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Location and research activities organization: could public/private cooperation be harmful?*

Marie-Laure Cabon-Dhersin[†], Emmanuelle Taugourdeau[‡]

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Abstract

This paper investigates the organization and the distribution of research activities between nearby public and private laboratories. In a three-stage game, the 'size', 'location' and 'research effort' are determined under the assumption that public spillovers depend on the location of the private laboratory. We compare two scenarios in which the research efforts are decided either cooperatively or non-cooperatively. We show that for particular levels of subsidy granted to the public lab, higher funding favors spatial proximity and increases the total research effort in the cooperative case, while it diminishes the total effort in the non-cooperative one. Moreover, compared with the non-cooperative case, research cooperation i) may increase the distance between the two laboratories, ii) makes the public laboratory smaller, iii) increases the total research effort, but iv) is detrimental to the payoff of the whole research sector.

Key words: research cooperation, spatial location, public subsidy.

Code JEL: C7, H2, L3, R3.

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1 Introduction

A recurrent challenge faced by European authorities is how to foster closer cooperation between universities and private firms to increase the production and dissemination of new knowledge (European-Commission, 2003, 2008). Regional knowledge clusters, competitiveness clusters, poles of excellence and/or science and technology parks are new institutional arrangements that aim to facilitate interactions between academia and industry (Link and Scott, 2007).

These initiatives to nurture competitiveness stem from theoretical approaches and empirical investigations on the advantages of R&D cooperation in the presence of knowledge spillover (i.e. knowledge produced by one agent that may contribute to the production of knowledge by the other agents without cost). Theoretical models of R&D cooperation show that the coordination of research efforts can help competitive firms internalize their knowledge spillover, increasing research efforts and social welfare (d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; De Bondt, 1997; Amir, 2000; Norman and Pepall, 2004). This argument can also be applied to research partnerships between firms and universities (Poyago-Theotoky, 2009; Poyago-Theotoky et al., 2002; Beath et al., 2003). Since knowledge has the properties of a public good, the outcomes of public sector research have often been put to good use by private sector firms (Jaffe, 1989; Audretsch et al., 2002; Autant-Bernard, 2001; Cohen et al., 1994; Veugelers and Cassiman, 2005). For instance, firms using emerging technologies (e.g. bio- and nano-technology) have a strong interest in public sector knowledge (Boufaden and Plunket, 2007).

Others studies in the geography of innovation have added an important spatial dimension to the discussion by showing that the positive effects also increase with proximity (Cooke, 2001; Furman et al., 2006; Autant-Bernard and LeSage, 2011). Geographical proximity that benefits the innovation process through knowledge spillover is frequently cited as an explanation for the emergence of cooperation between universities and firms (Audretsch, 1998; Varga, 2000).

What happens if we turn the question around though? Does research cooperation encourage firms to move closer to universities in order to benefit from public knowledge spillover? When the private sector learns from the public sector through knowledge spillover, which depends on geographical proximity, the incentive for the private firm is to anchor the cooperation at the same location in order to internalize the public externalities. Many cluster initiatives refer to Marshallian-type externalities. However, there is another force that acts in the opposite direction. Based on many examples in the medical, chemical and agricultural domains, we argue that public and private research actors are competitors in the production of knowledge and innovation. Geographical proximity can also enhance the rivalry between researchers from the two sectors. Recently, Bloom et al. (2013) and König et al. (2014) have shown that this competition effect (or rivalry effect) has a significant negative impact on a firm's payoff. This may constitute a negative competition externality. The race between public and private efforts in genome sequencing is an illustration of this type of competition (Car-raro and Siniscalco, 2003; Gisler et al., 2011). Focus group discussions have confirmed the existence of rivalry between researchers, which discourages sharing and may even lead some scientists to sabotage the work of competitors (Anderson et al., 2007). Recent studies in economic geography reveal how spatial proximity is an obstacle to a true win-win relationship and how proximity between researchers strengthens research competition (Ben Letaifa and Rabeau (2013) for an information and communication technology cluster in Canada, Broekel and Boschma (2012) for the Dutch aviation industry). In this setting, we investigate whether cooperation is more effective than non-cooperation in terms of research efforts and the payoff

that can be appropriated by research actors.

Another phenomenon that we want to highlight in this paper is the impact of geographical proximity on the size of the laboratories. Here, the 'size' of each laboratory evolves endogenously, the total number of researchers being fixed. This model also allows the distribution of researchers between teams to be determined, the stability of the teams being governed by standard coalition formation theory (d'Aspremont et al., 1983; Carraro and Siniscalco, 2003).

We define a research cluster (RC) as a set of nearby and cooperating public and private research organizations. On positive grounds, this paper studies the conditions required for such a RC to exist through a game theoretic formulation. We propose a three-stage game that involves a fixed number of researchers and two laboratories, one public and one private. In the first stage, each researcher chooses to work either in the public laboratory or in the private one. In the second stage, the private laboratory chooses its location and in the third stage, the members of the two laboratories choose their respective research efforts. Two scenarios are considered for the last stage: in the first, individuals chose their research efforts non-cooperatively and aim to maximize the payoff of their particular laboratory; in the second, researchers coordinate their activities so as to maximize the overall payoff of the two laboratories. This approach allows us to examine whether cooperation encourages spatial proximity and boosts the research efforts and gains of the two laboratories. We show that the results crucially depend on the magnitude of the subsidy granted to the public laboratory and on the research's commercial value. In our model, we realistically assume that the private laboratory's marginal benefit from the research effort is greater or equal to that of its public counterpart¹. To this end, we introduce the notion of a 'commodification capacity': a laboratory's ability to extract monetary value from research output.

Our approach incorporates some ideas previously developed in economic geography (Fujita and Thisse, 2002). A number of theoretical models describe how spillovers between firms shape the geography of production and innovation. They combine two different strands of literature: the theory of locational choice and the economics of innovation, dealing with spatial competition following Hotelling (Biscaia and Mota, 2012; Belleflamme et al., 2000; Gersbach and Schmutzler, 1999) or Cournot (Van Long and Soubeyran, 1998). Mai and Peng (1999) introduce cooperation between firms into the Hotelling spatial competition model in the form of innovation exchange through communication. Under the assumption that knowledge spillover depends on the firms' location, Piga and Poyago-Theotoky (2005) show that the distance between firms increases with the degree of product differentiation. In the same way, Van Long and Soubeyran (1998) conclude that firms agglomerate when the endogenous spillover is a convex (or linear) function of their separation. In our model, we assume that spillover occurs exclusively from the public to the private laboratory and we retain the assumed linear relationship between spillover and distance, but with only the private laboratory being able to move.

This article contributes to the field as follows. To the best of our knowledge, no existing spatial theoretical model explains the rationale for cooperative strategies between nearby public and private research organizations². On positive grounds, this paper is an attempt to fill this gap and provide a theoretical basis for empirical studies (Abramovsky and Simpson, 2011). First, we explore the subsidy conditions required for the two laboratories to exist: for a given asymmetry

¹In the public laboratory, knowledge gained from research efforts is considered a partial public good and cannot be fully "owned".

²An exception is the paper by Mukherjee and Ramani (2011), which explores the rationale for technology parks involving asymmetric firms.

in 'commodification capacities', the subsidy must lie between a minimum and a maximum level. The public laboratory only exists if the public subsidy is above a certain threshold, while too much subsidy prevents the formation of a private laboratory. For intermediate levels of subsidy and however the research is organized (i.e. cooperatively or non-cooperatively), the laboratories move closer and the public one grows as the subsidy is increased. Second, we compare the equilibrium outcomes under cooperative and non-cooperative scenarios given intermediate levels of subsidy. We show that cooperation (viz. coordinated research efforts in the two laboratories) outperforms non-cooperation in terms of the total research effort (particularly that of the private laboratory) but not in terms of the total payoff. The total effort is negatively affected by increases in public subsidy in the non-cooperative case, but positively affected in the cooperative one. Cooperation may however deter from spatial proximity and shrink the public laboratory.

The paper is organized as follows. Section 2 presents the model and Section 3 presents the results obtained under competitive and cooperative research effort. We compare and discuss the results in Section 4 before concluding.

2 The model

In our model, we study strategic interactions between private and public research laboratories that are competitors in the market of knowledge and innovation. This is typical of medical, chemical and agricultural research where public and private laboratories may pursue the same goals. Consider the Human Genome Project (molecular biology) for example; the public and private laboratories involved in this project aim to sequence the human genome by assembling a genetic and a physical map, and by sequencing model organisms³.

We can therefore assume that the same research may be undertaken in two independent laboratories, one in the private sector (*pr*) and one in the public sector (*pu*). We consider a fixed finite number N of researchers who may work either in the public laboratory (n), or in the private one ($N - n$) with $0 \leq n \leq N$. All the researchers are equally skilled.

