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# Consensus Opinion Model in Online Social Networks Based on Influential Users

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**ABSTRACT** A framework to consensus opinion model within a networked social group is put forward. The current research in opinion formation within the groups is largely based on the opinion aggregation of each user of the network. However, the consistency of users in aggregation, social power, and the impact of each individual user of the group for opinion formation are not considered. In this paper, we investigate a consensus opinion model in social groups based on the impact of influential users and aggregation methods. In order to reach the consensus model, we aggregate the users' opinions. To maintain consistency, we propagate the opinion through the users to reach an agreement. This propagation will consider the influential users' impacts that have a crucial effect on its process. A novel method is proposed to detect the influential users and opinion propagation based on them to derive the opinion toward the networked social group. In particular, we applied optimism and pessimism scores as the users' personality to discover the influential users in the network. Considering that, we propagate the opinion based on two facts: 1) the impact of influential users, derived by the presence of an extremely confident individual in the network and 2) the impact of neighbors, induced by the presence of the users who have a connection with the current user. Then, we proposed the opinion aggregation of the group induced by the weighted averaging operator and fuzzy techniques. In order to evaluate the validity of the method, we used enormous data sets of Epinions and Etsy which are signed and unsigned, respectively.

**INDEX TERMS** Influential users, link analysis, opinion propagation, voter model, fuzzy methods, consensus model, online social networks.

## I. INTRODUCTION

The online social networks (OSN) are growing and penetrating the people lives such that almost every person in the world has a membership at least in one of them. A user may refer to the web to get information about TV, songs, movie tickets, jobs, or even mates. As these decisions and the fundamental financial processes move to the web, there is growing economic inspiration to propagate opinion through the web. Social networking websites have facilitated a new style of communication and opinion propagation through the links between their users. Today, a tremendous amount of users take part in different social networks such as eBay, Twitter, Amazon, Facebook, Alibaba etc. and create many social links with other users of that network. The networks of interest are online shopping centers in which the websites provide product for users. In order to recommend a product or prepare

some information to a user, these websites gather the opinion of other users toward the current product and at the end provide a rate as an aggregated opinion of other users for that. This process which is named group decision making, recommendation system, opinion mining and etc. in other articles got a lot of attention in recent years. However, one of the challenging issues is specifying each user's effect on changing the opinion of other users of the network. We claim that each user has a different impact on others which affect a given users' general opinion and be reflected in other links she may initiate. It is in this way that opinion propagates through a network. The aim of this article is to propose an opinion consensus model in a networked social group based on link analysis. For instance, in Amazon which is an online shopping website, one of the long-term targets is to keep and also increase the number of customers which is guaranteed by making them pleased from what they buy (more customers end up with more selling). Generally, these online shopping websites have two kinds of data: (1) their products and 2) their

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users' network who can review the products. While a user reviews the products, one of the most situations regarding these websites is when she is searching for a product and she has a query to find the most proper one. In order to help the current user in her decision, these websites put the other users' reviews and also the aggregated rate of them for each product. Subsequently, the online shopping websites can propagate useful opinions through their customers by suggesting the users' opinion to each other. However, in order to convince the current user by others' opinions, they should know which opinions are useful and have the dominant impact to help her in the shopping. Accordingly, the websites can suggest and highlight the opinions of particular users who they know their opinions have a positive and constructive impact as the products review which will help and persuade the users to choose the product. Hence, in order to have a better outcome, these online shopping websites require to spread and propagate the efficient and positive opinions over the network of their users.

Generally, in a network, the users' opinions are based on the information they have. For instance, which city to travel, what kind of medicine may help? which candidate to vote for? and etc. which is characterized by users' opinion. Recently, the formation of the opinion in social networks attracted a lot of attention which represents its importance. It is shown that most of nowadays social interplays are formed from the opinions of the users [1]. Therefore, studying the dynamics of the opinions has tremendous advantages which according to that, a lot of researchers with different background have presented several models to verify the development of the opinion formation, propagation and aggregation from different points of view [2]–[4]. In online shopping websites, it is needed to create the users' preferences from their connections through evaluating their opinions and then aggregate them to a compact opinion to deduce a common rate [5], [6]. However, a group of users usually have the inconsistency problems due to distinct backgrounds and information on the encountered problem [7]–[13]. This means that the opinion aggregation among users cannot be solved simply by averaging. Preferably, the users need to reach a consensus which we call an agreement, before performing the aggregation process. This topic has enticed the interest of many researchers in this field and group decisions [14]–[18]. The consensus interplays of the group has been asserted to be an efficient procedure to decrease or even omit the inconsistency [19]–[23].

The available researches on the opinion formation are usually based on the users' connections and links which signifies that the opinion of the user's neighbors -who have a link with current user- can change or modify her opinion. However, the influences of the neighbors are not equal that is some of the neighbors have more effect on altering the current user's opinion. We call these users who have more reputation among other users and can be trusted more than usual ones as experts or influential users. It is worth mentioning that there are a few studies that investigate the impact of influential users in opinion formation. Current studies consider that the

experts of the network are specified before and they just need to aggregate their opinions. However, in most situations, they are not defined. Moreover, the links of a social network show the connection between users. In a signed network (a network in which each connection has a positive or negative sign), each connection indicates a trust or distrust bond between the two users and in the directed one the connections have a side from the source to the destination user. Also, if there is a connection between two users, they are each other's neighbor.

The aim of this study is to present a rate for each product and help the customers to have a better insight about that. To do so, we divided the process into three main parts (steps). Considering the network of an online shopping website, we first find the influential users (to have a more accurate opinion formation) and then propagate their opinion in the network (to omit the inconsistency) and later aggregate their opinion as final rate for each product to present it to the customers. In the current manuscript, we consider the effect of influential users in both signed and unsigned networks and propose a consensus opinion model for each of them. Suppose an online shopping website with its customers (users) and products. The users can make some connections with other ones and in this way form the network of the users. Also, they can add their opinions toward the products they bought or used (each user has some opinions for a limited number of products). Thus, there are two kinds of links, the first one is the link or connection between users and the second one is the link between users and the rated products by them. Here, we say  $U_1$  is neighbor of  $U_2$  if there is a link from  $U_2$  to  $U_1$ . Figure 1 shows our network of users and products. In this figure, there are two products  $P_1$  and  $P_2$  and four users. There is an opinion from  $U_4$  toward  $P_2$  but  $U_1$  has the opinion for both  $P_1$  and  $P_2$ . Also,  $U_1$  and  $U_4$  are the neighbors of  $U_2$ .

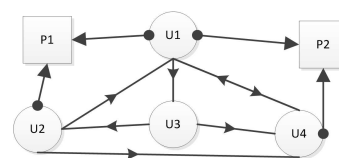


FIGURE 1. simple network of users and products.

We address the problem of consensus opinion formation by considering today online shopping centers. We considered four websites (Epinion, Amazon, Booking, and eBay) and assumed their network and users opinion adoption as our base for the model. In reality, the neighbors of a user have a direct effect on her opinion. Thus, the neighbors can convince her to take the same opinion as they have regarding a product and in this way, they propagate their opinions in the network. In particular, a user's opinion is formed by her own knowledge, the opinion of influential users and her neighbors. The user's knowledge can be specified by investigating in her profile and the impact of neighbors and the influential users can be observed by analyzing her connection to others (link analysis). In this manuscript, we investigate the links between users and considering the influential users and the user's

neighbors, we present a model for consensus opinion formation by three steps: finding the influential users, propagating the opinions to reach the common opinion and aggregating the opinions. The major contributions of the current study are as follows: (1) Introducing a ranking methodology to find the influential users in both signed and unsigned networks. (2) Proposing an opinion propagation method considering the influence of influential users. (3) Presenting an opinion aggregation method by taking into account the effect of influential users and opinion propagation to present the consensus opinion in signed and unsigned networks and consequently providing the final rating score of an online shopping product based on users' opinion.

The manuscript is organized as follows: in section II we present the related work in the literature about the opinion consensus models. In section III we describe our proposed models for influential users detection, opinion propagation and aggregation method. Then, in section IV we evaluate our work with two datasets. Finally, in section V we present the conclusion of our study and some guidelines as future works.

