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# Pairwise Markov fields for segmentation in astronomical hyperspectral images

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## Abstract

We consider the problem of segmentation in noisy, blurred astronomical hyperspectral images (HSI). Recent methods based on an hypothesis-testing framework handle the problem, but do not allow to use a prior on the result. This paper introduces a pairwise Markov field model, allowing the unsupervised Bayesian segmentation of faint sources in astronomical HSI. Results on synthetic images show that the segmentation methods outperform their state-of-the-art counterparts, and allow the detection at very low SNR. Besides, results on real images provide encouraging detections with respect to the application.

**Keywords:** Markov Random Fields, Pairwise Markov Fields, Bayesian Unsupervised Segmentation, Blurred Hyperspectral Image Segmentation

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## 1. Introduction

This paper considers the problem of astronomical Hyperspectral Image (HSI) segmentation, which consists in estimating the classes  $\mathbf{X}$  from an observation  $\mathbf{Y} = \mathbf{y}$ . This kind of images can be provided, *e.g.*, by the Multi-Unit Spectroscopic Explorer (MUSE) instrument [3]. Among others, the MUSE instrument targets the distant Universe to study very faint objects. Considering that these objects are blurred by a spread function and buried in the noise, both originating from the observational setting (instrument and atmosphere), their segmentation is particularly difficult.

The problem of faint source detection in astronomical HSI was recently addressed by dedicated methods within an hypothesis testing framework [7, 2, 1], in which the absence and presence of signal are two competing hypothesis. In [7], we accounted for spatial and spectral priors to perform a local contextual detection, and errors were handled locally with the expected Probability of False Alarm (PFA). In [2, 1], the authors use a matched-filter preprocessing to tackle the spatial structure, and the errors are globally handled through robust False Discovery Rate (FDR) control. Both methods perform locally, and the handling of the Field Spread Function (FSF) relies on contextual information. An alternative can be found in [18], where the authors use a point marked process modeling within a Monte Carlo Markov chain procedure to detect sources in astronomical HSI. This model however relies on the expected shapes of objects (ellipses), which does not matches the general cases of sources detection.

In a more general fashion, the detection can be seen as a segmentation task. For this purpose, a Markov field modeling

is relevant in order to bolster the regularity of the segmentation results. Numerous works were published for Bayesian HSI segmentation, using hidden Markov fields (HMF) models [23, 21, 14, 12, 15, 25]. Other works specifically handle astronomical multi-spectral images [22, 24]. However, as far as we know, there is no existing work on the Bayesian segmentation of faint objects in astronomical HSI handling the FSF. The FSF cannot be modeled within an HMF framework. This is why we turn, in this paper, to the pairwise Markov field (PMF) model [20] which is a generalization of HMF models.

The contributions of this paper are the following. We introduce a PMF model which allows the modeling and the unsupervised Bayesian segmentation of blurred images. This model is applied for the detection of signals in extremely noisy HSI (Section 2 and Section 3). The proposed model is evaluated on synthetic images, and it is shown that it outperforms the state-of-the-art methods from [7, 2, 1] shows that these latter are outperformed by the proposed method (Section 4). Finally, the segmentation of real MUSE images provide satisfying results with regards to the target application (Section 5).

## 2. Modeling

Along this paper, a random variable (resp. vector) is noted  $A$  (resp.  $\mathbf{A}$ ), its realization  $a$  (resp.  $\mathbf{a}$ ), and its distribution  $p(a)$  (resp.  $p(\mathbf{a})$ ).

Let  $\mathbf{Y} = (\mathbf{Y}_s)_{s \in \mathcal{S}}$  and  $\mathbf{X} = (X_s)_{s \in \mathcal{S}}$  be respectively the observation process and the class process, which has to be retrieved by the segmentation task.  $\mathcal{S}$  is the lattice of the sites  $s$  in the image, which is ruled by a 8-neighborhood system denoted  $(N_s)_{s \in \mathcal{S}}$  and associated to two-elements cliques. For all  $s \in \mathcal{S}$ ,  $X_s$  takes values in the finite set  $\Omega$  and  $\mathbf{Y}_s \in \mathbb{R}^\Lambda$ ,  $\Lambda$  being the number of spectral channels of  $\mathbf{Y}$ .

