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### ► To cite this version:

Adrien Cambier, Matthieu Chardy, Rosa Figueiredo, Adam Ouorou, Michael Poss. Optimizing the investments in mobile networks and subscriber migrations for a telecommunication operator. 2019. hal-02019853v1

**HAL Id: hal-02019853**

**<https://hal.science/hal-02019853v1>**

Preprint submitted on 14 Feb 2019 (v1), last revised 8 Jul 2020 (v4)

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# Optimizing the investments in mobile networks and subscriber migrations for a telecommunication operator

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## Abstract

We consider the context of a telecommunications company that is at the same time an infrastructure operator and a service provider. When planning its network expansion, the company can leverage over its knowledge of subscribers dynamic to better optimize the network dimensioning, therefore avoiding unnecessary costs. In this work, the network expansion represents the deployment and/or reinforcement of several technologies (e.g. 2G,3G,4G), assuming that subscribers to a given technology can be served by this technology or older ones. The operator can influence subscribers dynamic by subsidies. The planning is made over a discretized time horizon while some strategic guidelines requirements are demanded at the end of the time horizon. Following classical models, we consider that the behavior of customers follows an  $S$ -shape piecewise constant function. We propose a Mixed-Integer Linear Programming formulation and a heuristic algorithm for the multi-year planning problem. The scalability of the formulation and the quality of the heuristic are assessed numerically on real instances for a use-case with two generations.

*Keywords:* OR in Telecommunications, Mixed Integer Linear Programming, Capacity Expansion, Bass model.

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## 1. Introduction

Over the last few years, new bandwidth-consuming usages such as video streaming (see Table 1) have appeared, increasing the average monthly consumption by user, known as Average Usage per User. This phenomenon, correlated with an increase in the number of users, induces natural

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traffic growth. According to the Visual Networking Index of the IT and network company CISCO (CISCO, 2017), traffic will globally reach 49 Exabyte per month in 2020 with a compound annual growth rate of 47 %. This growth is particularly important in Africa (65%). Network expansion is necessary to support such traffic growth. Besides, the telecommunication market is a competitive one, constantly requesting a raise in service quality.

Service	2015	2018
Youtube videos viewed	2.78 M	4.3 M
Netflix hours watched	69444	266000
Instagram scrolling	38000	174000

Table 1: Number of usages of some services happening worldwide on the internet in 60 seconds (from DigitalInformationWorld (2018))

Whenever possible, telecommunication companies must hence satisfy the request of subscribers in speed and volume to remain competitive, which requires network investments (several billion € to improve the mobile network in the last six years, see Orange (2018)). Facing both needs of offering a satisfying service and of not exploding the investments, the operator does not want to under/over dimension its network.

In many countries, telecommunication companies are both infrastructure operators and service providers. As infrastructure operators, these companies are responsible for planning their network expansion. As service providers, they design the offers for users and have an influence on network traffic. The network expansion decisions can benefit from taking into account the subscribers dynamic, and vice-versa. Thus, operators wish to understand the behavior of subscribers in order to plan optimally the investments in new mobile generations. Having a deep and rigorous analysis of the demand evolution can be an advantage for an operator compared to others. This allows planning financial subsidies, e.g. cost reduction on a phone having access to the newest generation, in order to manage the network expansion and its market share.

	1G	2G	3G	4G	5G
Date	1980s	1990	2003	2009	2020
Speed	2.4 KB/s	64 KB/s	2 MB/s	1 GB/s	> 1 GB/s

Table 2: Evolution of speed through mobile generations (from Sharma (2013)).

Planning the network expansion is a process that is inherently multi-period since investments must be distributed along a couple of years. As often in such problems, it is more efficient to use a strategy that considers multiple years simultaneously. This is even more important in mobile networks because of the quick progress of mobile technology. This is illustrated in Table 2 that shows the speed increase through mobile generations. According to the forecasts of GSM<sup>1</sup> association (see GSMA (2018) for the detailed report), 4G will become the leading mobile network technology worldwide by number of connections (more than 3 billion) in 2019 while early commercial launches will start for 5G. This fast roll-out of mobile generations and progress of technology lead to a cyclic dependency between the subscriber and the network dynamics as investments in the network promote new subscriptions which in turn lead to new investments.

As we could expect, the network dynamic add important constraints that must be considered when planning investments related to new mobile generations. For example, dismantling one generation of a mobile network is not an easy option since operational teams are reluctant to abandon well-functioning (and robust) technologies for new ones without back-up. Moreover, several services may need old(er) technologies (machine-2-machine, roaming, ...). Hence, different technologies have to co-exist and operators have to maintain simultaneously up to three generations.

### 1.1. Mobile Master Plan

The points raised above motivate us to study the design of *multi-period master plans for mobile network* (Mobile Master Plans) which consists in deciding, for a given set of time points and in a given area served by telecommunication sites, how to invest in the evolution of network technologies regarding three aspects: densification, sites coverage extension and subscriptions upgrades. An *investment in densification* means adding new pieces of equipment (modules) of a given technology in order to increase the capacity of a site already covered by this technology. An *investment in coverage extension* means the installation of a given technology in telecommunication sites not covered so far by this technology. Finally, an *investment in user upgrades* means the proposition of financial subsidies to the subscribers in order to promote upgrades from their current offers to the newest technology available. As already mentioned, these three kinds of investment decisions must

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<sup>1</sup>GSM association is an originally-European trade body that represents the interests of mobile network operators worldwide.

be synchronized.

In order to be well-placed in operator rankings (ARCEP, 2018), strategic guidelines are decided by the operator at the end of time horizon and drive the investments in order to guarantee competitiveness (for instance: sites coverage, user coverage, experienced throughput). In practice, Mobile Master Plans are designed for a 5-year time horizon with decisions taken for each year in this period. Mobile Master Plans are driven by cost minimization while ensuring strategic targets over the whole time horizon.

## *1.2. Related literature*

In this work, we are interested in optimizing, over a time horizon, investments decisions related to network expansion and subscribers dynamics assuming the arrival of a new technology. Such problems have been treated in the literature but with other targets in mind and a focus on network investments.

### *1.2.1. Network expansion*

Capacity expansion problems in telecommunications networks have been studied for a long time in the integer programming community, see Knippel and Lardeux (2007); Gollowitzer et al. (2013) among many others, including the case of multi-period planning (Garcia et al., 1998; Gendreau et al., 2006; Kubat and Smith, 2001).

Closer to the context of mobile capacity expansion, Chardy et al. (2016) present an exact mixed-integer formulation and a heuristic method to solve a Mobile Master Plan in a restrictive framework. These models integrate the upgrade of subscribers thanks to subsidies. However, a limitation of the models studied in Chardy et al. (2016) is that the amount of subsidy offered to users is fixed and the number of users which upgrade is set as a variable, constrained only to be positive and upper bounded by the total number of users. The possibility of offering subsidies in order to increase users upgrades is hence not taken into account. Lim and Kim (2017) consider the transition of a generation to another but from a subscriber migration point of view only, applied for a South Korean network.

### *1.2.2. The Bass model*

Our Mobile Master Plan shall offer financial subsidies to the clients to incentive them to move to the newest technology. To model the effect of these subsidies, we rely on the well-known Bass

model from the marketing literature.