A crucial component of the model is the *physical distance* between the public and private laboratories. The location of the public laboratory is fixed exogenously and the distance between the two laboratories is determined only by the private one. Typically indeed, most public laboratories are hosted in universities, which cannot relocate (they are not geographically mobile). We introduce a variable $d \in [0, 1]$ that represents the distance between the two laboratories. The distance is minimal ($d = 0$) when the two laboratories share the same location, and maximal ($d = 1$) when they undertake their research activities independent of one another. While the researchers have identical abilities, the payoff of the public laboratory differs from that of the private one because the two organizations promote their research efforts differently:

1. The institutional arrangements governing the protection of knowledge differ between research sectors. Research efforts can generate knowledge externalities or spillovers. Nevertheless, in our model, knowledge externalities (or spillovers) occur in one direction only:

³Another example is the Tilapia project in which both public and private laboratories aim to produce genetically improved strains of these fish.

the research undertaken by the public laboratory may benefit the private laboratory at no cost, to an extent that depends on its location. Spillovers occur in one direction only because whereas private laboratories appropriate their research results through intellectual property rights, public laboratories produce knowledge that is publicly disseminated (via academic publications for instance). The existence of asymmetric externalities is consistent with the empirical literature (Jaffe, 1989; Audretsch et al., 2002; Cohen et al., 2002; Autant-Bernard, 2001; Cohen et al., 1994; Veugelers and Cassiman, 2005): spillovers from universities have been identified as a key source of firm innovation but the public research sector does not benefit from knowledge spillover.

The spillover function is proportional to $1 - d$. In line with Piga and Poyago-Theotoky (2005)⁴, we assume that the distance between the two laboratories, $d \in [0, 1]$, determines the size of the public spillover: the closer the laboratories, the greater the spillover. The spillover is null when $d = 1$ and maximum when $d = 0$ (laboratories in the same location). The existence of public spillovers can be a strong motivation for the private laboratory to move closer to the public one.

2. To deal with the incentive problem relating to the production of public goods with externalities in the public research sector, the public laboratory is funded by a fixed subsidy (i.e. independent of the research effort⁵) in the form of government payments to each researcher.
3. Because public research efforts cannot be owned by public laboratories in their entirety, we introduce the notion of a 'commodification capacity' that differs between laboratories. We call 'commodification capacity' a laboratory's ability to convert its research into monetary value. In practice, private laboratories commodify their research through patents that create a private incentive mechanism in the production of knowledge and prevent any disclosure of the patented results. The commodification of public research is less widespread and can take the form of prizes or research contracts. Realistically then, we assume that the marginal benefit achieved by the private laboratory from its research effort is equal to or greater than that achieved for the same research effort in the public laboratory.

According to these features, the payoffs of the public and private groups are expressed differently. The payoff of the private research laboratory is given by:

$$G^{pr} = \begin{cases} w \sum_1^N x_i^{pr} - \sum_1^N \left(\frac{1}{2} x_i^{pr} \right)^2 \equiv G_A^{pr} & \text{if } n = 0 & (1) \\ w \left(\sum_{n+1}^N x_i^{pr} + (1-d) \sum_1^n x_i^{pu} \right) - \sum_{n+1}^N \left(\gamma(1-d)^2 + \frac{1}{2} x_i^{pr2} \right) & \text{if } 1 \leq n \leq N-1 & (2) \end{cases}$$

⁴Our work differs from that of Piga and Poyago-Theotoky (2005), who consider a traditional Hotelling framework where transport costs are incurred by consumers.

⁵Research subsidies are a subject often discussed in the economic literature (Petraakis and Poyago-Theotoky, 2002; Klette et al., 2000). A common feature of these subsidies is that they are unconditional, that is, their level is not related to the research effort or the success of the research project.

where w is the marginal benefit yielded by research efforts in the private laboratory. The resources of the private laboratory depend on i) the collective efforts of its researchers, $\left(\sum_{n+1}^N x_i^{pr}\right)$, where x_i^{pr} is the research effort of an individual in the private laboratory, ii) the exploitation of spillover from the public laboratory's research, $(1-d)\sum_1^n x_i^{pu}$, where x_i^{pu} is the research effort of an individual in the public laboratory. Provided that $n \neq 0$, the amount of spillover depends both on the total research effort in the public laboratory, $\sum_1^n x_i^{pu}$, and on the distance between the two laboratories, $d \in [0, 1]$. The payoff also depends on the individual costs of effort, which are assumed to be quadratic, reflecting the diminishing returns of effort. There is an additional cost from the rivalry effect, namely that each researcher is engaged in a race with the researchers in the other laboratory. The term $(\gamma(1-d)^2)$ represents the rivalry cost per individual, which increases quadratically with the spatial proximity of the laboratories. The parameter $\gamma > 0$, assumed constant, associates a cost to this rivalry effect, which has been carefully documented by Anderson et al. (2007). Following Ben Letaifa and Rabeau (2013), we relate the rivalry effect to the spatial proximity of the two groups, reflecting the aggravating effect of contiguity⁶.

The payoff for the public research laboratory is:

$$G^{pu} = \begin{cases} ns + \frac{w}{a} \sum_1^N x_i^{pu} - \sum_1^N \frac{1}{2} (x_i^{pu})^2 \equiv G_A^{pu} & \text{if } n = N \\ ns + \frac{w}{a} \sum_1^n x_i^{pu} - \sum_1^n \left(\gamma(1-d)^2 + \frac{1}{2} x_i^{pu2} \right) & \text{if } 1 \leq n \leq N-1 \end{cases} \quad (3)$$

$$(4)$$

where s stands for the public subsidy per researcher and $\frac{w}{a}$ is the marginal benefit derived from the efforts of each researcher in a laboratory whose 'commodification capacity' is $a \geq 1$. (As mentioned above, this represents the laboratory's ability to commodify its research gains through prizes and research contracts). The public laboratory does not benefit from spillover but faces the same costs (effort and rivalry) as those of the private laboratory.

The strategic interactions between the two labs are driven by a three-stage game:

1. In the first stage, the N researchers are divided into two teams (n researchers in the public laboratory, $(N-n)$ in the private one) via a group formation game. We use the stability conditions of d'Aspremont et al. (1983) and Carraro and Siniscalco (2003) to calculate the equilibrium size of the laboratories:

Definition 1. *The stability of the research sector is governed by the following conditions:*

$$\frac{G^{pu}(n^*)}{n^*} = g^{pu}(n^*) \geq 0 \quad \text{and} \quad \frac{G^{pr}(n^*)}{N-n^*} = g^{pr}(N-n^*) \geq 0, \quad (5)$$

$$g^{pr}(n^*) \geq g^{pu}(n^* - 1) \quad \text{and} \quad g^{pu}(n^*) \geq g^{pr}(n^* + 1) \quad (6)$$

where $0 < n^* < N$ and g^{pu} (resp. g^{pr}) is the payoff of each researcher working in the public (resp. private) laboratory.

⁶Since we refer to the contiguity effect between the two groups, the rivalry effect depends neither on the number of researchers in the other group nor on their research efforts.

When these conditions are satisfied, no researcher wants to switch sectors. Assuming N is sufficiently large, conditions (5) and (6) can be approximated by the following equality:

$$g^{pu}(n^*) = g^{pr}(n^*) \quad (7)$$

2. In the second stage, the private sector laboratory chooses an optimal separation, d , from its public sector counterpart with $d \in [0, 1]$,
3. In the third stage, individual research efforts are chosen in each laboratory: x_i^{pu}, x_i^{pr} .

3 The outcome at equilibrium

In line with the theoretical literature on cooperative R&D and the formation of Research Joint Ventures, we distinguish two scenarios in the third stage:

- in the non-cooperative scenario, researchers compete and conduct their research independently: they chose their research effort so as to maximize the payoff of their laboratory.
- in the cooperative scenario, researchers coordinate their research efforts so as to maximize the overall payoff of the two laboratories. Each laboratory still commodifies the research effort separately.

Given the one-way spillover and the fixed location of the public laboratory, the distance between the two laboratories is chosen non-cooperatively by the private laboratory in both scenarios.

In this section we determine the non-cooperative and cooperative outcomes that we will compare in the next section. In both cases, the game is solved backward.

3.1 The non-cooperative scenario

In the non-cooperative (NC) scenario, the optimal level of effort is obtained by maximizing the payoff functions, Equations (2) and (4), with respect to x_i^{pr} and x_i^{pu} , respectively, treating the distance between the laboratories and the number of researchers in the two groups as constants. This yields

$$\frac{\partial G^{pr}}{\partial x_i^{pr}} = 0 \Rightarrow x_{i,nc}^{pr} = w \quad (8)$$

$$\frac{\partial G^{pu}}{\partial x_i^{pu}} = 0 \Rightarrow x_{i,nc}^{pu} = \frac{w}{a} \quad (9)$$

Individual efforts depend on the marginal benefit in each lab. Since the commodification capacity of the public laboratory is lower than that of the private one ($a \geq 1$), individuals usually contribute more in the latter .

