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Analysis of isolated solutions in a symmetric MEMS array during symmetry breaking event using NNM

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Summary. These works analyze isolated solutions in a symmetric 2-beam MEMS array designed for mass detection using Nonlinear Normal Modes (NNM) during symmetry breaking event. While symmetric, the system present two pure NNM and one mixed NNM. Then an added mass is dropped onto the MEMS array, the symmetry is broken and an isolated NNM is appearing. Particular behaviors on the isolated solutions depend on which type of NNM the isolated solution are linked to. The originality lies in the detection and analysis of isolated solution and NNM before and after symmetry breaking in a MEMS array using bifurcation calculation and continuation methods.

Keywords: MEMS array, isolated solutions, NNM, symmetry breaking, mass detection

1 Introduction

Current researches on mass sensing are essentially focused on a single MEMS resonator. A study by Nguyen et al. investigates alternative mass-sensing techniques using nonlinearities [1]. The multistability and frequency sweep, the resulting hysteresis cycle are used to provide a new mass sensing technique. Nevertheless array of resonant MEMS present complex dynamical behavior due to the nonlinear couplings induced by electrostatic forces. A paper by Gutschmidt and Goettlieb [2] analyzed arrays with electrical coupling. They focused on the n-beam dynamic behavior in the region of internal one-to-one, parametric and several internal three-to-one resonances corresponding to low, medium and large DC voltages. These nonlinear behaviors are complex to analyze and researches are currently working on methods that simplify the understanding of such phenomena. Hill et al. [3] analyze analytically the apparition of isolated solutions on backbone curves (NNMs). They show that symmetry breaking can generate isolated solutions supported by an isolated NNM. Kuether et al. worked on the Energy balance method [4] that allows to know the amplitude of forcing required to pass by the wanted point of a NNM. In this work, a symmetric 2-beam array is considered for mass detection purpose. When an added mass drops onto one of the moving beam of the MEMS array, the symmetry of the system is broken. This symmetry breaking event has been used in [5] for mass detection. Our work proposes to analyze the change in the nonlinear dynamics behavior of the MEMS array before and after symmetry breaking event using NNM, bifurcation analysis and continuation of bifurcation points in parallel with standard response curve. For some configuration of the system, isolated solutions, isolated and mixed NNM appear. Parametric methods of continuation that allows to catch the various isolated solutions and isolated NNM are presented. Moreover the results show that the merging behavior of the isolated solutions with the response curve depend greatly on which type of NNM the isolated solutions are found onto.

2 Array of two nanomechanical resonators

A 2-moving-beam array is considered, as represented in Fig. 1. The two beams 0 and 3 located at the ends of the array are clamped and serve only as electrostatic actuator. The two other beams 1 and 2 are clamped-clamped. All 4 beams are identical. l , b , h , I , g are respectively length, width, height, moment

of inertia of the beams, gap between two adjacent beams. Let E , ρ be the Young's modulus and the mass density.

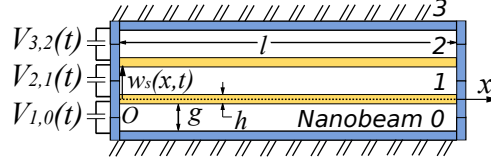


Figure 1: Array of two clamped-clamped M/NEMS beams.

Each beam is an electrostatic actuator for its adjacent beams. $V_{s,s+1} = V_{dc,s,s+1} + V_{ac,s,s+1} \cos(\Omega t)$ is the voltage applied between the successive beams s and $s + 1$ with V_{dc} , V_{ac} the continuous and alternative voltages. The equation of the beam s in bending is written as follows.

$$EI \frac{\partial^4 \tilde{w}_s(\tilde{x}, \tilde{t})}{\partial \tilde{x}^4} + \rho b h \frac{\partial^2 \tilde{w}_s(\tilde{x}, \tilde{t})}{\partial \tilde{t}^2} + \delta_i(s) \delta_{x0}(x) m \frac{\partial \tilde{w}_s}{\partial \tilde{t}^2} + c \frac{\partial \tilde{w}_s}{\partial \tilde{t}} - \left[\tilde{N} + \frac{E b h}{2l} \int_0^l \left(\frac{\partial \tilde{w}_s(\tilde{x}, \tilde{t})}{\partial \tilde{x}} \right)^2 d\tilde{x} \right] \frac{\partial^2 \tilde{w}_s(\tilde{x}, \tilde{t})}{\partial \tilde{x}^2} = \frac{\epsilon_0 b C_n}{2} \left[\frac{V_{s,s+1}^2}{(g + \tilde{w}_{s+1} - \tilde{w}_s)^2} - \frac{V_{s-1,s}^2}{(g + \tilde{w}_s - \tilde{w}_{s-1})^2} \right] \quad (1)$$