First studies on diffusion of innovation and new products appeared in the 60's, in a period of high economic growth and important innovations (television, etc). Everett Rogers published the diffusion of innovation theory in 1962 (see Rogers (2003)), based on the adoption curve of Figure 1. This curve presents the percentage of subscribers which adopt a new product during the time horizon. The curve assumes that the timing of a consumer's initial purchase is related to the number of previous buyers (imitation part) and enlightens different types of subscribers. The innovators are the easiest to convince and the laggards are those who adopt the last. External factors (marketing and attractiveness) due to decisions taken in the time horizon are not taken into account, as well as the generation effect (new generation replacing an older one).

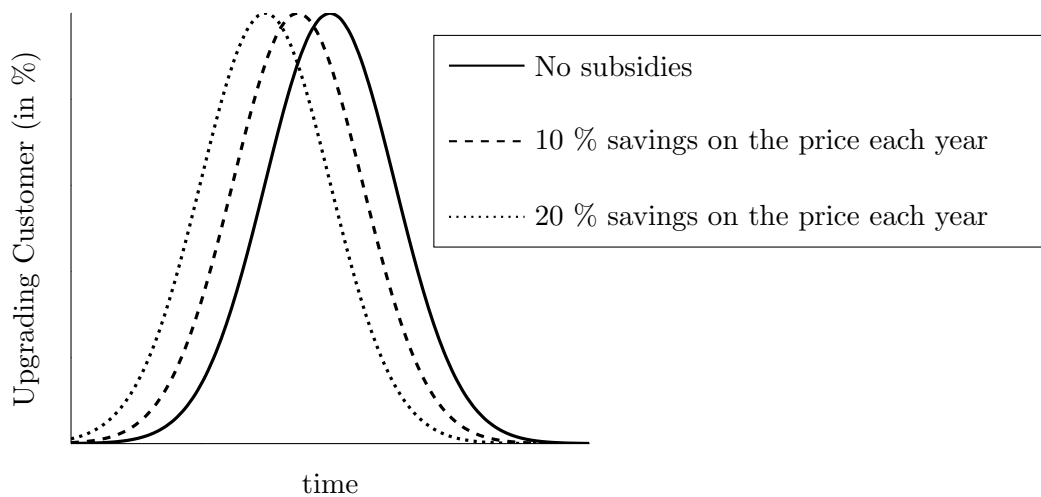


Figure 1: Curve of the diffusion of innovation and influence of subsidies

In 1969, Bass formalized Roger's model by using differential equations and later developed it to tackle some of the issues with external variables (see Bass (1995)) and generations (see Norton and Bass (1987)). This work considers marketing aspects by showing a left shift on the shape of adoptions curves when regular savings (equivalent to constant subsidies) on the price of a product are made, as illustrated in Figure 1. These models help the understanding of how subscribers react in a telecommunication market: see Section 5 in Bass (2004) for a discussion on 2G/3G upgrade; Lim and Kim (2017) for Bass model applied to the forecasting of the 5G upgrade; and Michalakelis et al. (2008) for an application to the Greek mobile market.

In order to adapt this formalism to the current telecommunication context, we consider two

important factors for modeling the adoption of a new technology. First, the percentage of users upgrading from older technologies is very sensitive to the price gap between the new technology and their current one. The second factor is the influence of network deployment which, from our knowledge, has not been taken into account. Indeed, subscribers upgrade more easily when they are sure to benefit from the new service, i.e. if the newest technology is deployed.

### 1.3. Contributions and structure of the paper

To the best of our knowledge, optimizing jointly the two dynamics (network and subscribers) under capacity and targeting constraints has not yet been studied. Hence, the main contribution of the manuscript is to provide a realistic model for the problem. Building on practical considerations, we come up with a non-linear non-convex Mixed-Integer Programming (MIP) formulation which is linearized using classical discretizations, making sense from the applied viewpoint. Our case study assesses in details the performance and solutions provided by the model on real instances. We also provide a simple decomposition matheuristic and compare its performance with the exact Mixed-Integer Linear Programming (MILP) formulation. Finally, we provide valid inequalities which prove successful in improving the performance of our MILP.

The remainder of this article is organized as follows. Section 2 introduces our Mobile Master Plan problem, for which a mixed integer formulation is provided in Section 3. A linear version of this formulation including a discrete modeling of the upgrade function is given in Section 4. This model is assessed numerically in Section 5 on real instances and compared with a heuristic based on year by year planning. Concluding remarks are given in Section 6.

## 2. Problem Description

We focus on the evolution of the mobile network on a global geographical area along a multi-period time horizon, whose periods are assumed to be equally-sized (typically 5 periods of one year each). We denote these time periods by  $t \in \mathcal{T} = \{1, \dots, \bar{t}\}$ , and we use “0” for denoting the time period directly anterior to the time horizon.

The whole area is served by telecommunication sites equipped with several mobile networks technologies. We thus consider a set  $\mathcal{G} = \{1G, 2G, \dots, NG\}$  of technologies and a set  $\mathcal{S} = \{1, \dots, N_S\}$  of telecommunication sites. As we have already mentioned, the network capacity on a site can be increased in two ways: by installing a technology on a site or by adding new modules of an already

installed technology. In this work, we assume that the oldest technology  $1G$  is installed in all sites at the beginning of the time horizon and that only the newest technology can be installed during the time horizon (focus on the investments towards the newest technology), with a cost by site of  $CA_{NG}$ . The binary parameter  $Z_{s,i}, s \in \mathcal{S}, i \in \mathcal{G} \setminus \{NG\}$ , is equal to 1 iff technology  $i$  is installed on site  $s$ . The binary parameter  $Z_{s,NG}^0, s \in \mathcal{S}$ , is equal to 1 iff the newest technology  $NG$  is installed at the beginning of the time horizon. We also assume that increasing the capacity with new modules is possible for all technologies with a cost  $CM_i$  for each added module of technology  $i \in \mathcal{G}$ . The initial number of modules, for each site and each technology, is denoted by  $M_{s,i}^0, s \in \mathcal{S}, i \in \mathcal{G}$ . For each technology, we have an upper bound on the number of modules by site denoted by  $NM_i, i \in \mathcal{G}$ . The capacity of a module, for each generation, is denoted by  $CAP_i, i \in \mathcal{G}$ .

We assume that each generation of set  $\mathcal{G}$  has its own offer with its own subscribers. The initial number of subscribers for each generation associated with each site is denoted by  $U_{s,i}^0$ . Subscribers to a technology cannot be served by a technology more recent than their offer (technical incompatibility) and are served by the nearest technology available in order to receive the best service. We denote by  $NU^t, t \in \mathcal{T}$  the percentage of incoming users at the beginning of each time period and  $P_i, i \in \mathcal{G}$  the proportion of incoming users subscribing to technology  $i \in \mathcal{G}$ . New subscribers are assumed to be assigned to sites proportionally to the number of subscribers at the end of the previous time period.

