Cooperation or non-cooperation in R&D: how should research be funded?

Marie-Laure Cabon-Dhersin, Romain Gibert

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Abstract

This article investigates two research funding policies in a cooperative and a non-cooperative R&D setting: subsidising private research (Spr) and subsidising public research (Spu). We show that R&D cooperation with subsidies (either Spr or Spu) always performs better than R&D cooperation with no subsidy. Furthermore, the Spr policy leads to better performance than the Spu approach does in terms of overall net surplus whether the firms cooperate or not in R&D. Nevertheless, comparing the two research funding policies for the same level of public spending shows that the Spu policy with R&D cooperation is in some cases more effective than the Spr policy, the latter becoming too costly for the government when spillovers are high.

Key words: R&D Cooperation, R&D spillovers, Knowledge public externalities, Subsidies, Public policy

Code JEL: C7, H2, H4, L3, L5, 03.

1 Introduction

Research and development activities benefit both the firms that undertake them and society as a whole because of the positive externalities produced. These externalities, also known as knowledge or technology spillovers, stem from the partial appropriation by rival firms of the knowledge generated by the investments of others. Although this is beneficial for society as a whole, the incentive for firms is to underinvest in R&D and adopt a free-rider strategy. The difference between the social and private benefits of innovation is widely acknowledged as a fact by both economists and politicians.

Governments have several public policy tools at their disposal to support R&D investments. Firstly of course, they can establish a judicial framework that protects innovation

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(patents, licences): firms are given a temporary monopoly during which to exploit their innovation. States can also offer financial support, directly (by funding selected firms) or indirectly (through fiscal incentives based on specific research criteria). Finally, cooperation agreements in R&D between potentially rival firms (horizontal agreements) increase R&D activity by internalising spillovers. In light of the social benefits of these deals, a large number of Western countries have weakened anti-trust regulations for firms signing R&D agreements (Treaty of Rome, 1957; exemption 85.3, National Cooperative Research and Production Act, 1984, 1993).

Over the past 15 years or so, political initiatives in many OECD countries have simultaneously combined several of these strategies to encourage innovation (Martin, 2016). Indeed, governments increasingly reserve subsidies for firms engaging in R&D collaboration. One example of this type of policy is the creation of *Pôles de compétitivité* (research clusters) in France since 2004, that bring firms and public bodies with similar research interests into close physical proximity to collaborate on R&D projects. In France between 2007 and 2011, 888 projects were funded in this way at a total cost of one billion euros, in partnership with local government agencies. However, there is nothing that would allow us to establish a priori that these political initiatives are always required to promote innovation. While there exists an empirical evidence supporting the impact of clusters on economic performance, there is scant evidence about the impact and role played by government cluster policies (Lehmann and Menter, 2017).

This article investigates from a theoretical point of view the effectiveness of a policy mix that combines cooperation and R&D funding policies. An interesting question concerns the choice of the type of research funding policy adopted by government depending on whether firms choose to cooperate or not in R&D.

Academic investigations in the industrial organisation literature have focussed on each R&D support measure separately. The most studied topic since the 1980s has probably been the mechanisms of R&D cooperation (Katz, 1986; d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; Brod and Shivakumar, 1997; Amir, 2000; Amir et al., 2002, 2003). This work indicates that below a certain level of technological appropriation (i.e. above a certain spillover threshold), cooperative R&D dominates over non-cooperative R&D activity. Furthermore, R&D investments increase with the amount of spillover when firms cooperate whereas it decreases when there is no cooperation. Analyses of the advantages of cooperation have been extended to include public-private partnerships (Poyago-Theotoky et al., 2002; Beath et al., 2003; Poyago-Theotoky, 2009). A number of empirical studies have shown that research produced by public laboratories or universities is beneficial for the private research sector (Jaffe, 1989; Cohen et al., 1994; Autant-Bernard, 2001; Audretsch et al., 2002; Veugelers and Cassiman, 2005; Bakhtiari and Breunig, 2018).

Secondly, a substantial strand of the literature has investigated public subsidies or fiscal incentives for innovation in the presence of R&D spillovers (Romano, 1989; Inci, 2009; Atallah, 2014; Heggedal, 2015). Among more recent empirical studies, Gelabert et al. (2009) have investigated the impact of public subsidies on R&D investments. The authors highlight the fact that public support for research helps firms that struggle to appropriate their research (i.e. those with high spillovers) more than it does those with a high level of appropriation (low spillovers). The latter thus use the subsidies to lower the cost of research that they would have in any case undertaken. This mechanism is known as an eviction effect. However, recent studies have not found clear evidence of a crowding-out effect (see, Becker (2015); Dimos and Pugh (2016)).

In spite of the current trend towards public policies encouraging the creation of research clusters as the primary means to stimulate innovation, there have been few investigations that bring together these two segments of the literature, namely research cooperation (between firms but especially between the public and private sectors) and subsidy allocation.[1] Hinloopen (1997, 2000, 2001) has investigated the theoretical effects of subsidies (financed by taxation) on R&D activity for cooperating and non-cooperating firms. On the basis of empirical work, Gussoni and Mangani (2010) argue that this effect is significantly stronger for cooperating firms in a context of low appropriation (high spillovers). However, the only theoretical work in which the role of public bodies in research is considered is the recent paper by Cabon-Dhersin and Taugourdeau (2017), which investigates the organization and the distribution of research activities between nearby public and private laboratories in a cooperative setting.