## II. RELATED WORKS

Social network analysis [24] is the studying of relationships between connected social entities such as members of a group. thereby, it permits us to investigate their structural properties like links, neighbors, centrality and etc. Many studies are interested in building a consensus model to face the problem in users and the group of them [5], [6], [14]–[18].

Bryson [25] suggested that in a group, consensus-relevant information is integrated with the preference data, and proposed consensus indicators and similarity measures for evaluating the level of consensus. Tanino [26] presented the certain use of fuzzy preference orderings in group decision making. They assumed that individual preferences can be considered as utility values. Then, they outlined two kinds of group fuzzy preference orderings that the level of diversity of opinions in the group seems as fuzziness. However, the users' opinions are not always considered as fuzzy sets. Herrera *et al.* [27] proposed a consensus model in groups under linguistic evaluations. Their model incorporates users consistency in decision making models, however, the opinions of users should be defined as a fuzzy set. In another study, Alonso *et al.* [5] proposed an implemented web-based consensus support system which can help the moderator in a consensus process using one of many types (linguistic, fuzzy and multi-granular linguistic) of incomplete preference relations. Herrera-Viedma *et al.* [15] presented a group consensus model with incomplete fuzzy preference relations. Their model uses two different types of measures in order to guide the consensus reaching method: (1) consistency and (2) consensus measures. Also, it provides a recommendation to experts in a discriminate manner.

In these studies, some of the researchers tried to find the consensus model without reaching the agreement between users which maintains the individual consistency of each user [14]–[18]. Some of them tried to reduce the

inconsistency but they assumed that the experts are defined before. Most of them investigated the consensus model for a group of users regardless of the impact of influential users in propagation. A user may have a tough personality and thus won't change her opinion easily or may have an unsophisticated personality to adapt her opinion quickly. On the other hand, in real life, the users make their opinions based on their links in networks (neighbors). Also, the researches conforming the influential users' detection and opinion propagation are described below.

### A. RESEARCHES ON INFLUENTIAL USERS DETECTION

In this part, we provide a state of the art of studies on link analysis and influential users' (experts) detection. Generally, the expert finding problem can be classified into two primary categories of research, (1) Authority ranking techniques [28]–[35] and (2) non-authority ranking techniques [36]–[43].

The first techniques are according to the link analysis to find the influential users. In fact, the authority ranking methods are established to rank the web-pages and verify the relationships and connections between users of a network. These methods are used when there is no information regarding the user's profile. For instance, Jurczyk and Agichtein [34] adjusted the HITS method [44] to find the authorities of users in question-answer groups. Kardan *et al.* [45] used PageRank method to detect experts in order to find whose knowledge should be shared in the social network. Later in [46] they extended the process of discovering the experts in online communities. Zhu *et al.* [30] used a new method based on Topical Random Surfer which is used to rank the web-pages, in order to propose an expert finding framework. To recognize the experts in question-answering forums, Bouguessa *et al.* [33] suggested to rank users based on the validity of their answers. Chen and He [28] suggested an integrated PageRank technique for the problem of maximization in order to choose the seeds in signed networks. Kong *et al.* [29] introduced a new PageRank scores based algorithm to compute the effect of authors on the author paper network.

On the other hand, the second introduced techniques are based on information extraction from the user's profile and activity. This category of ranking techniques lines up to discover the influential users using the information enclosed within the users' profiles, and analyze their activities as well as posts in a given network. For instance, Mimno and McCallum [42] proposed a method to define the reviewers' competence level based on papers. Deng *et al.* [39] elaborated a topical and hybrid methodology according to a devised weighted language for the task of influential user identification in the data set of Google and bibliography. Weng *et al.* [43] used the topical similarity among users to find the influential users in Tweeter. Also, Chen *et al.* [40] suggested a model for expert detection based on user activity analysis in rating the comments in question answering systems.

Furthermore, there are techniques that combine link and profile data to extend the accuracy of the detection process. For example, Guo *et al.* [41] introduced a technique in order to find the best related user regarding a particular subject by building users' profiles which is done by discovering their latent subjects and interests. Zhang *et al.* [38] suggested a propagation-based method which considers both person local and network information. The contribution of their work was combining relationships between persons with their local information to find the influential users. First, based on the local information of each person, they estimate its initial popular score then select the high ranked ones as candidates. Second, they proposed a propagation-based method, that propagates one's popular score to the other persons with whom he/she has relationships. Finally, the person who knows many popular on a subject or his name appears in many times with another popular person, then is considered as an expert on the subject (propagation theory). Balog *et al.* [47] proposed a probability model for popular identification using the users' topical profiles within multilingual systems. A new method to identify experts based on the overlapping community detection was proposed by Shahriari *et al.* [37]. Zhao *et al.* [36] used users similarity and matrix completion technique to fill the missing parts of the available information in question answering systems. Lu *et al.* [48] built a network graph from the question sessions and users' profiles. Then, using this graph, they proposed two popular identifying methods based on semantic language model and semantic propagation.

The presented personality feature used in ranking approaches belongs to the category of link analysis. The LAR methods, which are proposed in the related works, utilize the links between users in order to rank them. Nevertheless, each user has his own particularity of making links which affect significantly the link analysis. Consequently, this characteristic has the most direct impact on the ranking which was not considered in these related works. In this paper, we will consider the effect of users personality by using their opinions. Then classify them in order to detect the most influential ones.

## B. OPINION PROPAGATION STUDIES

User's opinion propagation is a very important step in our methodology. In this section, we present some scientific researches regarding opinion propagation.

Yang and Leskovec [49] proposed a linear influence model by displaying the global influence of a user based on the rate of diffusion through the network. In another study, Liang *et al.* [50] considered the effect of both the limited confidence and the impact radius of agents regarding the opinion dynamics. As a result, they discovered that heterogeneity has not always advance consensus. In addition, the relative size of the biggest opinion cluster can achieve its peak point under an optimal heterogeneity. Zhang *et al.* [51] concentrated on mining characteristics especially double propagation. They utilized two enhancements dependent on part-whole and 'no'

patterns in order to increase the recall. Then, they classified the extracted candidates' feature to enhance the accuracy of the best-ranked ones. Kou *et al.* [52] examined the opinion dynamics with different level of confidences in Hegselman and Krause method by classifying users under 3 groups namely, open-minded, moderate-minded, close-minded and on the basis of social differentiation theory. Thus, they partitioned the network into three sets without taking into account the impact of every user. Durret *et al.* [53] assumed a simple prototype of the users' network that each of them has an opinion of 0 or 1 which the opinions coevolve with network links. They discovered that because of the discrete evolution in the network's rewiring, a tiny shift in the model's dynamic leads to a massive alteration in a qualitative manner. In addition, Cha *et al.* [54] investigated the spread of the information in a social media website namely Flickr (image hosting service). One of their findings was that the information substituted between the users is considered for some of the chosen brands but with a noticeable postponement. Furthermore, they discovered that surprisingly, popular images will not diffuse within the network broadly. In another study, Shang [55] suggested a method for forming the opinion based on the confidence bound in multiple networks (a network that the links have non-permanent standards and distinct topologies). They noticed that in simple networks, because of the lack of fundamental data, aggregating the opinions will not provide an accurate formation and also, due to multiple entities in multiple networks, the opinion formation process cannot converge.

The above described studies centralized on distinct opinion propagation methods without considering the properties of the source and creator of the opinions (which is the users' impact). Particularly, the studies on the opinion formation of the users based on their links, including the model of Hegselman and Krause, apply the users' connections to their neighbors. However, the neighbors can have a distinct impact on opinion formation of the current user. For instance, the users are categorized to 3 distinct groups in [52] and then the opinion is formed for each of these groups regardless of the fact that each user can provide a distinct influence on opinion formation. In the current manuscript, we propose a consensus model by aggregating the opinions of users. In order to aggregate the opinions, we need to reach an agreement among users to maintain the consistency in which we use a propagation method. And finally, the propagation method is provided by considering the impact of influential users.

## III. OPINION FORMATION

The current section includes three different parts representing every single step to overcome the problem. In the first part, we present the influential users' detection model followed by the description of propagation method in the second part and finally the aggregating method will be presented in the last part.