As mentioned in the introduction, we assume in this work that the whole investments in user upgrades are made towards the newest technology  $NG$ . We assume that the upgrade mechanism modeling subscribers behavior depends only on two parameters. The first one is the value of the subsidy denoted by  $\sigma$ . The set of possible values taken by  $\sigma$  will be denoted by  $\mathcal{K}$ . The second one is the proportion of sites already covered by the  $NG$  technology, denoted  $\alpha$  in what follows. The function modeling the upgrade mechanism, denoted by  $f : \mathcal{K} \times [0, 1] \rightarrow [0, 1]$ , provides the proportion of subscribers reacting positively to the given subsidy  $\sigma$  under a given coverage  $\alpha$ . This function is assumed non-decreasing in both arguments. We hence consider that all subscribers are subsidized and react independently of their current offer.

As already pointed out, network and subscribers dynamics are linked. First, each network generation has to be dimensioned to handle the traffic demand pro subscriber  $D_i^t, i \in \mathcal{G}$ , defined as the 95% quantile of the possible demands occurring over the time period. Hence, the capacity has to



be sufficient to handle the resulting traffic 95% of the time. We assume that this traffic demand depends on the network serving the subscribers rather than their current offer. Second, telecommunications operators are ranked according to their performances. Therefore, we decide to focus on meeting satisfying levels for two key performances indicators: the proportion of sites covered by  $NG$  at the end of the time horizon, which is denoted by  $\alpha^{\bar{t}}$  and of averaged quality of experience to the corresponding subscribers. The averaged quality of experience is guaranteed by asking for a minimal proportion of subscribers being  $NG$  subscribers associated with  $NG$  sites. These indeed benefit from the new performances and have the maximum throughput. The thresholds required at the end of the time horizon associated with these two targeting indicators are respectively denoted by  $\underline{\alpha}$  and  $\underline{QoE}$ .

Decisions are taken over the time horizon. These decisions are installation of the  $NG$  technology, adding of modules (for all technologies), and the amount of the subsidy given to the subscribers from older technologies for upgrading to the  $NG$  technology. The problem defined in this work, denoted as the Mobile Master Plan Problem (MMPP), consists in finding the decisions which minimize network and subscribers investments while satisfying capacity and targeting constraints.

### 3. Mathematical Modeling

#### 3.1. Decision variables

We provide in this section a mixed-integer formulation for the problem described in Section 2. First, we define the set of decision variables used in our formulation. For  $t \in \mathcal{T} \cup \{0\}$ ,  $s \in \mathcal{S}$ , let us introduce the binary variable

$$z_{s,NG}^t = \begin{cases} 1 & \text{if the newest technology is installed at site } s \text{ at the end of time period } t \\ 0 & \text{otherwise} \end{cases}$$

For  $t \in \mathcal{T} \cup \{0\}$ ,  $s \in \mathcal{S}$ ,  $i \in \mathcal{G}$ , the integer variable  $m_{s,i}^t$  represents the total number of modules of generation  $i$  installed at site  $s$  at the end of time period  $t$ .

We also use the following continuous variables.

- For each  $t \in \mathcal{T} \cup \{0\}$ ,  $s \in \mathcal{S}$ ,  $i \in \mathcal{G}$ , let  $u_{s,i}^t$  denote the total number of subscribers to generation  $i$  in site  $s$  at the end of time period  $t$  and  $\bar{U}_{s,i}^t$  an upper bound on this quantity,

- For each  $t \in \mathcal{T}, s \in \mathcal{S}, i, j \in \mathcal{G}$ , let  $u_{s,i,j}^t$  denote the total number of subscribers to generation  $i$  served by generation  $j$  in site  $s$  at the end of time period  $t$ .

In addition, the notations  $\sigma^t$  and  $\alpha^t$  introduced in the previous section become optimization variables.

- For each  $t \in \mathcal{T}$ , let  $\sigma^t$  be the value of the subsidy, in k€, offered to subscribers of former technologies for upgrading to technology  $NG$  at the beginning of time period  $t$ ,
- For each  $t \in \mathcal{T}$ , let  $\alpha^t = \frac{\sum_{s \in \mathcal{S}} z_{s,NG}^{t-1}}{N_S}$  be the redundant variable that denotes the  $NG$  sites coverage at the beginning of the time period  $t$  (fraction of sites where  $NG$  is deployed at the end of time period  $t - 1$ ).

The upgrade function, representing the percentage of users reacting positively to a subsidy  $\sigma^t, t \in \mathcal{T}$  for a given coverage  $\alpha^t, t \in \mathcal{T}$ , is denoted by  $f(\sigma^t, \alpha^{t-1})$  and will be modeled explicitly in Section 4.

### 3.2. General Formulation

The MMPP can be modeled as follows:

$$\min \sum_{t \in \mathcal{T}} \sigma^t \times f(\sigma^t, \alpha^{t-1}) \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G} \setminus NG} u_{s,i}^{t-1} + \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G}} CM_i(m_{s,i}^{\bar{t}} - M_{s,i}^0) + \sum_{s \in \mathcal{S}} CA_{NG}(z_{s,NG}^{\bar{t}} - Z_{s,NG}^0) \quad (1)$$

$$\text{s.t. } m_{s,j}^t \leq NM_j Z_{s,j}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall j \in \mathcal{G} \setminus \{NG\}, \quad (2)$$

$$m_{s,NG}^t \leq NM_{NG} z_{s,NG}^t, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \quad (3)$$

$$z_{s,NG}^t \leq m_{s,NG}^t, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \quad (4)$$

$$m_{s,j}^t \geq m_{s,j}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall j \in \mathcal{G}, \quad (5)$$

$$u_{s,i}^t = \sum_{j \in \mathcal{G}: j \leq i} u_{s,i,j}^t, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \quad (6)$$

$$Z_{s,k} \sum_{i \in \mathcal{G}: i \geq k} \sum_{j \in \mathcal{G}: j \leq k} u_{s,i,j}^t = 0, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall k \in \mathcal{G} \setminus \{NG\}, \quad (7)$$

$$\sum_{j < NG} u_{s,NG,j}^t \leq \bar{U}_{s,NG}^t (1 - z_{s,NG}^t), \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \quad (8)$$

$$\sum_i D_j^t u_{s,i,j}^t \leq CAP_j m_{s,j}^t, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall j \in \mathcal{G}, \quad (9)$$

$$u_{s,i}^t = u_{s,i}^{t-1} + P_i NU^t \left( \sum_{k \in \mathcal{G}} u_{s,k}^{t-1} \right) - f(\sigma^t, \alpha^{t-1}) \times u_{s,i}^{t-1} \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \setminus \{NG\}, \quad (10)$$

$$u_{s,NG}^t = u_{s,NG}^{t-1} + P_{NG} NU^t \left( \sum_{k \in \mathcal{G}} u_{s,k}^{t-1} \right) + f(\sigma^t, \alpha^{t-1}) \times \sum_{i \in \mathcal{G} \setminus \{NG\}} u_{s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \quad (11)$$

$$\frac{\sum_{s \in \mathcal{S}} u_{s,NG,NG}^{\bar{t}}}{\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G}} u_{s,i}^{\bar{t}}} \geq \underline{QoE}, \quad (12)$$

$$\alpha^{\bar{t}} \geq \underline{\alpha}, \quad (13)$$

$$u_{s,i}^0 = U_{s,i}^0, \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{G}, \quad (14)$$

$$m_{s,i}^0 = M_{s,i}^0, \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{G}, \quad (15)$$

$$z_{s,NG}^0 = Z_{s,NG}^0, \quad \forall s \in \mathcal{S}, \quad (16)$$

$$m_{s,i}^t \in \mathbb{Z}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\}, \forall i \in \mathcal{G}, \quad (17)$$

