



**HAL**  
open science

## Magnetic devices analysis by Face FEM coupled with standard reluctance network

Anderson S Nunes, Olivier Chadebec, Gérard Meunier, Patrick Kuo-Peng

### ► To cite this version:

Anderson S Nunes, Olivier Chadebec, Gérard Meunier, Patrick Kuo-Peng. Magnetic devices analysis by Face FEM coupled with standard reluctance network. 20h Compumag Conference on the Computation of Electromagnetic Fields, 2015, Montréal, Canada. hal-02023239

**HAL Id: hal-02023239**

**<https://hal.science/hal-02023239>**

Submitted on 18 Feb 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Magnetic devices analysis by Face FEM coupled with standard reluctance network

Anderson S. Nunes<sup>1,4</sup>, Olivier Chadebec<sup>2,3</sup>, Gérard Meunier<sup>2,3</sup> and Patrick Kuo-Peng<sup>4</sup>

<sup>1</sup>Engineering Simulation & Scientific Software (ESSS), Rua Orlando Phillipi, n° 100, Edifício Techplan, 1° andar, Bairro Saco Grande, Florianópolis, CEP: 88032-700, Brasil, anderson@esss.com.br

<sup>2</sup>Univ. Grenoble Alpes, G2Elab, F-38000 Grenoble, France

<sup>3</sup>CNRS, G2Elab, F-38000 Grenoble, France

<sup>4</sup>GRUCAD/ UFSC - Universidade Federal de Santa Catarina, Florianópolis, CEP: 88040-970, Brasil

**A Finite Element Method (FEM) mesh is converted to a reluctance network through an original magnetostatic formulation based on face shape functions. This meshed reluctance network is coupled with an standard one, characterizing a 0D system. Both approaches are fully-compatible and the hybridized problem can be solve with a single circuit solver. The approach is tested in 2D on a magnetic circuit with an air gap and compared to classical FEM nodal formulation.**

*Index Terms*—Reluctance network, Finite Element Method (FEM), face shape functions, Face FEM

## I. INTRODUCTION

Optimizing electromagnetic devices can require a large amount of data that might be provided by numerical simulations. Many numerical methods are used to model electromagnetic devices, but the RNM (Reluctance Network Method) and FEM (Finite Element Method) are the most widely used for magnetostatic modeling.

The FEM is well known by its flexibility and generality, once the mathematical formulations are based on a mesh. Furthermore, it is noticeable the knowledge base available for this method, for instance [1]. However, it leads to an high number of degrees of freedom and so quite long computation times.

On the other hand, the RNM is one of the most primitive methods for magnetic modeling and its application is based on a reluctance network. This method has remained useful due to its coherent results obtained with low computational effort and low computational simulation time [2] and has been largely applied to model power transformers [3] [4]. This method is also largely applied to model rotating electrical machines [5][6][7] and transmission lines [8]. Nevertheless, it is important to notice that these applications are based on a reluctance network defined manually, that might imply an hard, long and non-general task.

In [9] is presented a methodology that couples nodal/edge FEM with external reluctances network. In this paper, we go a step forward by proposing a formulation fully-compatible with both numerical approaches and solved with a single 0D circuit solver.

Finally, the results of modeling a actuator with the classical nodal FEM and with the proposed methodology are compared.

## II. MAGNETOSTATIC FACE FEM FORMULATION

The magnetostatic fields might be described by the Ampère's (1) and Gauss (2) laws and the constitutive relation for magnetic materials (3).

$$\text{curl } \mathbf{H} = \mathbf{J} \quad (1) \quad \text{div } \mathbf{B} = 0 \quad (2) \quad \mathbf{B} = \mu \mathbf{H} \quad (3)$$

where the magnetic field  $\mathbf{H}$ , in  $A/m$ , is composed by the fields  $\mathbf{H}_0$ , due to an imposed current density source  $\mathbf{J}_0$ , and  $\mathbf{H}_m$  that can be obtained from the gradient of the magnetic scalar potential  $V_r$ , as presented in (5).