The algorithm 1 presents the pseudo code of the consensus opinion process which has three main steps. In step one,

**Algorithm 1** Consensus Opinion Model

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1: step1: Detecting the influential users
2: Personality features
3: Using POPRank to computing the users' rank
4: Evaluating the users' ranking by credibility
5: step2: Opinion propagation
6: for each user  $i [U_i]$  do
7:   Suppose the user's neighbors and their ranks
8:   Detach the neighbors with opinion in bounded confidence of  $U_i$ 
9:   Update the  $U_i$ 's opinion according to detached neighbors
10:   $p_i^{t+1} = p_i^t + \frac{1}{|N_i^t|} \sum \mu \frac{SP_j}{SP_i + SP_j} [p_j^t - p_i^t]$ 
11: end for
12: Using Fuzzy Majority Opinion to analyze the opinion propagation
13: step3: Users' opinion aggregation for each product
14: for each product  $i [product_i]$  do
15:   Consider all the users who has opinion toward  $product_i$ 
16:   Aggregate the users Opinion of current product
17: end for
18: Using influential users opinion to evaluate the provided consensus opinion

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we detect the influential users (by ranking them) followed by propagating the users' opinions considering the impact of found influential users in step two. Finally, in step three we aggregate the opinions to provide the consensus opinion of each product or group.

**A. INFLUENTIAL USERS DETECTION METHOD**

As discussed in the previous section, there are two main methods to find the influential users. In this manuscript, we consider the link analysis and present two values as the users' personality that can be applied to any ranking method to improve its efficiency. Later, this ranking will be used to find the influential users. We first present the baseline ranking procedure and the personality features. Then, in order to create our own ranking methodology, we apply the personality values namely Optimist and Pessimist to the presented ranking. In addition, the presented method is designed for both signed and unsigned networks.

**1) THE PAGERANK ALGORITHM**

The PageRank algorithm counts the number and also the quality of the connections to a website to specify its importance. This method was introduced to give a score to the web pages and rank them according to their links with other web pages. If we assume the web pages as the users, the PageRank can be used to rank them in a given unsigned network considering their connections [56]. Suppose there are  $P_1, P_2, \dots, P_N$  web pages to be ranked. The algorithm updates each web page

ranking score as:

$$PageRank(P_i) = \alpha \sum_{P_j \in M(P_i)} \frac{PageRank(P_j)}{L(P_j)} + (1 - \alpha) \frac{1}{N} \quad (1)$$

In this formula,  $M(P_i)$  is the collection of web pages that have one or more links to the current page ( $P_i$ ),  $\alpha$  is a damping factor which prevents the sinks (pages with no outgoing links) and  $L(P_j)$  is the amount of  $P_j$ 's outgoing links. The algorithm starts with an initial score for each web page and keeps updating them until it converges. In some researches, this algorithm is applied to the network of users considering the users and their connection instead of web pages and the hyperlinks. However, the algorithm does not verify from where these links are created (page's personality or behavior) and the sings of the link [57]. We considered all of the ranking algorithms and found that the PageRank is the most popular method to rank the users as a lot of new methods used it for their comparisons [57]. According to that, in this manuscript, we take into account the PageRank as the body of our ranking method and modify it to consider the sings of the links and also we apply the personality measures to enhance its efficiency.

**2) PERSONALITY - THE RANKING MEASURE**

Our proposed ranking method - POPRank - is based on a modified PageRank which uses the personality values of users in its update rule. We modify the method in order to be able to rank the users in the networks with signed (positive and negative) links. The personality measure includes two social science values, i.e. Optimism and Pessimism that are applied to the method to enhance its performance and precision.

**3) OPTIMISM AND PESSIMISM VALUES**

In a network of users, pessimist users are the ones who have negative attitude about the faced problems and create a more negative connection (distrust link) rather than positive ones (trust link) to others. In contrast, the optimist users have positive attitude regarding the faced problems and make more positive connection [57]. Considering the list of products and the rates of users to them, we estimate each user's optimism and pessimism values based on the rates they expressed toward the products. We use the same method as [58] to recognize the users' optimist and pessimist values. Later, these measures will be applied to our ranking method to rank the users and find the influential users (the ones who have higher ranks). The users' optimist and pessimist values are estimated as follows:

Assume we have  $N$  products  $I_1, I_2, \dots, I_N$ ,  $OptLow_i$  is the set of products that have low average rating values and rated by user  $u_i$ :

$$OptLow_i = \left\{ I_k | r_{ik} \neq 0 \wedge \bar{r}_k \leq \frac{(1+z)}{2} \right\} \quad (2)$$

where  $\bar{r}_k$  is the average rating values of the users to  $I_k$  and  $r_{ik}$  represents the rating value from user  $u_i$  to product  $I_k$ .

Considering the rate values in the range between 1 and  $z$ , we say the values in the range of  $[1, (1+z)/2]$  are low and the ones in the range of  $[(1+z)/2 + 1, z]$  are high values. Accordingly, the set of products that have low average values but assigned as high value by user  $u_i$  are:

$$OptHigh_i = \left\{ I_k | I_k \in OptLow_i \wedge r_{ik} > \frac{(1+z)}{2} \right\} \quad (3)$$

In the same way, the set of products that have high average rating values and rated by user  $u_i$  are as follows:

$$PessHigh_i = \left\{ I_k | r_{ik} \neq 0 \wedge \bar{r}_k > \frac{(1+z)}{2} \right\} \quad (4)$$

And the set of products that have high average values but assigned as low value by user  $u_i$  are:

$$PessLow_i = \left\{ I_k | I_k \in PessHigh_i \wedge r_{ik} \leq \frac{(1+z)}{2} \right\} \quad (5)$$

We say a user is rather optimistic if she has rated above the average. Considering that, the optimism value of the user  $u_i$  can be computed as  $Optimism_i = \frac{|OptHigh_i|}{|OptLow_i|}$ . Likewise, the pessimism score of this user can be estimated as  $Pessimism_i = \frac{|PessLow_i|}{|PessHigh_i|}$ . It is worth mentioning that to prevent the diverge in our ranking method, we normalize the optimist and pessimist values in the range from 0 to 1 and then apply them as personality coefficient to the method.

#### 4) THE POPRANK METHODOLOGY

In the main PageRank method, a link from the source page to the destination page is counted as the vote from the first one to the second page. In this way, the pages vote each other and the highest voted page can emerge. However, in order to have an accurate voting, the behavior of the source page that makes the connection should be considered as well. In POPRank each user is assumed as a web page and it tries to rank a specific user instead of web page considering the reputation of her neighbors (instead of source pages who has a link to destination web page). Particularly, POPRank finds the user's rank based on her links and the personality of her neighbors which is estimated by optimist and pessimist values. Considering a user, her neighbors and the links between them, the user will be ranked base on the vote of her neighbors. The concept of personality is used in the way that we decrease the vote influence of the neighbor who is an optimist and has a positive vote for the current user because an optimist user usually has positive votes. Accordingly, we increase the vote influence of a neighbor who is an optimist and has a negative vote for her because it is not common and there should be a meaningful reason behind that. The same strategy is followed for the pessimist neighbor. We observed this valuable information of optimist and pessimist users will enhance the ranking performance [57]. In a signed network, the update rule of POPRank is computed for positive and

negative subgraphs individually as below:

$$POPRank^+(U_i) = (1-\alpha)\frac{1}{N} + \alpha \sum_{U_j \in M(U_i)} \frac{PR^+(U_j)}{L^+(U_j)} \times Per_j \quad (6)$$

$$POPRank^-(U_i) = (1-\alpha)\frac{1}{N} + \alpha \sum_{U_j \in M(U_i)} \frac{PR^-(U_j)}{L^-(U_j)} \times Per_j \quad (7)$$

where  $L^+(U_j)$  is the user  $j$ 's number of positive out-going links and  $L^-(U_j)$  is her negative ones. According to the above formula, the POPRank update rule in an unsigned network is as follows:

$$POPRank(U_i) = (1-\alpha)\frac{1}{N} + \alpha \sum_{U_j \in M(U_i)} \frac{PR(U_j)}{L(U_j)} \times Per_j \quad (8)$$

The POPRank methodology performs the users' ranking by an initial ranking vector which eventually converges to the ultimate ranking values for each user of the network. In addition, a user of the network can be whether optimist or pessimist. According to this, the users' personality is recognized below:

$$Per_j = \max \{ Optimism_j, Pessimism_j \} \quad (9)$$

that is computed for user  $j$ . Finally, the rank of the users in the signed networks will be estimated as below:

$$POPRank(U_i) = POPRank^+(U_i) - POPRank^-(U_i) \quad (10)$$

while this computation is not needed for the unsigned networks and it is computed as equation 8. It is worth mentioning that due to the similarity of the update rule between the POPRank and the original PageRank, the convergence of the POPRank is guaranteed. Later in the opinion propagation algorithm, the above ranking scores of the users will be applied as their social power.

