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Contribution

A symmetric positive definite (SPD) matrix P solution to the Lyapunov equation $PA + A^T P = Q$, with Q an arbitrary symmetric negative definite (SND) matrix, gives rise to a quadratic Lyapunov function $v(x) = x^T Px$, whose existence proves that the origin is an exponentially globally stable fixed point of the linear system $\dot{x} = Ax$. Solving the Lyapunov equation for the linearization of a nonlinear system can also prove the exponential stability of its fixed points. However, in this case the stability is local and the Lyapunov equation provides no information about the region of attraction.

We present an interval version of the Lyapunov equation, which allows investigating a given Lyapunov candidate function for nonlinear systems inside an explicitly given neighborhood, leading to rigorous estimates of the domain of attraction (EDA) of exponentially stable fixed points. These results are developed in the context of uncertain systems that can be non-smooth, e.g., systems with saturations. In a second step, this EDA is used to define a nonlinear optimization problem whose solution gives rise to the largest EDA (LEDA) that can be inferred using the given Lyapunov function.

A comparison with the state of the art for computing EDA based on Lyapunov functions will be presented. In particular, the proposed process is shown to outperform the approach based on sum-of-squares [1] implemented in SMRSOFT [2].

Example

We consider the following linear system with saturated input, investigated in [3]: $\dot{x}_1 = x_2$ and $\dot{x}_2 = x_1 + 5\max\{-1, \min\{1, -2x_1 - x_2\}\}$. Linearizing the system at the stable fixed point 0 and solving the corresponding

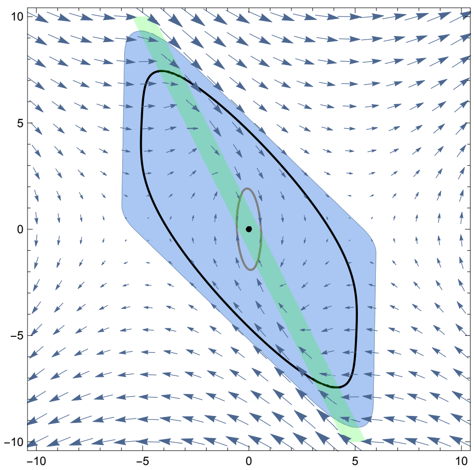

Figure 1: The vector field of the saturated linear system. In green, the non-saturated area. The following EDA are displayed: In blue the one from [3], which is the convex hull of two ellipsoids; the gray ellipsoid (resp. black curve) is the boundary of the LEDA computed with the quadratic (resp. degree 4) Lyapunov functions.

Lyapunov equation, we select the Lyapunov candidate function $v_2(u) = 1.28x_1^2+0.11x_1x_2+0.11x_2^2$. We also consider the degree 4 Lyapunov candidate function $v_4(x) = v_2(x) + \frac{1}{15}(x_1 + x_2)^4$, aiming at a larger EDA.

The results obtained are summarized in Figure 1: The LEDA obtained using the quadratic Lyapunov function is computed in $0.05s$ (the gray ellipsoid). It is smaller than the one obtained with the degree 4 Lyapunov function, which is computed in $0.2s$ (the black curve). The latter is smaller, but still competitive with the one computed in [3] (the blue area) using a technic dedicated to linear systems with saturations.

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