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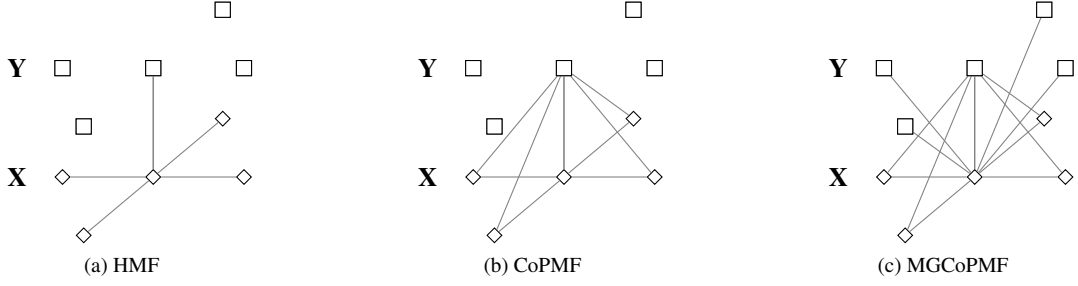


Figure 1: Dependency graph corresponding to the CoPMF models introduced in this paper. The graphs represent the distribution of  $(\mathbf{X}, \mathbf{Y})$  given the center node  $(X_s, Y_s)$ . Roughly speaking, eliminating a graph edge is equivalent to not account for a variable in one of the conditional distributions ruling the process  $(\mathbf{X}, \mathbf{Y})$ . The graph correspond to a local neighborhood, restricted to 4 sites for clarity purpose.

Within the pairwise Markov field (PMF) modeling [20], the pair  $(\mathbf{X}, \mathbf{Y})$  is a Markov field :

$$p(\mathbf{x}, \mathbf{y}) \propto \prod_{s \in S} p(x_s, y_s | \mathbf{x}_{N_s}, \mathbf{y}_{N_s}). \quad (1)$$

We assume that  $(\mathbf{X}, \mathbf{Y})$  is stationary and that  $\mathbf{Y}_s$  and  $\mathbf{Y}_{N_s}$  are independent given  $(X_s, \mathbf{X}_{N_s})$ :

$$p(x_s, y_s | \mathbf{x}_{N_s}, \mathbf{y}_{N_s}) = p(\mathbf{y}_s | x_s, \mathbf{x}_{N_s}) p(x_s | \mathbf{x}_{N_s}, \mathbf{y}_{N_s}) \quad (2)$$

The two resulting distributions are detailed in the following subsections.

### 2.1. Observation model

We assume the noise to be normally distributed:

$$p(\mathbf{y}_s | x_s, \mathbf{x}_{N_s}) \sim \mathcal{N}(\gamma_s, \Sigma); \quad (3)$$

where  $\gamma_s \in \mathbb{R}^\Lambda$  and  $\Sigma \in \mathbb{R}^{\Lambda \times \Lambda}$  are the mean and covariance matrix respectively.

The parameter  $\gamma_s$  models the convolution by the FSF  $\mathbf{f}^1$ :

$$\gamma_s = f_0 \mu_s + \sum_{r \in N_s} f_r \mu_r; \quad (4)$$

where  $\mu_s \in \mathbb{R}^\Lambda$  is the mean associated to the class of  $x_s$ . We assume in the remaining of this paper that  $\mathbf{f}$  is known prior to the segmentation, *e.g.* from instrumental calibration.

In this paper, the covariance matrix is designed to model the MUSE observations (cf. Section 5): it is pentadiagonal, with the same parameters for each spectral component:

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho_1 & \rho_2 & 0 & \dots & 0 \\ \rho_1 & \sigma^2 & \rho_1 & \rho_2 & \dots & 0 \\ \rho_2 & \rho_1 & \sigma^2 & \rho_1 & \rho_2 & \dots \\ 0 & \rho_2 & \rho_1 & \sigma^2 & \rho_1 & \rho_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \rho_2 & \rho_1 & \sigma^2 \end{bmatrix} \quad (5)$$

This configuration of  $\Sigma$  is set with respect to the application on MUSE observations (cf. Section 5), and can be adapted to other applications.

