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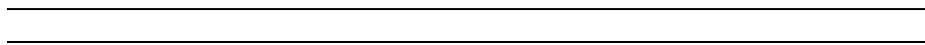
# Self-Motion conditions for a 3-PPPS Parallel Robot with Delta-Shaped Base

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# Self-Motion conditions for a 3-PPPS Parallel Robot with Delta-Shaped Base

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## Abstract

This paper presents the self-motion conditions of the 3-PPPS parallel robot with an equilateral mobile platform and an equilateral-shaped base. The study of the direct kinematic model shows that this robot admits self-motion of the Cardanic type as the 3-RPR planar parallel robot where the first revolute joint of each leg is actuated or the PamINSA parallel robot. This property explains why the direct kinematic model admits an infinite number of solutions in the center of the workspace but has never been studied until now. The condition of this singularity is described and the location of the self-motion in the workspace with respect to all the singularities is then presented. The quaternion parameters are used to represent the singularity surfaces and the self-motion conditions in the workspace.

*Keywords:* Parallel robots, 3-PPPS, singularity analysis, kinematics, self-motion.

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## 1. Introduction

It has been shown that by applying simplifications in parallel robot design parameters, self-motions of the mobile platform may appear [1, 2, 3]. Most of the time, the presence of this type of motion is not desired by robot designers. Indeed, a self-motion is defined as a finite mobility of the robot when all actuators are locked. This property should be avoided in most industrial cases because the knowledge of the pose of the mobile platform requires the presence of sensors attached to the actuated and passive joints. Unfortunately, this does not allow the pose of the mobile platform to be controlled in a known direction of the self-motion.

The best-known example is the 3-UPU robot. At first, the authors thought that the parasitic movement came from joint clearance [4], while later it was found that the movement was due to a particular singularity [5]. On the other hand, Bonev et al. demonstrated that all singular orientations of the popular 3-RRR spherical parallel robot design (known as the Agile Eye) correspond to self-motions [6], but arguably this design has the “best” spherical wrist. Among the motions followed by mobile

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16 platforms, the Cardanic motion can be found as self-motion for two robots in literature,  
 17 the 3-RPR parallel robot [7] and the PamInsa robot [8]. For the first robot, there is only  
 18 one set of actuated joint values to allow this motion to occur. For the second, this  
 19 property exists for any values of  $z$  but under the same conditions as the first example.

20 Most of the examples for a fully parallel 6-DOF manipulators can be categorized by  
 21 the type of their six identical serial chains namely UPS [9, 10, 11, 12, 13], RUS [14]  
 22 and PUS [15]. However for all these robots, the orientation of the workspace is rather  
 23 limited due to the interferences between the legs. To solve this problem, new parallel  
 24 robot designs with six degrees of freedom appeared recently having only three legs with  
 25 two actuators per leg. The Monash Epicyclic-Parallel Manipulator (MEPaM), called  
 26 3-PPPS is a six DOF parallel manipulator with all actuators mounted on the base [16].  
 27 Several variants of this robot have been studied were the three legs are made with three  
 28 orthogonal prismatic joints and one spherical joint in series. The first two prismatic  
 29 joints of each leg are actuated. In the first design, the three legs are orthogonal [17].  
 30 For this design, the robot can have up to six solutions to the Direct Kinematic Problem  
 31 (DKP) and is capable of making non-singular assembly mode change trajectories [17].  
 32 In [16], the robot was made with an equilateral mobile platform and an equilateral-  
 33 shape base. In [18], the robot was designed with an equilateral mobile platform and  
 34 a U-shaped base. For this design, the direct kinematic problem is simple and can be  
 35 solved with only quadratic equations.

36 The common point of these three variants is that the parallel singularity is independ-  
 37 ent of the pose of the end-effector. This paper presents the self-motion of the 3-PPPS  
 38 parallel robot derived from [16] with an equilateral mobile platform and an equilateral-  
 39 shaped base.

40 The structure of this article is as follows. The architecture of the manipulator and  
 41 its associated constraint equations are explained. Then, calculations for parallel singu-  
 42 larities and self-motions are presented. The article then concludes.

## 43 2. Mechanism Architecture

44 The robot under study is a simplified version of the MEPaM that has been developed  
 45 at the Monash University [16, 19]. This architecture is derived from the 3-PPSP that  
 46 was introduced earlier [20]. For this parallel robot, the three legs are identical and it  
 47 consists of two actuated prismatic joints, a passive prismatic joint and a spherical joint  
 48 (Fig. 1). The center of the circum-circle defined by the base points  $A_i$  forms the origin  
 49 of an orthogonal reference frame.