In this article, we will attempt to clarify the link between research funding policies and R&D cooperation between competing private firms involving the public research sector or not. To this end, we consider two research funding policies in a cooperative and a non-cooperative R&D setting. The first consists in providing an optimal subsidy for each unit invested by firms in private R&D (Spr). The second research policy (Spu) involves a public research sector body whose aim is to maximise social welfare. Instead of subsidising private sector research, the government funds public research optimally. Under the Spu approach, firms benefit freely from the public research body through public knowledge externalities that depend on a number of factors: geographical proximity, types of knowledge (basic vs applied research), and the capacity of firms to process and absorb external information. We will assume that the public and private research actors are not rivals in the production of innovation. They can operate in the same research areas but their motivations to produce knowledge (and the corresponding institutional arrangements) differ. We assume that the public research body does not generate a profit and can be seen as a means to support private innovation through the knowledge externalities it produces. Bio- and nano-technology companies are examples of firms that rely on public sector knowledge (Boufaden and Plunket, 2007).

In order to investigate the benefits and limits of these two research funding policies, we build on the seminal contribution of d'Aspremont and Jacquemin (1988) (AJ from now on). They consider a two-stage duopoly with homogeneous goods. In the first stage, the firms simultaneously set their R&D investments. The second stage involves Cournot competition in the product market. Although three different scenarios are considered in the AJ model, we retain only two: first, no cooperation in either of the two stages (NC scenario); second, cooperation in R&D and competition in the product market (C

[1]This point is emphasized by Marinucci (2012) in a survey on R&D cooperation.

scenario).[2] Following Kamien et al. (1992), R&D cooperation could involve cooperation in terms of cost sharing (a R&D cartel), information sharing (a RJV) or both (a RJV cartel). There is empirical evidence that one of the main objectives of R&D cooperation is to share the costs of R&D (Silipo, 2008), thus we will focus on R&D cartel (and RJV cartel when firms share the costs of R&D and their information completely). In our model, we introduce an active government which provides or not an optimal R&D subsidy before the R&D stage depending on whether firms decide to cooperate or not. In contrast with the AJ model, the choice to cooperate is not exogenously given and may emerge as the equilibrium of a multi-stage game as in Capuano and Grassi (2018); Marini et al. (2014). Thus, the time structure of the game unfolds as follow: in stage zero, firms decide whether to cooperate or not in R&D (signing an RJV agreement). In stage one, the government chooses to support or not either private sector research (Spr), or public sector research (Spu) through optimal R&D subsidies. In stage two, firms invest in R&D, while simultaneously (but only under the Spu approach), the public sector research sets its research effort so as to optimize social welfare. Firms play a standard Cournot game in the last stage. This theoretical framework allows the two research funding policies (Spr and Spu) to be compared directly in a cooperative or non-cooperative R&D setting (C and NC). It also enables us to investigate the effectiveness of R&D cooperation in the presence and absence of a research funding policy (Spr or Spu). The aim therefore is to understand what effect public funding policies sustaining R&D cooperation can have on the problem of underinvestment in innovation and social welfare.

In this context, three central questions are examined: The first is whether subsidising research (via Spr or Spu) yields better performance than does R&D cooperation with no subsidy. The second question is whether subsidising cooperative research (via Spr or Spu) is more effective than subsidising non-cooperative research. The third question is which of the Spr or the Spu policy is the most effective when firms decide to cooperate at equilibrium and when they do not.

The results of the analysis are that, regardless of whether the firms cooperate or not in R&D (NC or C), (i) the outcomes with research subsidies (Spr or Spu) are always better than without, (ii) social welfare is higher under the Spr than under the Spu policy, but (iii) for the same public expenditure, the Spu approach may be more effective in the cooperative scenario when spillovers are high. Comparing results in the cooperative and non-cooperative scenario for the two research policies (Spr and Spu) reveals that (iv) under the Spr policy, the outcomes in terms of R&D investment, consumer surplus and social surplus are the same whether the research subsidised is cooperative or not. Theses results suggest that sustaining R&D cooperation with high R&D subsidies to private sector research is a waste of public funds when spillovers are low. Nevertheless, R&D cooperation needs to be encouraged when spillovers are high under the Spu approach, which is more cost-effective.

The following section presents our analytical model, which is then used to obtain

[2] AJ also consider a third scenario (non-investigated in our analysis) in which firms cooperate in both stages.

equilibrium results for the two types of funding policies. Finally, in a third section, we compare and discuss the results before concluding.

2 The model

Consider two identical competing firms investing in upstream R&D, either cooperatively or non-cooperatively, to lower their production costs. They separately produce a homogeneous good. The demand side is described by a simple linear function:

$$P(Q) = a - Q$$

where the total level of production is $Q = q_1 + q_2$, with $Q \leq a$.