Replacing the values obtained for individuals' efforts into Equation (2) and (4) gives:

$$G_{nc}^{pr} = \frac{N - n_{nc}}{2} w^2 + n_{nc} \frac{w^2}{a} (1 - d_{nc}) - \gamma(N - n_{nc})(1 - d_{nc})^2$$

and

$$G_{nc}^{pu} = n_{nc} s + \frac{n_{nc}}{2} \left(\frac{w}{a}\right)^2 - \gamma n_{nc} (1 - d_{nc})^2$$

The private laboratory chooses the optimal distance from its public counterpart, d_{nc} , by maximizing its payoff function with respect to this distance.

$$\frac{\partial G_{nc}^{pr}}{\partial d_{nc}} = 0 \iff \underbrace{2\gamma(N - n_{nc})(1 - d_{nc})}_{\text{rivalry effect}} = \underbrace{n_{nc} \frac{w^2}{a}}_{\text{spillover effect}} \quad (10)$$

The optimal distance is the one that equalizes the rivalry and spillover effects.

$$d_{nc} = 1 - \frac{w^2}{2a\gamma} \frac{n_{nc}}{N - n_{nc}} < 1 \quad (11)$$

with $\frac{\partial d_{nc}}{\partial n_{nc}} < 0$. Equation (10) shows that when the number of researchers in the public laboratory increases, the private laboratory benefits from higher knowledge spillover and lower rivalry costs because there are fewer researchers present. Therefore, to balance the two effects, the private laboratory moves closer to the public one. Equation (11) can also be expressed in terms of the ratio of the respective numbers of researchers in the two laboratories:

$$\frac{n_{nc}}{N - n_{nc}} = (1 - d_{nc}) \frac{2a\gamma}{w^2} \quad (12)$$

Lemma 1. *For a given n_{nc} , the optimal distance d_{nc} verifies the following relationships:*

$$\begin{cases} \text{If } \frac{n_{nc}}{N} \geq \frac{2a\gamma}{2a\gamma + w^2}, & d_{nc} = 0 \\ \text{If } \frac{n_{nc}}{N} < \frac{2a\gamma}{2a\gamma + w^2}, & (1 - d_{nc}) = \frac{w^2}{2a\gamma} \frac{n_{nc}}{N - n_{nc}} \in (0, 1) \end{cases}$$

Taking into account the two previous stages of the game, the non-cooperative payoffs of the two laboratories become:

$$G_{nc}^{pu} = n_{nc} g_{nc}^{pu}(d_{nc}) = n_{nc} \left(s + \frac{w^2}{2a^2} - \gamma(1 - d_{nc})^2 \right) \quad (13)$$

$$G_{nc}^{pr} = (N - n_{nc}) g_{nc}^{pr}(d_{nc}) = (N - n_{nc}) \left(\frac{w^2}{2} + \gamma(1 - d_{nc})^2 \right) \quad (14)$$

We can now solve the first stage of the game. According to definitions (5) and (6), the equilibrium condition in the NC scenario is as follows:

- When all the researchers want to leave the private laboratory to join the public one:

$$g_{nc}^{pu}(d_{nc}) > g_{nc}^{pr}(d_{nc})$$

Hence,

$$n_{nc}^* = N \quad \text{and} \quad N - n_{nc}^* = 0$$

- When no researcher wants to switch from one laboratory to the other:

$$g_{nc}^{pu}(d_{nc}^*) = g_{nc}^{pr}(d_{nc}^*)$$

$$\Leftrightarrow s + \frac{1}{2} \left(\frac{w}{a} \right)^2 - \gamma(1 - d_{nc})^2 = \frac{1}{2}w^2 + \frac{n_{nc}}{N - n_{nc}} \frac{w^2}{a} (1 - d_{nc}) - \gamma(1 - d_{nc})^2 \quad (15)$$

Combining Equations(15) and (12) yields:

$$(1 - d_{nc})^2 = \frac{s - \frac{w^2}{2} \left(1 - \frac{1}{a^2}\right)}{2\gamma} \quad (16)$$

Hence,

$$1 \leq n_{nc}^* \leq N - 1$$

- When all the researchers want to leave the public laboratory to join the private institution:

$$g_{nc}^{pu}(d_{nc}) < g_{nc}^{pr}(d_{nc})$$

Hence,

$$n_{nc}^* = 0 \quad \text{and} \quad N - n_{nc}^* = N$$

Determining the equilibrium of the whole game, we obtain the following proposition:

Proposition 1. *In the NC scenario, the subgame perfect Nash equilibrium is such that:*

1. *If the public subsidy s is too low ($s \leq \underline{s}$), only the private research laboratory exists: $n_{nc}^* = 0$.*
2. *If the public subsidy s is neither too high ($s < \bar{s}$) nor too low ($s > \underline{s}$), the research laboratories coexist with:*

$$n_{nc}^* = N \frac{2a\gamma(1 - d_{nc}^*)}{2a\gamma(1 - d_{nc}^*) + w^2} \in [1, N - 1]$$

and

$$d_{nc}^* = 1 - \frac{1}{2a} \sqrt{\frac{2a^2s - w^2(a^2 - 1)}{\gamma}} \in (0, 1)$$

3. *If the public subsidy is such that $s = \bar{s}$, the laboratories coexist with:*

$$n_{nc}^* = N \frac{2a\gamma}{2a\gamma + w^2} < N$$

and the private laboratory chooses to share the public laboratory's location:

$$d_{nc}^* = 0$$

4. *If the public subsidy s is too high ($s > \bar{s}$), only the public research laboratory exists: $n_{nc}^* = N$.*

With

$$\underline{s} = \frac{1}{2} \left(\frac{w}{a} \right)^2 (a^2 - 1) \quad \text{and} \quad \bar{s} = \frac{1}{2} \left(\frac{w}{a} \right)^2 (a^2 - 1) + 2\gamma.$$

Proof: The proof is straightforward.

Proposition 1 shows that when the public subsidy is too low, an individual's payoff is always higher in the private laboratory than in the public one. There is no incentive to join the public team and all researchers move to the private laboratory. Conversely, when the public subsidy is too high, the mechanism is reversed and researchers shun the private laboratory. The two research laboratories can coexist provided the public subsidy is neither too low nor too high. In this case, there is an interior solution that determines the equilibrium proportion of researchers that join the public laboratory, n_{nc}^* , and the optimal distance chosen by the private laboratory, d_{nc}^* . The level of the public subsidy that maximizes the proximity of the two laboratories, $d_{nc}^* = 0$, is \bar{s} .

3.2 The cooperative scenario

In the cooperative scenario considered here (denoted by an underscore c), the researchers of the two laboratories coordinate their research efforts : they choose their efforts so as to maximize the laboratories' joint payoff: $G_c^{tot} = G_c^{pu} + G_c^{pr}$. This gives:

$$\frac{\partial G_c^{tot}}{\partial x_i^{pr}} = 0 \Rightarrow x_{i,c}^{pr} = w \quad (17)$$

$$\frac{\partial G_c^{tot}}{\partial x_i^{pu}} = 0 \Rightarrow x_{i,c}^{pu} = w \left(1 - d + \frac{1}{a} \right) \quad (18)$$

In the cooperative scenario, since spillover benefits the private lab, the efforts of researchers in the public laboratory depend positively on the proximity of the private laboratory; however, these efforts only become higher than those of researchers in the private laboratory if $d < \frac{1}{a}$.

Using the optimal values given by Equations (17) and (18) for the research efforts, we rewrite the payoff functions for both laboratories (Equations (2) and (4)):

$$G_c^{pu} = n_c \left(s + \frac{w^2}{2} \left(\frac{1}{a^2} - (1 - d_c)^2 \right) - \gamma(1 - d_c)^2 \right) \quad (19)$$

$$G_c^{pr} = \frac{N - n_c}{2} w^2 + n_c w^2 (1 - d) \left(1 - d_c + \frac{1}{a} \right) - (N - n_c) \gamma (1 - d_c)^2 \quad (20)$$

In the second stage, the private laboratory non-cooperatively chooses its optimal separation from the public laboratory, d_c , by maximizing its payoff function ⁷.

$$\frac{\partial G_c^{pr}}{\partial d_c} = 0 \iff \underbrace{2\gamma(N - n_c)(1 - d_c)}_{\text{rivalry effect}} = \underbrace{n_c w^2 \left(1 - d_c + \frac{1}{a} \right) + n_c w^2 (1 - d_c)}_{\text{spillover effect}} \quad (21)$$

The optimal distance is still the one that equalizes the marginal benefit of knowledge spillover and the marginal cost of rivalry. In addition to the effects already mentioned for the NC case, the benefits of knowledge spillover are enhanced here because the public researchers' efforts increase when the private laboratory is close (second term in the spillover effect). We obtain:

$$d_c = 1 - \frac{n_c w^2}{2a((N - n_c)\gamma - n_c w^2)} < 1 \iff \frac{n_c}{N} < \frac{\gamma}{w^2 + \gamma} \quad (22)$$

⁷The concavity of the function is verified in Appendix 1.

and

$$\frac{\partial d_c}{\partial n_c} = -\frac{w^2}{2a} \frac{N}{((N - n_c)\gamma - n_c w^2)^2} < 0 \quad (23)$$

Equation (21) shows that a larger number of public researchers increases the spillover effect and diminishes the rivalry effect. Since the optimal location of the private laboratory is the one that balances these two effects, the distance between the private and the public laboratory decreases with the number of researchers in the public team. Equation (22) can also be rewritten in terms of the ratio of the respective numbers of researchers in each lab:

$$\frac{n_c}{N - n_c} = \frac{2\gamma(1 - d_c)}{w^2(2(1 - d_c) + \frac{1}{a})} \quad (24)$$

Analyzing the private payoff functions yields the following lemma:

Lemma 2. *For a given n_c , the optimal distance d_c verifies the following properties:*

$$\begin{cases} \text{If } \frac{n_c}{N} \geq \frac{2a\gamma}{2a\gamma + w^2(1+2a)}, & d_c = 0 \\ \text{If } \frac{n_c}{N} < \frac{2a\gamma}{2a\gamma + w^2(1+2a)}, & (1 - d_c) = \frac{n_c w^2}{2a((N - n_c)\gamma - n_c w^2)} \in (0, 1) \end{cases}$$

Proof: Appendix 1

Taking into account the two previous stages of the game, the cooperative payoff functions become

$$G_c^{pu} = n_c \left(s + \frac{w^2}{2} \left(\frac{1}{a^2} - (1 - d_c)^2 \right) - \gamma(1 - d_c)^2 \right) \quad (25)$$

$$\begin{aligned} G_c^{pr} &= (N - n_c) \left(\frac{w^2}{2} + \frac{2a\gamma(1 - d_c)^2}{2a(1 - d_c) + 1} \left((1 - d_c) + \frac{1}{a} \right) - \gamma(1 - d_c)^2 \right) \\ &= (N - n_c) \left(\frac{w^2}{2} + \gamma(1 - d_c)^2 \left(\frac{1}{2a(1 - d_c) + 1} \right) \right) \end{aligned} \quad (26)$$

for the public and the private laboratory, respectively.