50 We assume an origin  $A_i$  for each leg and the radius of circum-circle is considered as  
 51 2. The coordinates of  $A_i$  are given by:

$$\mathbf{A}_1 = [2, \rho_{1y}, \rho_{1z}]^T \quad (1)$$

$$\mathbf{A}_2 = [-1 - \sqrt{3}\rho_{2y}/2, \sqrt{3} - \rho_{2y}/2, \rho_{2z}]^T \quad (2)$$

$$\mathbf{A}_3 = [-1 + \sqrt{3}\rho_{3y}/2, -\sqrt{3} - \rho_{3y}/2, \rho_{3z}]^T \quad (3)$$

52 The coordinates of the point  $C_1$  are  $\rho_{1x}$ ,  $\rho_{1y}$  and  $\rho_{1z}$ , wherein the last two are actuated.  
 53 The coordinates of  $C_2$  and  $C_3$  are obtained by a rotation around the  $z$  axis by  $2\pi/3$  and

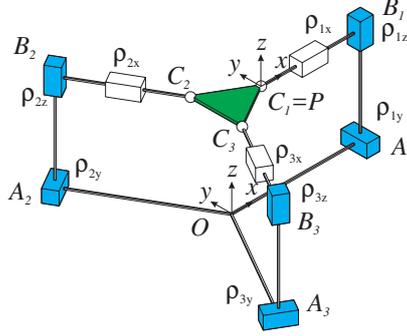


Figure 1: A scheme for the 3-PPPS parallel robot with the actuated prismatic joints in blue, the passive joints in white and the mobile platform drawn in green with  $x = 1/\sqrt{3}, y = 0, z = 0, q_1 = 1, q_2 = 0, q_3 = 0, q_4 = 0$

54  $-2\pi/3$  respectively and the coordinates are as follows:

$$\mathbf{C}_1 = [\rho_{1x}, \rho_{1y}, \rho_{1z}]^T \quad (4)$$

$$\mathbf{C}_2 = [-\rho_{2x}/2 - \sqrt{3}\rho_{2y}/2, \sqrt{3}\rho_{2x}/2 - \rho_{2y}/2, \rho_{2z}]^T \quad (5)$$

$$\mathbf{C}_3 = [-\rho_{3x}/2 + \sqrt{3}\rho_{3y}/2, -\sqrt{3}\rho_{3x}/2 - \rho_{3y}/2, \rho_{3z}]^T \quad (6)$$

55 The origin on the mobile equilateral platform is similar to the parameters used in [18],  
56 meaning on the mobile platform.

$$\mathbf{V}_1 = [0, 0, 0]^T \quad (7)$$

$$\mathbf{V}_2 = [-\sqrt{3}/2, 1/2, 0]^T \quad (8)$$

$$\mathbf{V}_3 = [-\sqrt{3}/2, -1/2, 0]^T \quad (9)$$

57 Generally, in the robotics community, Euler or Tilt-and-Torsion angles are used to rep-  
58 resent the orientation of the mobile platform [17]. These methods have a physical  
59 meaning, but there are singularities to represent certain orientations. The unit quater-  
60 nions gives a redundant representation to define the orientation but at the same time, it  
61 gives a unique definition for all orientations. In addition, the associated rotation matrix  
62 is algebraic, which simplifies the use of the Siropa library [21] and Gröbner basis [22].  
63 The rotation matrix  $\mathbf{R}$  is described by:

$$\mathbf{R} = \begin{bmatrix} 2q_1^2 + 2q_2^2 - 1 & -2q_1q_4 + 2q_2q_3 & 2q_1q_3 + 2q_2q_4 \\ 2q_1q_4 + 2q_2q_3 & 2q_1^2 + 2q_3^2 - 1 & -2q_1q_2 + 2q_3q_4 \\ -2q_1q_3 + 2q_2q_4 & 2q_1q_2 + 2q_3q_4 & 2q_1^2 + 2q_4^2 - 1 \end{bmatrix} \quad (10)$$

64 With  $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$  and  $q_1 \geq 0$ . We can write the coordinates of the mobile