### 2.2. Convolutional pairwise Markov fields

In this paper, we investigate two possible choices for the second term of (2) :

- in the first case, we assume that  $X_s$  and  $\mathbf{Y}_{N_s}$  are independent given  $\mathbf{X}_{N_s}$ , and that  $\mathbf{X}$  has a Markov field distribution. We choose to use an Ising potential as it is a widespread prior for image regularization [4]:

$$p(x_s | \mathbf{x}_{N_s}, \mathbf{y}_{N_s}) = p(x_s | \mathbf{x}_{N_s}) \propto \exp \left( - \sum_{s' \in N_s} \alpha (1 - 2\delta_{x_{s'}}(x_s)) \right) \quad (6)$$

where  $\delta_{x_{s'}}$  is the Kronecker function of  $x_{s'}$  and  $\alpha$  is a model parameter controlling the “granularity” of the realization  $\mathbf{x}$ . This model will be hereafter referred as “Convolutional Pairwise Markov Field” (CoPMF).

- In the second case, we choose to penalize the gaps between the spectral mean at the site  $s$ , and the observed spectra in the neighboring sites within  $N_s$ . To do so, we add a Gaussian term to (6), yielding:

$$p(x_s | \mathbf{x}_{N_s}, \mathbf{y}_{N_s}) \propto \exp \left[ - \sum_{s' \in N_s} \alpha (1 - 2\delta_{x_{s'}}(x_s)) - \frac{1}{2} \left( \gamma_s - \frac{1}{|N_s|} \sum_{s' \in N_s} \mathbf{y}_{s'} \right)^\top \Sigma^{-1} \left( \gamma_s - \frac{1}{|N_s|} \sum_{s' \in N_s} \mathbf{y}_{s'} \right) \right] \quad (7)$$

where  $|\cdot|$  is the cardinal function. This model will hereafter be referred as “Multivariate Gaussian CoPMF” or MGCoPMF.

Fig. 1 illustrates the dependency graphs yielded by the two models, in comparison with the classical HMF model. The two models will be evaluated in Section 4.

## 3. Unsupervised Bayesian segmentation

### 3.1. Bayesian segmentation

We compare two estimators for segmentation, namely the Maximum A Posteriori (MAP) [13] :

$$\hat{\mathbf{x}}^{\text{MAP}} = \arg \max_{\omega \in \Omega^{|\mathcal{S}|}} p(\mathbf{X} = \omega | \mathbf{Y} = \mathbf{y}); \quad (8)$$

<sup>1</sup>Note that since  $(\mathbf{X}, \mathbf{Y})$  is stationary,  $\mathbf{f}$  does not depend on  $s$ .

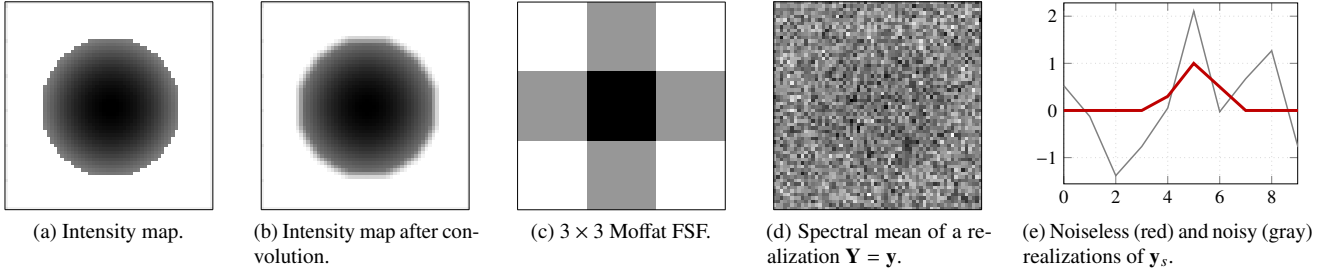


Figure 2: Synthetic image formation.

and the Maximum Posterior Mode (MPM) [16]:

$$\forall s \in \mathcal{S}, \hat{x}_s^{\text{MPM}} = \arg \max_{\omega \in \Omega} p(x_s = \omega | \mathbf{Y} = \mathbf{y}). \quad (9)$$

The exact distributions  $p(\mathbf{x}|\mathbf{y})$  is known only up to a constant. However, the segmentation remains possible, using *e.g.* adaptations of Iterated Conditional Modes (ICM) [4] for the MAP and Marroquin's algorithm [16] for the MPM. These methods rely on simulation algorithms, such as the Gibbs sampling [13].