$$z_{s,NG}^t \in \{0, 1\}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\}, \quad (18)$$

$$u_{s,i}^t \geq 0, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\}, \forall i \in \mathcal{G}, \quad (19)$$

$$u_{s,i,j}^t \geq 0, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \forall j \in \mathcal{G}, \quad (20)$$

$$\sigma^t \in \mathcal{K}, \quad \forall t \in \mathcal{T}. \quad (21)$$

The objective function (1) minimizes both subscribers migrations costs and network investments. It involves three types of costs: (i) costs incurred by the subsidies offered (user upgrades); (ii) costs incurred by the installation of new modules for increasing the capacity (densification); and (iii) costs incurred by the deployment of the newest technology (coverage extension). Constraints (2)–(5) are the network dynamic constraints, in charge of setting the number of modules. Constraints (2) and (3) define the upper bounds on the numbers of modules for each technology installed at each site. These constraints also ensure that if the corresponding technology is not installed, no modules can be added. Constraints (4) impose that when the newest technology is installed at a site at least one module of this technology be installed. Constraints (5) impose that the total numbers of modules be non-decreasing during the time horizon for each technology and site.

Constraints (6)–(9) are the network dimensioning constraints, in charge of making the link between the network dynamic and the subscriber dynamic. Constraints (6) state that the subscribers to a technology cannot be served by more recent technologies. Constraints (7) impose that the nearest older technology installed is used. Constraints (8) are the counterpart of (7) for the newest technology. Constraints (9) are the capacity constraints: for each site, the installed capacity has to be sufficient for providing services for all users located at this site and having to be served by this technology.

Constraints (10)–(11) refer to the subscribers dynamic. They define the total number of subscribers to a technology at each site and each time period, taking into account incoming subscribers and current subscribers who decide to upgrade to the newest generation, thanks to subsidies and coverage improvements. Constraints (10) are related to all generations but the newest, which is modeled by (11). Constraints (12) and (13) are the targeting constraints and refer to the end of time horizon. Constraint (12) ensures that the threshold of subscribers covered by the latest technology is met. The indicator is proportional to the quality of experience which measures the percentage of users having access to the new technology throughput. Constraint (13) imposes that the threshold on the number of sites served by the latest technology is met. Constraints (14)–(16) refer to the initial conditions. Finally, constraints (17)–(21) define the domain of the variables.

The next section complements the above formulation by detailing function  $f$  and set  $\mathcal{K}$ .

#### 4. Upgrade function and discretization

As we have mentioned, the upgrade function is non-decreasing in both the subsidy amount,  $\sigma$ , and the proportion of sites covered by  $NG$   $\alpha$ . We discretize the later and partition the interval  $[0, 1]$  into  $C$  smaller intervals  $[L_c, U_c[$ , and define  $\mathcal{C} = \{1, \dots, C\}$ . Specifically, we define a set of function  $f_c(\sigma)$  such that  $f(\alpha, \sigma) = f_c(\sigma)$  for all  $\alpha \in [L_c, U_c[$  and  $c \in \mathcal{C}$ . We further model each  $f_c(\sigma)$  as an  $S$ -shape function, see the example from Figure 2 where the discretization of the coverage involves four ranges:  $[0, 0.25[$  (x axis),  $[0.25, 0.50[$  (plain),  $[0.50, 0.75[$  (dashed) and  $[0.75, 1]$  (dotted).

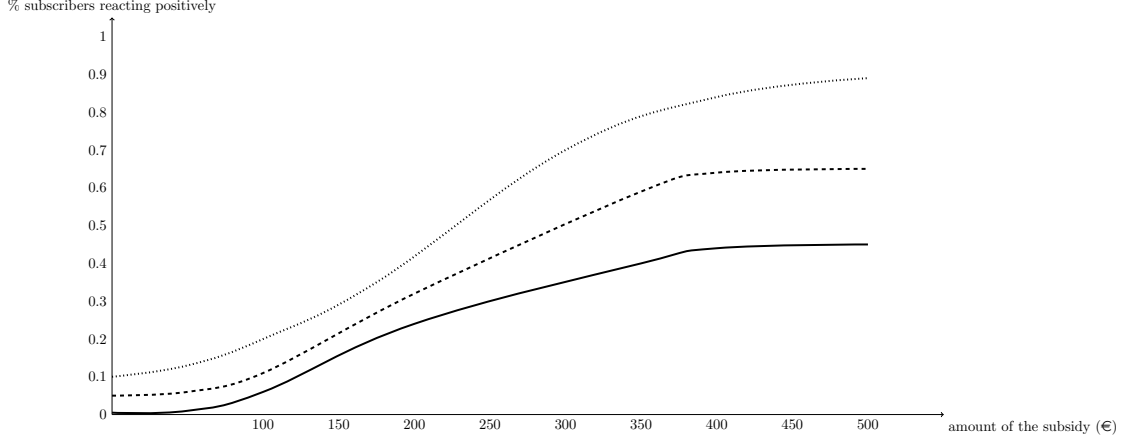


Figure 2: Example of upgrade function

We further discretize each function  $f_c$  by noticing that, in practice, subsidies are usually taken from a small set of pre-defined values, e.g. multiple of 100 €. Hence, we define the set of possible choices of subsidies as  $\mathcal{W} \subset \mathcal{K}$  and consider in what follows the restriction of each function  $f_c$  to  $\mathcal{W}$ . To simplify the formulations that follow, we also define  $f_{w,c} = f_c(w)$ . Hence,  $f_{w,c}$  is the percentage of subscribers that react positively when subsidy  $w$  is offered and the coverage belongs to the range  $[L_c, U_c]$ .

We must now incorporate functions  $f_c$  to our MILP. In this aim, we define a binary variable  $\delta_{w,c}^t$ , for each  $t \in \mathcal{T}, w \in \mathcal{W}, c \in \mathcal{C}$ , taking value equal to one iff  $\sigma^t = w$  and  $\alpha^{t-1} \in [L_c, U_c]$ . The objective function from Section 3.2 can be rewritten as follows.

$$\sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} SUB_w \sum_{c \in \mathcal{C}} f_{w,c} \delta_{w,c}^t \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G} \setminus \{NG\}} u_{s,i}^{t-1} + \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G}} CM_i (m_{s,i}^{\bar{t}} - M_{s,i}^0) + \sum_{s \in \mathcal{S}} CANG (z_{s,NG}^{\bar{t}} - Z_{s,NG}^0) \quad (22)$$

Also, constraints (10) and (11) are written as:

$$u_{s,i}^t = u_{s,i}^{t-1} + P_i NU^t \left( \sum_{k \in \mathcal{G}} u_{s,k}^{t-1} \right) - \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}} f_{w,c} \delta_{w,c}^t u_{s,i}^{t-1}, \quad \forall i \in \mathcal{G} \setminus \{NG\}, \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \quad (23)$$

$$u_{s,NG}^t = u_{s,NG}^{t-1} + P_{NG} NU^t \left( \sum_{k \in \mathcal{G}} u_{s,k}^{t-1} \right) + \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}} f_{w,c} \delta_{w,c}^t \sum_{i \in \mathcal{G} \setminus \{NG\}} u_{s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \quad (24)$$