#### 5) RANKING SCORES VERIFICATION BASED ON THE CREDIBILITY

Mei et al. [59] showed that the trust between users can emerge based on two main quantities, i.e. 1) the similarity and 2) the familiarity. In fact, the users can trust others as they look like (they are similar) or know (they are familiar) each other. Accordingly, these two quantities estimate the users' trust score in a given network that indicates the credibility of the users. Taking this into account, the credibility of the users is characterized by their similarity or familiarity. In the current article, the concept of similarity is considered to assess the value of the users' credibility. In this domain, some researches used the users' similarity concept in order to estimate their credibility and then concluded that popular users have a higher credibility score [60], [61]. In addition, they found that there is a direct relationship between the users' ranks and their credibility values. Hence, one way to verify the users' ranking, is analyzing their credibility values. In our

methodology, we rank the users and in order to verify this ranking, we check if the higher ranked users have better credibility score rather than others. The credibility of a user is a measure based on her neighbors' votes which implies her trust score. This measure is computed based on the votes of neighbors and also their validity as the following equation:

$$Credibility(U_i) = \frac{1}{|M^i(U_i)|} \sum_{U_p \in M^i(U_i)} W_{U_p U_i} \cdot Sim(U_p, U_i) \cdot Credibility(U_p) \quad (11)$$

where  $M^i(U_i)$  is the users who have a link to user  $U_i$  (her neighbors) and  $W_{U_p U_i}$  points the weight of the link from the user  $U_p$  toward the user  $U_i$ . Many methods are presented to estimate the distance of two users in a network. The similarity of two users can be defined based on their distance in the network, that is, more similar users have less distance with each other. In a trust network (e.g. Epinions), the similarity of a user depends on her neighbors and usually she trusts equivalent users more than others [62]. In this manuscript, we apply the *Jaccard Distance* to find the users' similarity as follows:

$$Sim(U_p, U_i) = \frac{|F_p \cap F_i|}{|F_p \cup F_i|} \quad (12)$$

where  $F_p \cap F_i$  is the set of similar neighbors between users  $U_p$  and  $U_i$  and  $F_p \cup F_i$  is their all neighbors.

## B. OPINION PROPAGATION METHODOLOGY

Many researchers studied the opinion dynamics and its propagation as we discussed, and proposed different algorithms for modeling the opinion propagation. Among these algorithms, the Voter method [63], [64] became the pioneer opinion propagation method and several researchers applied it as the basis of their models.

The original Voter model consists of a mathematical calculation of opinion formation in which the voters are users of a network. Considering that each user has an opinion, a randomly chosen user assumes the opinion of one of her neighbors in each time step  $t$  (the neighbor will affect the user's opinion). There are three main methods for Voter opinion propagation:

1) **S - Sznajd**: This method will be applied when the opinion of the users are discrete (for instance +1 or -1). In Sznajd, two users will be selected randomly and if they have the same opinion, their neighbors will assume that opinion.

2) **D - Deffuant**: The Deffuant method will be applied to the networks which the users' opinions are continuous (for instance [0, 1]). In this method, the user and one of her randomly chosen neighbors will update their opinions if their opinions are close enough. In other words, they will interact if their opinions have a little difference which is defined as confidence bound. Confidence bound of user  $u$  with opinion  $o$  is any of her neighbor whose opinion is  $|o - o'| < \epsilon$ .

3) **KH - Karause and Hegselman**: This method, same as Deffuant will be applied to the opinions with continuous values. Each time  $t$ , a user will be chosen by random

and her opinion will be updated based on the opinions of her neighbors in the user's confidence bound. More precisely, the user  $U_i$  with  $x_i$  opinion will update it based on the following equation in each step of  $t$ :

$$x_i(t+1) = x_i(t) + \frac{\mu}{N_i} \sum [x_j(t) - x_i(t)] \quad (13)$$

where  $N_i$  is the collection of current user's neighbors and the  $\mu$  is convergence argument which is defined in the range of [0, 1].

The proposed opinion propagation methodology is motivated by baseline method namely Voter and will be presented in the following section. The method applies the link analysis to suggest our devised method of the propagation in both signed (a network with a positive or negative amount for its connections) and unsigned networks. As long as the Voter model applies the users' neighbors and the connection among them (link analysis), we consider it as our proposed formation and propagation standard. Furthermore, in our experiments, we will verify the performance of the proposed methodology with the results of Voter model.

## 1) OPIU: OPINION PROPAGATION EXPLOITING INFLUENTIAL USERS

Users of a network often update and change their opinions based on three main sources: 1) the current opinion (from their primary viewpoint), 2) the influential user opinion (famous or popular users), and 3) their neighbors (who have a link with them). In the proposed OPIU model, we just consider the users' links to propagate the opinions. As we described before, current studies on opinion propagation methods did not verify the effect of popular users (we call them influential) as well as the personality of the neighbors. In reality, the neighbors have a different impact on propagation nevertheless, most of the existing methods do not consider this fact and apply their effect on the propagation equally. The OPIU methodology applies the impact of influential users in propagation and alters (updates) the value of the users' opinion based on that. In addition, for each user, this updating procedure detects and then applies her neighbors' impact as each of them has a distinct influence. Below, as an introduction to our propagation model, we discuss the items that make the users update their opinions in general.

## 2) OPINION PROPAGATION AND THE SOCIAL INFLUENCE

The procedure of updating the users' opinions caused by their connections and also the social interactions with other users is called social influence [65]. The theory of social influence created to distinguish the opinion propagation in a network of users who have a connection and interact with each other. Specifically, the researchers introduced two main opinion factors for social influence namely 1) the impact of influential users and 2) the impact of the majority. The first one is regarding the users who are expert and know things better than other usual users and the second one is derived from the existence of a community who its members



have a similar opinion. In reality, the update of the users' opinion take place regarding several social elements e.g. users' amount of confidence, social status, personality, social power, and the users' credibility [66] which is inspired from the social science observations. In our OPIU methodology, both described social influence factors will be considered and applied. Particularly, the influential users' impact is used to propagate the opinion among users and the majority impact is used as a criterion to evaluate the performance of the propagation model (see section III-B.5).

### 3) USERS' INFLUENCE IN OPINION PROPAGATION

According to the researches on social influence, if two users are connected and their opinion difference is negligible (within the bounded confidence), they will probably interact and this will affect their opinions. The studies indicated that there is a strong desire among the people sharing similar opinions to reinforce their confidence after interacting with each other [67]. Subsequently, for each user of the network, the OPIU model selects a subset of current users' neighbors who lay on her bounded confidence and then updates her opinion based on these neighbors. As discussed before, each neighbor has a different impact on the opinion formation of the current user. For instance, some of them contain social powers which means they possess more influence. In order to detect this power of the users in the network, we employed the ranking method [57] explained in III-A which ranks the users and gives a score to each of them. In other words, the users who are ranked higher (the ones who have more score), have more social power and subsequently have more impact on others. These high ranked users are influential users of the network.

### 4) THE OPIU FORMULATION

The POPRank sorts and finds the influential users of the given network. In fact, it gives a score to every single user of the network. Later, this score will be applied and used as the social power of the users in OPIU. Below is the description of the OPIU formulation:

Suppose  $G$  is a network with directed links and there is a product  $P$  which the empirical user has an opinion about that. Based on what we discussed before, her opinion regarding  $P$  is a mixture of three main sources:  $Opinion_{User \rightarrow P} = Function\{FK, NO, IU\}$ . Here,  $FK$  implies the user's former knowledge,  $NO$  shows her neighbors opinions and  $R$  indicating the opinion of influential users of the network. The strategies regarding the modularization of the opinion propagation are (i) information impacts [68] and (ii) direct benefit impacts [69]. In the second strategy, the value of accepting an opinion increases according to the number of current user's neighbors who are conforming it (which means if more neighbors follow that opinion the user will have more profit to conform it). In our proposed OPIU we use this strategy and update the users' opinions based on their links. In addition, we apply the following effects of the users' link

in the propagation formula: (i) the neighbor effect and (ii) the influential user effect.