- At equilibrium, researchers do not switch between laboratories:

$$\begin{aligned} g_c^{pu}(n_c^*) &= g_c^{pr}(n_c^*) \\ \Leftrightarrow s + \frac{1}{2} \left(\frac{w}{a} \right)^2 - \frac{w^2}{2} (1 - d_c)^2 - \gamma(1 - d_c)^2 &= \frac{1}{2} w^2 + \frac{n_c}{N - n_c} \frac{w^2}{a} (1 - d_c) (a(1 - d_c) + 1) - \gamma(1 - d_c)^2 \end{aligned} \quad (27)$$

with $(1 - d_c)$ given by Equation (22) and the number of researchers working in the public laboratory satisfies:

$$1 \leq n_c^* \leq N - 1$$

- When all the researchers want to join the public laboratory:

$$g_c^{pu}(n_c) > g_c^{pr}(n_c) \quad \forall n_c$$

Hence, $n_c^* = N$.

- When all the researchers want to join the private laboratory:

$$g_c^{pu}(n_c) < g_c^{pr}(n_c) \quad \forall n_c$$

Hence, $n_c^* = 0$.

We can deduce the following proposition:

Proposition 2. *In the cooperative scenario, the subgame perfect Nash equilibrium is such that:*

1. *If the public subsidy s is too low ($s \leq \underline{s}$), only the private research laboratory exists: $n_c^* = 0$.*
2. *If the public subsidy s is neither too high ($s < \bar{s}$) nor too low ($s > \underline{s}$), the two laboratories coexist with:*

$$n_c^* = N \frac{2a\gamma(1 - d_c)}{2a(1 - d_c)(\gamma + w^2) + w^2} < N$$

and

$$d_c^* \in (0, 1)$$

3. *If the public subsidy is such that $s = \bar{s}$, both research laboratories coexist with:*

$$n_c^* = N \frac{2a\gamma}{2a\gamma + w^2(1 + 2a)} < N$$

and the private laboratory chooses to share the public laboratory's location (i.e. pure Research Cluster):

$$d_c^* = 0$$

4. *If the public subsidy s is too high ($s > \bar{s}$), only the public laboratory exists: $n_c^* = N$.*

with

$$\underline{s} = \frac{1}{2} \left(\frac{w}{a} \right)^2 (a^2 - 1) \quad \text{and} \quad \bar{s} = \frac{1}{2} \left(\frac{w}{a} \right)^2 (a^2 - 1) + \frac{w^2(2a + 1) + 4\gamma(a + 1)}{2(2a + 1)}.$$

Proof: Appendix 2

Proposition 2 shows that, as for the NC case, the coexistence of a public and a private laboratory requires public subsidies that are neither too high nor too low. Note that the subsidy level that maximizes the proximity (i.e. $d_c^* = 0$) and thereby leads to a pure research cluster is similar to the threshold in the non-cooperative case for $w^2 = \frac{4a\gamma}{2a+1}$

Before comparing the equilibria of both scenarios, it is important to point out that:

- 1) The rivalry effect still exists under the cooperative scenario because even if the researchers coordinate their efforts, the laboratories remain in competition. The coordination of research efforts only allows competing laboratories to internalize knowledge spillovers. Moreover, they commodify their research results differently. Note also that the rivalry effect is affected by the way the research is organized through the distance and the size of the laboratories that emerge.

2) The spillover effect also depends on the way the research is organized but the two scenarios are similar from the private laboratory's perspective. This is because of the one-way spillovers. On the other hand, the knowledge externality, which is ignored by the public laboratory in the non-cooperative case, is internalized when researchers choose their individual research efforts so as to maximize the overall payoff (i.e. in the cooperative scenario)⁸.

4 Comparative results

In this section, we proceed to comparisons between the cooperative and non-cooperative cases. Let us start by comparing the locations chosen by the private laboratory for a given number of researchers in the public team.

Lemma 3. *In the cooperative equilibrium, the private laboratory chooses to be closer to the public laboratory when comparing with non-cooperative equilibrium, given a fixed number of researchers in each laboratory $n = n_{nc} = n_c$,*

$$\begin{aligned} (i) \quad & 0 \leq d_c \leq d_{nc} < 1 \\ (ii) \quad & \frac{\partial d_{nc}}{\partial n_{nc}} < \frac{\partial d_c}{\partial n_c} < 0. \end{aligned}$$

These results are explained by comparing Equations (10) and (21). In the cooperative setting, spillover is reinforced by the distance-dependence of researcher effort in the public laboratory. As a result, the rivalry and spillover effects are balanced at a closer distance when the laboratories cooperate.

In order to provide guidelines for research policies, we consider the impact of the public subsidy at equilibrium. Let us denote $X^{pr} = (N - n)x^{pr}$ and $X^{pu} = nx^{pu}$ the total research efforts of the private and public laboratories and $X^{tot} = X^{pr} + X^{pu}$ the collective research effort of the two groups.

Proposition 3. *A rise in the public subsidy, with $s \in (\underline{s}, \min(\bar{s}, \bar{\bar{s}})]$, has the following effects:*

⁸Our setting can be reframed as a "mixed duopoly" involving private and public firms: the non-cooperative case of our model corresponds to the situation in which the two private actors maximize their own payoff (only one benefits from spillovers), whereas the cooperative scenario corresponds to a mixed market structure in which the public laboratory aims to maximize the overall payoff, while the private laboratory maximizes its own payoff only (see Gil-Moltó et al. (2011)).

| | <i>NC case</i> | <i>C case</i> |
|--|---|---------------|
| $\frac{\partial n^*}{\partial s}$ | > 0 | > 0 |
| $\frac{\partial d^*}{\partial s}$ | < 0 | < 0 |
| $\frac{\partial X^{pu*}}{\partial s}$ | > 0 | > 0 |
| $\frac{\partial X^{pr*}}{\partial s}$ | < 0 | < 0 |
| $\frac{\partial X^{tot*}}{\partial s}$ | < 0 | > 0 |
| $\frac{\partial g_{nc}^{pu*}}{\partial s}$ | > 0 | > 0 |
| $\frac{\partial g_{nc}^{pr*}}{\partial s}$ | > 0 | > 0 |
| $\frac{\partial G^{pu*}}{\partial s}$ | > 0 | > 0 |
| $\frac{\partial G^{pr*}}{\partial s}$ | $\left\{ \begin{array}{l} < 0 \\ > 0 \end{array} \right. \dagger$ | $?$ |
| $\frac{\partial G^{tot*}}{\partial s}$ | > 0 | > 0 |

$$\dagger \begin{cases} < 0 & \forall s \in \left[\underline{s}, \underline{s} + w^2 + \frac{w^4}{a^2\gamma} - \frac{w^3}{a^2\gamma} \sqrt{2a^2\gamma + w^2} \right] \\ > 0 & \forall s \in \left[\underline{s} + w^2 + \frac{w^4}{a^2\gamma} - \frac{w^3}{a^2\gamma} \sqrt{2a^2\gamma + w^2}, \underline{s} + 2\gamma \right] \end{cases}$$

Proof: Appendix 3

Increasing the public subsidy increases the number of researchers employed at equilibrium in the public laboratory. Following a rise in the public subsidy, the individual payoff of researchers working in the public laboratory increases. This encourages more researchers to join the public team, which also increase the individual payoff of researchers in the private laboratory through greater spillover. When the balance between the individual payoffs in the private and public sector is reestablished by an increase in the number of public researchers, transfers between laboratories stop.

The distance between the laboratories chosen by the private one ensures that the rivalry and spillover effects are balanced. A larger number of public researchers leads the private laboratory to move closer to the public group (from Equation (23) in the cooperative case, and similarly in the non-cooperative one).

Since it encourages more researchers to join the public laboratory, a higher public subsidy increases the total research effort of the public laboratory (X^{pu}), an effect that is strengthened in the cooperative case because the public researchers contribute more individually. The trend is opposite in the private laboratory (X^{pr}): since the researchers' individual efforts do not depend on the distance from the public laboratory (and thus, on the number of researchers working therein), a reduction in research personnel has a negative impact on the total private research effort.

The impact of higher public subsidy on the total research effort (X^{tot}) depends on which of these two effects dominates. In the non-cooperative setting, the negative trend of the private laboratory's effort dominates and the total effort decreases. Conversely, in the cooperative scenario, the positive trend of the public laboratory's effort dominates so that the total research effort increases.

In combination, the effects of a rise in the public subsidy on the distance between the laboratories, the distribution of researchers between the two, and their efforts, mean that the individual payoffs increase in both labs. However, the public laboratory's payoff and the total payoff always increase while the private payoff may increase or decrease depending on the subsidy level. In the non-cooperative setting, the private laboratory's payoff is low (high) when the public

subsidy is low (high). Moreover, when a is low, the private laboratory's payoff only decreases for a narrow range of s values, meaning that the private payoff is more likely to increase. The numerical results in Table 1 confirm these two effects.