We linearize the products between binary variable  $\delta_{w,c}^t$  and continuous variable  $u_{s,i}^{t-1}$  using a classical

method (Fortet, 1960). Consequently, the MOMP can be formulated as the following MILP.

$$\begin{aligned} \min \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} SUB_w \sum_{c \in \mathcal{C}} f_{w,c} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G} \setminus NG} \pi_{w,c,s,i}^t &+ \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G}} CM_i(m_{s,i}^{\bar{t}} - M_{s,i}^0) \\ &+ \sum_{s \in \mathcal{S}} CAN_G(z_{s,NG}^{\bar{t}} - Z_{s,NG}^0) \end{aligned} \quad (25)$$

s.t. (2) – (9), (12) – (13), (14) – (16)

$$\begin{aligned} u_{s,i}^t &= u_{s,i}^{t-1} + P_i NU^t \left( \sum_{k \in \mathcal{G}} u_{s,k}^{t-1} \right) \\ &- \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}} f_{w,c} \pi_{w,c,s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \setminus \{NG\}, \end{aligned} \quad (26)$$

$$\begin{aligned} u_{s,NG}^t &= u_{s,NG}^{t-1} + P_{NG} NU^t \left( \sum_{k \in \mathcal{G}} u_{s,k}^{t-1} \right) \\ &+ \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}} f_{w,c} \sum_{i \in \mathcal{G} \setminus \{NG\}} \pi_{w,c,s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \end{aligned} \quad (27)$$

$$\sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}} \delta_{w,c}^t \leq 1, \quad \forall t \in \mathcal{T}, \quad (28)$$

$$\sum_{w \in \mathcal{W}} \delta_{w,c}^t - 1 \leq U_c - \alpha^t, \quad \forall t \in \mathcal{T}, \forall c \in \mathcal{C}, \quad (29)$$

$$1 - \sum_{w \in \mathcal{W}} \delta_{w,c}^t \geq L_c - \alpha^t, \quad \forall t \in \mathcal{T}, \forall c \in \mathcal{C}, \quad (30)$$

$$\pi_{w,c,s,i}^t \leq \delta_{w,c}^t \bar{U}_{s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \setminus \{NG\}, \forall w \in \mathcal{W}, \forall c \in \mathcal{C}, \quad (31)$$

$$\pi_{w,c,s,i}^t \leq u_{s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \setminus \{NG\}, \forall w \in \mathcal{W}, \forall c \in \mathcal{C}, \quad (32)$$

$$\pi_{w,c,s,i}^t \geq u_{s,i}^{t-1} - (1 - \delta_{w,c}^t) \bar{U}_{s,i}^{t-1}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \setminus \{NG\}, \forall w \in \mathcal{W}, \forall c \in \mathcal{C}, \quad (33)$$

$$m_{s,i}^t \in \mathbb{Z}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\}, \forall i \in \mathcal{G}, \quad (34)$$

$$z_{s,NG}^t \in \{0, 1\}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\}, \quad (35)$$

$$u_{s,i}^t \geq 0, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\}, \forall i \in \mathcal{G}, \quad (36)$$

$$u_{s,i,j}^t \geq 0, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \forall j \in \mathcal{G}, \quad (37)$$

$$\delta_{w,c}^t \in \{0, 1\}, \quad \forall t \in \mathcal{T}, \forall w \in \mathcal{W}, \forall c \in \mathcal{C}, \quad (38)$$

$$\pi_{w,c,s,i}^t \in \{0, 1\}, \quad \forall t \in \mathcal{T}, \forall w \in \mathcal{W}, \forall c \in \mathcal{C}, \forall s \in \mathcal{S}, \forall i \in \mathcal{G}. \quad (39)$$

Constraints (28) ensure that at most one subsidy from the set  $\mathcal{W}$  is offered at each time period. Constraints (29) and (30) ensure that, for each time period, variables  $\delta_{w,c}^t$  are set according to the coverage at the previous time period. Constraints (29) (respectively (30)) set all  $\delta$  related to a range at 0 if the coverage is greater (resp. smaller) than the upper (resp. lower) bound of the range. Constraints (31) – (33) are the typical linearizations of the products of a binary variable with a continuous one. Constraints (34) –(39) define the domain of the variables.

Preliminary computational experiments on small instances with two generations have shown that the solutions of linear relaxation present variables  $z$  and  $\delta$  fractionary. Consequently, we reinforce the model by proposing the following valid inequalities.

$$z_{s,NG}^t \leq \sum_{t' > t} z_{s,NG}^{t'}, \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \cup \{0\} \quad (40)$$

$$1 - \sum_w \delta_{w,c}^t \geq \sum_w \delta_{w,c'}^{t'}, \forall t \in \mathcal{T} \forall c \in \mathcal{C}, \forall c' < c, \forall t' > t \quad (41)$$

Constraints (40) state that if we have installed the newest technology NG on a site, we should have it in the future while constraints (41) say that if we are at a given range of coverage, we cannot be in a lower range of coverage in the future.

## 5. Case study for 3G and 4G

The purpose of this case study is two-fold. First, we assess the scalability of the exact MILP model. Second, we observe the behavior of the solutions in terms of costs, including when the investment expenses are smoothed along the time horizon.

### 5.1. Instances and platform

Numerical tests are performed on instances of French telecommunication operator Orange in the French area of Brittany representing a case study with two network generations: 3G and 4G. The full area contains 1112 sites: 717 equipped only with 3G technology and 395 equipped with both technologies. We create a set of smaller instances, out of this large instance, in order to have different scenarios characterized by the number of sites and the 4G initial coverage: rural scenarios where the 4G technology is initially installed on 17% of the sites, mid-dense scenarios where this proportion is equal to 34% and urban scenarios where it is equal to 68 %.

The real data includes the number of modules and subscribers for each site. Others values of this case study are realistic values taken from telecommunication equipment sellers. Each site can carry a maximum of four 3G modules (carrier) of 5MHz with a capacity of 3 Mbps and a cost of 3 k€ each and a maximum of five 4G modules of 10 MHz (bandwidth) with a capacity of 25 Mbps and a cost of 16 k€ each. The cost for adding the 4G technology on a site is 75 k€. As for subsidies hypothesis, we take 10 discrete values for the discretization of the amount of subsidy offered,  $\sigma \in [0, 100, 150, 200, 250, 300, 350, 400, 450, 500]$ €. We also use four levels of 4G sites coverage: *low*, *medium low*, *medium high* and *high* respectively refer to ranges (in %)  $[0, 25[$ ,  $[25, 50[$ ,  $[50, 75[$  and  $[75, 100]$ . The curves from Figure 2 are hence discretized into the table of values Table 3, which constitutes our reference upgrade function. Two other upgrades functions are also

Coverage level \ Subsidies (in €)	0	100	150	200	250	300	350	400	450	500
<i>low</i>	0	0	0	0	0	0	0	0	0	0
<i>medium low</i>	0.5	5	12	21	30	40	42	43	44	45
<i>medium high</i>	5	10	20	30	40	50	60	62	64	65
<i>high</i>	10	20	33	45	58	70	80	83	87	90

Table 3: Reaction of the customers (in %) on reference markets for given subsidies and coverage levels.

considered for adapting to very technology-reluctant markets and technology-friendly markets. We ask for final target objectives of  $\underline{QoE} = 80\%$  for the quality of experience and  $\underline{\alpha} = 70\%$  for the 4G sites coverage. We optimize on the typical time horizon of five years discretized in five time periods of one year. The unit of the objective value is k€.