Following Kamien et al. (1992), the *effective* level of R&D, X_i, corresponds to the sum of its own R&D-output level, x_i, and part of firm j's, βx_j, with $\beta \in [0, 1]$ being the level of spillover. If $\beta = 0$, there is no free flow of information from firm i to firm j, whereas $\beta = 1$ implies that the firms share all their results. The *effective* level of R&D available to firm i is thus:

$$X_i = x_i + \beta x_j, \qquad i \neq j, i, j \in \{1, 2\}$$

This *effective* level of R&D reduces firm i's constant marginal cost c (process innovation),

$$C_i(q_i, x_i, x_j) = (c - x_i - \beta x_j)q_i, \qquad i \neq j, i, j \in \{1, 2\}$$

with $a > c > 0$ and $x_i + \beta x_j \leq c$.

Note that the R&D cost function is quadratic (leading to diminishing returns to R&D investment): $\gamma \frac{x_i^2}{2}$, with $\gamma > 0$.[3].
The profit function of firm i (in the absence of any funding for research) can be written:

$$\pi_i(q_i, q_j, x_i, x_j) = (a - q_i - q_j)q_i - (c - x_i - \beta x_j)q_i - \gamma \frac{x_i^2}{2} \tag{1}$$

with $i \neq j, i, j \in \{1, 2\}$

[3]The R&D cooperation models proposed by d'Aspremont and Jacquemin (1988) and Kamien et al. (1992) (KMZ) have been compared several times. In contrast with the KMZ model, AJ's does not consider diminishing returns for research when calculating spillovers, which are outside the research process. This tends to make the R&D process in the AJ model more productive and to increase the level of the equilibrium results. One can switch between the two models by replacing γ with $(1 + \beta)\gamma$ (see Amir (2000) pp. 1030-31, Hinloopen (2000)). Changing the R&D cost structure qualitatively does not affect any of the conclusions presented here. The corresponding calculations are available from the authors upon request.

2.1 The two research funding policies

We now introduce the two research funding policies. The first involves directly funding private research by allocating a subsidy for each unit of R&D output (Spr). In the second, public laboratories are subsidised (Spu) to stimulate private R&D through the knowledge externalities of the public research.

- *Subsidising private sector research: the Spr policy*

  This policy consists in subsidising the R&D-output level of each firm.[4] By spending $\gamma \frac{x_i^2}{2}$ on R&D, a firm can lower its costs by x_i, due to its own research spending and by an additional amount βx_j. Under the Spr approach, the government subsidizes the R&D level of each firm. Each firm receives a uniform subsidy s for each unit of R&D output: $S(x_i) = s.x_i$.

  Thus, the firms' profits include the subsidies received for R&D:

$$\pi_i(q_i, q_j, x_i, x_j) = P(Q)q_i - C_i(q_i, x_i, x_j) - \gamma \frac{x_i^2}{2} + s.x_i \quad i \neq j, i, j \in \{1, 2\} \qquad (2)$$

  The government subsidy is calculated so as to maximise the social welfare function. Social welfare, defined as the sum of consumer surplus, $CS = \frac{1}{2}Q^2$ and the firms' profits, net of the total amount paid to the private sector research by the government, is:

$$SW = \underbrace{\frac{Q^2}{2}}_{\text{Consumer surplus}} + \underbrace{(\pi_i + \pi_j)}_{\text{Producer surplus}} - \underbrace{s.(x_i + x_j)}_{\text{Cost of the R\&D subsidies}} \qquad (3)$$

  Given that the subsidy is simply a transfer from the government to the firms, its level vanishes from the welfare function (Equation 3). However, these subsidies affect social welfare and equilibrium outputs indirectly through the R&D level of each firm, which depends directly on the subsidy level.

- *Subsidising public sector research: the Spu policy*

[4] In practice, governments can offer firms direct support via grants, loans or procurement or can use fiscal incentives, such as R&D tax incentives (R&D tax credits, R&D allowances, reductions in R&D workers' income taxes and social security contributions, and accelerated depreciation of R&D capital). In our model, we do not distinguish between these different R&D and innovation support tools. We assume that the subsidy is provided based on the R&D output as in Hinloopen (1997, 2000, 2001); Gil-Molto et al. (2011); Gil-Moltó et al. (2018). We could consider the case in which the government subsidizes R&D expenditure $\left(s\gamma \frac{x_i^2}{2}\right)$, but this would not change our equilibrium results: the subsidy term disappears from the social welfare function. The relevant calculations are available from the authors upon request.

Here, we consider a policy of support for public research.[5] To this end, we introduce a public research body supplying a research output x_{PU}. By spending x_{PU}, the authorities can lower the firms' costs by αx_{PU} with $\alpha \in (0,1)$: the firms benefit indirectly from the public subsidy through the knowledge externalities αx_{PU} generated by the public research sector (see Equation 4). Firms and the public research body have different motivations: profit according to the market allocation mechanism for the private research sector; social welfare for the public research sector, its research being treated as a public good and benefiting the private sector. In a R&D stage, the public research body chooses a level of research x_{PU} to maximize welfare whereas the private firms decide on their R&D to maximize profits. This set-up is similar to some of the models used in the mixed markets literature (Gil-Molto et al., 2011; Gil-Moltó et al., 2018; Kesavayuth and Zikos, 2013).