Suppose  $G^t$  is a graph of the given directed network at the time  $t$ :

$$G_{ij}^t = \begin{cases} 1 & \text{if } i \text{ considers } j \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

where  $G_{ij}^t = 1$  denotes that the  $user_i$  is able to have an opinion by  $user_j$ . As a matter of fact, we supposed the user is linked with herself ( $G_{ii}^t = 1$ ). In addition, the graph is asymmetric due to the directed links of the network ( $G_{ij}^t \neq G_{ji}^t$ ). Furthermore, we considered that each user is able to verify her neighbors opinions and also her own one (so later she will update her opinion based on these two factors). The users' opinion vector at the moment of  $t$  is  $P^t = (p_1^t, p_2^t, \dots, p_n^t)$ , where  $p_i^t$  indicates the  $user_i$ 's opinion at  $t$  moment (each time has its own opinion vector). Suppose  $dif_{ij}^t = |p_i^t - p_j^t|$  is the opinion discord of the  $user_i$  and  $user_j$  in time  $t$  where  $|p_i^t - p_j^t|$  is the absolute value of  $p_i^t - p_j^t$ . Obviously, we have  $dif_{ii}^t = 0$  and  $dif_{ij}^t = dif_{ji}^t$ . Moreover, the influence weight from  $user_i$  to  $user_j$  is as follows:

$$W_{ij}^t = \begin{cases} 1 & \text{if } dif_{ij}^t \leq \epsilon \text{ and } G_{ij}^t = 1 \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

in which  $\epsilon$  is defined as the level of confidence and for all users  $w_{ii}^t = 1$  for  $\epsilon \geq 0$ . The user's opinion at the moment of  $t$  is computed based on her own opinion and the average opinions of her neighbors who are in her confidence bound. Then, in each time step, the vector of the users' opinions  $P$  will be estimated as follow:

$$P_i^{t+1} = \sum_{j=1}^n \frac{w_{ij}^t}{\sum_{a \in A} w_{ia}^t} p_j^t \quad (16)$$

This vector stops its update when the convergence of  $P$  is met. The convergence measure is defined as  $\sum_{i=1}^n (p_i^{t+1} - p_i^t)^2 \leq \omega$  in which  $\omega$  is generally a negligible positive value such as  $10^{-4}$ . In addition, the influential users can affect the opinions of normal users easily, however, due to their strong personality they tend to keep their own opinions when facing the other users' opinions hence, others can affect them rarely. Considering the network of users that consists of a number of influential users and each user's social power (SP), the updating of each user's opinion at time step  $t$  is defined as follows:

$$p_i^{t+1} = \begin{cases} p_i^t + \frac{1}{|N_i^t|} \sum \mu \frac{SP_j}{SP_i + SP_j} [p_j^t - p_i^t], & N_i^t \neq \emptyset \\ p_i^t, & \text{Otherwise} \end{cases} \quad (17)$$

where  $N_i^t = \{j \mid |p_j^t - p_i^t| \leq \epsilon_i\}$  is the set of  $user_i$ 's neighbors who are in her bounded confidence,  $|N_i^t|$  is the the number of elements in  $N_i^t$  and the SP is a normalized score that is computed according to the users' ranks by POPRank [70].

We want to propagate the opinion through the users who are in the same group and in our case the consumers who have

an opinion toward a specific product. Accordingly, for each product, we have a group of users. As long as these users can read each others opinion, we say they are all connected (there is a link between each user). Later, considering each product forms a group, the above formula is used to propagate the opinion toward the users of each group.

5) THE CONCEPT OF FUZZY MAJORITY OPINION (FMO)

The notion of the majority opinion is exerted to verify the performance of opinion propagation (see part IV-B). In this part, we first describe the estimation of the FMO and later, we apply it to OPIU in order to inspect its performance.

The main methods to estimate the majority opinions are: 1) aggregation methods and 2) fuzzy procedure [71]. In this article, the second one is utilized as it enables us to have more accurate verification of the opinion propagation due to its groups and memberships considerations. Suppose the group of  $A = a_1, \dots, a_n$  consisting the users' opinions toward a product. Indeed, the description of the FMO involves the knowledge about the similarity between the provided values. Considering  $M$  as a sub-group of  $A$ , the primary objective is determining if this sub-group has the majority opinion. The  $M$  contains the majority opinion if its members are homologous and its number of members is not less than a certain amount. Suppose  $MOP(M)$  shows the level of majority opinion formation by members of  $M$  with homologous contents. Accordingly,  $MOP(M) = QUN(\frac{|M|}{n}) \wedge Sim(M)$  in which  $\wedge$  implies the min operator and  $Sim(M)$  is equal to  $Min_{a_i, a_j \in E} [Sim(a_i, a_j)]$ . Then,  $Opi(M) = Average(M) = \frac{\sum_{a_i \in M} a_i}{|M|}$  is the opinion of the members in  $M$ . According to these definitions, the FMO of a collection of opinions specifies as follows:

$$FMO = \bigcup_{M \subseteq A} \left\{ \begin{matrix} MOP(M) \\ Opi(M) \end{matrix} \right\} \quad (18)$$

The value of  $MOP(M)$  represents the  $Opi(M)$ 's rank of being a majority opinion. In fact, it ranks each sub-group based on the majority opinion. We considered two similarity relation based on [71] and [72], respectively:

$$Sim(a_i, a_j) = \begin{cases} 1 & \text{if } |a_i - a_j| < \sigma \\ \frac{2\sigma - |a_i - a_j|}{\sigma} & \text{if } \sigma < |a_i - a_j| < 2\sigma \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

$$Sim(a_i, a_j) = \begin{cases} 1 & \text{if } |a_i - a_j| < 2 \\ \frac{1}{2}(4 - |a_i - a_j|) & \text{if } 2 < |a_i - a_j| < 4 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

note that  $\sigma$  is the standard deviation of  $a_1, \dots, a_n$ . Furthermore, the quantity function  $QUN$  is defined based on the following fuzzy set which is limited

from 0 to 1 ( $[0,1]$ ) [70], [71]:

$$QUN(x) = \begin{cases} 0 & \text{if } x \leq 0.4 \\ 5(x - 0.4) & \text{if } 0.4 < x \leq 0.6 \\ 1 & \text{otherwise} \end{cases} \quad (21)$$

C. AGGREGATING METHOD

As the last step of the proposed method to achieve the consensus opinion model, we propose the aggregation method. To do so, we introduce three methods namely, Lehner-Wanger, ordered weighted averaging and Fuzzy majority. When the users of a company or any group face a problem, a decision should be adopted. In this time, the interaction of the users will take place which means that the users will negotiate and swap their knowledge and consult the causes of that specific problem. Then, they will act based on the decided opinion or pass the group decision to the manager and she will act based on that. In an online shopping center, the users rated a product form a group and their opinions toward the product will be aggregated and presented as the final opinion. This opinion is the overall rate of users which helps the other consumer who has an inquiry about the current product. Thomas and Fink [73] evaluated three distinct patterns namely: 1) the independent pattern, 2) the rational pattern and 3) the consensus pattern. In the first one, each member of the group tries to overcome the faced problem without the help of others, in the second one, when one of the users finds the solution, the group will adopt that, and the last one considers the groups' tendency through all of the members identically. They found that although the last pattern is non-complex and easy to compute but it has a better and more precise performance rather than the other ones.

We implied that to provide a consensus opinion within a group, we need the users to negotiate and update their opinions based on their relations. This update is done by the proposed opinion propagation method. The reason is based on the concept of mutual respect between the users, every single user of the group must reconsider her primary opinion [74]. The respect between users arises from knowing each other, having a connection, being an influential user or from the manner and personality of the users. Apart from the concept of mutual respect, in most situations, it is reasonable to consult with other users and combine the opinions to have a better one. This will assure the consistency of the group. The consistency is an explanation and a reason for aggregation, since rejecting the aggregation is equal to disregarding and ignoring all of the other users [75]. In other words, rejecting the combination and aggregation of the opinions will come to unjustified religiosity or bigotry [76].