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_m \quad (4) \quad \mathbf{H}_m = -\nabla V_r \quad (5)$$

Thus, applying (5) in (4) and integrating the resulting equation along a domain  $\Omega$ , it is possible to obtain (6), for which  $\mathbf{W}$  is the face interpolation function.

$$\int_{\Omega} \mathbf{W}_i \cdot \mathbf{H} d\Omega + \int_{\Omega} \mathbf{W}_i \cdot \nabla V_r d\Omega = \int_{\Omega} \mathbf{W}_i \cdot \mathbf{H}_0 d\Omega \quad (6)$$

Since the magnetic induction  $\mathbf{B}$ , in  $T$ , is given by (7), the left side of (6) is rewritten as (8).

$$\mathbf{B} = \sum_{j=1}^{n_f} \mathbf{W}_j \Phi_j \quad (7)$$

$$\int_{\Omega} \mathbf{W}_i \cdot \mathbf{H} d\Omega = \sum_{j=1}^{n_f} \int_{\Omega} (v \mathbf{W}_i \cdot \mathbf{W}_j d\Omega) \Phi_j \quad (8)$$

where  $j$  and  $i$  are the faces index,  $n_f$  is the number of faces,  $v$  is the magnetic reluctivity, and  $\Phi$  is the magnetic flux.

Applying the divergence theorem in the second term of (6) and evaluating it along two adjacent elements  $\Omega_a$  and  $\Omega_b$ , leads to (9). Taking into account that the normal component of the function  $\mathbf{W}$  is constant along the face shared by  $\Omega_a$  and  $\Omega_b$ , its second term becomes null and it can be rewritten as (10).

$$\int_{\Omega_a + \Omega_b} \mathbf{W}_i \cdot \nabla V_r d\Omega = \oint_{\Gamma_a - \Gamma_b} V_r \mathbf{W}_i \cdot \mathbf{n} d\Gamma - \int_{\Omega_a + \Omega_b} V_r (\nabla \cdot \mathbf{W}_i) d\Omega \quad (9)$$

$$\int_{\Omega} \mathbf{W}_i \cdot \nabla V_r d\Omega = - \int_{\Omega_a + \Omega_b} V_r (\nabla \cdot \mathbf{W}_i) d\Omega \quad (10)$$

This equation can be split in terms of  $\Omega_a$  and  $\Omega_b$ , where the flux direction is defined from  $\Omega_a$  to  $\Omega_b$ . Then, considering that  $\nabla \cdot \mathbf{w}_i$  is the inverse of the element volume, it is possible to obtain (11).

$$\int_{\Omega} \mathbf{W}_i \cdot \nabla V_r d\Omega = V_{r_a} - V_{r_b} \quad (11)$$

The third term of (6) is the magnetic field due to an imposed current density  $\mathbf{J}_0$ , which can be obtained using Biot-Savart law, for instance.

Finally, lets rewrite (6) as the following matrix system

$$[\mathfrak{R}] [\Phi] - [\Delta V_{Mean}] = [H_0] \quad (12)$$

where  $[\mathfrak{R}]$  is a reluctance matrix,  $[\Phi]$  is the unknown flux matrix,  $[\Delta V_{Mean}]$  is the magnetic potential jump between two reluctances and  $[V_0]$  contains the  $fmm$  sources.

Besides the matrix system is stated, (2) is not solved yet. Nevertheless, the  $\mathbf{B}$  free divergence is constrained solving (12) as a circuit system, where the Kirchoff's current law is imposed. It suggests that the solution can be obtained by the use of a 0D circuit solver.

### III. MODEL AND RESULTS

In order to compare the results obtained with the proposed methodology, a simple magnetic circuit with an air gap is analyzed using the well established nodal first order FEM, based on magnetic vector potential  $\mathbf{A}$ . The magnetic induction distribution is presented in Fig. 1.

