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# Electro-mechanical system control based on observers

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**Abstract.** The monitoring of the gear behavior is becoming major concerns in several industries for improve production and safety. For this reason, in this article, an electro-mechanical modeling is developed in order to simulate a gear element driven by an asynchronous motor. The electrical part, which is an induction motor, is modeled by using the Kron's model while the mechanical part, which is a single stage gear element, is accounted for by a torsional model. The mechanical model that simulates the pinion-gear pair is obtained by reducing the degree of freedom of the global spur or helical gear system. The electrical and mechanical state variables are coupling in order to obtain a unique differential system that describes the dynamics of the electro-mechanical system. The global coupled electro-mechanical model can be characterized by a unique set of non-linear state equation. The aim of this work is to apply the control based on observers in order to supervise the electrical and mechanical behavior of the electro-mechanical system from only its inputs and its measurements outputs (sensors outputs).

**Keywords:** Gears transmission, Observers, Simulation, Asynchronous motor

## 1 Introduction

Electro-mechanical systems such as mechanical gear transmission driven by induction motors are commonly used in many industrial applications. For this reason, many techniques and tools of control and diagnostics, such as vibration and sound signal analysis (Baydar & Ball 2001, Tan et al. 2007), analysis of stator currents (Feki 2012), have traditionally been used to supervise the gear element behavior and to detect faults (Chaari et al. 2008). These techniques are advantageous by their reduced cost and present a high reliability. However, they are sensitive to the positioning of the sensors and in some applications, they present technical difficulties to implement sensors in rotating parts or hostile environment. Although gear monitoring by vibration signal analysis and current stator analysis are still widely used, a new method of electro-mechanical system control and monitoring based on observers will be presented in this paper.

In general, for technical and economic reasons, the state of the system is not completely accessible. Indeed, the complexity of the technical feasibility as well as prohibitive costs for the implantation of several sensors can considerably

reduce the number of states measured. In this case, the state vector size is greater than the output vector size. However, under some conditions of existence, the state can be reconstructed using an observer Larroque (2008).

The paper will be organized as follows: a) in section 2, a single stage gear element is accounted for by using torsional model, b) in section 3, an electrical modeling of the induction motor is presented, c) the electro-mechanical coupling is developed in section 4, d) the section 5 is dedicated to implement the used observer and e) the simulation results of the electro-mechanical system dynamic behavior are presented in section 6.

## 2 Mechanical modeling

The modeling of the mechanical part (see Fig.1) contains only the gear element. To do this, we are based on the modeling in (Feki et al. 2012) but reduce the global model of 36 degrees of freedom to 2 degrees of freedom. A driving torque

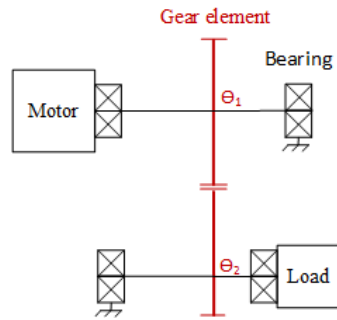


Fig. 1: Electro-mechanical system

$C_m$  is applied to the pinion gear and a load torque  $C_r$  is applied to the wheel gear (attached to output shaft). The system modeling is accounted for by two degrees of freedom, which correspond to the torsional components. Using Euler-Lagrange equation, the motion equation of the torsional model of the gear element is obtained as follows (1):

$$M\ddot{x} + C\dot{x} + K(t, x)x = F, \quad (1)$$

where

- $M$ : is the mass matrix composed by the polar moment of inertia for the gear  $k$ ,
- $K(t, x)$ : is stiffness matrix that depends on the state vector and on the time,
- $C$ : is the Rayleigh model damping,
- $F = [C_m \quad C_r]^t \in \mathbb{R}^2$ : external forces,

–  $x = [\theta_1 \ \theta_2]^t \in \mathbb{R}^2$ : represents the two degrees of freedom vector.