The firm's profit function is now given by

$$\pi_i(q_i, q_j, x_i, x_j) = P(Q)q_i - (c - x_i - \beta x_j - \alpha x_{PU})q_i - \gamma\frac{x_i^2}{2} \quad i \neq j, i, j \in \{1, 2\} \quad (4)$$

with $c \geq x_i + \beta x_j + \alpha x_{PU}$.

Instead of subsidising private sector research, the government funds the public sector research in a welfare-optimal manner. This subsidy is calculated so as to maximise the social welfare function:

$$SW = \underbrace{\frac{Q^2}{2}}_{\text{Consumer surplus}} + \underbrace{(\pi_i + \pi_j)}_{\text{Producer surplus}} - \underbrace{s.(x_{PU})}_{\text{Funding of public research}} \quad (5)$$

Research costs under the Spu policy are treated differently from those under the Spr policy. Under the Spr approach, the subsidy cancels out when aggregating (Equation 3). This implies that the subsidy has no direct effect on social welfare. In contrast, under the Spu approach, the subsidy has a direct effect on welfare (Equation 5). The motivation of the public sector is very different from that of the private sector research: public laboratories set their research level in order to optimally support the private sector; the returns of public expenditures are ignored[6] and the research cost corresponds to the cost of the R&D subsidy (public spending) which will depend on the spillover level (β) and the knowledge externalities (α) at the equilibrium.

In order to investigate the effectiveness of the two research funding policies (Spr and Spu), we consider a benchmark case without subsidies (NS policy). The NS approach

[5] When governments support public research sector, the public money spent on R&D goes to universities and public research institutes (PRIs).

[6] This amounts to considering that the government funding public research does not evaluate the effectiveness of this research in terms of costs.

leads to similar equilibrium results of those obtained by d'Aspremont and Jacquemin (1988) (AJ model).[7] However, in contrast with the AJ model, the choice to cooperate is not exogenously given and may emerge as the equilibrium of a multi-stage game (see the timing of the game below). Table 1 summarizes the results of the NS policy (first column) to facilitate comparisons between the different policies.

2.2 The time structure of the game

To study the effects of the two research funding policies, in a cooperative and non-cooperative R&D setting, on innovation, firm profitability and welfare, we consider a multi-stage game with observable actions. The choice to cooperate is endogenous and can emerge as an equilibrium of the whole game. The time structure of the game is as follows (see also Fig.1):

- At stage $t = 0$, firms decide whether to cooperate or not in R&D. An R&D cooperation is profitable if each firm earns a higher profit than they would if they did not cooperate. By signing an R&D agreement, the firms commit to coordinating their R&D levels in order to maximize the sum of overall profits.

- At stage $t = 1$, the government chooses to support the research sector or not. By deciding to support research, it can (optimally) either subsidise private sector research (Spr), or subsidise public sector research (Spu).

- At stage $t = 2$, conditional on the availability or not of subsidies for private (Spr) or public (Spu) research, the firms choose how much to invest in R&D based on the possibility or not of cooperation. In the case of cooperative research, the firms form a R&D cartel and the level of spillovers is not affected by their cooperation.[8] Under the Spr approach, the two firms simultaneously set their R&D activities (x_{Spr}) either cooperatively or non-cooperative in the absence of the public research sector. Under the Spu approach, the two firms decide how much to invest in R&D, x_{Spu}, while simultaneously, the public laboratory sets its research output, x_{PU}, so as to optimise social welfare.

- At stage $t = 3$, the firms engage in Cournot competition. Firms determine their production non-cooperatively, conditional on the subsidy and their research activity (including that of the public sector under the Spu policy).

[7] As a reminder, the higher the spillover level is, the higher the cooperative effort in R&D is; conversely, non-cooperative R&D output decreases with β, which highlights the disincentive effect of technology leaks on research investments. R&D cooperation induces better performance (in terms of R&D levels, consumer surplus, profit and welfare) than R&D non-cooperation provided the spillovers are sufficiently high ($\beta > 0.5$).

[8] A RJV cartel is then a R&D cartel with $\beta = 1$. In this case, firms coordinate their R&D efforts and share their information completely. In our model, this form of cooperation can also be considered.

We assume that the two firms, the public research sector and the government know how their actions will affect those of all the other actors in the next stages of the game, which requires subgame perfection. We solve the game by backward induction for each research policy in a cooperative and non-cooperative R&D setting. The game is reduced to three stages when the subsidy level is zero (NS policy). As mentioned above, the well-known results (in terms of R&D-output level, profit, consumer surplus and welfare) for this case are listed in Table 1.

