

A simple turbulent model for compressible flows

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Introduction

This work is a sequel of some previous work on one and two-equation turbulent models for single phase compressible flows (B. Audebert, C. Berthon, F. Coquel, A. Forestier, S.Gavrilyuk, X. Louis, E. Xeuxet, [1,2,3,4,6,8] among others...).

The main objective is to get a meaningful and cheap conservative model for turbulent industrial *unsteady* applications (with little information on boundary and/or initial conditions for the turbulent kinetic energy) .

Another goal is to obtain a rough way in order to account for Reynolds stress tensors in two-phase flow two-fluid models.

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Governing equations

In the following, $\rho(\mathbf{x}, t)$, $\mathbf{U}(\mathbf{x}, t)$, $P(\mathbf{x}, t)$ and $K(\mathbf{x}, t)$ respectively denote the mean density, the mean velocity, the mean pressure and the turbulent kinetic energy.

The governing set of equations is:

$$\begin{cases} \partial_t (\rho) + \nabla \cdot (\rho \mathbf{U}) = 0 ; \\ \partial_t (\rho \mathbf{U}) + \nabla \cdot (\rho \mathbf{U} \otimes \mathbf{U} + (P + 2K/3)\mathbf{Id}) = \nabla \cdot (\sigma^{\mathbf{v}}) ; \\ \partial_t (\rho E) + \nabla \cdot (\mathbf{U}(\rho E + P + 2K/3)) = \nabla \cdot (\sigma^{\mathbf{v}} : \mathbf{U}) . \end{cases} \quad (1)$$

Where we note :

$$\rho E = \rho (P, \rho) + \rho \mathbf{U} \cdot \mathbf{U} / 2 + K \quad (2)$$

and also:

$$K = K_0 \rho^{5/3} \quad (3)$$

Main properties of this set of PDEs in a 1D framework

We introduce the condensed form of (1) which reads:

$$\partial_t (W) + B(W)\partial_x (W) = 0 \quad (4)$$

and introduce $c(P, \rho)$ such that:

$$c^2(P, \rho) = \left(\frac{P}{\rho^2} - \partial_\rho (\epsilon(P, \rho)) \right) / (\partial_P (\epsilon(P, \rho)))$$

Property 1 (Hyperbolicity):

- In a 1D framework, system (1) is strictly hyperbolic:

$$\lambda_1 = U \quad , \quad \lambda_2 = U - \tilde{c} \quad , \quad \lambda_3 = U + \tilde{c} \quad (5)$$

where : $(\tilde{c})^2 = c^2 + 10K\tau/9$.

- The 1-field is LD and the 2 – 3- fields are GNL.

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Entropy inequality

We define $S(P, \rho)$ such that:

$$c^2(P, \rho) \partial_P (S(P, \rho)) + \partial_\rho (S(P, \rho)) = 0$$

Property 2 (Entropy inequality):

We note:

$$\begin{aligned} \eta &= \rho \text{Log}(S) \\ f_\eta &= \rho \text{Log}(S) U \end{aligned} \tag{6}$$

the entropy-entropy flux pair.

Solutions $W(x, t)$ of (1) comply with:

$$\partial_t (\eta) + \nabla \cdot (f_\eta) \leq 0 . \tag{7}$$

Jump conditions

Owing to the conservative form, we get jump conditions at once; noting σ the speed of the travelling discontinuity separating two physical states W_l and W_r , we have:

$$-\sigma[W]_l^r + [F(W)]_l^r = 0 \quad (8)$$

or:

$$\begin{aligned} [\rho v]_l^r &= 0 \\ \rho_l \rho_r ([v]_l^r)^2 &= [P + 2K/3]_l^r [\rho]_l^r \\ \rho v \left([\epsilon(P, \rho) + K\tau]_l^r + (\bar{P}_{l,r} + 2\bar{K}_{l,r}/3)[\tau]_l^r \right) &= 0 \end{aligned} \quad (9)$$

setting :

$$v = U - \sigma$$

and also:

$$[\psi]_l^r = \psi_r - \psi_l, \quad (\psi)_{l,r} = (\psi_r + \psi_l)/2.$$

Jump conditions (remark)

We focus on a γ -perfect gas EOS (with $\gamma > 1$). We recall that in that case, a shock wave separating two states Z_r and Z_l is such that:

$$(\beta)^{-1} \leq \frac{\rho^*_r}{\rho^*_l} \leq \beta$$

where: $\beta = \frac{\gamma+1}{\gamma-1}$.

We can derive the exact form of the EOS for the mean variables, that is:

$$p(P, \rho) = P/(\gamma - 1). \quad (10)$$

What about the shock wave separating mean variables W_l, W_r ?

Actually, the answer is, as expected:

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Solution of the 1D Riemann problem

Once more, we focus on a γ -perfect gas EOS (with $\gamma > 1$).

Property 3 (Solution of the 1D RP):

The Riemann problem associated with initial states $W(x < 0, t = 0) = W_L$ and $W(x > 0, t = 0) = W_R$ admits a unique self-similar solution:

$$W(x, t) = \omega(x/t)$$

with no vacuum occurrence, provided that initial left and right states $W_{L,R}$ are such that:

$$U_R - U_L < h(S_L, P_L) + h(S_R, P_R)$$

where:

$$h(S_{L,R}, P_{L,R}) = \int_0^{P_{L,R}} \frac{1}{\gamma P} \left(\gamma S_{L,R} (P/S_{L,R})^{\frac{\gamma-1}{\gamma}} + \frac{10K_0}{9} (P/S_{L,R})^{\frac{2}{3\gamma}} \right)^{1/2} dP$$

Intermediate states $\rho_{1,2}$ and $P_{1,2}$ are in the physical range.

Summary and perspectives

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- A simple turbulent model for single-phase flow applications, even if complex EOS are involved, with an interesting behaviour in shock waves;
- Approximate solutions using Finite Volume methods: exact Godunov scheme or approximate Godunov scheme (VFRoe-ncv using $U, P + 2K/3, S$ variables [5]);
- The solution of the 1DRP enables to derive suitable boundary conditions for computational purposes;
- A comparison with recent models for shallow water flows, proposed by G.L. Richard and S. Gavrilyuk, might be of interest [9,10];
- An extended version of the Baer-Nunziato model (Int. J. Multiphase Flow, 1986) has been recently derived (hyperbolic system, entropy inequality, unique jump conditions in single fields, see [7]).

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