### 3.2. Parameter estimation

In an unsupervised context, the unknown parameters  $\theta = \{\mu, \sigma, \rho_1, \rho_2, \alpha\}$  are estimated prior to segmentation with dedicated methods. It is possible to use, *e.g.*, methods inspired from the Expectation-Maximization (EM) [9, 17], the Stochastic EM (SEM) [6] or the Iterative Conditional Estimator (ICE) [8] algorithms. For robustness reasons [19, 11], we use an adaptation of the SEM method with, at a given step  $q$ :

- simulate  $\mathbf{x}^{(q)}$  along  $p_{\theta(q-1)}(\mathbf{x}|\mathbf{y})$
- estimate  $\theta^{(q)}$  from  $(\mathbf{x}^{(q)}, \mathbf{y})$  with the estimators described below.

Let  $\mathbf{w} = (w_s)_{s \in \mathcal{S}} = \mathbf{x} * \mathbf{f}$  be the convolution of  $\mathbf{x}$  by the FSF  $\mathbf{f}$ . For  $\mu$  and  $\Sigma$ , we use the following maximum likelihood estimators:

$$\hat{\mu} = \frac{\sum_{s \in \mathcal{S}} w_s \mathbf{y}_s}{\sum_{s \in \mathcal{S}} w_s^2} \quad (10)$$

$$\hat{\Sigma} = \frac{1}{|\mathcal{S}| - 1} \sum_{s \in \mathcal{S}} (\mathbf{y}_s - w_s \hat{\mu})(\mathbf{y}_s - w_s \hat{\mu})^\top \quad (11)$$

The proof for (10) is given at the end of the paper, and the derivation of (11) is straightforward. Thanks to the function invariance property of the maximum likelihood estimator [5], the estimators of  $\sigma, \rho_1, \rho_2$  are obtained by :

$$\hat{\sigma} = \sqrt{\frac{1}{\Lambda} \sum_{i=1}^{\Lambda} \hat{\Sigma}_{i,i}}; \quad \hat{\rho}_1 = \frac{1}{\Lambda-1} \sum_{i=1}^{\Lambda-1} \hat{\Sigma}_{i,i+1}; \quad \hat{\rho}_2 = \frac{1}{\Lambda-2} \sum_{i=1}^{\Lambda-2} \hat{\Sigma}_{i,i+2}. \quad (12)$$

Finally,  $\alpha$  is estimated with the least-squares estimator of [10]. To sum up, the Bayesian unsupervised detection within blurred and noisy HSI is made possible by both CoPMF and MGCoPMF models, using either the MPM or the MAP es(timators).

## 4. Numerical results

### 4.1. Settings and alternatives

In this section, we evaluate the method for detection of faint sources in hyperspectral images, with  $\Omega = \{0, 1\}$ . The numerical experiments are conducted on HSI containing  $60 \times 60$  spectra and  $\Lambda = 10$  spectral channels. The noiseless image is a 2D Gaussian where the faintest intensities are set to zero (cf. Fig. 2a). The intensities are then convolved by a  $3 \times 3$  Moffat FSF (cf. figures 2c, 2b and also [7]). Finally,  $\mu$  contains 3 non-zero coefficients, as shown in Fig. 2e.

Six methods are evaluated:

1. the proposed CoPMF model with the MAP (8) ;
2. ————— MPM (9);
3. the proposed MGCoPMF model with the MAP (8);
4. ————— MPM (9);
5. the detection method introduced in [7], based on an hypothesis-testing framework. We use the same experimental settings proposed as in [7] concerning the choice of the catalog and the  $P_{\text{FA}}$ , and use the same FSF as in the synthetic images ;
6. the detection method proposed in [2] and extended in [1]. We perform the preprocessing with the same FSF as in the images, and set the FDR value to 0.1.