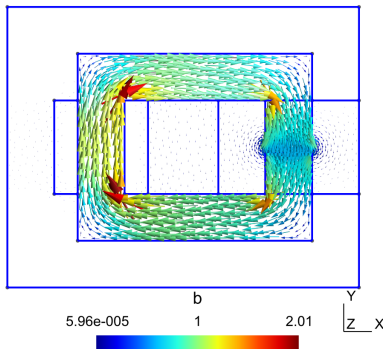


Fig. 1: Magnetic induction distribution obtained with classical FEM

Then, this model is reduced to a 0D model, i.e most of the magnetic circuit is modeled by an classical reluctance network and just the air gap region is meshed (i.e. the region where the equivalent reluctance network is not so easy to define), as presented in Fig. 2. Then the problem is solved by a circuit solver and magnetic induction is interpolated using (7), resulting in Fig. 3.

The maximum magnetic induction in the air gap obtained with the proposed methodology is 0.625 T and with the classical FEM is 0.606 T, that represents a difference of 3.13%.

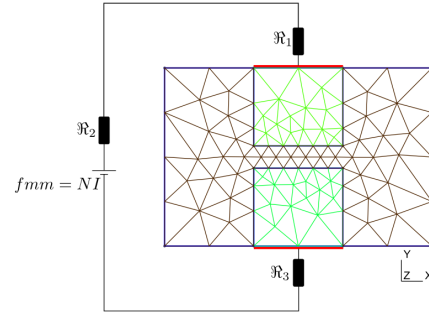


Fig. 2: Networks coupling

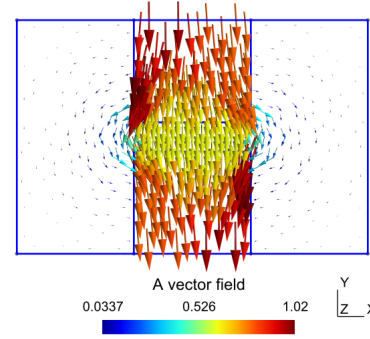


Fig. 3: Magnetic induction distribution along the air gap region, obtained with Face FEM.

### IV. CONCLUSION

This paper has presented a methodology capable to convert a FEM mesh into a reluctance network, allowing its easy coupling with a classical network obtained analytically. The original model was reduced to a 0D system keeping coherent results.

### REFERENCES

- [1] J. P. A. Bastos and N. Sadowski, *Magnetic Materials and 3D Finite Element Modeling*, 2013.
- [2] J. K. Sykulski, *Computational Magnetics*, ser. International journal of numerical modelling, J. K. Sykulski, Ed., 1995.
- [3] X. M. Lopez-Fernandez, A. S. Rodriguez, D. S. Revenga, and J. Turowski, "Stray losses control in shell type transformers. part ii: Method of three-dimensional network solution," *Proc. of Advanced Research Workshop on Transformers (ARWtr2007)*, pp. 69–75, 2007.
- [4] A. Soto, D. Souto, J. Turowski, X. M. Lopez-Fernandez, and D. Couto, "Software for fast interactive three-dimensional modeling of electromagnetic leakage field and magnetic shunts design in shell type transformers," in *2008 18th International Conference on Electrical Machines*, 9 2008, pp. 1–6.
- [5] D. Petrichenko, "Contribution à la modélisation et à la conception optimale des turbo-alternateurs de faible puissance," Ph.D. dissertation, 2007.
- [6] M. Fukuoka, K. Nakamura, and O. Ichinokura, "A method for optimizing the design of spm type magnetic gear based on reluctance network analysis," in *2012 XXth International Conference on Electrical Machines*, 9 2012, pp. 30–35.
- [7] Y. Tang, T. E. Motoasca, J. J. H. Paulides, and E. a. Lomonova, "Analytical modeling of flux-switching machines using variable global reluctance networks," in *2012 XXth International Conference on Electrical Machines*, 9 2012, pp. 2792–2798.
- [8] D. Bormann and H. Tavakoli, "Reluctance network treatment of skin and proximity effects in multi-conductor transmission lines," *IEEE Transactions on Magnetics*, vol. 48, no. 2, pp. 735–738, 2 2012.
- [9] P. Dular, J. Gyselinck, T. Henneron, and F. Piriou, "Dual finite element formulations for lumped reluctances coupling," *IEEE Transactions on Magnetics*, vol. 41, no. 5, pp. 1396–1399, 5 2005.