65 platform using the previous rotation matrix as:

$$\mathbf{C}_i = \mathbf{R}\mathbf{V}_i + \mathbf{P} \quad \text{where} \quad \mathbf{P} = [x, y, z]^T \quad (11)$$

66 Thus, we can write the set of constraint equations with the position of  $C_i$  in both refer-  
67 ence frames by the relations:

$$\rho_{1y} = y \quad (12)$$

$$\rho_{1z} = z \quad (13)$$

$$(2q_1q_4 - x)\sqrt{3} + 2q_1^2 + 3q_2^2 - q_3^2 - y - 2\rho_{2y} = 1 \quad (14)$$

$$-\sqrt{3}q_1q_3 + \sqrt{3}q_2q_4 - q_1q_2 - q_3q_4 + \rho_{2z} = z \quad (15)$$

$$(2q_1q_4 + x)\sqrt{3} - 2q_1^2 - 3q_2^2 + q_3^2 - y - 2\rho_{3y} = -1 \quad (16)$$

$$-\sqrt{3}q_1q_3 + \sqrt{3}q_2q_4 + q_1q_2 + q_3q_4 + \rho_{3z} = z \quad (17)$$

### 68 3. Constraint equations and self-motion conditions

69 The main problem is to find the location of the mobile platform by looking for the  
70 value of the passive prismatic joints  $[\rho_{1x}, \rho_{1y}, \rho_{1z}]$  as proposed in [23]. The distances  
71 between any couple of points  $C_i$  are given by:

$$\|\mathbf{C}_1 - \mathbf{C}_2\| = \|\mathbf{C}_1 - \mathbf{C}_3\| = \|\mathbf{C}_2 - \mathbf{C}_3\| = 1 \quad (18)$$

72 It was shown in [16] that using dialytic elimination [24], the Eq. 18 could be reduced  
73 into a univariate fourth degree polynomial in  $\rho_{1x}$ . Thus, the direct kinematic model  
74 can accommodate up to four solutions. However, when we want to solve the DKP  
75 with  $\rho_{iy} = 0$  and  $\rho_{iz} = 0$  for  $i = 1, 2, 3$ , we have an infinite number of solutions, which  
76 correspond to the self-motion. This result remains the same if we set  $\rho_{1z} = \rho_{2z} = \rho_{3z}$ .  
77 Assuming that this motion is in a plane parallel to the plane  $(Oxy)$ , we write the system  
78 coefficients for  $\rho_{iz} = 0$  with  $i = 1, 2, 3$ . The univariate fourth degree polynomial in  $\rho_{1x}$   
79 then degenerates and a quadratic polynomial equation is obtained which is given by:

$$\begin{aligned} & 9(\rho_{1y} + \rho_{2y} + \rho_{3y})^2 \rho_{1x}^2 + \\ & 6\sqrt{3}(\rho_{1y} + \rho_{2y} + \rho_{3y})^2 (\rho_{2y} - \rho_{3y}) \rho_{1x} + \\ & (\rho_{1y} + \rho_{2y} + \rho_{3y})^2 (\rho_{1y}^2 + 2\rho_{1y}\rho_{2y} + 2\rho_{1y}\rho_{3y} + 4\rho_{2y}^2 - 4\rho_{2y}\rho_{3y} + 4\rho_{3y}^2 - 3) = 0 \end{aligned} \quad (19)$$

80 In this form, all the terms of the equation cancel each other out when  $(\rho_{1y} + \rho_{2y} + \rho_{3y}) =$   
81  $0$ .

82 In general, the analysis of the Gröbner basis is done by using the *InfiniteEquations*  
83 function from the *Siropa* library. Three equations characterize the locus where the

84 constraint equations are rank deficient.

$$\rho_{1z} - \rho_{2z} = 0 \quad (20)$$

$$\rho_{1y} + \rho_{2y} + \rho_{3y} = 0 \quad (21)$$

$$3\rho_{1z}^4 - 8\rho_{1z}^3\rho_{2z} - 4\rho_{1z}^3\rho_{3z} + 6\rho_{1z}^2\rho_{2z}^2 + 12\rho_{1z}^2\rho_{2z}\rho_{3z} - 12\rho_{1z}\rho_{2z}^2\rho_{3z} + \rho_{2z}^4 - 4\rho_{2z}^3\rho_{3z} + 12\rho_{2z}^2\rho_{3z}^2 - 8\rho_{2z}\rho_{3z}^3 + 2\rho_{3z}^4 \quad (22)$$