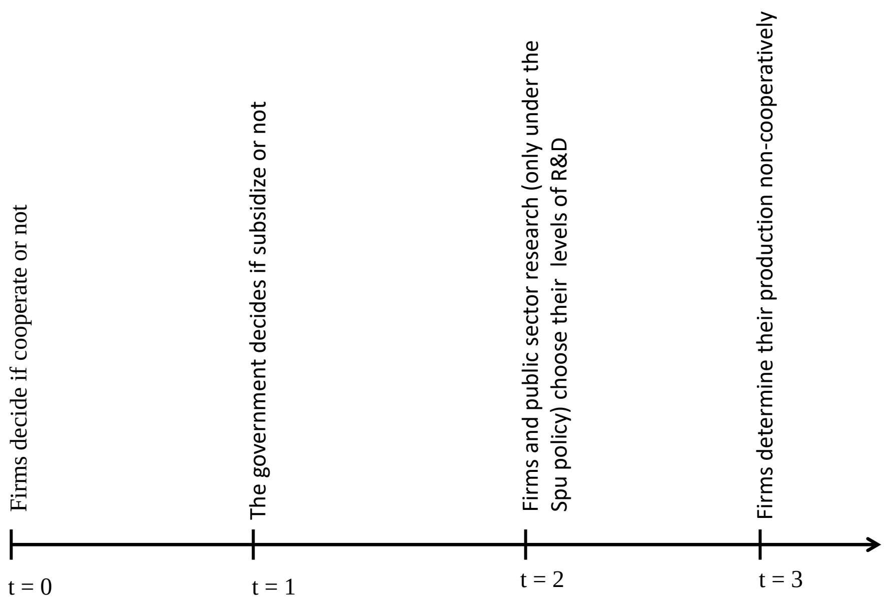

Figure 1: The time structure of the game

3 The equilibrium outcomes

In this section, we present the equilibrium outcomes for the two research policies as they arise in the non-cooperative and cooperative R&D subgames (the last stage of production remaining non-cooperative). This allows direct comparisons for each subgame of the performance of the different research funding policies (Spr, Spu and NS policies). Finally, considering the whole game, it also enables us to compare the cooperative and non-cooperative equilibrium results and to deduce whether firms decide to cooperate in R&D or not (at $t = 0$).

3.1 The non-cooperative subgame

If firms do not cooperate in R&D, there are three possible cases to consider: either the government chooses to not fund research ($s = 0$), or it adopts one of the two research funding policies (Spr or Spu). In the two last cases, it chooses the optimal R&D subsidy. Under each research funding policy, firms (and the public research sector under Spu) simultaneously and independently choose their levels of R&D and then engage in Cournot competition in the last stage.

We first present the equilibrium outcomes for the Spr policy before considering more extensively the Spu policy.

3.1.1 The Spr policy

Following Hinloopen (1997, 2000, 2001), it is easy to show that, in case of the Spr policy, the optimal level of subsidy is the following:

$$s_{Spr}^{NC} = \frac{3\gamma\beta}{4.5\gamma - 2(1+\beta)^2}(a-c) \tag{6}$$

The government chooses the value of the R&D subsidy that maximizes welfare (Equation 3):

$$SW_{Spr}^{NC} = 4(q_{Spr}^{NC})^2 - \gamma(x_{Spr}^{NC})^2$$

Substituting equilibrium R&D and quantities into the social welfare objective function (given in Appendix A) and solving the FOC with respect to s [9], we find the equilibrium R&D subsidy:

$$\frac{\partial SW_{Spr}^{NC}}{\partial s} = 8\underbrace{\frac{\partial q_{Spr}^{NC}}{\partial s}}_{>0} q_{Spr}^{NC} - 2\gamma \underbrace{\frac{\partial x_{Spr}^{NC}}{\partial s}}_{>0} x_{Spr}^{NC}$$

The subsidy increases social welfare by increasing the equilibrium output (q_{Spr}^{NC}) and thus consumer surplus. However, the subsidy decreases social welfare because of the social cost of the increase in R&D activity. The optimal level of subsidy balances out these two effects. This gives the values x_{Spr}^{NC}, Q_{Spr}^{NC}, π_{Spr}^{NC}, and SW_{Spr}^{NC} at the non-cooperative subgame-perfect Nash equilibrium. These are listed in Table (1).

3.1.2 The Spu policy

In the last stage of the game (solved by backward), each firm i chooses the output level that maximises its objective function (Equation 4), accounting for its rival's output and the public research R&D:

[9] The second order condition is that $4.5\gamma > 2(1+\beta)^2$.

$$q_i(x_i, x_j, x_{pu}) = \frac{a - c + (2-\beta)x_i + (2\beta - 1)x_j + \alpha x_{PU}}{3} \tag{7}$$

with $\quad i \neq j, i, j \in \{1, 2\}$

We can now carry out equilibrium calculations in the non-cooperative R&D stage and thus deduce the optimal subsidy.

In the R&D stage, we simultaneously determine the R&D investments of the two firms (for a given public research level) and the research activity of the public laboratory (for a given level of private R&D investment).

First of all, the profit equation from the previous stage,

$$\pi(x_i, x_j, x_{PU}) = (q_i(x_i, x_j, x_{PU}))^2 - \frac{\gamma}{2}x_i^2, \quad i \neq j, i, j \in \{1, 2\}$$

can be used to determine the level of R&D that maximises the two firms' profits.

