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## DRAFT - DETC2018-85799

### GENERALIZATION OF CONSERVATIVE NONLINEAR NORMAL MODES TO THE NON-CONSERVATIVE EQUATION OF MOTION

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#### ABSTRACT

*This work presents a method to calculate non-conservative Nonlinear Normal Modes (NNMs) based on the conservative NNMs. A specific fictive force is added to the initial non-conservative equation of motion in order to insure the energy balance of the damped nonlinear equation. By doing so, conservative equations corresponding to Phase and Energy resonances are obtained. Therefore non-conservative NNMs can be computed with conservative NNMs calculation methods. Finally, the proposed method allows to calculate all non-conservative NNMs branches with their stability and bifurcation points.*

#### INTRODUCTION

In the literature methods to compute conservative NNM can be found [1]. In the paper [2], Harmonic Balance Method (HBM), a specific phase condition and Lagrange multipliers are used to compute periodic solutions. The quadratic eigenvalue problem obtained by Hill's method is shifted to calculate stability and bifurcation points. Some researches worked on the influence of damping onto the NNM. In the paper [3], an Energy balance is applied to conservative NNMs to explore the connection between NNMs and damped frequency responses. Malte Krack proposed to insure energy balance by introducing a fictional negative damping [4]. In this work, Phase and Energy resonances are computed by introducing a fictional force to insure the energy balance without modifying the invariant manifold.

#### Equation of motion

A nonlinear dynamical system with 2 degrees of freedom is considered [1]. Let  $\mathbf{x}(t)$  the displacement vector and  $\gamma = 5e - 2$ .

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{f}_{\text{nl}}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{f}$$
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{C} = \gamma\mathbf{K} = \gamma \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \mathbf{f}_{\text{nl}} = \begin{bmatrix} \frac{1}{2}x_1^3(t) \\ 0 \end{bmatrix} \quad (1)$$

with  $\mathbf{M}$ ,  $\mathbf{K}$ ,  $\mathbf{C}$  the generalized mass, damping and stiffness matrices,  $\mathbf{f}_{nl}$  the vector of nonlinear forces. The HBM method is applied to (1) as in [2].  $\mathbf{X}$ ,  $\mathbf{Fnl}$  represent the Fourier coefficient of the displacement vector and the vector of nonlinear forces.  $\otimes$  corresponds to the Kronecker tensor product and  $\nabla$  represents the time derivative operator in the frequency domain. The equation of motion in frequency domain is obtained.

$$\begin{aligned}\mathbf{R}(\mathbf{X}, \omega) &= \mathbf{Z}(\omega)\mathbf{X} + \mathbf{Fnl}(\mathbf{X}) = \mathbf{F} \\ \mathbf{Z}(\omega) &= \omega^2 \nabla^2 \otimes \mathbf{M} + \omega \nabla \otimes \mathbf{C} + \mathbf{I}_{2H+1} \otimes \mathbf{K} \\ \nabla &= \text{diag}(\mathbf{0}, \nabla_1, \dots, \nabla_j, \dots, \nabla_H) \quad \text{with} \quad \nabla_j = j \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\end{aligned}\quad (2)$$

## Resonances

For usual damping of a few percents, the Phase and Energy resonances give almost the same results. However for higher damping values, the obtained NNMs are different and the calculation method has to be chosen appropriately. This work deals with small and high damping, but for length consideration only the case where damping is small is dealt within this extended abstract.

### Phase resonance

Let  $\xi_\phi$  be one of the damped NNMs of the system (2) corresponding to the phase resonance. The phase  $\phi$  between the periodic solution  $\mathbf{X}$  and the damped NNM  $\xi_\phi$  is calculated (3).  $Im(\mathbf{R})$  and  $Re(\mathbf{R})$  representing the corresponding imaginary and real part of Eqn. (2). In order to have a phase resonance,  $\xi_\phi^T(Re(\mathbf{R}))$  has to be equal to zero and  $\xi_\phi^T(Im(\mathbf{R})) \neq 0$ . Since  $\xi_\phi \neq 0$ , the equation corresponding to the Phase resonance is obtained (4).  $\mathbf{X}_{\xi_\phi}$  and  $\omega_{\xi_\phi}$  representing the Fourier coefficients and the frequency of the damped NNM  $\xi_\phi$ . One can note that Eqn. (4) corresponds to the classical equation used to calculate undamped-unforced NNMs [1]. By identifying Eqns. (2) and (4), the force vector that has to be applied to the system in order for the corresponding forced response to pass through the considered NNM point is calculated (5).