All methods are evaluated under a varying signal-to-noise ratio (SNR), defined as

$$\text{SNR} = 10 \log_{10} \left( \frac{\|\bar{\mu}\|_2^2}{\Lambda \sigma^2} \right); \quad (13)$$

where  $\bar{\mu}$  the product of  $\mu$  by the average of non-zero intensities in the intensity map. An insight of the results for the six evaluated method is provided in Fig. 3.

### 4.2. MGCoPMF and CoPMF

In Fig. 4, we report the performances of the CoPMF-based methods in terms of false classification, false positive and false negative rates. These results yield the following findings:

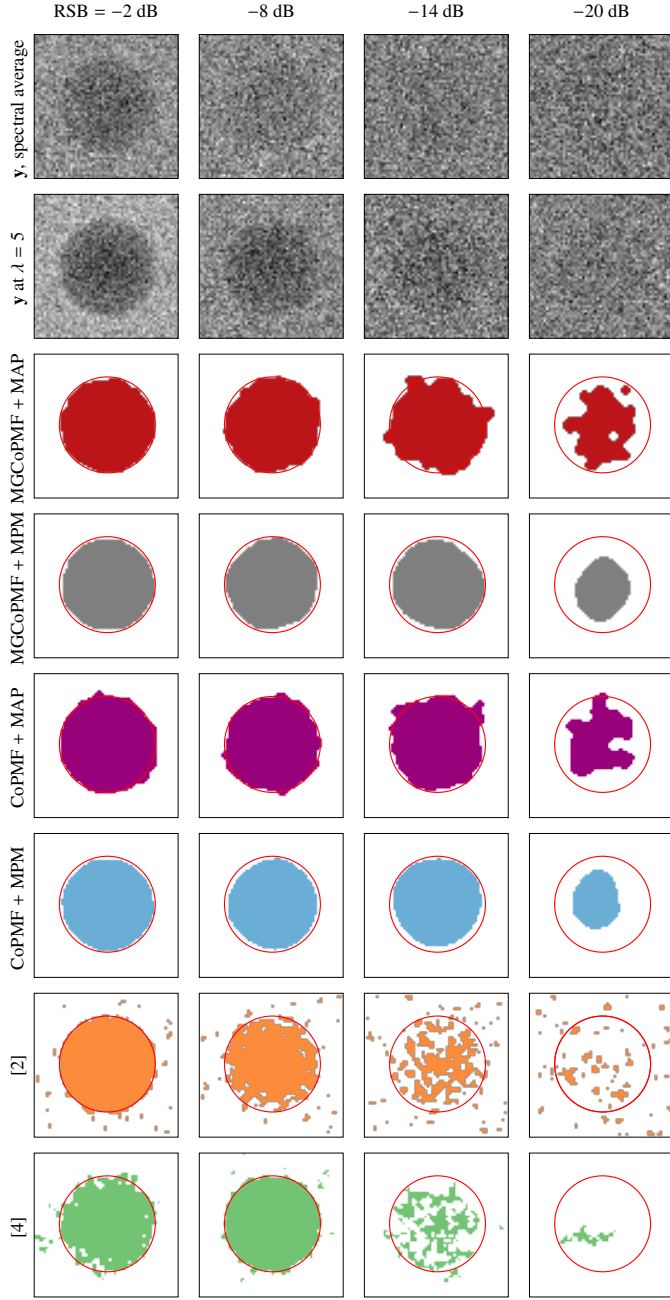


Figure 3: Results insights under low SNR for synthetic images. Red circles represent the ground truth (cf. Fig. 2a).