The stiffness matrix can be expressed in terms of a structural vector  $V(M_i)$  (Maatar & Velez 1996) by using the following form  $K(t, x) = k(t)V(M_i)V(M_i)^t$ , with  $k(t)$  the stiffness simulated by a square waveform. Developing (1), we obtain the space representation of the gear element:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & I_d \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix}, \quad (2)$$

with  $I_d$  the 2nd order identity matrix.

### 3 Electrical modeling

The motor equations are obtained by using the Kron's transformation model. The key idea of this transformation is to translate the three-phases quantities ( $abc$ ) of the motor to an equivalent two-phases quantities (see Fig.2).

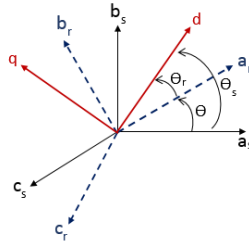


Fig. 2: Park transformation plan.

The stator variables are obtained by  $\theta = \theta_s = \omega_s t = (a_s, d)$  while the rotor variables are calculated by  $\theta = \theta_r = \omega_{sl} t = (a_r, d)$ .

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = T_{2/3}(\theta_s) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}, \quad (3)$$

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = T_{2/3}(\theta_r) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}. \quad (4)$$

The voltage equations of the three stator and rotor phases are given by (5) and (6).

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \phi_{as} \\ \phi_{bs} \\ \phi_{cs} \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} v_{ar} \\ v_{br} \\ v_{cr} \end{bmatrix} = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \phi_{ar} \\ \phi_{br} \\ \phi_{cr} \end{bmatrix}. \quad (6)$$

The flux equations can be written as follows:

$$[\phi_{abcs}] = L_{ss} [i_{abcs}] + L_{sr} [i_{abcr}], \quad (7)$$

$$[\phi_{abcr}] = L_{rr} [i_{abcr}] + L_{rs} [i_{abcs}], \quad (8)$$

where

$$\begin{aligned} - L_{ss} &= \begin{bmatrix} l_s & m_s & m_s \\ m_s & l_s & m_s \\ m_s & m_s & l_s \end{bmatrix} : \text{stator inductance matrix,} \\ - L_{rr} &= \begin{bmatrix} l_r & m_r & m_r \\ m_r & l_r & m_r \\ m_r & m_r & l_r \end{bmatrix} : \text{rotor inductance matrix,} \\ - L_{sr} &= L_{rs}^t = m_{sr} \begin{bmatrix} \cos(\theta) & \cos(\theta + \frac{2\Pi}{3}) & \cos(\theta - \frac{2\Pi}{3}) \\ \cos(\theta + \frac{2\Pi}{3}) & \cos(\theta) & \cos(\theta + \frac{2\Pi}{3}) \\ \cos(\theta + \frac{2\Pi}{3}) & \cos(\theta - \frac{2\Pi}{3}) & \cos(\theta) \end{bmatrix} : \text{mutual inductance matrix between stator and rotor,} \end{aligned}$$

with the constants parameters:

- $l_s$  (respectively  $l_r$ ): self-inductance of the stator (rotor),
- $m_s$  (respectively  $m_r$ ): mutual-inductance between the stator phases (the rotor phases),
- $m_{sr}$ : the maximum value of mutual inductances between stator and rotor phases.

Applying the Kron's transformation for (5),(6),(7) and (8), we obtain:

$$\begin{cases} v_{ds} = R_s i_{ds} + \frac{d}{dt} \phi_{ds} - \omega_s \phi_{qs}, \\ v_{qs} = R_s i_{qs} + \frac{d}{dt} \phi_{qs} + \omega_s \phi_{ds}, \end{cases} \quad (9)$$

$$\begin{cases} v_{dr} = R_r i_{dr} + \frac{d}{dt} \phi_{dr} - \omega_{sl} \phi_{qr} = 0, \\ v_{qr} = R_r i_{qr} + \frac{d}{dt} \phi_{qr} + \omega_{sl} \phi_{dr} = 0, \end{cases} \quad (10)$$

$$\begin{cases} \phi_{ds} = L_s i_{ds} + L_m i_{dr}, \\ \phi_{qs} = L_s i_{qs} + L_m i_{qr}, \end{cases} \quad (11)$$