85 If we substitute the Eq. 20 in the Eq. 22, we have

$$\rho_{3z} - \rho_{2z} = 0 \quad (23)$$

86 Finally, the self-motion conditions can be written as

$$\rho_{1z} = \rho_{2z} = \rho_{3z} \quad (24)$$

$$\rho_{1y} + \rho_{2y} + \rho_{3y} = 0 \quad (25)$$

87 With these conditions, the robot becomes similar to the 3-RPR parallel robot for which  
 88 its self-motion produces a Cardanic motion as described in [7] for the 3-RPR parallel  
 89 robot and in [8] for the PamInsa robot. Geometrically, the self-motion could be de-  
 90 scribed when the motion axes of passive joints intersect at a single point and an angle  
 91 of  $2\pi/3$  is formed between each axis. An example is shown in Fig. 2 where the green  
 92 mobile platform has one assembly mode. Since the self-motion corresponds to singu-  
 93 lar configurations, the objective now will be to locate these singularities with respect  
 94 to other singularities.

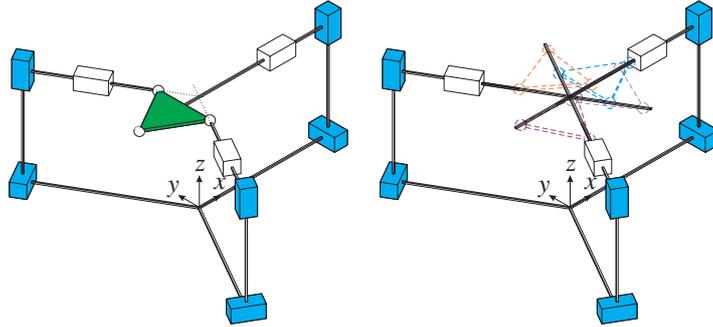


Figure 2: Set of postures to describe the Cardanic motion starting from the “home” pose

#### 95 4. Singularity Analysis and self-motion locus

96 The singular configurations of the 3-PPPS robot have been studied in several articles  
 97 with either a parametrization of orientations using Euler angles or Quaternions [18].

98 Serial and parallel Jacobian matrices can be calculated by differentiating the constraint  
 99 equations with respect to time [25, 26, 27]. These Jacobian serial and parallel matrices  
 100 must satisfy the following relationship:

$$\mathbf{A}\mathbf{t} + \mathbf{B}\dot{\boldsymbol{\rho}} = 0 \quad (26)$$

101 where  $\mathbf{t}$  is the twist of the moving platform and  $\dot{\boldsymbol{\rho}}$  is the vector of the active joint veloc-  
 102 ities. According to the leg topology of the 3-PPPS robot, there is no serial singularity  
 103 because the determinant of the  $\mathbf{B}$  matrix does not vanish. Using the same approach as  
 104 in [19], we can determine the matrix  $\mathbf{A}$  and its determinant can be factorized as follows:

$$(q_1^2 - q_2^2 - q_3^2 + q_4^2)(q_1 - q_4)(q_1 + q_4) = 0 \quad (27)$$

105 To represent this surface, we eliminate  $q_1$  thanks to the relation on the quaternions

$$(q_2^2 + q_3^2 + 2q_4^2 - 1)(2q_2^2 + 2q_3^2 - 1) = 0 \quad (28)$$

106 One of the surfaces represents a cylinder and the other an ellipsoid. In general, if the  
 107 system is plotted, infinite cylindrical trace will be obtained. In our case, the limits of  
 108 cylinder should have an intersection with the spherical quaternions. With the help of  
 109 *Plot3D* function of the Siropa library, this representation is made possible by respecting  
 110 the inequality condition  $q_1^2 + q_2^2 + q_3^2 + q_4^2 \leq 1$ [21] and is shown in Fig. 3.