- the four methods yield very close averaged error rates, providing for all of them less than 10% error rate for SNR > -14 dB. In most cases, the MGCOPMF with the MAP yields slightly better results than the other methods, and for both MGCOPMF and MPM, the MAP estimator yields smaller error rates than the MPM estimator;
- these two estimators have however distinct behaviors in terms of false positive rates. On one hand, the MAP provides less false negatives than any other methods (less than 10% when RSB > -14 dB), at the cost of having small, but non-zero false positive rates. On the other hand,

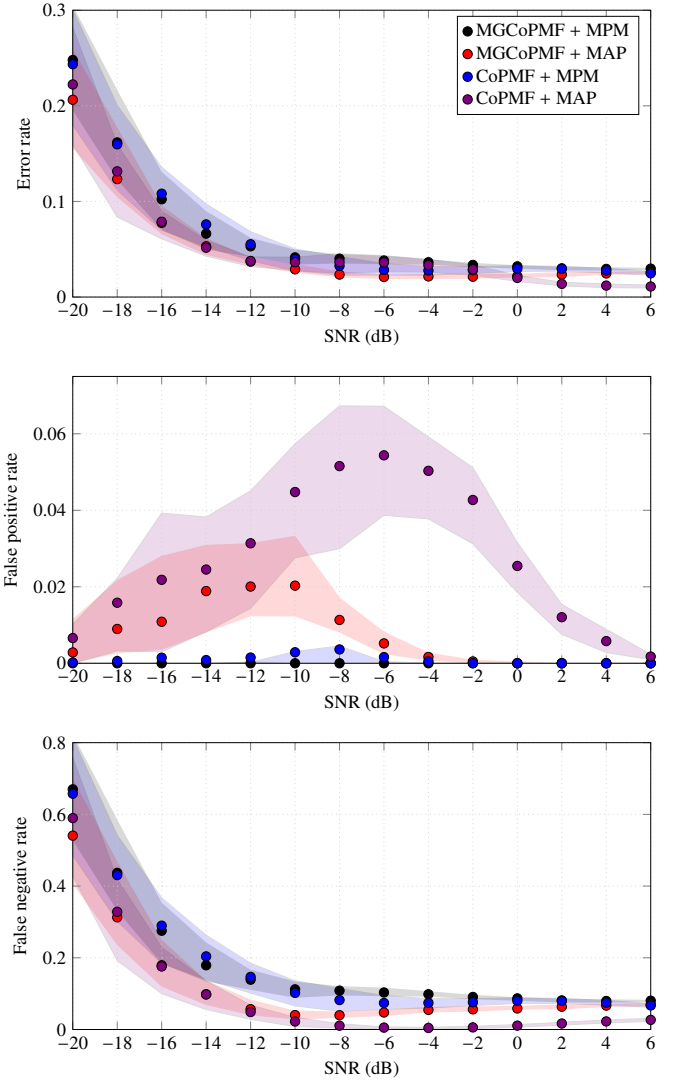


Figure 4: Average performances of the MGCOPMF and CoPMF models using the MAP (8) and the MPM (9). Each point is an average results, and the shaded regions are between the first and fourth quartiles. The results were obtained with 100 realizations  $\mathbf{Y} = \mathbf{y}$ .

the MPM provides almost no false alarm (less than 3% at all SNR), but has a non-zero false negative rate. We interpret this observation as an insight of the stability of the MPM criterion with respect to the MAP in practical implementation.

- in average, the MGCOPMF yields better results with either the smaller error rates for the MGCOPMF with the MAP, or the smallest error rate with zero-valued false alarms for the MGCOPMF with the MPM.

#### 4.3. Comparison with state-of-the-art alternatives

Fig. 5 reports the performance comparison of the MGCOPMF model with [7] and [2, 1]. The investigation of these results reveals the following points:

- the CoPMF segmentations yield, at all SNR, the best error rate with respect to the alternatives, with similar error rates for the MAP and MPM estimators;