In the cooperative setting, since the equations governing the private payoff cannot be solved analytically, we proceed to simulations, whose results are presented in Table 1. These simulations show that the private laboratory's payoff tends to increase with the subsidy level except when the commodification capacity is high and the subsidy level and the rivalry parameter (γ) are both low. Indeed, as Table 1 shows, following a rise in s , the increase in n_c^* is greater when the difference in commodification capacities is large (high a). When γ is high, this leads to a big decrease in the number of researchers in the private sector that exceeds the positive impact of a higher s on individual payoffs. When γ is low, the rivalry effect is small and there are few researchers in the public laboratory (because there is no need for high spillover). Moreover, n_c^* varies little when s increases so that the increase in the individual payoffs dominates the decrease in the number of private researchers.

We now compare the non-cooperative and cooperative equilibrium outcomes:

Proposition 4. *In the interval $s \in (\underline{s}, \min(\bar{s}, \bar{\bar{s}})]$:*

- i) $n_{nc}^* > n_c^*$
- ii)
$$\left\{ \begin{array}{l} d_{nc}^* < d_c^* \\ d_{nc}^* > d_c^* \end{array} \right. \text{ if } \left\{ \begin{array}{l} \gamma > \frac{w^2(2a+1)}{4a} \\ \gamma < \frac{w^2(2a+1)}{4a} \end{array} \right. \text{ and } s < \underline{s} + \frac{\gamma w^4}{2a^2(2\gamma-w^2)^2} < \min(\bar{s}, \bar{\bar{s}})$$

$$\left\{ \begin{array}{l} d_{nc}^* > d_c^* \\ d_{nc}^* < d_c^* \end{array} \right. \text{ if } \left\{ \begin{array}{l} \gamma > \frac{w^2(2a+1)}{4a} \\ \gamma < \frac{w^2(2a+1)}{4a} \end{array} \right. \text{ and } s > \underline{s} + \frac{\gamma w^4}{2a^2(2\gamma-w^2)^2} < \min(\bar{s}, \bar{\bar{s}})$$
- iii) $x_{nc}^{pr} = x_c^{pr}$ and $x_{nc}^{pu} < x_c^{pu}$
 $X_{nc}^{tot}(n_{nc}^*, d_{nc}^*) < X_c^{tot}(n_c^*, d_c^*)$
- iv) $g_{nc}^{pu} = g_{nc}^{pr} > g_c^{pu} = g_c^{pr}$
- v) $G_{nc}^{pu}(n_{nc}^*, d_{nc}^*) > G_c^{pu}(n_c^*, d_c^*)$
 $G_{nc}^{tot}(n_{nc}^*, d_{nc}^*) > G_c^{tot}(n_c^*, d_c^*)$

where $X^{tot} = X^{pu} + X^{pr} = nx^{pu} + (N - n)x^{pr}$.

Proof: Appendix 4

From Proposition 4, research cooperation limits the number of researchers in the public laboratory. Equations (15) and (27) provide a clear-cut explanation. The distribution of researchers is such that their payoffs in the private and public laboratories match. An individual's public payoff is always lower in the cooperative case than it is in the non-cooperative one: higher levels of individual public research effort lead to higher costs. Conversely, the number of public researchers required to achieve a given level of spillover is smaller in the cooperative case. These two effects act in concert to reduce the number of public researchers in the cooperative case.

The private laboratory's optimal choice of location is less clear-cut and depends on the levels of the rivalry cost parameter (γ) and the public subsidy. On the one hand, the rivalry parameter affects the rivalry effect in Equations (10) and (21). Knowing that $n_{nc}^* > n_c^*$, the rivalry effect is lower in the non-cooperative case, which implies that the laboratories are closer. On the other hand, the public subsidy has a positive effect on the number of public researchers (see Proposition 3) in both settings. The difference between the non-cooperative and the cooperative cases comes from the additional spillover effect in the latter. This effect tends to draw the laboratories closer together and increases with n . Contrary then to the first effect, this spillover effect tends to make the distance between the labs smaller in the cooperative scenario. In combination, the result of these two effects is that the distance between the two laboratories is lower in the cooperative case when the rivalry parameter is low, whatever the level of subsidy. When the rivalry parameter is high however, the outcome depends on the subsidy level: if s is high enough, the additional spillover in the cooperative case created by the high number of public researchers better compensates for the higher rivalry effect, so $d_c^* < d_{nc}^*$. If the subsidy is low, there is less extra spillover in the cooperative case, so $d_c^* > d_{nc}^*$.

For a given level of public subsidy, $s \in (\underline{s}, \min(\bar{s}, \bar{\bar{s}})]$, the research efforts of individuals in the public laboratory are higher when the two laboratories cooperate than when they do not: in the cooperative setting, public researchers internalize the positive effect of knowledge spillover on the private payoff. Since spillover occurs in one way only, private researchers produce the same effort whether the setting is cooperative or not. At the aggregate level, the total research effort is driven by the private research effort, X^{pr} , which is higher in the cooperative scenario because more researchers work in the private lab.

By internalizing the spillover effect in the cooperative setting, public researchers increase their individual efforts, which in turn increases the cost of effort, thereby reducing the public payoff. The smaller number of public researchers also reduces the gain of the public sector. This trend drives the outcome in terms of total gain, which, instead of cooperative choices of effort, is lower in the cooperative setting than in the non-cooperative one. This result is partly explained by the fact that the proximity of the laboratories is chosen by the private sector.

The level of the private payoff cannot be determined analytically. However, we can state that $G_{nc}^{pr}(n_{nc}^*, d_{nc}^*) < G_c^{pr}(n_c^*, d_c^*)$ for any $\gamma < \frac{w^2(a-2)}{2a}$ (see Appendix 3 d-3). For $\gamma > \frac{w^2(a-2)}{2a}$, we proceed to simulations. The simulations presented in Table 1 confirm that the private laboratory's payoff is higher in the cooperative case than in the non-cooperative one for $\gamma > \frac{w^2(a-2)}{2a}$, which is the case for $a = 4$ in our example. This means that the effect of a higher number of private researchers dominates the individual payoff effect, such that the private lab benefits from the cooperative setting. In this case, the private laboratory benefits greatly from a rise in the number of researchers it employs.

Finally, compared with the non-cooperative scenario, although cooperative behavior increases the total research effort, the overall payoff of the two laboratories decreases.

| | | | G_{nc}^{pr*} | d_{nc}^* | n_{nc}^* | G_c^{pr*} | d_c^* | n_c^* |
|----------------|-----------|-----------|----------------|------------|------------|--------------|---------|---------|
| $\gamma = 150$ | $a = 1.5$ | $s = 200$ | 75802 | 0.46 | 190 | 87628 | 0.54 | 90 |
| | | $s = 300$ | 77785 | 0.21 | 236 | 89084 | 0.32 | 101 |
| | | $s = 400$ | 81855 | 0.02 | 262 | 90905 | 0.15 | 106 |
| | $a = 4$ | $s = 200$ | 63958 | 0.80 | 190 | 78955 | 0.84 | 110 |
| | | $s = 300$ | 45160 | 0.39 | 323 | 79811 | 0.46 | 117 |
| | | $s = 400$ | 43411 | 0.16 | 358 | 80257 | 0.25 | 122 |
| $\gamma = 300$ | $a = 1.5$ | $s = 200$ | 65489 | 0.62 | 232 | 78685 | 0.63 | 142 |
| | | $s = 300$ | 65072 | 0.44 | 279 | 79902 | 0.45 | 160 |
| | | $s = 400$ | 67241 | 0.31 | 305 | 81956 | 0.31 | 168 |
| | $a = 4$ | $s = 200$ | 55264 | 0.86 | 232 | 72640 | 0.86 | 142 |
| | | $s = 300$ | 35609 | 0.57 | 361 | 67178 | 0.56 | 185 |
| | | $s = 400$ | 33501 | 0.41 | 390 | 67533 | 0.38 | 192 |
| $\gamma = 500$ | $a = 1.5$ | $s = 200$ | 57705 | 0.70 | 264 | 70217 | 0.70 | 187 |
| | | $s = 300$ | 55982 | 0.57 | 310 | 70806 | 0.55 | 210 |
| | | $s = 400$ | 57111 | 0.46 | 334 | 72692 | 0.43 | 220 |
| | $a = 4$ | $s = 200$ | 48689 | 0.89 | 264 | 63710 | 0.89 | 187 |
| | | $s = 300$ | 29426 | 0.67 | 385 | 56169 | 0.63 | 241 |
| | | $s = 400$ | 27296 | 0.54 | 411 | 56315 | 0.48 | 250 |

Table 1: Private sector payoff in non-cooperative and cooperative research ventures with a public laboratory (simulations are made with Mathematica for $w = 20$).