$$\phi = \arctan\left(\frac{\xi_\phi^T(Im(\mathbf{R}))}{\xi_\phi^T(Re(\mathbf{R}))}\right) \quad (3)$$

$$\begin{aligned}\mathbf{Z}_\phi(\omega_{\xi_\phi})\mathbf{X}_{\xi_\phi} + \mathbf{Fnl}(\mathbf{X}_{\xi_\phi}) &= \mathbf{0} \\ \mathbf{Z}_\phi(\omega_{\xi_\phi}) &= \omega_{\xi_\phi}^2 \nabla^2 \otimes \mathbf{M} + \mathbf{I}_{2H+1} \otimes \mathbf{K}\end{aligned}\quad (4)$$

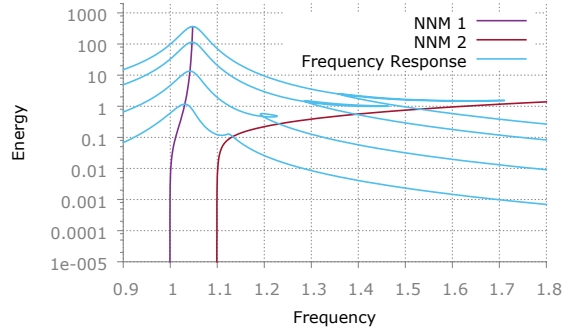
$$\mathbf{F}_\phi = \omega_{\xi_\phi} (\nabla \otimes \mathbf{C}) \mathbf{X}_{\xi_\phi} \quad (5)$$

### Energy resonance

Let  $\xi_E$  be one of the damped NNM corresponding to the energy resonance with  $\mathbf{X}_{\xi_E}$  and  $\omega_{\xi_E}$  the corresponding Fourier coefficient and frequency. The energy resonance takes place at a point  $(\mathbf{X}_{\xi_E}, \omega_{\xi_E})$  where the derivative of the energy along the frequency is null. By performing the calculation, Eqn.(6) is obtained. Energy resonance takes the influence of damping into account through a fictional stiffness. By identifying Eqns. (2) and (6), the force vector that has to be applied to the system in order for the corresponding forced response to pass through the considered NNM point is calculated (7).

$$\mathbf{Z}_\phi(\omega_{\xi_E})\mathbf{X}_{\xi_E} + \mathbf{Fnl}(\mathbf{X}_{\xi_E}) - \frac{1}{2} \left( \mathbf{I}_{2H+1} \otimes \mathbf{M}^{-1} \mathbf{C}^2 \right) \mathbf{X}_{\xi_E} = \mathbf{0} \quad (6)$$

$$\mathbf{F}_E = \left( \omega_{\xi_E} \nabla \otimes \mathbf{C} + \frac{1}{2} \mathbf{I}_{2H+1} \otimes \mathbf{M}^{-1} \mathbf{C}^2 \right) \mathbf{X}_{\xi_E} \quad (7)$$



**FIGURE 1:** Frequency responses with the NNMs

Since Eqns. (4) and (6) are autonomous, methods for conservative NNMs computation [2] are used to compute NNMs of the system Fig. 1. Using periodic solutions of the first NNM to calculate fictional forces, the corresponding frequency responses are plotted. For small  $\gamma$ , the Phase and Energy resonances cannot be distinguished from one another. As expected, frequency responses only respond greatly to the NNM used to calculate the fictional force.

## Conclusion

A generalization of conservative NNMs to non-conservative equations of motion has been presented. By using conservative equations of motion to describe the Phase and Energy resonances from non-conservative equations of motion, classical methods for NNMs calculation can be used to compute damped NNMs, their stability and bifurcations points. Moreover, fictional force allows specific NNM to be targeted by frequency responses. Phase and Energy resonances are almost superposed for small damping and separate from one another for higher damping.

## REFERENCES

- [1] L. Renson, G. Kerschen, and B. Cochelin. Numerical computation of nonlinear normal modes in mechanical engineering. *Journal of Sound and Vibration*, 364:177 – 206, 2016.
- [2] C. Grenat, S. Baguet, R. Dufour, and C-H Lamarque. Bifurcation analysis of nonlinear normal modes with the harmonic balance method. In *ENOC 2017, 9th European Nonlinear Dynamics Conference*, Budapest, Hungary, June 2017.
- [3] R.J. Kuether, L. Renson, T. Detroux, C. Grappasonni, G. Kerschen, and M.S. Allen. Nonlinear normal modes, modal interactions and isolated resonance curves. *Journal of Sound and Vibration*, 351:299–310, 2015.
- [4] M. Krack. Nonlinear modal analysis of nonconservative systems: extension of the periodic motion concept. *Computers & Structures*, 154:59–71, 2015.