The solution to this equation is unique and symmetric[10] since the two firms adopt the same behavior at equilibrium, i.e. $x_i = x_j = x_{Spu}$. The best-response function is thereby:

$$x_{Spu}^{NC} = \frac{(2-\beta)(a - c + \alpha x_{PU})}{4.5\gamma - (1+\beta)(2-\beta)} \tag{8}$$

Simultaneously, the welfare-optimal public research level for a given x_{Spu} satisfies:

$$\frac{\partial SW}{\partial x_{PU}} = 8\frac{\partial q}{\partial x_{PU}}q - s = 0$$

The best-response function of the public research body is then:

$$x_{PU} = \frac{9}{8\alpha^2}s - \frac{a-c}{\alpha} - \frac{(1+\beta)}{\alpha}x_{Spu} \tag{9}$$

The R&D-output levels in the third stage are obtained from the best-response functions (Equations (8) and (9)), as a function of the subsidy level:

$$x_{Spu}^{NC} = \frac{(2-\beta)}{4\alpha\gamma}s \tag{10}$$

$$x_{PU}^{NC} = \frac{4.5\gamma - (1+\beta)(2-\beta)}{4\alpha^2\gamma}s - \frac{a-c}{\alpha} \tag{11}$$

We can see that the presence of a public body does not solve the problem of encouraging private innovation when the level of spillover increases:

$$\frac{\partial x_{Spu}^{NC}}{\partial \beta} < 0 \quad \forall \alpha \in (0,1), \quad \beta \in [0,1]$$

[10] The second-order condition is similar to the non-cooperative case in the Spr approach: $4.5\gamma > (2-\beta)^2$.

Substituting Equations (10) and (11) into Equation (7) with $x^{NC}_{Spu} = x_i = x_j$ yields the output level:

$$q^{NC}_{Spu} = \frac{3}{8\alpha}s \tag{12}$$

At end of the third stage, the profit of both firms as a function of the R&D subsidy is:

$$\pi^{NC}_{pr}(s) = \left(q^{NC}_{Spu}(s)\right)^2 - \frac{\gamma}{2}\left(x^{NC}_{Spu}(s)\right)^2 = \frac{4.5\gamma - (2-\beta)^2}{32\alpha^2\gamma}s^2 \tag{13}$$

This already shows that the profits and the firms' outputs and R&D levels all increase with the subsidy level and decrease as public externalities (α) increase. This latter tendency can be explained by the public sector's investment choices as a function of α. For the public sector indeed, externalities discourage (encourage) research levels if and only if the subsidy is high (low):

$$\frac{\partial x^{NC}_{PU}}{\partial \alpha} < 0 \quad \Leftrightarrow \quad s > \frac{2\alpha\gamma}{4.5\gamma - (2-\beta)(1+\beta)}(a-c) = \bar{s}_{NC}$$

Public externalities have a dual effect on the public research investment. The first effect, positive, reflects the aim of the public research sector (to maximise social welfare). This aim translates into supporting private R&D through the effect of public externalities on lowering production costs. The second effect, negative, reflects the social cost of the public R&D level, which increases with s:

$$\frac{\partial x^{NC}_{PU}}{\partial \alpha} = \underbrace{-\frac{9}{4\alpha^3}s}_{<0} \quad + \quad \underbrace{\frac{(a-c) + (1+\beta)x^{NC}_{Spu}}{\alpha^2}}_{>0}$$

Consequently, if the negative effect is stronger, public externalities (α) discourage public research activities. Thus, high subsidy levels ($> \bar{s}_{NC}$) lead to an investment incentive problem in the public research sector.

In the second stage of the game, the optimal subsidy level is obtained by maximising the welfare function (Equation 5), namely (as a function of s):

$$SW^{NC}_{Spu}(s) = 4\left(q^{NC}_{Spu}(s)\right)^2 - \gamma\left(x^{NC}_{Spu}(s)\right)^2 - s.x^{NC}_{PU}(s)$$

Maximising social welfare in terms of s gives, [11]

$$\frac{\partial SW}{\partial s} = 8\frac{\partial q^{NC}_{Spu}}{\partial s}q^{NC}_{Spu} - 2\gamma\frac{\partial x^{NC}_{Spu}}{\partial s}x^{NC}_{Spu} - s\frac{\partial x^{NC}_{PU}}{\partial s} - x^{NC}_{PU} = 0$$

[11] The second-order condition is $4.5\gamma > (2-\beta)(1+2.5\beta)$.

and thus:

$$s_{Spu}^{NC} = \frac{4\alpha\gamma}{4.5\gamma - (2-\beta)(1+2.5\beta)}(a-c) > \bar{s}_{NC} \tag{14}$$

The optimal subsidy is greater than $\bar{s}_{NC}$, which implies that α has a negative effect on public research investments.

Substituting the optimal subsidy into the preceding equations gives the equilibrium results for the SPNE of the non-cooperative subgame listed in Table (1).

3.2 The cooperative subgame

The cooperative subgame involves cooperation in R&D, while the production stage remains non-cooperative. We study the cooperative equilibrium with subsidies going to the private research sector (Spr policy), then with subsidies going to the public research sector (Spu policy).

3.2.1 The Spr policy

As before, the optimal subsidy level, s_{Spr}^{C} can be obtained by maximising the social welfare function depending on the cooperative equilibrium R&D and quantities developed in Appendix A: [12]

$$\frac{\partial SW_{Spr}^{C}}{\partial s} = 8\frac{\partial {q_s}^{C}}{\partial s}q_s^{C} - 2\gamma\frac{\partial {x_s}^{C}}{\partial s}x_s^{C} = 0$$

such that:

$$s_{Spr}^{C} = \frac{\gamma(1+\beta)}{4.5\gamma - 2(1+\beta)^2}(a-c) \tag{15}$$

All the values at equilibrium are listed in column 2 of Table (1). The following subsection investigates the Spu policy when firms decide to cooperate.