5 Conclusion

Only a few years ago, the conventional wisdom was that research clusters would foster public-private research linkages that would help firms internalize knowledge spillover from the academic sector, thereby increasing research efforts. The obsession of policy-makers in developed countries to 'create the next Silicon Valley' reveals the increased importance attributed to spatial proximity for innovative activities. However, public authorities cannot force firms and other research actors to cooperate or become neighbors. Cluster policies are complicated to implement and target (Duranton, 2007; Falck et al., 2010; Fontagné et al., 2013; Duranton et al., 2010). Thus, the first interesting question that arises concerns the conditions under which the private research sector chooses to move close to universities. If receptiveness to knowledge spillover depends on the distance from the knowledge source (as pointed out by several empirical studies), spatial concentration should always be observed. However, the propensity of innovative activities to cluster geographically varies between industries, countries, regions, etc. Another question concerns the performance of the research cluster: does cooperation in proximity (i.e. a research cluster) lead to better outcomes than non-cooperation between distant groups?

In this paper, we developed a three-stage game to better understand the endogenous formation of a research cluster. By determining equilibria for non-cooperative and cooperative research scenarios, we find the intermediate levels of subsidy that allow the two laboratories to coexist and the level of subsidy that maximizes the proximity of the two laboratories (i.e. pure research cluster in the cooperative setting).

One surprising result of this analysis is that the total payoff is lower in the cooperative setting

than it is in the non-cooperative one. A research cluster decreases the payoff of the public sector group, which benefits its private sector counterpart through knowledge spillover, but this does not compensate for the losses of the public sector. Moreover, our results confirm that the research effort is positively influenced by cooperation but not necessarily by spatial proximity. In the economic geography literature, many recent empirical studies show that spatial proximity is neither a necessary nor a sufficient condition for cooperation and innovation: too much proximity can harm innovative performance. This is known as the 'proximity paradox' (Broekel and Boschma, 2012; Lazzeretti and Capone, 2016). This paper provides a theoretical framework through which the role of proximity in innovation can be understood or debated. We show that the distribution of researchers between the public and private sector under cooperative and non-cooperative conditions crucially affects the distance between the laboratories and their output.

Note that these results have been obtained in the context of an interaction between just two laboratories, without considering the market of knowledge (the commodification capacity of the two laboratories for their research is governed by exogenous parameters). Second, the public subsidy is also taken to be an exogenous variable. It may be worth determining the level of public subsidy that maximizes social welfare. Third, an important issue in economic geography is the role and consequences of cluster size. One appropriate extension to our model would be to include more than one private laboratory. Our analysis can be seen as a first step toward understanding the formation of research clusters. New theoretical insights are still needed to better assess the performance of research clusters and the conditions necessary for their existence.

5.1 Appendix 1: Proof of Lemma 2

For the following proofs we use a change of variable: $D = 1 - d$ in order to simplify the calculations.

$$\begin{aligned}\frac{\partial G_c^{pr}}{\partial D_c} &= -2\gamma D_c (N - n_c) + n_c w^2 \left(2D_c + \frac{1}{a} \right) \\ \frac{\partial G_c^{pr}}{\partial D_c} &= 0 \iff D_c = \frac{n_c w^2}{2a(\gamma(N - n_c) - n_c w^2)} > 0 \iff n_c < N \frac{\gamma}{w^2 + \gamma} \\ D_c &= \frac{n_c w^2}{2a(\gamma(N - n_c) - n_c w^2)} < 1 \iff n_c < N \frac{2a\gamma}{w^2 + 2a(\gamma + w^2)} \text{ otherwise } D_c = 1 \implies d_c = 0 \\ \text{with } \frac{\partial^2 G_c^{pr}}{\partial D_c^2} &= -2\gamma(N - n_c) + 2n_c w^2 < 0 \iff n_c < N \frac{\gamma}{w^2 + \gamma}\end{aligned}$$

For $n_c > N \frac{\gamma}{w^2 + \gamma}$,

$$G_c^{pr}(0) = (N - n_c) \frac{w^2}{2} < (N - n_c) \frac{w^2}{2} - \gamma(N - n_c) + n_c w^2 \left(\frac{1+a}{a} \right) = G_c^{pr}(1)$$

5.2 Appendix 2: Proof of Proposition 2

According to Definitions (5) and (6), the equilibrium condition in the cooperative scenario is:

$$\frac{G_c^{pu}}{n_c} = \frac{G_c^{pr}}{N - n_c}$$

which reduces to

$$s + \frac{w^2}{2} \left(\frac{1}{a^2} - D_c^2 \right) = \frac{w^2}{2} + \frac{2\gamma D_c}{w^2(2D_c + \frac{1}{a})} w^2 D_c \left(D_c + \frac{1}{a} \right)$$

$$s + \frac{w^2}{2} \left(\frac{1}{a^2} - 1 - D_c^2 \right) = \frac{2\gamma D_c^2}{2(D_c + \frac{1}{a})} \left(D_c + \frac{1}{a} \right)$$

Let us analyze $\frac{G_c^{pu}}{n_c} - \frac{G_c^{pr}}{N - n_c}$

$$\begin{aligned}\frac{G_c^{pu}}{n_c} - \frac{G_c^{pr}}{N - n_c} &= s + \frac{w^2}{2} \left(\frac{1}{a^2} - 1 - D_c^2 \right) - \frac{2\gamma D_c^2}{(2D_c + \frac{1}{a})} \left(D_c + \frac{1}{a} \right) \\ &= \frac{1}{2a^2(2aD_c + 1)} \left((2a^2s + w^2(1 - a^2 - a^2D_c^2))(2D_c + \frac{1}{a}) - 4a^2\gamma D_c^2 \left(D_c + \frac{1}{a} \right) \right)\end{aligned}$$

Let us analyze

$$\begin{aligned}H(D_c) &= ((2a^2s + w^2(1 - a^2 - a^2D_c^2))(2aD_c + 1) - 4a^2\gamma D_c^2(aD_c + 1)) \\ &= D_c^3 2a^3(-w^2 - 2\gamma) + D_c^2 a^2(-w^2 - 4\gamma) + D_c 2a(2a^2s + w^2(1 - a^2)) \\ &+ (2a^2s + w^2(1 - a^2))\end{aligned}$$

$$H(0) = (2a^2s + w^2(1 - a^2)) > 0 \iff s > \frac{w^2(a^2 - 1)}{2a^2} = \underline{s}$$

$$\begin{aligned}H(1) &= 2a^3(-w^2 - 2\gamma) + a^2(-w^2 - 4\gamma) + 2a(2a^2s + w^2(1 - a^2)) \\ &+ (2a^2s + w^2(1 - a^2)) < 0\end{aligned}$$

$$\iff s < \frac{w^2(a^2 - 1)}{2a^2} + \frac{2a^3(w^2 + 2\gamma) + a^2(w^2 + 4\gamma)}{2a^2(1 + 2a)} = \bar{s}$$

$$H''(D_c) = 2D_c a^3 (-w^2 - 2\gamma) + 2a^2 (-w^2 - 4\gamma) < 0$$

$$H'(D_c) = 6D_c^2 a^3 (-w^2 - 2\gamma) + 2D_c a^2 (-w^2 - 4\gamma) + 2a (2a^2 s + w^2 (1 - a^2))$$

$$H'(0) = 2a (2a^2 s + w^2 (1 - a^2)) > 0 \iff s > \frac{w^2 (a^2 - 1)}{2a^2} = \underline{s}$$

$$H'(1) = 6a^3 (-w^2 - 2\gamma) + 2a^2 (-w^2 - 4\gamma) + 2a (2a^2 s + w^2 (1 - a^2)) < 0$$

$$\iff s < \underline{s} + \frac{w^2}{2} + \frac{w^2 + 2\gamma(2 + 3a)}{2a} = \hat{s}$$

with $\hat{s} > \bar{s}$ since

$$\frac{w^2}{2} + \frac{w^2 + 2\gamma(2 + 3a)}{2a} = \frac{w^2}{2} + \frac{w^2}{2a} + 2\gamma \frac{3a + 2}{2a + 1} > \frac{w^2}{2} + 2\gamma \frac{a + 1}{2a + 1}$$

In conclusion:

$$s < \frac{w^2 (a^2 - 1)}{2a^2} \implies \frac{G_c^{pr}}{N - n_c} > \frac{G_c^{pu}}{n_c} \text{ and } n = 0$$

$$s > \frac{w^2 (a^2 - 1)}{2a^2} + \frac{2a^3 (w^2 + 2\gamma) + a^2 (w^2 + 4\gamma)}{2a^2 (1 + 2a)} \implies \frac{G_c^{pr}}{N - n_c} < \frac{G_c^{pu}}{n_c} \text{ and } n_c = N$$

$$\frac{w^2 (a^2 - 1)}{2a^2} < s < \frac{w^2 (a^2 - 1)}{2a^2} + \frac{2a^3 (w^2 + 2\gamma) + a^2 (w^2 + 4\gamma)}{2a^2 (1 + 2a)} = \bar{s} \text{ implies } n_c^* \in (0, N)$$

and $H(D_c) > (<) 0$ for $D_c < (>) D_c^*$.

Moreover we have

$$\bar{s} > \underline{s} \iff w^2 > \frac{4a\gamma}{2a + 1}$$

Appendix 3: Proof of Proposition 3

i) The non-cooperative scenario:

According to Equation (7), equilibrium between the two laboratories is reached in the non-cooperative scenario if and only if

$$s + \frac{1}{2} \left(\frac{w}{a} \right)^2 = \frac{1}{2} w^2 + 2\gamma (D_{nc})^2$$

with $D_{nc} = 1 - d_{nc}$.

