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Diagrammatic approaches in computational musicology: some theoretical and philosophical aspects

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Abstract. Despite a long historical relationship between mathematics and music, the use of diagrammatic approaches in computational musicology is a relatively recent phenomenon. Within the different branches of formal methods in music analysis, the so-called "transformational" paradigm has progressively shifted from an object-oriented to a graph-theoretical and categorical approach. Both graph theory and category theory make large use of diagrams which enable the description of the inner relationships of musical structures. In the categorical framework recently proposed by the authors, whose results are summarized and discussed in this abstract, musical transformations are viewed as natural transformations between chords represented as labelled graphs with vertices corresponding to the notes and arrows corresponding to musical transpositions and inversions operations. The diagrammatic approach also provides a very powerful conceptual tool that can have crucial theoretical implications for music cognition. We discuss this aspect by showing some deep connections between transformational music analysis and some mathematically-oriented directions in developmental psychology and cognition (such as Halford and Wilson's neostructuralist approach, Ehresmann and Vanbremeersch's Memory Evolutive Systems, Phillips and Wilson's Categorical Compositionality, Fauconnier and Turner's Conceptual Blending and its structural extension proposed by Goguen) and epistemology (Gaston-Granger's "objectal" and "operational" duality).

Keywords: Graphs, Category Theory, Transformational Analysis, Computational Musicology, Epistemology, Cognitive Sciences.

1 The transformational approach in music analysis

Within the different formal approaches in transformational music analysis, neo-Riemannian theory occupies a singular position by stressing a "dualistic" perspective on the Tone System based on inversive relations between major and minor chords. After

a first algebraic formalization by David Lewin through the concept of *Generalized Interval System* [9], neo-Riemannian theory and analysis have developed more and more sophisticated tools by showing their deep categorical roots [10, 5].

According to this music-theoretical and analytical paradigm, there are three ways of transforming a major chord into a minor chord by preserving two common notes: the R transformation (as "relative"), that changes for example the C major chord into the A minor chord; the P transformation (as "parallel") that changes the C major chord into the C minor chord; the L transformation (as "Leading-Tone operator"), changing the C major into the E minor chord. In a categorical framework, the neo-Riemannian operations are viewed as *natural transformations* between major and minor chords represented as labelled graphs with vertices corresponding to the notes and arrows corresponding to transposition and inversion operations [12, 13, 14]. By definition a transposition by h semi-tons is an operation indicated by T_h that sends a generic element x of the cyclic group of order 12 (i.e. a pitch-class in the musical set-theoretical terminology) into $x+h$ (modulo 12). Similarly, one may define the generic inversion operation indicated by I_k that sends a pitch-class x into $k-x$ (always modulo 12). The diagrammatic approach to neo-Riemannian operations is shown in Fig. 1.

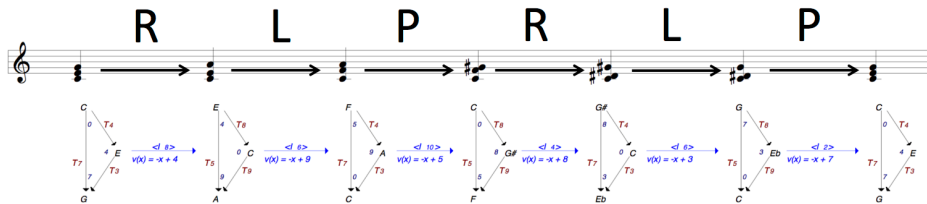


Fig. 1. The sequence of six neo-Riemannian transformations constituting the hexagonal cycle that generates the Tonnetz by musical transpositions. We have associated to the chord sequence the underlying diagrammatic framework displaying the transformations between major and minor chords viewed as labelled graphs with vertices corresponding to the notes and arrows corresponding to musical transposition. Although inversions I_k do not play any role in this case, one may find so-called "negative isographies" $\langle I_k \rangle$ acting on the arrows T_k of the corresponding labelled graphs (for example, the first negative isography $\langle I_8 \rangle$ transforms T_7 into T_5 , T_4 into T_8 and T_3 into T_9). Each negative isography is accompanied with the transformation acting on the vertices of the labelled graph (in the previous case, for example, the notes C, E and G are transformed respectively in the notes E, C and A via the map $-x+4$).

2 Towards a diagrammatic and category-based music cognition

Following some previous attempts at developing a category-based approach to creativity [2], the diagrammatic formalization of musical structures provides a very powerful conceptual tool that can have crucial theoretical implications for cognitive sciences and mathematical psychology. Combining the transformational approach, with a

computational perspective has some crucial theoretical implications for cognitive sciences and mathematical psychology, as one may see by analyzing some mathematically-oriented directions in developmental psychology and cognition [8, 3, 11, 4, 6]. From an epistemological point of view, transformational analysis provides an instantiation, in the music domain, of Gilles-Gaston Granger's articulation between the "objectal" and the "operational" dimensions [7]. This duality was considered by the French epistemologist as the foundational basis for the very notion of "concept" in philosophy.

The use of diagrams within a categorical framework enables precisely to explain the shift from the object-oriented perspective provided by tradition musical *Set Theory* [1] to a "relational" approach in transformational music analysis [14].

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