1) LEHNER-WANGER AGGREGATING METHOD

The method of Lehrer-Wagner is one of the outstanding models introduced for opinion aggregation [76]. The model tries to estimate the value of  $x$  (a product) based on the opinion  $o_i$  of each user of the group. The value of  $x$  could be the aggregated opinions of the users toward a product. The main

idea of this method is giving the users the insight about the competence of every other user so they distinguish which one with which ratio can interfere in their opinion (mutual respect). Accordingly, the  $w_{ij}$  indicates the impact ratio of user  $j$  on user  $i$ 's opinion (this will be used as a weight coefficient). Here, the concept of explained mutual respect will be applied to update the opinion of the users. To do so, we first define the weight matrix  $W$  which implies the impact ratio of the users on each other:

$$W = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1N} \\ w_{21} & w_{22} & \dots & w_{2N} \\ \dots & \dots & \dots & \dots \\ w_{N1} & w_{N2} & \dots & w_{NN} \end{pmatrix} \quad (22)$$

where  $N$  implies the number of users. The elements of the  $W$  are positive quantity and normalized in which  $\sum_{j=1}^N w_{ij} = 1$  (for each row). Considering  $W$  as the weight coefficient vector and  $o$  as the users' opinion vector, the updated opinion vector of the users will be:

$$W.o = \begin{pmatrix} w_{11}o_1 & w_{12}o_2 & \dots & w_{1N}o_N \\ w_{21}o_1 & w_{22}o_2 & \dots & w_{2N}o_N \\ \dots & \dots & \dots & \dots \\ w_{N1}o_1 & w_{N2}o_2 & \dots & w_{NN}o_N \end{pmatrix} \quad (23)$$

In general, since the elements of the above matrix are different (there will be a unique  $Wo$  matrix for each user), the method cannot propel to the consensus. However, later they showed with some assumptions, the matrix will converge to the infinitive one that the rows are equal (so that  $(Wo)_i = (Wo)_j$ ). In this case, the users will eventually reach the consensus regarding  $x$  by duplication of the average process in extreme. This method is similar to the ordered average weighting (see next section).

## 2) THE ORDERED WEIGHTED AVERAGE (OWA)

The method of OWA is presented in [77] to aggregate the group's decision. The author introduced a kind of factor to aggregate the values and entitled it as ordered weighted aggregation and then investigated its characteristics. In general, the OWA is an operator among some terms (that could be the opinion of the users). There are two common operators for OWA, namely (i) AND operator and (ii) OR operator. In the first one, all the terms should be true and in the second one, at least one term should be true to reach the consensus.

An OWA operator is a mapping  $F : R^n \rightarrow R^n$  and has a related weight vector  $W$  with the following characteristic:

$$\sum_{j=1}^n w_j = 1, w_j \in [0, 1] \quad (24)$$

and also

$$F(o_1, \dots, o_n) = \sum_{j=1}^n w_j b_j \quad (25)$$

where  $b_j$  represents the  $j$ th greatest  $o_j$ . The OWA, changes and reorders the elements according to their content.

Furthermore, the weight vector is connected to the specific place instead of being connected to a specific element. It is worth mentioning that OWA presents a simple method to reorder the elements with a non-linear computation. Consider the vector  $B$  as the ordered elements (which is titled as ordered element vector) and  $W^T$  as the weight vector of the elements. The OWA aggregation can be represented as follows:

$$F_w(a_1, \dots, a_n) = W^T B \quad (26)$$

The operator of OWA presents and gives a precious aggregation procedure which is characterized by a weight vector [78]. Among common operators for OWA, there are other ones such as the average, max, min and median of the elements. The operator can be defined as appropriate to the situation. For instance, consider there is a war and some sources regarding the number of enemy troops approaching. The officer should know how many enemy troops are coming to stand against them, so he should combine the sources of the information. As long as underestimating the enemies could be costly, it is better to use max operator.

## 3) FUZZY MAJORITY AGGREGATION

On one hand, as described above, the Lehner-Wanger will not always proceed to a consensus opinion model and it needs a promising constraint. On the other hand, the aggregation carried out by OWA is related to the values of the weight vector  $W$  which can be simply the average of opinions ( $w_k = (\frac{1}{n})$ ). Therefore, the weight vector plays a crucial role and can significantly change the performance. One way to acquire the  $W$  vector is associating the OWA with a linguistic quantifier represented by the fuzzy subset (which we called it  $Q$  in III-B.5) in the range of  $[0, 1]$ . Here, we use the described fuzzy majority in III-B.5 which tries to find the majority opinion among the users as their aggregated one. According to this interpretation, the majority opinion will be defined as a fuzzy set (instead of an amount) which gives us an excess information regarding the majority opinion. In this approach, the weights related to the aggregation are defined as follows:

$$w_j = Q(\frac{k}{n}) - Q(\frac{k-1}{n}) \quad \text{for } k = 1, \dots, n \quad (27)$$

Here, for each of the existing product, we observe the users who have an opinion toward it (who rated the current product). These users form a group. Then, we use the fuzzy majority opinion to present the aggregated opinion of this group. In order to evaluate the model, we compare the aggregated opinion with expert satisfaction [79], [80]. Using our method, each group generate an aggregated score toward the product. In order to assess this score's validity and performance, we compare the similarity between the aggregated opinion with the opinion of the most prominent expert in that group (the expert should be ranked high in POPRank). Ultimately, if the provided consensus opinion is changed in a way that gets near to the expert opinion, we can conclude that the aggregation method is satisfying the expert.

#### IV. THE EVALUATIONS

In this part, we analyze and evaluate the proposed opinion consensus model considering the influential users' detection and opinion propagation using actual networks. We divided the evaluations into two distinct sections: 1) presenting the data-sets and our observations and 2) evaluating the performance of the proposed methodology. The solution for the consensus opinion model has three main phases: finding the influential users, propagating the opinion considering the influential users and aggregating the opinions to provide the consensus opinion. In our experiments, we try to discuss each of these phases.

##### A. THE DATA-SETS

In order to measure the performance of our study, we utilized signed and unsigned data-sets i.e. Epinions and Etsy, respectively (both are directed). The Epinions data-set is obtained from the Stanford data collection and the Etsy data-set is gathered by our devised web-scraping.<sup>1</sup>

##### 1) THE EPINIIONS DATA-SET:

This data-set is based on the trust relationship of the users of the Epinions.com website. The users are usually the consumers and can rate the product of this website. Based on the users' preferences, they can trust or distrust each other (the links have the values of +1 or -1). The Epinions data-set includes 131, 828 users and 841, 372 links or connections between them. It is worth mentioning that for simplicity, we pruned the users without connections. Finally, we reached to a network with 49, 289 users and 507, 592 links for this data-set.

##### 2) THE ETSY DATA-SET:

Etsy is the global market-place for unique and creative (handmade) products which is crawled from "https://www.Etsy.com" website. It includes a broad domain of products consisting universe or special ones, unique handcrafted pieces, vintage jewels and etc. Also, based on the online verification (Hoover), the main rivals of this online shopping website are eBay, Craigslist and Amazon. The data-set has two main distinct information: 1) the graph and the connection of the users and 2) and the user rates on the goods (which implies their opinions). Similar to Epinions data-set, the range of the users' rates are integers in the interval of [1,5]. This website has several divisions for the products and for simplicity, we picked one group of the products (the home). In addition, the crawler is devised in C# programming language which fetches the data (the network of users and their opinions). We used a computer equipped with corei7 and 16GB of RAM which lasted for one month to gather the data (caused by the massive number of connections). At the end, we gathered a data-set with 239, 237 users and 4, 618, 783 connections. Like Epinions, the pruning was performed to

erase the users without the connection. The properties of both data-sets are given in table 1.