The total derivative of the above expression with respect to s is:

$$\frac{\partial D_{nc}^*}{\partial s} = \frac{1}{4\gamma D_{nc}} > 0$$

from $n_{nc} = \frac{2a\gamma D_{nc}}{2a\gamma D_{nc} + w^2} N$, we have

$$\frac{\partial n_{nc}^*}{\partial s} = \frac{\partial n_{nc}^*}{\partial D_{nc}} \frac{\partial D_{nc}}{\partial s} > 0$$

since $\frac{\partial n_{nc}^*}{\partial D_{nc}} = \frac{2aN\gamma w^2}{(2a\gamma D_{nc} + w^2)^2} > 0$.

Since $X_{nc}^{pu*} = n_{nc}^* \frac{w}{a}$, $X_{nc}^{pr*} = (N - n_{nc}^*)w$ and $X_{nc}^{tot*} = X_{nc}^{pu*} + X_{nc}^{pr*}$, we have

$$\frac{\partial X_{nc}^{pr*}}{\partial s} < 0, \quad \frac{\partial X_{nc}^{pu*}}{\partial s} > 0 \quad \text{and} \quad \frac{\partial X_{nc}^{tot*}}{\partial s} = w \left(\frac{1}{a} - 1 \right) \frac{\partial n_{nc}^*}{\partial D_{nc}} \cdot \frac{\partial D_{nc}^*}{\partial s} < 0$$

We deduce the effect on G_{nc}^{pu*} and on G_{nc}^{pr*} :

$$\frac{\partial G_{nc}^{pu*}}{\partial s} = \frac{\partial n_{nc}^*}{\partial s} g_{nc}^{pu*} + \frac{\partial g_{nc}^{pu*}}{\partial s} n_{nc}^* > 0 \quad \text{since} \quad \frac{\partial g_{nc}^{pu*}}{\partial s} = 1 - 2\gamma D_{nc}^* \frac{\partial D_{nc}^*}{\partial s} = \frac{1}{2}$$

$$\frac{\partial G_{nc}^{pr*}}{\partial s} = -\frac{\partial n_{nc}^*}{\partial s} g_{nc}^{pr*} + \frac{\partial g_{nc}^{pr*}}{\partial s} (N - n_{nc}^*)$$

with $\frac{\partial g_{nc}^{pr*}}{\partial s} = 2\gamma D_{nc}^* \frac{\partial D_{nc}^*}{\partial s} > 0$

we obtain

$$\frac{\partial G_{nc}^{pr*}}{\partial s} = \frac{\partial n_{nc}^*}{\partial s} \left(\frac{w^2}{a} D_{nc}^* + \gamma D_{nc}^{2*} - \frac{1}{2} w^2 \right)$$

Replacing $D_{nc}^* = 1 - d_{nc}^*$ with the value obtained from Proposition 1 gives:

$$\frac{\partial G_{nc}^{pr*}}{\partial s} > 0 \Leftrightarrow s > \underline{s} + w^2 + \frac{w^4}{a^2\gamma} - \frac{w^3}{a^2\gamma} \sqrt{2a^2\gamma + w^2} \quad (28)$$

As a result:

$$\begin{aligned} \frac{\partial G_{nc}^{pr*}}{\partial s} < 0 \quad \forall s \in \left[\underline{s}, \underline{s} + w^2 + \frac{w^4}{a^2\gamma} - \frac{w^3}{a^2\gamma} \sqrt{2a^2\gamma + w^2} \right] \quad \text{and} \\ \frac{\partial G_{nc}^{pr*}}{\partial s} > 0 \quad \forall s \in \left[\underline{s} + w^2 + \frac{w^4}{a^2\gamma} - \frac{w^3}{a^2\gamma} \sqrt{2a^2\gamma + w^2}, \underline{s} + 2\gamma \right] \end{aligned}$$

At equilibrium, $g_{nc}^{pu*} = g_{nc}^{pr*}$, and

$$G_{nc}^{tot*} = (N - n_{nc}^*)g_{nc}^{pr*} + n_{nc}^*g_{nc}^{pu*} = N g_{nc}^*$$

The total derivative of the above expression with respect to s is:

$$\frac{\partial G_{nc}^{tot*}}{\partial s} = n_{nc}^* \frac{\partial g_{nc}^{pu*}}{\partial s} + (N - n_{nc}^*) \frac{\partial g_{nc}^{pr*}}{\partial s} > 0$$

since $\frac{\partial g_{nc}^{pu*}}{\partial s} > 0$ and $\frac{\partial g_{nc}^{pr*}}{\partial s} > 0$.

ii) The cooperative scenario:

The total derivative of Expression (29) with respect to s is:

$$\begin{aligned} 1 &= \frac{\partial D_c}{\partial s} \left(w^2 D_c + \frac{8a^2\gamma D_c^3 + 10a\gamma D_c^2 + 4\gamma D_c}{(2aD_c + 1)^2} \right) \\ \frac{\partial D_c}{\partial s} &= \frac{(2aD_c + 1)^2}{D_c(w^2(2aD_c + 1)^2 + 8a^2\gamma D_c^2 + 10a\gamma D_c + 4\gamma)} > 0 \end{aligned}$$

From (23) we deduce $\frac{\partial n_c}{\partial s} > 0$.

We have $\frac{\partial X_c^{pr*}}{\partial s} < 0$ since individuals' efforts are not distance-dependent in the private sector and $\frac{\partial X_c^{pu*}}{\partial s} > 0$ since $X_c^{pu*} = n_c^*(\frac{w}{a} + wD_c^*)$ and both D_c^* and n_c^* increase with s .

With $X_c^{pu*} = n_c^*(w + \frac{w}{a})$, $X_c^{pr*} = (N - n_c^*)w$ and $X_c^* = X_c^{pu*} + X_c^{pr*} = Nw + n_c^*\frac{w}{a}$, we have

$$\frac{\partial X_c^*}{\partial s} > 0$$

We deduce the effect on G_c^{pu*} and on G_c^{pr*} :

$$\frac{\partial G_c^{pu*}}{\partial s} = \frac{\partial n_c^*}{\partial s} g_c^{pu*} + \frac{\partial g_c^{pu*}}{\partial s} n_c^* > 0 \text{ since}$$

$$\frac{\partial g_c^{pu*}}{\partial s} = 1 - (w^2 + 2\gamma)D_c^* \frac{\partial D_c^*}{\partial s} = \frac{2\gamma(aD_c^* + 1)}{w^2(2aD_c^* + 1)^2 + 8a^2\gamma D_c^{*2} + 10a\gamma D_c^* + 4\gamma} > 0$$

$$\frac{\partial G_c^{tot*}}{\partial s} = \frac{\partial n_c^*}{\partial s} g_c^{pu*} + \frac{\partial g_c^{pu*}}{\partial s} n_c^* - \frac{\partial n_c^*}{\partial s} g_c^{pr*} + \frac{\partial g_c^{pr*}}{\partial s} (N - n_c^*)$$

Since $g_c^{pu*} = g_c^{pr*}$ it reduces to

$$\frac{\partial G_c^{tot*}}{\partial s} = \frac{\partial g_c^{pu*}}{\partial s} n_c^* + \frac{\partial g_c^{pr*}}{\partial s} (N - n_c^*)$$

with $\frac{\partial g_c^{pr*}}{\partial s} = \frac{2\gamma D_c^*}{(2aD_c^* + 1)^2} \frac{\partial D_c^*}{\partial s} > 0$ and $\frac{\partial g_c^{pr*}}{\partial s} (N - n_c^*) = 2\gamma D_c^* \frac{aD_c^* + 1}{(2aD_c^* + 1)^2} \frac{\partial D_c^*}{\partial s} > 0$.

We deduce $\frac{\partial G_c^{tot*}}{\partial s} > 0$.

Appendix 4: Proof of comparisons

We restrict the comparisons to the interval $[\underline{s}, \min[\bar{s}, \bar{\bar{s}}]]$ which enables the existence of both laboratories in each case.

a) Comparison of the number of researchers in the cooperative and the non-cooperative setting.

From Equation (12) and (24) we have

$$\frac{n_{nc}^*}{N - n_{nc}^*} = \frac{2a\gamma D_{nc}^*}{w^2} \text{ and } \frac{2a\gamma D_c^*}{w^2(2aD_c^* + 1)} = \frac{n_c^*}{N - n_c^*}$$

and

$$\left(\frac{n_{nc}^*}{N - n_{nc}^*} \right)^2 - \left(\frac{n_c^*}{N - n_c^*} \right)^2 = \left(\frac{2a\gamma}{w^2} \right)^2 \left(D_{nc}^{*2} - \left(\frac{D_c^*}{(2aD_c^* + 1)} \right)^2 \right)$$

At the non-cooperative equilibrium, we have:

$$D_{nc}^{*2} = \frac{s - \frac{w^2}{2a^2}(a^2 - 1)}{2\gamma}$$

and at the cooperative equilibrium we have

$$s - \frac{w^2}{2a^2}(a^2 - 1) = D_c^{*2} \frac{w^2}{2} + \frac{2\gamma D_c^{*2}(aD_c^* + 1)}{2aD_c^* + 1} \quad (29)$$

Combining the two relations we obtain:

$$\left(\frac{D_{nc}^*}{D_c^*}\right)^2 = \frac{1}{2\gamma} \left(\frac{w^2}{2} + \frac{2\gamma(aD_c^* + 1)}{2aD_c^* + 1}\right) \quad (30)$$

Then

$$\left(\frac{n_{nc}^*}{N - n_{nc}^*}\right)^2 - \left(\frac{n_c^*}{N - n_c^*}\right)^2 = \left(\frac{2a\gamma}{w^2} D_c^{*2}\right)^2 \left(\frac{w^2}{4\gamma} + \frac{aD_c^* + 1}{2aD_c^* + 1} - \frac{1}{(2aD_c^* + 1)^2}\right) > 0$$

This immediately implies that $n_{nc}^* > n_c^*$.

b) Comparison of the distance between the two laboratories in the cooperative and the non-cooperative setting.