$$\begin{cases} \phi_{dr} = L_r i_{dr} + L_m i_{ds}, \\ \phi_{qr} = L_r i_{qr} + L_m i_{qs}, \end{cases} \quad (12)$$

where  $L_s$  (respectively,  $L_r$ ) represents the stator synchronous inductance ( respectively, rotor synchronous inductance) and  $L_m$  is the magnetizing (synchronous) inductance. The advantage of this transformation is the fact that we get a constant mutual inductance and that along an axis, the fluxes depend only to the rotor and stator currents. Relations (9) and (10) detail the electro-magnetic

behavior of the asynchronous machine written in non a linear differential equations form. These equations can be described in matrix space representation by choosing the space vector  $z(t)$  composed by the both stator currents and the both rotor fluxes of the motor:

$$\dot{z}(t) = Az(t) + BU(t), \quad (13)$$

with  $z(t) = [i_{ds} \ i_{qs} \ \phi_{dr} \ \phi_{qr}]^t \in \mathbb{R}^4$ ,  $U(t) = [v_{ds} \ v_{qs}]^t \in \mathbb{R}^2$ : the input vector,  $A$ ,  $B$  the state, input matrices given by:

$$A = \begin{bmatrix} -\left(\frac{1}{T_s\sigma} + \frac{1}{T_r} \frac{1-\sigma}{\sigma}\right) & \omega_s & \frac{1-\sigma}{\sigma} \frac{1}{L_m T_r} & \frac{1-\sigma}{\sigma} \frac{\omega_m}{L_m} \\ -\omega_s & -\left(\frac{1}{T_s\sigma} + \frac{1}{T_r} \frac{1-\sigma}{\sigma}\right) & -\frac{1-\sigma}{\sigma} \frac{\omega_m}{L_m} & \frac{1-\sigma}{\sigma} \frac{1}{L_m T_r} \\ \frac{L_m}{T_r} & 0 & -\frac{1}{T_r} & w_{sl} \\ 0 & \frac{L_m}{T_r} & -w_{sl} & -\frac{1}{T_r} \end{bmatrix}, \quad (14)$$

$$B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (15)$$

where the constants parameters are explained below:

$$- \sigma = 1 - \frac{L_m^2}{L_s L_r}, \quad T_s = \frac{L_s}{R_s}, \quad T_r = \frac{L_r}{R_r}.$$

The electromagnetic torque is represented by (16), with  $p$  the number of pole-pairs.

$$C_{em} = \frac{pL_m}{L_r} (\phi_{dr} i_{qs} - \phi_{qr} i_{ds}). \quad (16)$$

## 4 Electro-mechanical coupling

The aim of the electro-mechanical coupling is to obtain a space representation that includes the mechanical system with two degrees of freedom and the induction motor equations (see Fig.1). This modeling allows to implement control methods to supervise the dynamic behavior of the system. The electro-magnetic torque given by (16) is regarded as the input of the mechanical part and the small vibrations caused by the gear element act on the speed rotation of the asynchronous motor. The coupled system leads to a first order differential system of the form:

$$\dot{\zeta}(t) = A_c(t, \zeta)\zeta(t) + Gu, \quad (17)$$

where  $\zeta(t) = [i_{ds} \ i_{qs} \ \phi_{dr} \ \phi_{qr} \ \theta_1 \ \theta_2 \ \dot{\theta}_1 \ \dot{\theta}_2]^t$ : the global state vector,  $A_c = \begin{bmatrix} A & 0 \\ P & H \end{bmatrix}$  is the state matrix, with  $A$  the state matrix associated to the

electric part expressed in (14),  $P \in \mathbb{R}^{(4 \times 4)}$  is the coupling matrix between the electric and mechanical part depending to the electromagnetic torque donated by

$$(16) \text{ and } H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -M^{-1}K & -M^{-1}C & & \end{bmatrix} \text{ is the matrix associated to the gear modeling}$$

$$, G = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & & \\ & & & M^{-1} \\ 0 & 0 & & \end{bmatrix} \text{ is the input matrix and } u = \begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ C_r \end{bmatrix} \text{ the input vector.}$$

The evolution of the system is obtained by introducing the control based on observers to monitor the dynamic behavior of the electro-mechanical system.