The computations have been made on a server of 16 processors Intel Xeon of CPU 5110 and clocked at 1.6 GHz each. The code has been written in Julia 0.5.0, and the solver used is CPLEX 12.6 (default branch-and-bound algorithm). The time limit for MIP solving is set to 1800 seconds.

## 5.2. Results for algorithmic tests

Our objectives in this section are to assess the scalability of our formulation and to test its sensitivity to the type of areas. We will hence focus on 15 instances considering five sizes between 25 and 125 sites and the three types of areas mentioned above. The upgrade function used here is the reference one (see Table 3). Instances are displayed in the two left columns of Table 4, column “ $N_S$ ” standing for the number of sites and column “area” standing for the type of area.



The solutions of the linear relaxation and of the MIP are computed with and without the valid inequalities. In Table 4, results without the valid inequalities are displayed under “MMMP without ((40)-(41))” while results with the valid inequalities are displayed under “MMMP with ((40)-(41))”. The following indicators are used for both formulations. The linear relaxation is stored in column “relax”. Column “sol” stands for the value of the solution obtained by CPLEX within the time limit, labeled with a “\*” if the branch-and-bound procedure converges. The solver computation time, in seconds, is given in column “time”. Column “f-gap” (final gap) represents the gap between the solution found and the best lower bound found while “r-gap” (root gap) stands for the gap between the solution found and the linear relaxation.

Table 4: Scalability test for (MMPP): 5 sizes, 3 density.

Instance		(MMMP) without ((40)-(41))					(MMMP) with ((40)-(41))				
$N_S$	area	relax	sol	time	f-gap	r-gap	relax	sol	time	f-gap	r-gap
25	rural	1525	2278*	437	0,00	33,07	1599	2278*	143	0,00	29,83
	mid-dense	1160	1928*	897	0,00	39,81	1244	1928*	233	0,00	35,48
	urban	405	1298*	63	0,00	68,82	677	1298*	45	0,00	47,83
50	rural	3031	4532	1800	5,79	33,13	3172	4532	1800	5,80	30,01
	mid-dense	2346	3972	1800	6,31	40,94	2537	3972	1800	3,44	36,12
	urban	853	2624*	517	0,00	67,48	1418	2624*	279	0,00	45,94
75	rural	4555	6920	1800	13,63	34,18	4770	6859	1800	7,07	30,45
	mid-dense	3455	6008	1800	15,12	42,50	3726	5807	1800	5,88	35,83
	urban	1271	3944	1800	2,53	67,78	2112	3944*	1641	0,00	46,45
100	rural	6077	9178	1800	13,32	33,78	6368	9141	1800	13,61	30,34
	mid-dense	4714	8076	1800	18,80	41,63	5095	7841	1800	8,66	35,02
	urban	1615	5113	1800	7,85	68,42	2696	5038	1800	1,84	46,47
125	rural	7541	11450	1800	15,44	34,13	7894	11424	1800	15,47	30,90
	mid-dense	5874	10370	1800	17,45	43,36	6343	10087	1800	13,79	37,12
	urban	1895	6107	1800	7,38	68,97	3212	6104	1800	4,44	47,37

We observe that the formulation with the valid inequalities has a tighter relaxation. Adding

the valid inequalities also improves the computation time (up to 75 % better) when the branch-and-bound procedure converges for both formulations. We also observe that the convergence of the urban instance of 75 sites is achieved only when including the valid inequalities. Finally, we see, for instances who do not converge within the time-limit, an improvement on the final gap found by the branch-bound procedure when the valid inequalities are added. For following tests and observations, we will hence focus on the formulation with the valid inequalities.

First, referring to scalability, we see that the branch-and-bound procedure converges to optimality only for five instances: the three instances with 25 sites and the urban instances of 50 and 75 sites. The final gap remains below 7 % for instances until 75 sites but is greater than 15 % for the rural instance of 125 sites. The branch-and-bound procedure proves hence to be adequate for small sizes but cannot handle for solving larger instances, which leads us to propose a time-decomposition heuristic solution method (see Section 5.4).

Second, we focus on the sensitivity to the type of area. We notice that the problem is easier in urban areas, which can be explained by the decisions on coverage extension needed to satisfy the strategic targets: having more sites already covered by the newest technology results indeed in fewer decisions to take. This enlightens that the problem difficulty is strongly correlated with the coverage extension. We notice that these resulting coverage extension investments also have a significant financial impact. For instance, the optimal solution for the rural instance of 25 sites is 75 % more expensive than the optimal solution for the urban instance.

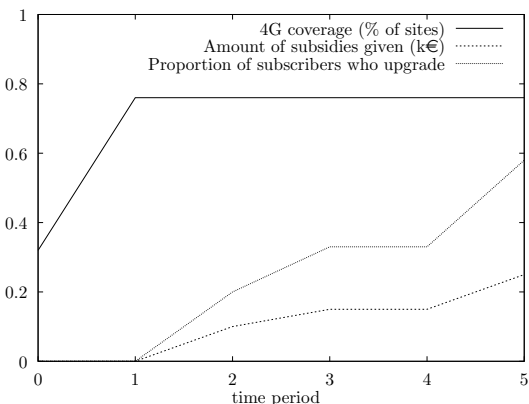


Figure 3: Evolution of the coverage, subsidies decisions and reactions.

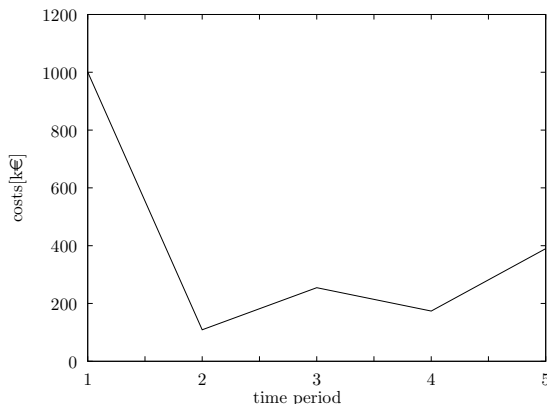


Figure 4: Evolution of the costs.

Moreover, we aim to analyze the behavior of an optimal solution over the time horizon. We

thus draw the features of the solution for the 25 sites mid-dense instance in Figure 3. The 4G sites coverages, the amount of subsidies given and the reaction of the subscribers are plotted. On the 4G sites coverage curve, we can notice that the 4G sites coverage at the end of the time horizon is 76 % and that this value is not reached progressively throughout the time horizon. Indeed, the 4G sites coverage at the end of the first year is already equal to this value. This fast deployment is made to benefit from more reactions to subsidies thanks to coverage improvements (for instance, three times more in the second year thanks to the change from coverage range *medium low* to range *high*). This enlightens the financial interest for the operator in quickly having a network of good quality. However, it results also in large budget variations, with the first year costing up to twice the others years as we can see in Figure 4. These important variations do not match with the financial context of a telecommunication operator as investments should be distributed along the whole time horizon. This observation leads us to consider the case where the costs are smoothed over the time horizon (see Section 5.3).