3.2.2 The Spu policy

Under the Spu policy, the firms choose the R&D level that maximises the sum of profits accounting for the public research output:

$$\Pi_{Spu}^{C} = \sum_{i=1}^{2}\left\{\left(\frac{a-c+(2-\beta)x_i+(2\beta-1)x_j+\alpha x_{PU}}{3}\right)^2 - \frac{\gamma}{2}x_i^2,\right\} \tag{16}$$

$i \neq j, i, j \in \{1,2\}$.

Solving the game as before, we obtain:[13]

[12]The second-order condition is similar to the non-cooperative case: $4.5\gamma > 2(1+\beta)^2$.

[13]The second-order condition is similar to the one in the cooperative case under Spr: $4.5\gamma > (1+\beta)^2$.

$$x_{Spu}^{C} = \frac{(1+\beta)\left((a-c)+\alpha x_{PU}\right)}{4.5\gamma-(1+\beta)^2} \tag{17}$$

Equations (17) and (9), with a private R&D level x_{Spu}^{C}, give the R&D outputs as a function of the subsidy, s:

$$x_{Spu}^{C}(s) = \frac{(1+\beta)}{4\alpha\gamma}s \tag{18}$$

$$x_{PU}^{C}(s) = \frac{4.5\gamma-(1+\beta)^2}{4\alpha^2\gamma}s - \frac{a-c}{\alpha} \tag{19}$$

The level of output at the end of the third stage is then:

$$q_i(x_{Spu}^{C}, x_{PU}^{C}) = \frac{(a-c)+(1+\beta)x_{Spu}^{C}+\alpha x_{PU}^{C}}{3} \tag{20}$$

with $i = \{1, 2\}$

Comparing the results with and without cooperation (Equations 10 and 18) shows that the effect of the subsidy on the private research sector is stronger in the cooperative scenario (C), provided the spillovers are high enough:

$$\frac{\partial x_{Spu}^{C}}{\partial s} > \frac{\partial x_{Spu}^{NC}}{\partial s} > 0 \quad \text{if} \quad \beta > 0.5$$

However, from Equations 11 and 19, we obtain that the effect is opposite for the public research output:

$$\frac{\partial x_{PU}^{NC}}{\partial s} > \frac{\partial x_{PU}^{C}}{\partial s} > 0 \quad \text{if} \quad \beta > 0.5$$

For a given α, the subsidy has a greater effect on private R&D levels when the firms cooperate in R&D than when they do not, and less of an effect on public research activities when the spillover level is high. This policy therefore has opposite effects on the public and private sectors: when it favours private research (high spillovers) it hinders public research and vice versa.

For the public research output, the result is the same as in the non-cooperative subgame: public externalities discourage R&D investments if and only if the subsidy is above a certain threshold:

$$\frac{\partial x_{PU}^{C}}{\partial \alpha} < 0 \quad \Leftrightarrow \quad s > \frac{2\alpha\gamma}{4.5\gamma-(1+\beta)^2}(a-c) = \bar{s}_c$$

This threshold above which the incentive for the public sector is to reduce its R&D level is higher the stronger the public externalities are. The reasoning is similar to the one followed for the non-cooperative subgame. The threshold subsidy is higher in the cooperative case, $\bar{s}_c > \bar{s}_{nc}$, if and only if $\beta > 0.5$. Otherwise, $\bar{s}_c < \bar{s}_{nc}$.

Equations (20), (18) and (19), can be used to show that the output levels are the same as in the non-cooperative subgame (Equation 12), even though the research investments differ:

$$q^C_{Spu}(s) = \frac{3}{8\alpha}s$$

Here, we see that the public research effort compensates for the drop in private output between the NC and C cases, thereby maximising the total net output.

At the end of the second stage, the expression for the two firms' profits as a function of the R&D subsidy is:

$$\pi^C_{Spu}(s) = (q^C_{Spu}(s))^2 - \frac{\gamma}{2}(x^C_{Spu}(s))^2 = \frac{4.5\gamma - (1+\beta)^2}{32\alpha^2\gamma}s^2 \tag{21}$$

For a same level of subsidy s,

$$\pi^{NC}_{Spu}(s) > (<)\pi^C_{Spu}(s) \quad \text{if} \quad \beta > (<)0.5$$

which can be explained by a higher level of private investment in the cooperative R&D case when the spillovers are high enough (> 0.5) for a same output level ($q^C_{Spu}(s) = q^{NC}_{Spu}(s)$).

We can therefore calculate the optimal subsidy in terms of social welfare,[14]

$$\frac{\partial SW}{\partial s} = 8\frac{\partial q^C_{Spu}(s)}{\partial s}q^C_{Spu}(s) - 2\gamma\frac{\partial x^C_{Spu}(s)}{\partial s}x^C_{Spu}(s) - s\frac{\partial x_{PU}(s)}{\partial s} - x_{PU}(s) = 0$$

which is solved by:

$$s^C_{Spu} = \frac{4\alpha\gamma}{4.5\gamma - 1.5(1+\beta)^2}(a-c)$$

Substituting the optimal subsidy into the relevant equations gives the SPNE of the cooperative subgame shown in Table (1).