## 5 Observer form

A state observer is a control method (Perruquetti & Barbot 2002) that gives an estimate of the internal state of the real system, only from the measurements given by the sensors and the real input of the system. The observers are used in order to control the behavior of systems, to detect the faults or to identify the unknown parameters of systems (Oueder 2012). In our case, four differentiators (Ghanes et al. 2017) are used to estimate the drift of both currents of the asynchronous motor and the displacements of the gear element. Assuming that  $[s_1, \dots, s_8] = [\dot{i}_{ds}, \dot{i}_{ds}, \dot{i}_{qs}, \dot{i}_{qs}, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]$ , the observer equations are written in following form:

$$\begin{cases} \dot{\hat{s}}_i = \hat{s}_{i+1} + k_1 \mu_j |e_i|^\alpha \text{sign}(e_i), \\ \dot{\hat{s}}_{i+1} = k_2 \alpha \mu_j^2 |e_i|^{2\alpha-1} \text{sign}(e_i), \\ e_i = s_i - \hat{s}_i, \end{cases} \quad (18)$$

where

- $e_i, i = \{1, 3, 5, 7\}$  are the output estimation errors,
- $k_1, k_2$  are constants acting on the stability of the system,
- $\mu_j, j = 1, 2$  are positive constants, the first one associated to the electric model and the second one is related to the mechanical part.

## 6 Simulation and Results

In this simulation, a spur gear system is considered. The main characteristics of the gear are given in table 1 and the motor parameters are shown in table 2.

Parameters	value
Module (mm)	4
Tooth number of Pinion	21
Tooth number of wheel	31
Face width (mm)	10
Pressure angle	20

Parameters	value
Stator resistance $R_s$ ( $\Omega$ )	9.163
Rotor resistance $R_r$ ( $\Omega$ )	5.398
Stator inductance $L_s$ (H)	0.115
Rotor inductance $L_r$ (H)	0.0943
Magnetizing inductance $L_m$ (H)	0.0943
Number of pole-pairs $p$	1

As it can be noticed, the applied observer gives a good performances (see Fig .3, 4, 5 and 6). In these figures, the estimated states converge, in finite time, to the real quantities states. Fig .3 and Fig. 4 represent the two stator currents expressed in  $(dq0)$  frame. They show that the periodic recurrence of gears meshing frequency  $t_m$  is regained in the electrical states.

