Extreme and rare events in hydrodynamical and optical systems

Giovanna Tissoni, Eric Simonnet

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Abstract Rare and extreme events are ubiquitous in nature and society, spanning from magnetic storms or particularly violent earthquakes, to market crashes or oceanic rogue waves. Different viewpoints are therefore possible: coming from two different communities, we will try here to bring together our ways to look at these events. After a very brief review of some recent results in our respective scientific domains, that is, non-equilibrium/ statistical physics and nonlinear/dissipative optical systems, we draw some perspectives to develop a new approach using genetic algorithms to calculate extreme events in nonlinear dissipative optical systems.

1 Introduction

Nowadays, the modeling of complexity, in biology, geophysics or hydrodynamical turbulence for instance, involves systems with a huge number of degrees of freedom (d.o.f.) together with stochastic parametrisations. These parametrisations are needed, in general, to handle the physics at the microscopic/subgrid scale and to reduce the number of d.o.f. like in the Earth climate system, which involves multi-scale dynamics from a few meters to thousands of kilometers. Even more challenging, these systems are gener-

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ally far from equilibrium. Therefore, deterministic tools such as dynamical systems and bifurcation theory are not sufficient to handle these aspects. Nonequilibrium statistical physics appears to be the natural framework to study these phenomena.

It is often the case that physical systems exhibit phase transitions related to the existence of metastable states. The typical scenario is a system at equilibrium that exhibits (random) small-amplitude fluctuations. From time to time, some of these fluctuations might not be small and "push" the known (local) dynamics into another regime. These noise-induced transitions generally occur over a very short length of time. For many physical systems, it is particularly crucial to understand and predict these phase transitions, as they may have dramatic impacts to our societies.

The main aspect of this research is to use recent algorithmic tools for studying extreme and rare events in physics. These tools are not only able to compute the probability of these events but they also provide the physics associated to them. Even more importantly, they are able to overcome the so-called "curse of dimensionality" by handling systems with many d.o.f. These ideas were first proposed by J. Von Neumann (Manhattan project, unpublished) and developed in [1]. The first mathematical developments were obtained much later [2–5] and many others. The first successful applications in complex systems are very recent [6,7].

Roughly speaking, the idea is to mimic the evolution of species by performing Darwinian selections on the system dynamics, in a controlled (unbiased) way. These types of algorithms essentially perform a large number of mutations and selections (branching) by cloning the system dynamics. This is the reason they have sometimes been called *genetic algorithms* although they bear several different names (multilevel splitting, go-with-the-winner, rare event - large-deviation algorithms, etc.). In summary, these approaches are able to sample the system tail probability very efficiently. It thus allows us to compute rare or/and extreme events (see [7] for computing extremes), no matter the dimension of the system.

2 Phase transitions for atmospheric jets

In the recent years, it has been conjectured that a global climate change has occurred on Jupiter in the past century, which can be traced back to the 30's and the sudden disappearance of one of the atmospheric jets (similar to the North-Atlantic"jet stream" on Earth) [8]. In this work, we investigate phase transitions in a simple turbulent model of (barotropic) atmospheric jets. We are able to observe rare events corresponding to the absorption of a west-

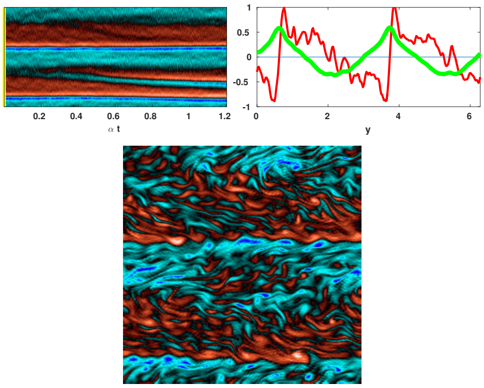

Fig. 1 Phase transition for an atmospheric jet (nucleation): Hovmöller diagram (space vs. time) of the vorticity (left panel), zonal velocity and vorticity (right panel, green and red curves resp.). The probability to observe this phenomenon is of the order 10^{-7} in this case. The lower panel is a snapshot of the vorticity 2-D field.

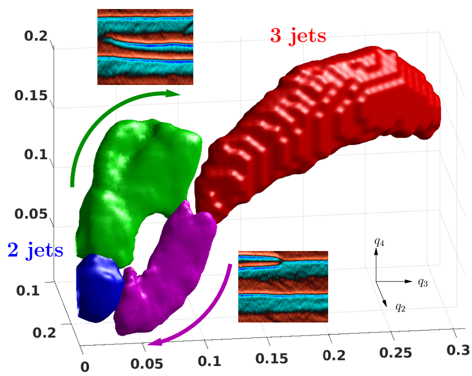

Fig. 2 Finite-noise instanton concentrated along the most probable transition path between two-jet states and three-jet states.

ward jet into an eastward jet (coalescence) as well as the spontaneous creation of new eastward jets (nucleation). The statistics and the corresponding physics of these rare events are computed using a genetic algorithm called adaptive-multilevel splitting. This algorithm is exponentially faster than traditional tools (e.g., Monte-Carlo sampling) and allow for robust statistics of very low probability events. Figure 1 illustrates the creation of a new eastward jet. In addition, we can show that these transitions are controlled by an underlying instanton in the weak-noise limit (see Fig. 2). Therefore, it suggests that only specific fluctuations along the instanton can yield such transitions and that predictors can be defined.