### 5.3. Results for business-oriented tests

In this section, we assess the impact of several business-oriented scenarios from both algorithmic and financial perspectives:

- smoothing the costs over the time horizon,
- considering three upgrade functions: the reference upgrade used in the algorithmic tests, the technology-reluctant and technology-friendly upgrades.

For these purposes, and in order to have a sufficient expected number of optimal solutions (according to the algorithmic tests) for assessing financial aspects, we consider a set of 21 instances made of:

- for the 25 sites instances: the three types of areas and the three types of markets (9 instances),
- for the 50 to 125 sites instances: the mid-dense area and the three types of markets (12 instances).

These 21 instances are presented in Table 5. The type of market is displayed in column “Upgrade ”: “T.F markets”, “Ref. markets” and “T.R. markets” standing, respectively, for technology-friendly,

reference and technology-reluctant markets. In order to smooth the costs, we introduce a cost equilibrium set of constraints which bounds the budget fluctuations by a percentage  $p$ . Let the budget spent in year  $t \in \mathcal{T}$  be denoted by

$$B_t = \sum_{w \in \mathcal{W}} SUB_w \sum_{c \in \mathcal{C}} F_{w,c} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G} \setminus N_G} \pi_{w,c,s,i}^t + \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{G}} CM_i (m_{s,i}^t - m_{s,i}^{t-1}) + \sum_{s \in \mathcal{S}} CA_{NG} (z_{s,NG}^t - z_{s,NG}^{t-1}), \forall t \in \mathcal{T} \quad (42)$$

The objective function becomes equal to  $\sum_{t \in \mathcal{T}} B_t$ . The cost equilibrium set of constraints can be written as follows:

$$(1 - p) \times \frac{\sum_{t' \in \mathcal{T}} B_{t'}}{T} \leq B_t \leq (1 + p) \times \frac{\sum_{t' \in \mathcal{T}} B_{t'}}{T}, \quad \forall t \in \mathcal{T} \quad (43)$$

The constraints enforce all period expenses to lie between  $(1 - p)$  and  $(1 + p)$  times the quotient of the total expenses over the time horizon by the number of time periods.

Results for both formulations in Table 5 are displayed respectively below “(MMMP)” and “(MMMP + cost equilibrium)”. The indicators provided are the same as in Section 5.2. The last column “overcost” gives the resulting overcost (the relative gap in % between the solutions values without and with the cost equilibrium set of constraint).

On a computational point of view, adding the cost equilibrium constraints hardens the problem, increasing solution times and final gaps, especially for instances with a high number of sites. For example, considering the 125 sites instances, final gaps are three to four times higher with the additional set of constraints.

Table 5: Scalability of cost equilibria.

Instance		(MMMP)				(MMMP) + cost equilibria				Overcost	
$N_S$	area	Upgrade	sol	time	f-gap	r-gap	sol	time	f-gap	r-gap	
25	rural	T.F. markets	2019*	174	0,00	35,57	2417	1800	2,05	47,62	19,71
		Ref. markets	2278*	143	0,00	29,83	2485*	289	0,00	36,75	9,08
		T.R. markets	2492*	556	0,00	27,86	2681*	159	0,00	33,30	7,59
25	mid-dense	T.F. markets	1664*	128	0,00	43,55	2027*	331	0,00	55,94	21,81
		Ref. markets	1928 *	233	0,00	35,48	2068*	307	0,00	38,41	7,26
		T.R. markets	2143*	651	0,00	32,49	2265*	419	0,00	36,97	5,69
25	urban	T.F. markets	948*	14	0,00	69,27	970*	76	0,00	84,52	2,34
		Ref. markets	1298*	45	0,00	47,83	1311*	114	0,00	66,12	1,04
		T.R. markets	1521*	20	0,00	38,69	1546*	71	0,00	49,94	1,62
50	mid-dense	T.F. markets	3372	1800	3,49	44,59	3970	1800	3,87	55,45	17,75
		Ref. markets	3972	1800	3,44	36,12	4311	1800	3,98	43,50	8,53
		T.R. markets	4398	1800	3,36	31,86	4704	1800	2,39	37,28	6,96
75	mid-dense	T.F. markets	4862	1800	5,63	42,65	5919	1800	17,47	55,01	21,74
		Ref. markets	5807	1800	5,88	35,83	6407	1800	18,32	44,06	10,33
		T.R. markets	6333	1800	6,55	30,79	6948	1800	16,72	37,84	9,72
100	mid-dense	T.F. markets	6151	1800	7,62	42,54	8350	1800	21,12	57,02	35,76
		Ref. markets	7841	1800	8,66	35,02	8964	1800	20,15	45,43	14,32
		T.R. markets	8663	1800	8,63	30,57	9260	1800	15,16	36,04	6,89
125	mid-dense	T.F. markets	8277	1800	8,70	43,22	11337	1800	27,79	60,57	36,97
		Ref. markets	10087	1800	13,79	37,12	10990	1800	20,99	44,56	8,95
		T.R. markets	10913	1800	8,14	31,47	12291	1800	20,64	40,07	12,62

On a financial point of view, we consider the 25 sites instances in order to discuss optimal solutions. For the 25 sites urban area instance with the upgrade for the reference markets, we plot in Figure 5 the counterpart of Figure 3 when requiring the costs to be smoothed. We see on these curves that the range of coverages *high* is reached in three years in the solution with cost equilibrium instead of one without cost equilibrium (see Figure 3). This has an impact on the upgrade investments, which become higher as we can see on the figures. By comparing the previous instance with the other 25 sites instances for reference markets in rural and urban areas, we see the influence of the initial density on the overcost resulting from the cost smoothing. This effect can be seen on Table 5, column “overcost” and rows *Ref.markets*: 8 % for rural instances, 7 % for mid-dense and only 1 % for urban ones. The needed investments for reaching the upper range of coverages are indeed higher when starting from low initial 4G sites coverage, leading to a higher gap between solutions with and without cost equilibrium. We hence are able to quantify the overcost to get business-fit solutions and see that this overcost is limited in urban instances.

We notice that there seems to be no algorithmic sensitivity to the choice of the upgrade function. In what concerns the financial sensitivity, we notice that, as expected since more upgrade investments are needed, the cost is higher on reluctant markets. For instance, the cost for the instance of 25 sites and, 34 % initial 4G sites coverage with the upgrade for reluctant markets is 50 % more expensive than the instance with the upgrade for technology-friendly markets.

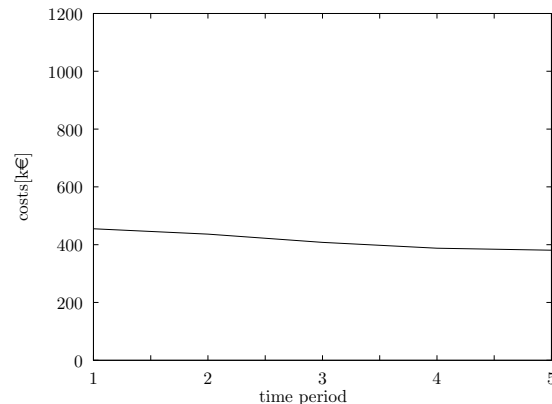
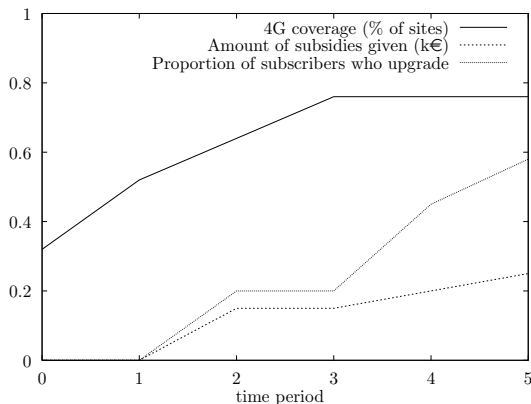


Figure 5: Evolution of the coverage, subsidies decisions and reactions (with cost equilibria constraints). Figure 6: Evolution of the costs (with cost equilibria constraints).