From Equation (30), we can say that

$$\frac{D_{nc}^*}{D_c^*} > 1 \iff \frac{w^2}{4\gamma} + \frac{aD_c^* + 1}{2aD_c^* + 1} > 1 \quad (31)$$

$$\iff D_c^* < \frac{w^2}{2a(2\gamma - w^2)} \text{ for } w^2 \leq \frac{4a\gamma}{2a + 1} \quad (32)$$

$$\text{and always true for } w^2 > \frac{4a\gamma}{2a + 1} \quad (33)$$

Let us check the sign of $\frac{G_c^{pu}}{n_c} - \frac{G_c^{pu}}{N - n_c}$ for $D_c = \frac{w^2}{2a(2\gamma - w^2)}$

$$\frac{G_c^{pu}}{n_c} - \frac{G_c^{pr}}{N - n_c} = s + \frac{w^2}{2} \left(\frac{1}{a^2} - 1 - D_c^2\right) - \frac{2\gamma D_c^2}{(2D_c + \frac{1}{a})} \left(D_c + \frac{1}{a}\right) \quad (34)$$

Replacing D_c in this expression gives

$$\frac{G_c^{pu}}{n_c} - \frac{G_c^{pr}}{N - n_c} = s + \frac{w^2}{2} \left(\frac{1}{a^2} - 1\right) - \frac{\gamma w^4}{2a^2(2\gamma - w^2)^2}$$

and

$$\frac{G_c^{pu}}{n_c} - \frac{G_c^{pr}}{N - n_c} < 0 \iff s < \underline{s} + \frac{\gamma w^4}{2a^2(2\gamma - w^2)^2} = \tilde{s}$$

Let us check if $\tilde{s} < \bar{s}$ or $\tilde{s} > \bar{s}$ ($w^2 \leq \frac{4a\gamma}{2a+1}$ implies that $\min[\bar{s}, \hat{s}] = \bar{s}$) with

$$\tilde{s} = \underline{s} + \frac{\gamma w^4}{2a^2(2\gamma - w^2)^2} \text{ and } \bar{s} = \underline{s} + \frac{2a^3(w^2 + 2\gamma) + a^2(w^2 + 4\gamma)}{2a^2(1 + 2a)}$$

$$\tilde{s} - \bar{s} = \gamma w^4 - a^2 (2\gamma - w^2)^2 \left(w^2 + \frac{4\gamma(a + 1)}{(1 + 2a)}\right)$$

Let us analyze the function

$$Z(w^2) = \gamma(w^2)^2 - a^2 (2\gamma - w^2)^2 \left(w^2 + \frac{4\gamma(a + 1)}{(1 + 2a)}\right) \text{ for } w^2 \leq \frac{4a\gamma}{2a + 1}$$

we obtain: $Z\left(\frac{4a\gamma}{2a+1}\right) = 0$ and $Z'(w^2) = 2\gamma w^2 + a^2(2\gamma - w^2)\left(3w^2 + \frac{2\gamma}{2a+1}\right) > 0$ which implies $\tilde{s} < \bar{s}$.

As a result:

- For $\gamma < \frac{w^2(2a+1)}{4a}$ (which implies $\bar{s} < \bar{\bar{s}}$), $D_{nc}^* > D_c^*$ i.e. $d_{nc}^* < d_c^* \forall s \in [\underline{s}, \bar{s}]$
- For $\gamma > \frac{w^2(2a+1)}{4a}$ (which implies $\bar{s} > \bar{\bar{s}}$) then $D_{nc}^* > D_c^*$ i.e. $d_{nc}^* < d_c^*$ for $s \in [\underline{s}, \tilde{s}]$ and $D_{nc}^* < D_c^*$ i.e. $d_{nc}^* > d_c^*$ for $s \in [\tilde{s}, \bar{s}]$

c) Comparison of research efforts in the cooperative and the non-cooperative setting.

$$X_c^{tot*} - X_{nc}^{tot*} = w \left(n_{nc}^* \left(1 - \frac{1}{a} \right) - n_c^* \left(d_c^* - \frac{1}{a} \right) \right)$$

Since $d_c^* < 1$ and $n_c^* < n_{nc}^*$,

$$X_c^{tot*} - X_{nc}^{tot*} > 0$$

d) Comparison of payoffs in the cooperative and the non-cooperative setting.

d-1 Total payoff

The total payoff in the non-cooperative scenario is:

$$G_{nc}^{tot*} = n_{nc}^* g_{nc}^{pu}(n_{nc}^*, d_{nc}^*) + (N - n_{nc}^*) g_{nc}^{pr}(n_{nc}^*, d_{nc}^*)$$

The total payoff in the cooperative scenario is:

$$G_c^{tot*} = n_c^* g_c^{pu}(n_c^*, d_c^*) + (N - n_c^*) g_c^{pr}(n_c^*, d_c^*)$$

At equilibrium $g_c^{pu}(n_c^*, d_c^*) = g_c^{pr}(n_c^*, d_c^*)$ and $g_{nc}^{pu}(n_{nc}^*, d_{nc}^*) = g_{nc}^{pr}(n_{nc}^*, d_{nc}^*)$.

Thus,

$$G_c^{tot*} = N g_c(n_c^*, d_c^*) \quad \text{and} \quad G_{nc}^{tot*} = N g_{nc}(n_{nc}^*, d_{nc}^*)$$

Hence,

$$\begin{aligned} G_c^{tot*} > G_{nc}^{tot*} &\iff g_c(n_c^*, d_c^*) > g_{nc}(n_{nc}^*, d_{nc}^*) \\ \iff s + \frac{w^2}{2} \left(\frac{1}{a^2} - D_c^{*2} \right) - \gamma D_c^{*2} &> \frac{1}{2} w^2 + 2\gamma D_{nc}^{*2} - \gamma D_{nc}^{*2} \end{aligned}$$

Let us determine $g_c^{pr}(n_c^*, d_c^*) - g_{nc}^{pr}(n_{nc}^*, d_{nc}^*)$ from (14) and (26), we have

$$g_c^{pr}(n_c^*, d_c^*) - g_{nc}^{pr}(n_{nc}^*, d_{nc}^*) = D_c^{*2} \left(\left(\frac{2(aD_c^* + 1)}{2aD_c^* + 1} - \gamma \right) - \left(\frac{D_{nc}^*}{D_c^*} \right)^2 \right) \quad (35)$$

Using Relation (30) we obtain:

$$g_c^{pr}(n_c^*, d_c^*) - g_{nc}^{pr}(n_{nc}^*, d_{nc}^*) = \gamma D_c^{*2} \left(\frac{-aD_c^*}{2aD_c^* + 1} - \frac{w^2}{4\gamma} \right) < 0 \quad (36)$$

so that $G_{nc}^{tot*} > G_c^{tot*}$.

d-2 Public payoff

From above we know that we have $g_{nc}^{pu}(n_{nc}^*, d_{nc}^*) > g_c^{pu}(n_c^*, d_c^*)$ and since $n_{nc}^* > n_c^*$ we immediately obtain $G_{nc}^{pu*} > G_c^{pu*}$.

d-3 Private payoff

The relationship $n_{nc}^* > n_c^*$ implies that $N - n_{nc}^* < N - n_c^*$.
using Conditions (12) and (24), we obtain:

$$N - n_{nc}^* < N - n_c^* \iff D_{nc}^* > \frac{D_c^*}{2aD_c^* + 1}$$

Let us denote $\Omega(D_{nc}) = \frac{G_c^{pr*}}{G_{nc}^{pr}} = \frac{w^2(2aD_c^*+1)+2\gamma D_c^{*2}}{w^2(2aD_{nc}^*+1)+2\gamma aD_c^*} \frac{w^2+2a\gamma D_{nc}}{w^2+2\gamma D_{nc}^2}$ and evaluate $\Omega(D_{nc})$ for $D_{nc} = \frac{D_c^*}{2aD_c^*+1}$

$$\Omega\left(\frac{D_c^*}{2aD_c^*+1}\right) = \frac{\frac{w^2}{2} + \frac{\gamma D_c^{*2}}{(2aD_c^*+1)^2}}{\frac{w^2}{2} + \frac{\gamma D_c^{*2}}{(2aD_c^*+1)}} > 1$$

Moreover, we have:

$$\frac{\partial G_{nc}^{pr}}{\partial D_{nc}} = \frac{Nw^2}{2(w^2 + 2a\gamma D_{nc})^2} (2a\gamma D_{nc}^2 - aw^2 + 2D_{nc}w^2) < 0 \text{ for } \gamma < \frac{w^2(a-2)}{2a}$$

so that $\frac{\partial \Omega(D_{nc})}{\partial D_{nc}} < 0$ for $\gamma < \frac{w^2(a-2)}{2a}$ and $G_c^{pr*} > G_{nc}^{pr*}$. For $\gamma > \frac{w^2(a-2)}{2a}$ we cannot conclude. We proceed to simulations.

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