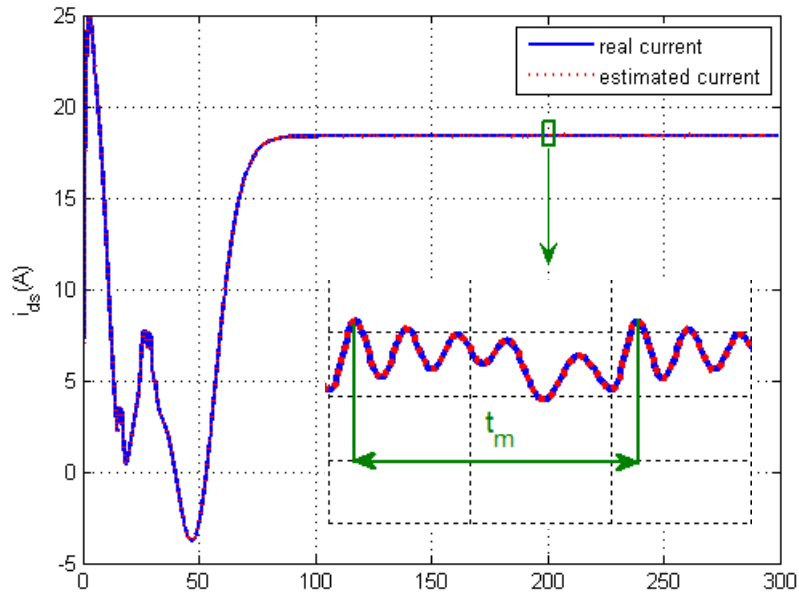


Fig. 3: State  $\zeta_1$  and its estimate



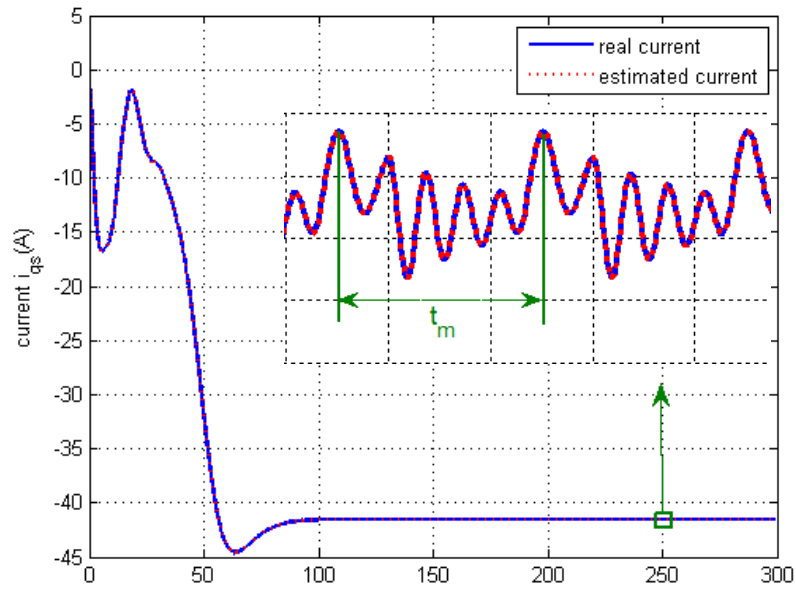
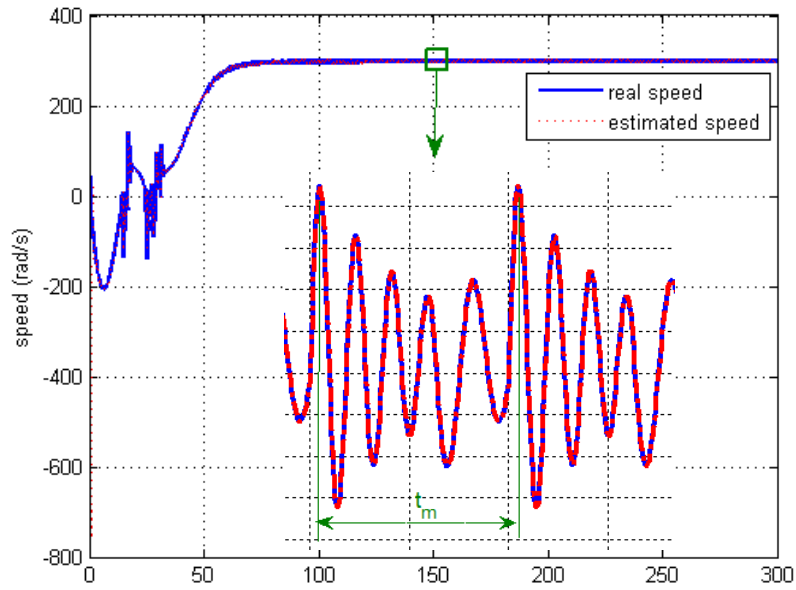
Fig. 4: State  $\zeta_2$  and its estimateFig. 5: State  $\zeta_7$  and its estimate

Fig. 5 illustrates that the rotational speed of the pinion is of the order to  $300 \text{ rad/s}$ . This value represents the average meshing speed added to the small vibrations of the gear.

The last figure (Fig. 6) displays the evolution of the error transmission given by the equation  $R_{b1}\theta_1 + R_{b2}\theta_2$  (where  $R_{b1}, R_{b2}$  are, respectively, the base radii of the pinion and the wheel).