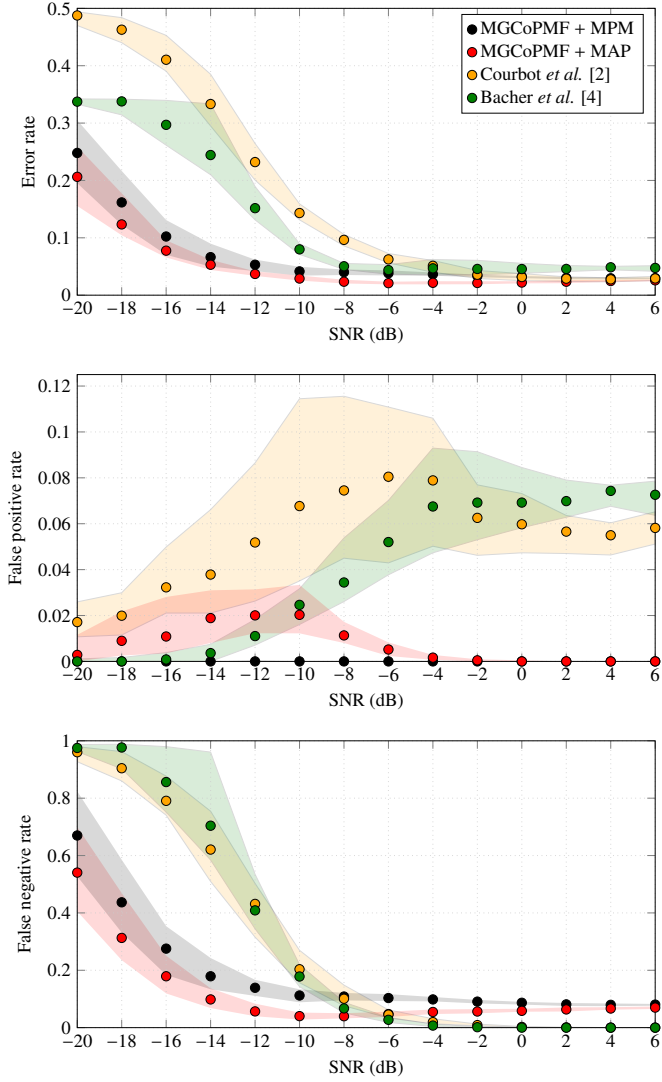


Figure 5: Results for the methods from [7], from [2, 1] and with the MAP (8) and the MPM (9) within the MGCoPMF introduced in this paper (the red and black curves are the same as in Fig. 4). The legend is the same as in Fig. 4.

- the method from [1] yields fairly high error rate when  $\text{SNR} < -8$  dB, due to an excess of non-detection required for the FDR control;
- the method from [7] provides no clear advantage over the other detection methods. This is partly caused by the use of the  $3 \times 3$  pixels FSF from the simulations.

## 5. Real MUSE images

Detection in astronomical hyperspectral images may reveal spatially extended, spectrally-located light emission called Lyman-alpha, tracing the circum-galactic medium (CGM). The CGM may be occurring in the vicinity of known galaxies, making possible to extract from a MUSE observation (see [3]) a smaller HSI centered spatially and spectrally on the emission of interest. Such HSI may be contaminated by brightest objects presenting a “continuum” emission. These contamination are

