



## Erratum: A review on the systematic formulation of 3-D multiparameter full waveform inversion in viscoelastic medium

Pengliang Yang, Romain Brossier, Ludovic Métivier, Jean Virieux

### ► To cite this version:

Pengliang Yang, Romain Brossier, Ludovic Métivier, Jean Virieux. Erratum: A review on the systematic formulation of 3-D multiparameter full waveform inversion in viscoelastic medium. *Geophysical Journal International*, 2018, 212 (3), pp.1694-1695. 10.1093/gji/ggx493 . hal-02009744

HAL Id: hal-02009744

<https://hal.science/hal-02009744>

Submitted on 13 Oct 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

## Erratum: A review on the systematic formulation of 3-D multiparameter full waveform inversion in viscoelastic medium

by Pengliang Yang,<sup>1</sup> Romain Brossier,<sup>1</sup> Ludovic Métivier<sup>1,2</sup> and Jean Virieux<sup>1</sup>

<sup>1</sup> Univ. Grenoble Alpes, ISTerre, F-38000 Grenoble, France. E-mail: pengliang.yang@univ-grenoble-alpes.fr

<sup>2</sup> Univ. Grenoble Alpes, CNRS, Laboratoire Jean Kuntzmann (LJK), F-38000 Grenoble, France

Erratum of the paper ‘A review on the systematic formulation of 3-D multiparameter full waveform inversion in viscoelastic medium’, by Yang *et al.*, published in Geophys. J. Int. (2016) 207, 129–149.

The following equations were erroneously displayed in this paper and should be read as follow.

The gradient of the misfit function with respect to model parameters  $\mathbf{m}$  is the same as the gradient of the Lagrangian at the saddle points considering  $\mathbf{w}$  and  $\mathbf{m}$  are independent variables when performing derivatives:

$$\frac{\partial \mathbb{L}}{\partial \mathbf{m}} = \left\langle \bar{\mathbf{w}}, \frac{\partial F(\mathbf{m}, \mathbf{w})}{\partial \mathbf{m}} \right\rangle_T \Leftrightarrow \frac{\partial \chi}{\partial \mathbf{m}} = \left\langle \bar{\mathbf{w}}, \frac{\partial F(\mathbf{m}, \mathbf{w})}{\partial \mathbf{m}} \right\rangle_T \quad (61)$$

$$\begin{aligned} \left\langle \frac{\partial \chi}{\partial \mathbf{m}}, \delta \mathbf{m} \right\rangle_{\Omega} &= \int_{\Omega} d\mathbf{x} \delta \rho \left( \int_0^T dt \bar{\mathbf{v}}^{\dagger} \partial_t \mathbf{v} \right) \\ &+ \sum_{I=1}^6 \sum_{J=I}^6 \int_{\Omega} d\mathbf{x} \delta C_{IJ} \left( \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} \frac{\partial C^{-1}}{\partial C_{IJ}} \left( \partial_t \boldsymbol{\sigma} - \mathbf{f}_{\sigma} + (C :: \Gamma) \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell} \right) \right) + \sum_{I=1}^6 \sum_{J=I}^6 \int_{\Omega} d\mathbf{x} \delta C_{IJ} \left( \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} C^{-1} \frac{\partial (C :: \Gamma)}{\partial C_{IJ}} \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell} \right) \\ &+ \sum_{\ell=1}^L \sum_{I=1}^6 \sum_{J=I}^6 y_{\ell} \int_{\Omega} d\mathbf{x} \delta Q_{IJ}^{-1} \left( \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} C^{-1} \left( C :: \frac{\partial \Gamma}{\partial Q_{IJ}^{-1}} \right) \boldsymbol{\xi}_{\ell} \right), \end{aligned} \quad (66)$$

$$\begin{aligned} \frac{\partial \chi}{\partial \rho} &= \int_0^T dt \bar{\mathbf{v}}^{\dagger} \partial_t \mathbf{v}, \\ \frac{\partial \chi}{\partial C_{IJ}} &= \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} \frac{\partial C^{-1}}{\partial C_{IJ}} \left( \partial_t \boldsymbol{\sigma} - \mathbf{f}_{\sigma} + (C :: \Gamma) \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell} \right) + \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} C^{-1} \frac{\partial (C :: \Gamma)}{\partial C_{IJ}} \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell}, \\ \frac{\partial \chi}{\partial Q_{IJ}^{-1}} &= \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} C^{-1} \left( C :: \frac{\partial \Gamma}{\partial Q_{IJ}^{-1}} \right) \left( \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell} \right), \quad \text{with } \left( \frac{\partial C :: \Gamma}{\partial C_{IJ}} \right)_{ij} = \begin{cases} (Q_{IJ}^{-1})_{ij}, & \text{if } ij = IJ, JI \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (69)$$