with $s = 1, 2$. Small added mass m is dropped on the beam i at the location x_0 . Let ϵ_0 , C_n be the dielectric constant and fringing the coefficient respectively. \tilde{N} represents the lineic load along the x -axis. The following voltages are taken, see Tab. 1.

V_{dc10}	V_{ac10}	V_{dc21}	V_{ac21}	V_{dc32}	V_{ac32}
3	0.1	3	0	3	-0.1

Table 1: Applied voltages.

Since the resonators of the 2-beams-array are identical, the beams 1, ..., 2 have the same eigenmodes. Therefore Galerkin method is used to withdraw the spatial dependence from the equation of motion, see Eq. (1). To compute the solution of the MEMS array, the Harmonic Balance Method (HBM) is used which consists in researching a periodic solution as a truncated Fourier series. Then the alternate frequency-time (AFT) is used for the calculation of nonlinear part of the equation of motion. The NNM are then computed on a conservative equation of motion [6]. This equation being autonomous, a phase condition is added and the equation of motion regularized to ensure the computation of an unique solution. Then the stability and bifurcation points of the NNM is calculated using a regularized quadratic eigenvalue problem and augmented system. An Energy balance is performed to calculate the corresponding amplitude of the force necessary to pass by the corresponding point of the NNM [4].

2.1 Before symmetry breaking event

The NNM and the response curve of the system are computed for a null added mass, see Fig. 2. The denomination pure and mixed NNM below are defined as in [7] [8]. The system exhibit two pure NNM initialized using the Linear Normal Modes frequency and modal shape. One can see, on the NNM that a Branch Point (BP) appears on the second pure NNM initialized at the Frequency 22.35. The Branch point, also known as symmetry breaking point, is symptomatic of a symmetric system. Beyond the BP, the pure NNM becomes unstable and two new stable mixed NNM, that does not respect the symmetry of the system, appear. A mixed NNM has a modal shape mixing modal shapes of the pure NNM of the system. The given voltages (see Table 1), excite the first pure NNM and the two mixed NNM. One can see on the response curve that the first pure NNM behaves as expected, while the second pure NNM does not react at all. Additionally, isolated solutions are present on each mixed NNM. They are found and initialized using the energy balance method. The two mixed NNM being isolated from the excited

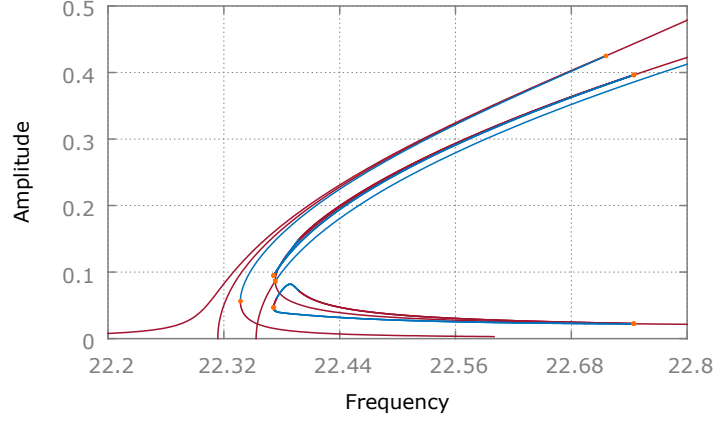


Figure 2: Beam 1 and 2 for an null added mass, Stable (red), Unstable (blue), Bifurcation (orange).

NNM, the merging of the isolated solutions with the response curve can not occur with the current shape of excitation 1.