Although it is too early to relate this theoretical work with [8] we expect in the near future to consider more realistic models exhibiting the observed phenomenology. This is joint work with Freddy Bouchet at the Physics lab. ENS Lyon (in preparation).

3 Rogue waves in a laser with saturable absorber

In the recent years, extreme events in optics have been attracting a lot of interest, originating from the seminal paper by Solli *et al.* [9], due to the well-known analogy between optics and hydrodynamics, where rogue wave formation and prediction is a priority field of investigations. A huge body of literature has been blooming on this subject, studying rogue waves for many different optical systems (for a review, see [10–12] and references therein).

Optical fibers and fiber lasers are systems of choice for the analysis of optical rogue waves due to their natural longitudinal extension both for the conservative and the dissipative case. Dissipative rogue waves in laser devices have been also studied, and semiconductor systems have emerged as experimentally convenient test beds for the analysis of extreme phenomena. For instance, low dimensional semiconductor systems in which the wave envelope is severely constrained by boundary conditions served to demonstrate that the emergence of rogue events can be associated to an external crisis in a chaotic regime [13], thus showing the deterministic character of these extreme events.

Very recently, extreme events were studied both experimentally and numerically [14] in the intensity emitted by a monolithic broad-area vertical cavity surface emitting laser (VCSEL) with a saturable absorber with a linear pump (which reduces to one the transverse dimensions), and spatio-temporal chaos is claimed to be at the dynamical origin of extreme events.

Here we show numerical results about extreme events occurring in the field intensity emitted by a monolithic broad-area VCSEL with an intra–cavity saturable absorber [15], as the one used in the experiments on cavity solitons [14]. The system is displayed in the left panel of Fig. 3. The model we use is a set of 4 PDEs for the (complex) slowly varying envelope of the intra-cavity electric field, and the carrier populations in the active and in the passive material. The control parameters are the pump parameter μ in the active medium, and the ratio r between the nonradiative lifetimes in the active and passive materials (for the model and the parameters we refer the reader to [15]).

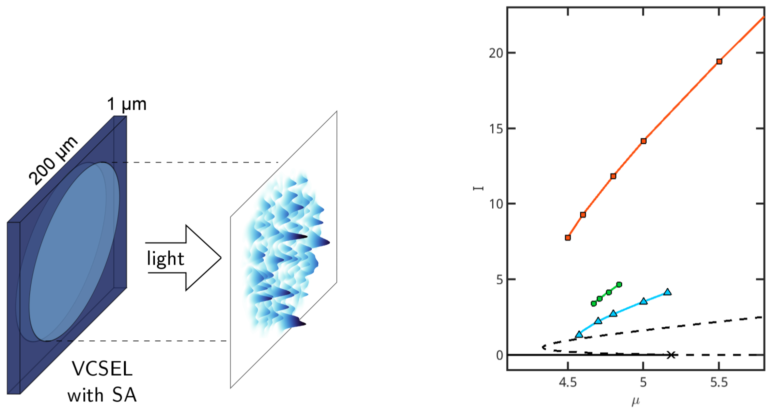

Fig. 3 Left panel: scheme of the the system in study. Right panel: branches of solutions displayed for $r = 1$ as a function of the control parameter μ (injected current in the active medium, see text): homogenous stationary solution (dashed black line), stationary cavity solitons (blue line and triangles), time averaged maximum intensity of the turbulent state (orange line and squares) and of chaotic solitons (green line and circles). Note that the laser threshold is at $\mu_{th} = 5.18$ (subcritical bifurcation).

In the right panel of Fig. 3 we show that below the lasing threshold the system may present multiple solutions, such as stationary cavity solitons (also called localised structures), oscillating or chaotic solitons and a global "turbulent" solution where the light intensity oscillates aperiodically in space and time, together with the trivial non-lasing solution. The turbulent solution survives above threshold, where it is the only attractor of the system. This solution has been associated (for the 1D case and for a slightly different set of parameters) to spatio-temporal chaos [14].

When the system is emitting on the "turbulent" solution, we perform a statistical treatment on the full set of 3D data of field intensity as a function of space and time. In contrast with previous literature about optical rogue waves in spatially extended systems [14,16], we developed a numerical method for the individuation of the spatio-temporal maxima of the transverse field intensity in which each maximum appearing in the space profile is counted as an “event”only when its peak intensity reaches the maximum value also in time. This method allows a comparison, for example, with the hydrodynamical definition of “significant wave height”, corresponding to the mean value of the wave height (from trough to crest) of the highest third of the waves.

