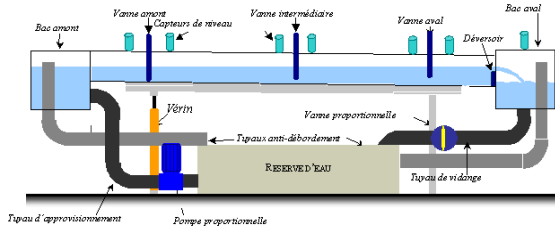


Fig. 4. Pilot channel of Valence

Simulations gave satisfactory results for a single reach (cf. (Dos-Santos *et al.*, 2004)) and for the multireach cases, too. Then, the proposed control

law was implemented on the Valence channel (LCIS/INPG, France). This pilot channel is an experimental process (length=8 m, width=0.1 m) with a rectangular basis, a variable slope and with three gates (three reaches and an overflow). In both cases, Simulink is used.

parameters	$B$ (m)	$L$ (m)	$K$	slope $^0/00$ ( $m^{1/3}s^{-1}$ )	$Q_{max}$ ( $m^3s^{-1}$ )
values	0.1	7	97	1.6	0.009
parameters	$\mu_{UG1}$	$\mu_{UG2}$	$\mu_{UG3}$	$\mu_{DG}$	
values	0.6	0.65	0.73	0.66	

Table 1. Parameters of the channel of Valence

### 5.1 Simulation : Infiltrations

The case of one reach is treated, and infiltrations are considered with  $ql = -0.001 dm^2 \cdot s^{-1}$  by unit length. It stands for  $0.1 mm \cdot s^{-1}$  by  $dm^2$  or  $3.6 dm \cdot h^{-1}$  by  $dm^2$ . The aim of the simulation is to compare the effect when infiltrations are taking into account on the model ( $ql \neq 0$  in (7) called AI) or not (infiltrations are considered as perturbations  $ql = 0$ , called SI).

The reference is to stay at equilibrium  $r(t) = 1.16 dm$ , and initial conditions are the following:

$$z_e(0) = 0.95 dm, q_e(0) = 3 dm^3 \cdot s^{-1}.$$

The model including the infiltrations is the bet-

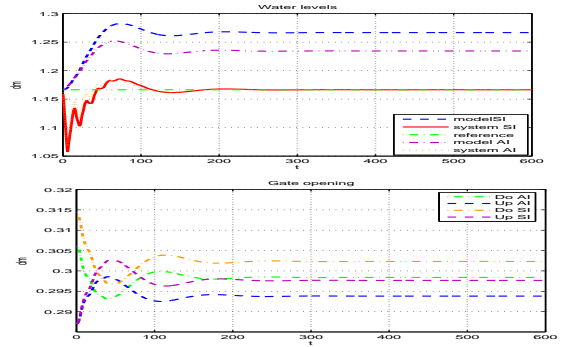


Fig. 5. Comparison of the models with (AI) or without infiltrations (SI)

ter. Nevertheless, in both cases the system tracks the reference asked quite similarly. The difference is stressed on the variations of the gates opening, the model AI (with infiltrations) seems more suitable. The variations are less importants than for the second model, allowing to manage other kinds of perturbations.

On the next simulation, rain and infiltrations are coupled ( $2 cm^3 s^{-1}$  when  $t = 690s$ ), initial conditions are the same, and the infiltrations too. The results obtained show the suitability of this approach, experimentations have so been realized on the micro-channel.

### 5.2 Experimentation: two reaches

For this experimentation, the aim is to show that the conditions (24) and (25) are sufficient but

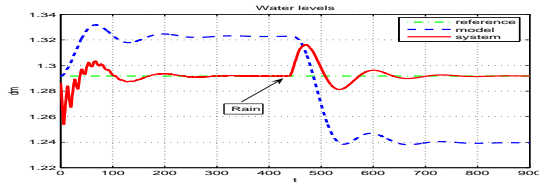


Fig. 6. Rain and infiltrations

not necessary. Indeed, the synthesis parameters are equal to  $\alpha_i = 2$ ,  $\alpha_p = 0$  with  $\alpha_{i,max} \simeq 0,73$ ,  $\alpha_{p,max} \simeq 0,65$ . Initial conditions are:  $q_e = 1 \text{ dm}^3 \cdot \text{s}^{-1}$ ,  $z_{e1}(0) = 1.22 \text{ dm}$ ,  $z_{e2}(0) = 1.02 \text{ dm}$ . References are for (each with a length of  $3.5 \text{ dm}$ ):

- the first reach,  $r_0 = 1.28 \text{ dm}$ :

$$r(t) = r_0 \text{ dm when } 0s \leq t \leq 85s$$

$$r(t) = 1.2 * r_0 \text{ when } 90s \leq t \leq 330s$$

$$r(t) = 0.9 * r_0 \text{ when } 330s \leq t \leq 475s$$

$$r(t) = 1.1 * r_0 \text{ when } 480s \leq t.$$

- and for the second reach,  $r_L = 1.077 \text{ dm}$ :

$$r(t) = r_L \text{ dm when } 0s \leq t \leq 160s$$

$$r(t) = 0.76 * r_L \text{ when } 160s \leq t \leq 320s$$

$$r(t) = 0.9 * r_L \text{ when } 325s \leq t.$$

Even if  $\alpha_i \gg \alpha_{i,max}$  and that the variations are greater than  $\pm 20\%$ , the error between the model and the system is less than  $10\%$ , and the system tracks the references on both reaches. Experimental results show too that this approach

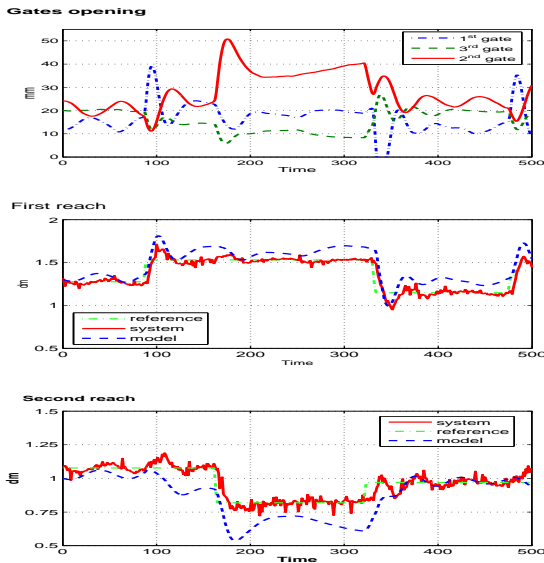


Fig. 7. Gate opening & Water levels

is suitable for the regulation. Indeed, given a control space of  $\pm 20\%$  around the equilibrium state, the results are satisfactory. Beyond those  $\pm 20\%$ , the error between the system and the reference (and the model too) can increase dramatically. It seems important to develop necessary conditions for the closed loop stability.

## 6. CONCLUSION

The direct approach developed here, seems suitable for the irrigation channel regulation. Previous

theoretical results have been extended to a more general class of hyperbolic equations, which can be written as  $A(x) = A_e(x)\partial_x + B_e(x)$ , for a system such that (19)-(21). They are applied to the multireach case for which lateral perturbations of the water flow are added with success. Spatial evolution of the parameters allow to manage in a better way the perturbations, and to transpose it to real situation. Simulation and experimentation results are encouraging for network applications.

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