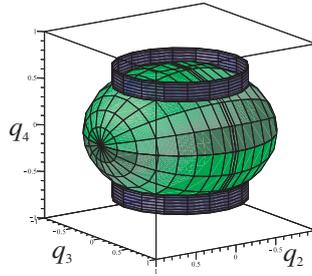


Figure 3: Parallel singularity of the 3-PPPS robot with quaternion representation

111 By writing the conditions from Eq. 24 and Eq. 25 with the constraint equations  
 112 defined in Eqs. 12-17, the Gröbner basis elimination method makes it possible to obtain

113 a set of equations that depends on  $q_2$ ,  $q_3$  and  $q_4$ .

$$q_2 q_4 = 0 \quad (29)$$

$$q_3 q_4 = 0 \quad (30)$$

$$q_2 (q_2^2 + q_3^2 - 1) = 0 \quad (31)$$

$$q_3 (q_2^2 + q_3^2 - 1) = 0 \quad (32)$$

$$q_4^3 - q_4 = 0 \quad (33)$$

$$q_2^4 + (q_3^2 + q_4^2 - 1) q_2^2 + q_3^2 q_4^2 = 0 \quad (34)$$

114 The solutions of the Equations 29-34 are

$$[q_1 = 0, q_2 = 0, q_3 = 0, q_4 = 1] \quad (35)$$

$$[q_1 = 0, q_2 = 0, q_3 = 0, q_4 = -1] \quad (36)$$

$$[q_1 = 1, q_2 = 0, q_3 = 0, q_4 = 0] \quad (37)$$

$$[q_1 = -1, q_2 = 0, q_3 = 0, q_4 = 0] \quad (38)$$

$$[q_1 = 0, q_2 = 0, q_3 = 1, q_4 = 0] \quad (39)$$

$$[q_1 = 0, q_2 = 0, q_3 = -1, q_4 = 0] \quad (40)$$

$$[q_1 = 0, q_2 = q_2, q_2^2 + q_3^2 = 1, q_4 = 0] \quad (41)$$

115 The postures defined in Eqs. 35-38 are non-singular postures of the robot. If we  
 116 substitute these values in Eq. 27, we obtain  $-1$  or  $1$ . Only the set of postures from  
 117 Eqs. 39-41 are a subset of the singularity surface defined in Eq. 27. This means that  
 118 the self-motions conditions obtained by studying the direct kinematic are necessary but  
 119 not sufficient conditions.

120 Its graphical representation is a unit radius circle centered at the origin of the plane  
 121  $(0 \ q_2 \ q_3)$  as shown in Fig. 4. This circle lies on the surface of parallel singularity and  
 is shown in Fig. 3.

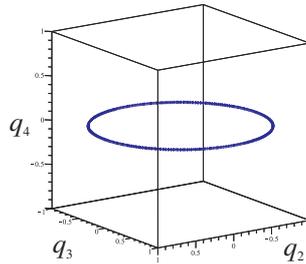


Figure 4: Location of self-motions in the workspace.

122 The same solution can be obtained if we add the singularity constraints from Eq. 27,  
 123 the self-motion conditions from Eq. 24-25 and the constraint equation from Eqs. 12-17  
 124 during the procedure elimination to have the self-motion locus as a function of  $q_2$ ,  $q_3$   
 125

126 and  $q_4$ .

127 As the singularity does not depend on the pose of the mobile platform, the self-  
128 motion exists for an infinite number of poses of the mobile platform. This phenomenon  
129 may not be found if numerical methods are used to solve the DKP because the rela-  
130 tions ( $\rho_{1y} + \rho_{2y} + \rho_{3y} = 0$ ) and  $\rho_{1z} = \rho_{2z} = \rho_{3z}$  are never fully satisfied. This is what  
131 happens when we use Matlab or Scilab which uses a floating number representation for  
132 computation.

## 133 5. Conclusions and Perspectives

134 In this article, we have studied the parallel robot 3-PPPS to explain the presence  
135 of self-motions. This motion is a Cardanic motion that has already been studied on the  
136 3-RPR parallel robot or the PamInsa robot. Self-motions can often explain the prob-  
137 lems of solving the DKP when using algebraic methods. The calculation of a Gröbner  
138 base makes it possible to detect this problem but the characterization of this motion  
139 for robots with six degrees of freedom is difficult because of the size of the constraint  
140 equations when we mix the pose and the orientation of the mobile platform. Other  
141 robots, such as the CaPaMan at the University of Cassino have the same type of singu-  
142 larity despite the phenomenon never being studied. The objective of future works will  
143 be to identify architectures with passive prismatic articulations connected to the mobile  
144 platform which are capable of having the same singularity conditions for more general  
145 shapes of the base and mobile platforms. Such results can be useful for design engi-  
146 neers to avoid the use of such architectures that comes from optimization procedures  
147 where the numerical functions cannot detect the issues.

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