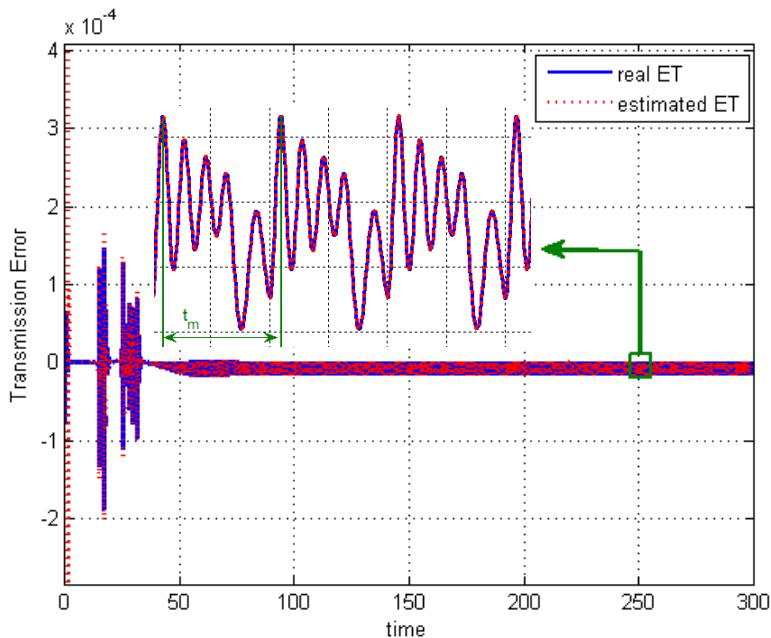


Fig. 6: Real and estimated transmission error

All results confirm the good convergence of the applied observer. This convergence has appeared in the frequency content and the amplitude of the electro-mechanical system behavior signals. Meaning that the evolution of the real states and those of the estimated quantities are perfectly confused in the frequency study. The observer gives a rapid and accurate convergence for all system states.

## 7 Conclusion

In this paper, an electro-mechanical coupling of the gear transmission system driven by a asynchronous motor has been studied. The monitoring of this model is obtained by using the control based on observers. Further work is in progress in order to implement other types of observers with presence of gear faults and variation of the sensors noise.

## Bibliography

- Baydar, N. & Ball, A. (2001), ‘A comparative study of acoustic and vibration signals in detection of gear failures using wigner-ville distribution’, *Mechanical Systems and Signal Processing* **15**, 1091–1107.
- Chaari, F., Baccar, W., Abbas, M. S. & Haddar, M. (2008), ‘Effect of spalling or tooth breakage on gearmesh stiffness and dynamic response of a one-stage spur gear transmission’, *European Journal of Mechanics-A/Solids* **27**(4), 691–705.
- Feki, N. (2012), Modelisation electro-mecanique de transmissions par engrenages : Applications à la detection et au suivi des avaries, PhD thesis. These de doctorat dirigée par Clerc, Guy et Vex, Philippe Mécanique Lyon, INSA 2012.  
**URL:** <http://www.theses.fr/2012ISAL0041/document>
- Feki, N., Clerc, G. & Vex, P. (2012), ‘An integrated electro-mechanical model of motor-gear units?applications to tooth fault detection by electric measurements’, *Mechanical Systems and Signal Processing* **29**, 377–390.
- Ghanes, M., Barbot, J.-P., Fridman, L. & Levant, A. (2017), A second order sliding mode differentiator with a variable exponent, in ‘American Control Conference (ACC), 2017’, IEEE, pp. 3300–3305.
- Larroque, B. (2008), Observateurs de systèmes linéaires: application à la détection et localisation de fautes, PhD thesis, Institut National Polytechnique de Toulouse.
- Maatar, M. & Vex, P. (1996), ‘An analytical expression for the time-varying contact length in perfect cylindrical gears: some possible applications in gear dynamics’, *Transactions of the ASME-R-Journal of Mechanical Design* **118**(4), 586–588.
- Oueder, M. (2012), Synthèse des observateurs pour les systèmes non linéaires, PhD thesis, Université de Caen.
- Perruquetti, W. & Barbot, J.-P. (2002), *Sliding mode control in engineering*, CRC Press.
- Tan, C. K., Irving, P. & Mba, D. (2007), ‘A comparative experimental study on the diagnostic and prognostic capabilities of acoustics emission, vibration and spectrometric oil analysis for spur gears’, *Mechanical Systems and Signal Processing* **21**(1), 208–233.