#### 5.4. Computational tests on larger instances

For solving large instances in shorter amounts of time, we have designed a heuristic based on a year by year planning. We use the same formulation as before, however with only one period ( $T = 1$ ). We solve the five years problem iteratively (year by year) with the values of variables  $u^1$ ,  $m^1$  and  $z^1$  becoming parameters  $U^0, M^0$  and  $Z^0$  in the next run. This allows us to obtain good solutions in shorter amounts of time. However, this model leads to optimize the four first years without the target constraints on quality of experience and coverage. Preliminary tests have shown that it could lead to infeasibility since the subscribers upgrade are delayed until the last period. In order to avoid this, we decide to test the model with intermediary target objectives for the quality of experience, see Algorithm 1. Notice that this heuristic does not apply when asking for costs to be smooth. With the aim to assess the heuristic quality, we use the 15 instances of the algorithmic tests and 4 instances corresponding to a French territorial division (called department). Instances are displayed in Tables 6 and 7. Columns have the same meaning than in previous tables, with in addition for the departments the name of the department “Department” and its initial 4G sites coverage “ $\alpha^0$ ” in % (replacing the type of area).

Results are displayed in Table 6 and Table 7, in the columns below “heuristic”. Columns “sol” and “time” stand respectively for the value of the solution and the computation time of the algorithm. Column “gap MIP” stands for the relative gap (in %) between the value of the best solution found by the branch-and-bound procedure and the heuristic solution value.

---

**Algorithm 1** Algorithm for year by year solving.

---

**for**  $t \in \{1, \dots, \bar{t}\}$  **do**

solve (MMMP( $U^0, M^0, Z^0$ ) + constraint( $\frac{\sum_{s \in S} u_{s, N_G, N_G}^t}{\sum_{s \in S} \sum_{i \in \mathcal{G}} u_{s, i}^t} \geq \underline{QoE}^t = \frac{t}{T} * \underline{QoE}$ ))

$U^0 \leftarrow \text{getvalue}(u)$

$M^0 \leftarrow \text{getvalue}(m)$

$Z^0 \leftarrow \text{getvalue}(z)$

**end for**

---

Table 6: Results of heuristic and MIPstart methods.

Instance		heuristic			MIPstart				
$N_S$	area	sol	time	gap MIP	sol	time	f-gap	o-gap	gap MIP
25	rural	2793	6	22,60	2278*	235	0,00	29,83	0,00
	mid-dense	2389	1	23,90	1928*	134	0,00	35,48	0,00
	urban	1459	10	12,41	1298*	50	0,00	47,83	0,00
50	rural	5418	2	19,55	4532	1800	3,92	30,01	0,00
	mid-dense	4958	2	24,81	3972	1800	2,94	36,12	0,00
	urban	3494	6	33,19	2624*	182	0,00	45,94	0,00
75	rural	8002	8	16,66	6837	1800	5,88	30,22	-0,32
	mid-dense	7057	3	21,54	5807	1800	5,77	35,83	0,00
	urban	4740	1	20,16	3944	1800	1,22	46,45	0,00
100	rural	10648	5	16,49	9102	1800	7,49	30,04	-0,43
	mid-dense	9715	8	23,90	7841	1800	8,95	35,02	0,00
	urban	6002	2	19,16	5037	1800	1,92	46,47	0,00
125	rural	13430	3	17,56	11452	1800	15,75	31,06	0,25
	mid-dense	11712	15	16,10	10092	1800	14,61	37,15	0,05
	urban	7301	2	19,61	6101	1800	5,01	47,35	0,00



Table 7: Results for departments instances.

Instance			MIP					heuristic					MIPstart				
Department	$N_S$	$\alpha^0$	sol	time	f-gap	o-gap	sol	time	gap MIP	sol	time	f-gap	o-gap	gap MIP			
Finistère	210	36	16196	1800	17,05	44,86	18995	1831	17,28	15549	1800	14,21	37,05	-4,00			
Côtes d'Armor	149	29	12394	1800	14,97	39,97	14532	43	17,25	12093	1800	11,02	33,53	-2,43			
Morbihan	168	38	14720	1800	21,28	49,07	15353	20	4,30	13195	1800	9,06	36,66	-10,36			
Île-Et-Vilaine	214	43	15979	1800	19,04	50,64	17414	17	8,98	15322	1800	11,43	41,62	-4,11			

We observe a gap between the values of the best solution found with the MIP and the solution found by the heuristic of around 20-25% for 25 sites (when the optimal solution of the MIP is available). The same gap stays between 4 % and 18 % for the four French departments.

We use the solution obtained by the heuristic as an initial solution (MIPstart) for the MIP. Results are presented in Tables 6 and 7, in the columns below “MIPstart”. Columns “sol” and “time” stand, respectively, for the solving time and the value of the best solution. Column “gap MIP” stands for the relative gap (in %) between the values of the best MIP solution found by the branch-and-bound procedure without MIPstart and the one with, see Table 4. Note that the negative values correspond to improvements with the MIPstart method. The improvements are particularly important in the four department instances. For the department of Morbihan, we notice that the heuristic solution (obtained in 20 seconds) is only 4% more expensive than the solution provided by the MIP. This good quality of the solution allows the MIPstart method to find a solution 10 % better within the time-limit.

## 6. Conclusion

We have introduced a problem of multi-year investments planning for a telecommunication operator. Encompassing several real aspects faced by operators, our problem consists in optimizing network and subscribers dynamic under capacity and targeting constraints. In particular, we have modeled the fraction of subscribers’ adopting a new technology as depending on the coverage of that technology. In addition, the operator can provide subsidies to incentive the subscribers to migrate faster to that technology. We have provided a non-linear non-convex MIP formulation for this problem which we linearize with classical discretization, making sense from the applied viewpoint. Computational tests have been made for a real 3G/4G case-study. The efficiency of the valid inequalities in improving the performances has been underlined. We also provide a simple decomposition matheuristic which runs much faster than the exact MIP. The gaps between MIP and heuristic solutions underline that the problem is not time-decomposable. However, using the heuristic solution as an initial solution/MIPstart for the MIP formulation allow us to obtain significant improvements. Our results also illustrate the effect of imposing additional constraints, needed from the operational viewpoint. For instance, we have modeled the possibility for the operator of smoothing its investments along the time-horizon, which allow us to quantify the overcost due

to smoothing policies. This overcost is significant, especially in mid-dense and rural areas. This could push operators to reconsider their investment policies and decide to do an important initial investment in order to make savings on the whole time-horizon.

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