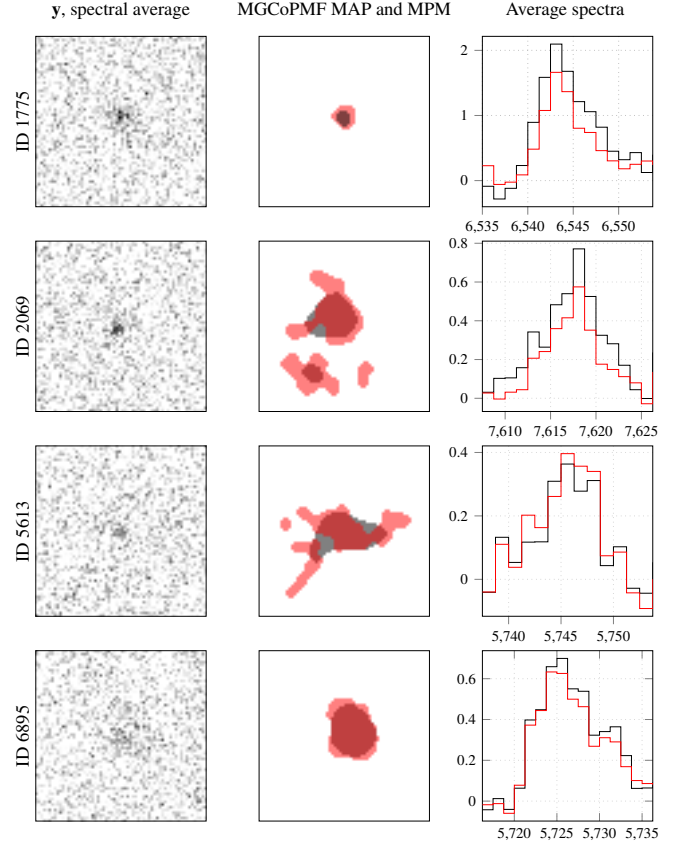


Figure 6: Results on 4 real MUSE images. Left : spectral average of  $\mathbf{Y} = \mathbf{y}$  (inverse video) after the spectral median subtraction has removed the brightest objects. Center : detection results with the MGCoPMF model, using the MPM (gray) and MAP (red) criteria. Right : average spectra in the two detected regions, with intensities in  $10^{-20} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$  as a function of the wavelength (in Ångström).

efficiently removed by a spectral median filtering prior to the segmentation.

Fig. 6 illustrates the unsupervised segmentation of 4 MUSE HSI. In the absence of ground truth, the qualification of the results must be conducted thoroughly by experts. Nevertheless, two points are worth mentioning :

- in most cases, the MAP-based segmentation yields a larger detected region than its MPM counterpart. This is consistent with the results obtained on synthetic images;
- the inspection of the average spectra reveals that their shape is consistent with a possible Lyman-alpha emission. Besides, the spatial shapes make likely the presence of an extended emission wider than a FSF blur on a point source, bolstering the hypothesis of the detection of an extended Lyman-alpha emission.

## 6. Conclusion

In this paper, we introduced the CoPMF and the MGCoPMF model, allowing to handle the convolution in the Markovian modeling of images. This model was specified for the application to detection in astronomical hyperspectral images, and the

unsupervised segmentation in this context was presented. Then, comparative results with state-of-the-art method indicate that the CoPMF methods outperform their state-of-the-art alternatives, with in addition complimentary results provided by the MAP and MPM estimators. We finally presented results on real MUSE images, showing that the both segmentation method are robust to a real-world use.

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## Proof : maximum likelihood estimator of $\mu$

Let us derive the expression of the maximum likelihood estimator (MLE) of  $\mu$ , stated in (10). The log-likelihood is written:

$$\log \ell(\mathbf{x}, \mathbf{y}; \mu, \Sigma) = c - \frac{1}{2} \sum_{s \in \mathcal{S}} (\mathbf{y}_s - w_s \mu)^\top \Sigma^{-1} (\mathbf{y}_s - w_s \mu). \quad (14)$$

where  $c = -\frac{|\mathcal{S}|}{2} \log((2\pi)^\Lambda \det(\Sigma))$ . The MLE of  $\mu$ , noted  $\hat{\mu}$ , verifies:

$$\left. \frac{\partial \log \ell(\mathbf{x}, \mathbf{y}, \mu, \Sigma)}{\partial \mu} \right|_{\mu=\hat{\mu}} = 0 \quad (15)$$

which can be rewritten using (14) as:

$$\begin{aligned} 0 &= -\frac{1}{2} \frac{\partial}{\partial \mu} \left( \sum_{s \in \mathcal{S}} \mathbf{y}_s^\top \Sigma^{-1} \mathbf{y}_s - 2 w_s \mu^\top \Sigma^{-1} \mathbf{y}_s + w_s^2 \mu^\top \Sigma^{-1} \mu \right) \Big|_{\mu=\hat{\mu}} \\ 0 &= -\frac{1}{2} \sum_{s \in \mathcal{S}} -2 w_s \Sigma^{-1} \mathbf{y}_s + 2 w_s^2 \Sigma^{-1} \hat{\mu}. \end{aligned} \quad (16)$$

Assuming that  $\sum_{s \in \mathcal{S}} w_s^2 \neq 0$ , a left-hand product by  $\Sigma$  yields:

$$\hat{\mu} = \frac{\sum_{s \in \mathcal{S}} w_s \mathbf{y}_s}{\sum_{s \in \mathcal{S}} w_s^2}. \quad (17)$$

□

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