$$\begin{aligned} \frac{\partial \chi}{\partial \rho} &= \int_0^T dt \bar{\mathbf{v}}^{\dagger} \partial_t \mathbf{v}, \\ \frac{\partial \chi}{\partial C_{IJ}} &= - \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} C^{-1} \frac{\partial C}{\partial C_{IJ}} C^{-1} D^T \mathbf{v} + \int_0^T dt \bar{\boldsymbol{\sigma}}^{\dagger} C^{-1} \frac{\partial (C :: \Gamma)}{\partial C_{IJ}} \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell}, \\ \frac{\partial \chi}{\partial Q_{IJ}^{-1}} &= \int_0^T dt (D^T \bar{\mathbf{u}})^{\dagger} \left( C :: \frac{\partial \Gamma}{\partial Q_{IJ}^{-1}} \right) \left( \sum_{\ell=1}^L y_{\ell} \boldsymbol{\xi}_{\ell} \right), \end{aligned} \quad (70)$$

$$\Gamma = \begin{bmatrix} \frac{\rho\alpha^2 Q_\alpha^{-1}}{\rho\alpha^2} & \frac{\rho\alpha^2 Q_\alpha^{-1} - 2\rho\beta^2 Q_\beta^{-1}}{\rho\alpha^2 - 2\rho\beta^2} & \frac{\rho\alpha^2 Q_\alpha^{-1} - 2\rho\beta^2 Q_\beta^{-1}}{\rho\alpha^2 - 2\rho\beta^2} & 0 & 0 & 0 \\ \frac{\rho\alpha^2 Q_\alpha^{-1} - 2\rho\beta^2 Q_\beta^{-1}}{\rho\alpha^2 - 2\rho\beta^2} & \frac{\rho\alpha^2 Q_\alpha^{-1}}{\rho\alpha^2} & \frac{\rho\alpha^2 Q_\alpha^{-1} - 2\rho\beta^2 Q_\beta^{-1}}{\rho\alpha^2 - 2\rho\beta^2} & 0 & 0 & 0 \\ \frac{\rho\alpha^2 Q_\alpha^{-1} - 2\rho\beta^2 Q_\beta^{-1}}{\rho\alpha^2 - 2\rho\beta^2} & \frac{\rho\alpha^2 Q_\alpha^{-1} - 2\rho\beta^2 Q_\beta^{-1}}{\rho\alpha^2 - 2\rho\beta^2} & \frac{\rho\alpha^2 Q_\alpha^{-1}}{\rho\alpha^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\rho\beta^2 Q_\beta^{-1}}{\rho\beta^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\rho\beta^2 Q_\beta^{-1}}{\rho\beta^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\rho\beta^2 Q_\beta^{-1}}{\rho\beta^2} \end{bmatrix} \quad (86)$$

$$\frac{\partial \Gamma}{\partial Q_\alpha^{-1}} = \begin{bmatrix} 1 & \frac{\alpha^2}{\alpha^2 - 2\beta^2} & \frac{\alpha^2}{\alpha^2 - 2\beta^2} & 0 & 0 & 0 \\ \frac{\alpha^2}{\alpha^2 - 2\beta^2} & 1 & \frac{\alpha^2}{\alpha^2 - 2\beta^2} & 0 & 0 & 0 \\ \frac{\alpha^2}{\alpha^2 - 2\beta^2} & \frac{\alpha^2}{\alpha^2 - 2\beta^2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \frac{\partial \Gamma}{\partial Q_\beta^{-1}} = \begin{bmatrix} 0 & \frac{-2\beta^2}{\alpha^2 - 2\beta^2} & \frac{-2\beta^2}{\alpha^2 - 2\beta^2} & 0 & 0 & 0 \\ \frac{-2\beta^2}{\alpha^2 - 2\beta^2} & 0 & \frac{-2\beta^2}{\alpha^2 - 2\beta^2} & 0 & 0 & 0 \\ \frac{-2\beta^2}{\alpha^2 - 2\beta^2} & \frac{-2\beta^2}{\alpha^2 - 2\beta^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (87)$$

$$\partial_t \xi_\ell^{ij} + \omega_\ell \dot{\xi}_\ell^{ij} = \omega_\ell \dot{\epsilon}_{ij} \Rightarrow \dot{\xi}_\ell^{ij} = -\frac{1}{\omega_\ell} \partial_t \xi_\ell^{ij} + \dot{\epsilon}_{ij}, \quad (B8)$$

The online version of this paper has been corrected. The publisher apologise for these errors.