**TABLE 1. The properties of etsy and epinions [81] data-sets.**

Properties	Etsy	Epinions
Total number of crawled users	239.237	131.828
Total number of links	4.618.783	841.372
Number of users after pruning	72.528	49.289
Number of links after pruning	1.914.852	507.592
Number of positive links	-	434.694
Number of negative links	-	72.898
Number of total Products	24.362	139.738
Number of opinions	200.148	664.824

##### B. THE EXPERIMENTS AND THE OBSERVATIONS

To detect the influential users in the given network we applied the proposed POPRank algorithm. To do so, we used the personality features and computed each users optimist and pessimist scores. Then, the POPRank methodology is used to rank the users of the network. We verify the ranking of users in POPRank by comparing it with two other baseline ranking methods namely Prestige and PageRank [57] using the credibility values. As we described before, the users' credibility is introduced to measure the ranking validity. According to the definition of it, the credibility and the users' ranks have a mutual impact on each other meaning that more credible users have more score (superior rank) and low ranked users are less credible (have lower credibility) and etc. Subsequently, the three named methods will be evaluated by the credibility values. Each method ranks the users and nominates some users as the highest ranked ones (and as each method detected influential users) for different percentages. The credibility of these users will be added for each method and then compared with other methods to verify which one nominated the most credible users. The one who nominated the more credible users ranked the users correctly and found the influential users better than the other methods. Figures 2a and 2b show the values of users' credibility for each method with different percentages of top nominated influential users for both data-sets. As it is indicated, the POPRank nominated users are more credible rather than the other two methods. This implies the fact that applying the personality of the users can effectively enhance the influential users' detection. Moreover, the Spearman correlation is performed on the nominated users for each method. We observed that this correlation for PageRank, Prestige, and POPRank are 16.09%, 5.73%, and 19.50% for Epinions data-set and 10.51%, 3.68%, and 13.74% for Etsy data-set, respectively which again proves the performance of POPRank. In other words, the personality of users has a positive impact on ranking the users and finding the influential ones of the network.

After finding the influential users, we need to propagate the opinions (here the rates of users toward the products)

<sup>1</sup>gathered from <https://snap.stanford.edu/data/>

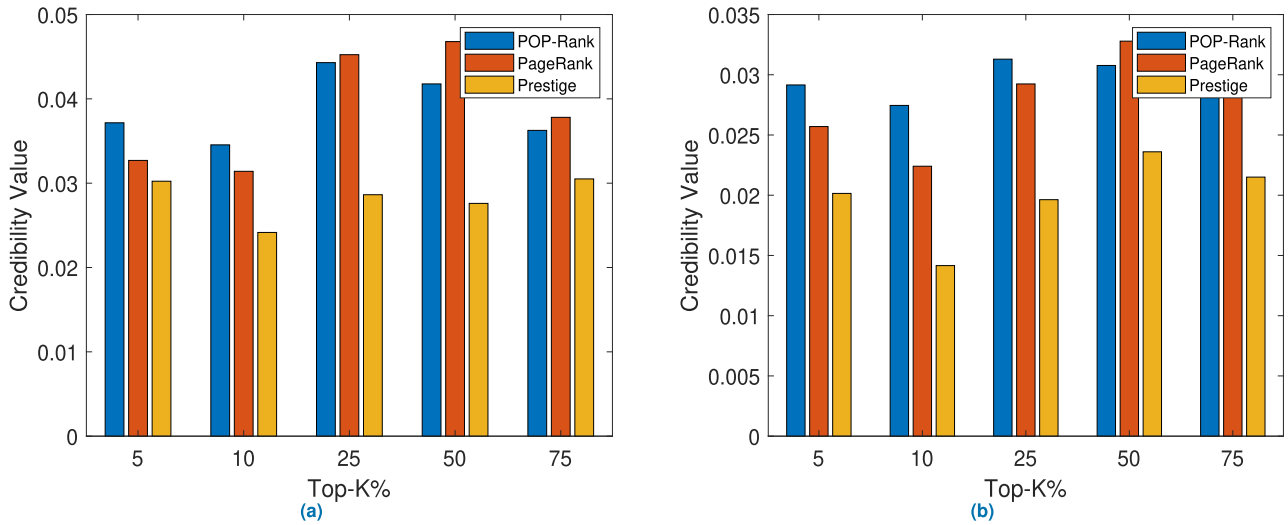


FIGURE 2. The credibility value of different ranking methods - the X axis implies to the various percentages of top detected users. (a) The Epinions data-set. (b) The Etsy data-set.

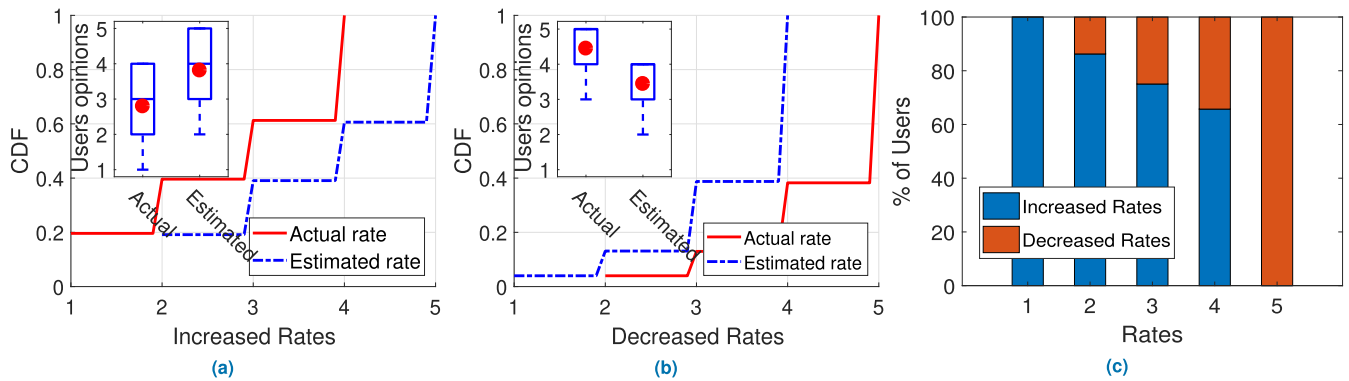


FIGURE 3. The Epinions data-set. (a) Increased rates' CDF. (b) Decreased rates' CDF. (c) The increased and decreased rates' percentage.

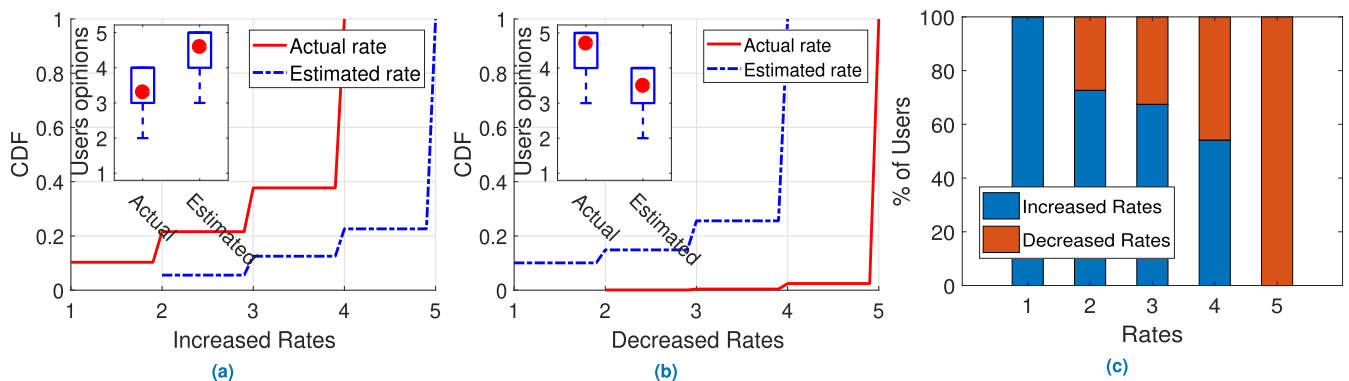


FIGURE 4. The Etsy data-set. (a) Increased rates' CDF. (b) Decreased rates' CDF. (c) The increased and decreased rates' percentage.

considering the influence of influential users in order to maintain the consistency. To do so, the proposed opinion propagation OPIU is used. We observed that some of the users' opinions did not change during executing the OPIU method. Among the changed ones, we have two categories:

(i) the opinions that increased and (ii) the ones that decreased after propagation. Figures 3a, 3b, 4a and 4b illustrate these two categories to compare the actual opinions (rates) with the estimated ones for both data-sets. These results imply that the users who submitted low rates for the products will make their

rates lower and the ones who have high rates will change to higher rates.