The results of the statistical analysis are shown in Fig. 4, where the thresholds for RW are defined as follows:

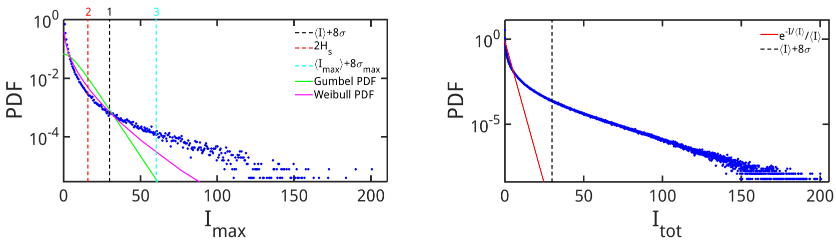

Fig. 4 (a) PDF of the spatio-temporal maxima for a numerical simulation lasting 25 ns, for $\mu = 7$ and $r = 2.5$. The green and magenta lines are respectively the Gumbel and Weibull distributions computed from the mean and standard deviation of the data. The three vertical dashed lines indicate three different definitions of rogue wave thresholds, defined in the text as thresholds 1,2 and 3 (see the legend, and the text). (b) PDF of all the values explored by the intensity during the simulation in each point of the transverse plane. Black dashed vertical line: threshold for rogue waves (same as threshold 1 in (a)). Here $\mu = 5$ and $r = 2.4$. The presence of very heavy tails is clearly visible, and RW exist according to all the threshold definitions.

Threshold 1: the mean intensity, averaged on every point of the transverse plane and every instant in time, plus 8 times the standard deviation. This is the definition most commonly used for studying optical rogue waves in spatially extended systems [16,17].

Threshold 2: two times the significant wave height H_s, defined as the average of the highest third of the spatio-temporal maxima values. This is the typical hydrodynamic definition, and permits one to get rid of a possible global increase of the average value, that would not correspond to a freak wave. The typical number of “events" detected during a simulation lasting $25ns$ is around 6×10^5.

Threshold 3: average of spatio-temporal maxima values plus 8 times the standard deviation.

We performed a statistical analysis of all the data (spatio-temporal maxima or total intensity) varying systematically the pump parameter μ and the carrier lifetimes ratio r, and we could conclude that rogue waves are most probable below the lasing threshold (in the multistability region, see Fig.3) towards the lowest values of μ (leftmost part of the turbulent branch) and for high values of r, meaning fast saturable absorbers.

Finally, in Fig. 5 we show an example of how a rogue wave looks like in the transverse plane, together with its temporal profile.

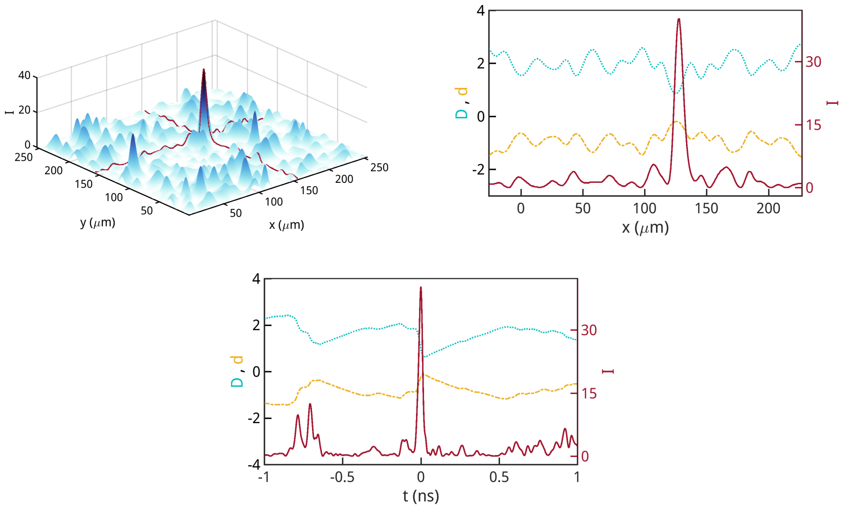

Fig. 5 Example of a rogue wave in the transverse plane (upper left panel) and its spatial (upper right panel) and temporal (lower panel) profiles, shown for the variables I (solid red line), D (dotted blue line) and d (dash dotted yellow line). Parameters are $\mu = 4.8, r = 2.2$.

4 Conclusion

Based on our expertise, both in optical systems and the computation of rare and extreme events, we would like to analyze spatially extended optical cavities in various regimes. In particular, genetic algorithms similar to the one used for studying atmospheric jet phase transitions, could potentially give very large-ensemble statistics of optical rogue waves. One of the key points is to compare these results with known experiments taking place in our laboratory and in various places over the world (New Zealand, Scotland, and France). This study will provide a rigorous formalism for optical rogue waves and will also give tools for predicting their appearance and in particular to detect instanton-driven regimes. This approach based on non-equilibrium statistical physics is completely new and could be extended to many area of physics due to its predictive character.

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