2.2 After symmetry breaking event

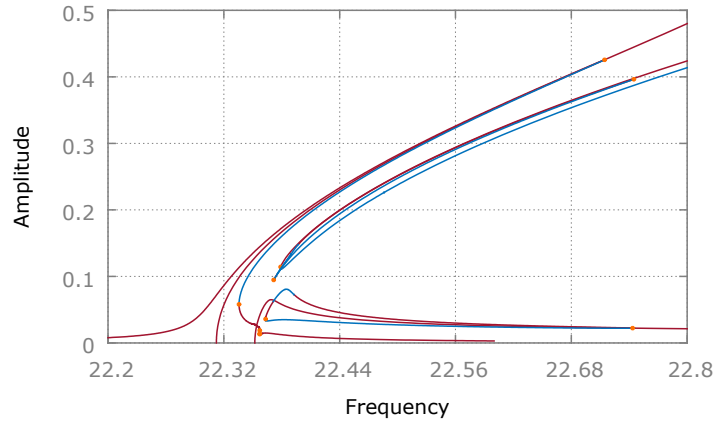


Figure 3: Beam 1 for an dimensionless added mass of $1e - 4$, Stable (red), Unstable (blue), Bifurcation (orange).

After the drop of a dimensionless added mass of $1e - 4$ onto the beam 1, the symmetry of the system is broken. The corresponding NNM of the system are computed in Figs. 3,4. The dynamical behavior of the system is modified, the BP onto the second pure NNM does no longer exist. The drop of the added mass has broken the symmetry and therefore the BP becomes an imperfect bifurcation point. The second pure NNM and the two mixed modes existing before the symmetry breaking event become one pure NNM and one isolated NNM. Such a behavior for beam 1 can be seen in Fig. 3. Moreover the system now being non-symmetric has different modal shape on each beam. The second pure NNM is composed by the previous lower energy part of the second pure NNM and lower energy mixed NNM. While the upper part of the second pure NNM and mixed NNM form now the isolated NNM. On the second beam 4, the left part of the previous second pure NNM and the mixed NNM form the second pure NNM after symmetry breaking. In the same way, the right part of the previous second pure NNM and mixed NNM compose the isolated NNM. The isolated NNM are found and initialized using Limit Point continuation of NNM. Moreover, the second pure NNM, now composed partially by a previous mixed NNM, responds to the current excitation. Therefore the isolated solution found onto the second

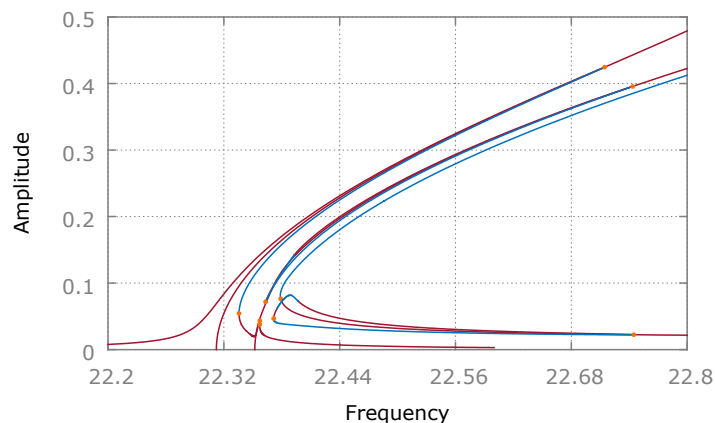


Figure 4: Beam 2 for a dimensionless added mass of $1e - 4$, Stable (red), Unstable (blue), Bifurcation (orange).

pure NNM is supported by an excited NNM. Consequently, for a high enough amplitude of voltages with respect to the current shape of excitation, the isolated solution supported by the second pure NNM will merged with the response curve. On the other hand, the isolated solution found onto the isolated NNM is totally isolated from any excited NNM. Hence, the isolated solution will remain isolated even for other amplitude and shape of excitation. The analysis of the complex dynamical behavior of an array of two nanomechanical resonators beams explain well the symmetry breaking phenomena used for mass detection in [5].

3 Conclusion

Multiple complex dynamical behavior, such as isolated NNM and isolated solutions with different merging behaviors, have been found in the considered array of two nanomechanical resonators. The merging of isolated solutions present in such system have been characterized depending on which NNM it can be found. Method that allows to find and initialize isolated solutions and isolated NNM are presented. As a conclusion, the results found with the presented method explain and analysis complex dynamical behavior found in a 2-beam MEMS before and after symmetry breaking used for mass detection. This research represents an increment towards the comprehension and modeling of resonator arrays for application in mass sensing.

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