Figures 3c and 4c demonstrate how much percentage of rates are increased or decreased for each existing rates (1 to 5). Note that we just considered the rates that are changed. Moreover, as long as we just verified the changed rates, it is logical that the rates from 1 are just increased and the rates from 5 are just decreased (we can not have the rate 0 or 6). It can be seen that most of the rates' alterations are non-descending (from lower ones to higher rates) which means that the tendency of being positive is more than negative among users. Also, we can see that the changes in users who have the rate 2 are more increased than the ones who have the rate 4. This indicates that there is more tendency to increase the rate for users who have lower rates. Table 2 shows how much percentage of users' rates are altered to other ones for both data-sets. We observed that most of the changes are around the previous rate. For instance, a user who has rate of 3 tends to change it to 2 or 4 rather than 1 and 5. This table indicates the changes in rates are normally smooth and users avoid to have a big jump in their opinions. Accordingly, we can conclude that the users tend to change their opinions to the ones that are surrounding their current opinions rather than the ones which are far.

**TABLE 2.** Percentage of rates that changed to other rates for both datasets.

From \ To	Data-set	1	2	3	4	5
1	Epinions	0	2	0.3	0	0
	Etsy	0	1	0.5	1	2
2	Epinions	10.5	0	3.5	0.3	0
	Etsy	4	0	0.5	1	2
3	Epinions	0.2	11.2	0	11.5	0.3
	Etsy	0	4.5	0	1.5	3.5
4	Epinions	0	0.5	11.5	0	26.7
	Etsy	0.2	0.3	6	0	21
5	Epinions	0	0.1	0.4	21	0
	Etsy	3	2	4	42	0

The above experiments are for propagating the opinions through the whole network. The users rated the same product form a group. In our case, we propagate the opinion in each group. In other words, for each product, the opinion propagates among its users. Then, the Fuzzy Majority Opinion is used to evaluate the opinion propagation. Considering a group, the members' opinions can lead us to the majority opinion. In many research such as [65], the researchers showed that in a network, the users' opinion will be actuated to the users' majority one. We apply this fact to measure the evaluation of our proposed opinion propagation method. In other words, to verify the performance of OPIU we investigate if the new computed opinions by OPIU are leading to the majority ones. Eventually, we compare the baseline opinion propagation model (i.e. Voter) with OPIU by properties of FMO. To do so, for each product we compare

the OPIU estimated opinion with the FMO estimated one. We apply two similarity relation function to have a different comparison in our experiments. The similarity relation functions *MO1* and *MO2* are described in equations 19 and 20, respectively.

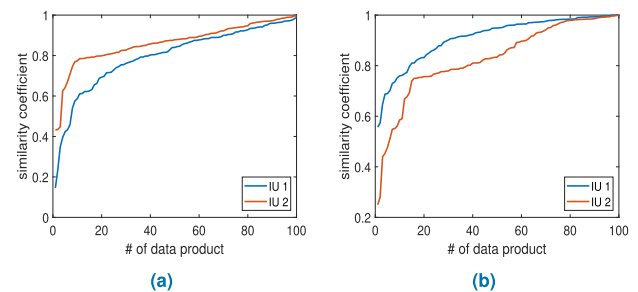
Table 3 indicates the mean square error (MSE) of both Voter and OPIU methods regarding FMO for Epinions and Etsy data-sets. Also, note that Mode of a set is the number that occurs most frequently. When we applied the methods, there were some opinions which remained unchanged. The above computing is performed towards the changed opinions (regardless of the unchanged ones). This result confirms the performance of OPIU in opinion propagation.

**TABLE 3.** The MSE of different propagation methods for both data-sets.

Data-set	Propagation Method	MO1	MO2	Mode
Epinions	OPIU	0.89	0.78	1.43
	Voter Model	1.21	1.18	1.85
Etsy	OPIU	0.97	0.87	1.33
	Voter Model	1.57	1.41	1.69

In the last step, we aggregate the opinions for each product. We used two methods for aggregating namely Ordered Weighted Average and Fuzzy Majority Aggregation. As we discussed before, we used expert satisfaction to evaluate the aggregation methods. To verify the performance of the aggregation methods, for each product we compared the similarity between the aggregated opinion with the opinion of the two most prominent influential users in each group. To do so, we performed two aggregations OWA and FMO in both Etsy and Epinions data-sets. For both data-sets, we first chose the top 10% of influential users and then determined the products that they rated. Among these products, we chose one hundred products to analyzed the aggregation performance. Each product has a number of users who have opinions towards them. Hence, for each product, we found the two best influential users (*IU1* and *IU2*) and compared their opinion with the aggregated one.

Figures 5 and 6 show the similarity of different aggregation methods with the two chose influential users for Epinions and Etsy data-sets respectively. These figures show that for most of the products, the similarity value of aggregated opinion



**FIGURE 5.** Aggregation in Epinions data-set. (a) OWA aggregation. (b) FMO Aggregation.

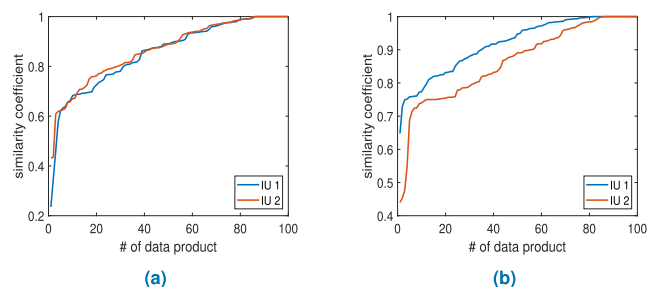


FIGURE 6. Aggregation in Etsy data-set. (a) OWA aggregation. (b) FMO aggregation.

and the opinion of the influential user who rated the current product is high. In other words, the aggregated opinion is near to the opinion of the influential users. This confirms the performance of aggregated methods. However, the performance of OWA and FMO are different. Table 4 illustrates the MSE of FMO and OWA methods with two influential users (IU1 and IU2) for both data-sets.

TABLE 4. MSE of different aggregation methods.

Data-set	OWA		FMO	
	IU1	IU2	IU1	IU2
Epinions	0.89	1.09	0.79	0.64
Etsy	0.96	0.62	0.99	0.59

We observed that in general, the performance of FMO is better than OWA however, it does not mean that FMO always makes better aggregation value. For example, the MSE of the OWA method for IU1 in Etsy is slightly better than FMO which makes the OWA better, however, from the results we can say FMO has better performance rather than OWA. As long as we used expert satisfaction to evaluate the aggregation, we cannot verify it correctly when the influential user is wrong or inconsistent. This explains the low similarity of influential users and aggregated methods in some products.

### V. CONCLUSION

In the current study, we proposed a consensus opinion approach that has been specially designed to model opinion formation in social groups. The aim is presenting a mechanism for online shopping centers to provide a comprehensive information for their users regarding each product of their websites. Assuming the social power and different impact of each individual user in the network, we proposed a consensus model. To do so, we first identified the influential users using personality of them in the network. Then, in order to maintain the consistency, the users need to negotiate their opinions in which we proposed a new method of propagation regarding the impact of influential users. Finally, we aggregated the opinion by OWA and fuzzy methods to present the consensus opinion. We performed several experiments in signed and unsigned networks. The proposed POPRank algorithm demonstrated the personality features can efficiently enhance the rankings and consequently, finding the

influential users in different social networks. Furthermore, we observed that most of the users like to increase their opinions rather than decreasing them during the propagation period. Finally, in general, the fuzzy aggregation method performed better than OWA which proves that FMO suit best to our context. On the other hand, we used the expert satisfaction to evaluate the performance of aggregation and we should point out that we are not able to perform a precise evaluation of the aggregation models if the opinions of influential users are wrong. As a future work, we investigate the incorporation of a mechanism to evaluate the performance of aggregation regardless of the opinion of the expert satisfaction.

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