3.3 Preliminary comparisons for each subgame

At this stage of the analysis, we can advance the following proposition:

[14] The second-order condition is $4.5\gamma > 1.5(1+\beta)^2$.

Proposition 1 *At* $t = 1, \quad \forall \beta \in (0, 1]$

$$x_{Spr}^{NC} > x_{Spu}^{NC} > x_{NS}^{NC}, \quad and \quad x_{Spr}^{C} > x_{Spu}^{C} > x_{NS}^{C}$$

$$Q_{Spr}^{NC} > Q_{Spu}^{NC} > Q_{NS}^{NC}, \quad and \quad Q_{Spr}^{C} > Q_{Spu}^{C} > Q_{NS}^{C}$$

$$\pi_{Spr}^{NC} > \pi_{Spu}^{NC} > \pi_{NS}^{NC}, \quad and \quad \pi_{Spr}^{C} > \pi_{Spu}^{C} > \pi_{NS}^{C}$$

$$SW_{Spr}^{NC} > SW_{Spu}^{NC} > SW_{NS}^{NC}, \quad and \quad SW_{Spr}^{C} > SW_{Spu}^{C} > SW_{NS}^{C}$$

Proof: See Appendix B.

In the two subgames (NC and C), subsidizing research (under Spr or Spu) leads to better performance than the benchmark case without subsidies (NS policy) does when externalities are positive. When there are no spillovers and no public externalities, government intervention is unnecessary if firms compete in R&D (see lemma 1). The optimal R&D subsidy increases with the spillover (public externality) rate under the Spr (Spu) policy. For the two research funding policies, regardless of whether firms cooperate or not in R&D, the optimal R&D subsidy increases the R&D, output, profits and net total surplus. Proposition 1 is explained by the incentives for investing in R&D. Whatever the research policy adopted, the optimal R&D subsidy leads the firms (and the public sector research under Spu) to choose the optimal research level, which is always socially beneficial. On the other hand, the Spr policy seems preferable in terms of total surplus (consumer and producer surplus) compared with the Spu policy. The following section compares the two public policies when firms decide to cooperate or not at $t = 0$.

Lemma 1 *At t=1,*

(i) $s_{Spr}^{NC,C} > 0$ *if* $\beta \in (0, 1]$ *and* $s_{Spu}^{NC,C} > 0$ *if* $\alpha \in (0, 1]$

(ii) $s_{Spr}^{NC} = 0, s_{Spr}^{C} > 0$ *if* $\beta = 0$ *and* $s_{Spu}^{NC,C} = 0$ *if* $\alpha = 0$

(iii) $\frac{\partial s_{Spr}^{NC,C}}{\partial \beta} > 0$ *and* $\frac{\partial s_{Spu}^{NC,C}}{\partial \alpha} > 0$

Proof: The proof is straightforward (see Table (1)). □

As proposed by Atallah (2014), it may be interesting to compare the cost of the subsidy (SC) (or of public spending) for each policy in the two subgames (i.e. the non-cooperative and the cooperative one). By comparing the amount of public aid provided by the government under both policies in both subgames, we obtain that:

Lemma 2 *At t=1,* $\forall \alpha, \beta \in (0, 1]$,

(i)

$$SC_{Spr}^{NC} = s_{Spr}^{NC}.(2x_{Spr}^{NC}) > SC_{Spu}^{NC} = s_{Spu}^{NC}.x_{PU}^{NC}$$

(ii)

$$SC^C_{Spr} = s^C_{Spr}.(2x^C_{Spr}) > SC^C_{Spu} = s^C_{Spu}.x^C_{PU}$$

Proof: Appendix C. □

Lemma 2 states that whether the firms cooperate or not, directly encouraging private R&D investment via an optimal R&D subsidy is more costly for the government than is supporting the public research sector to generate knowledge externalities for the private research sector.

3.4 The cooperative equilibrium of the whole game

At $t = 0$, firms compare the profits associated with the two SPNE of the alternative subgames (i.e. the non-cooperative and the cooperative one). An R&D cartel (or RJV cartel when $\beta = 1$) is profitable if firms earn a profit higher than they would if they did not cooperate. By signing an R&D agreement, firms commit to coordinate their R&D activity so as to maximize the sum of profits. This leads to the following proposition:

Proposition 2 *At $t = 0$,*

(i) $\forall \beta \in (0, 0.5)$*, R&D cooperation can only emerge as part of the SPNE under the Spr policy:*

$$s^C_{Spr} > s^{NC}_{Spr} \quad \textit{and} \quad \pi^C_{Spr} > \pi^{NC}_{Spr}$$
$$s^C_{Spu} < s^{NC}_{Spu} \quad \textit{and} \quad \pi^C_{Spu} < \pi^{NC}_{Spu}$$

(ii) $\forall \beta \in (0.5, 1]$*, R&D cooperation can only emerge as part of the SPNE under the Spu policy:*

$$s^C_{Spr} < s^{NC}_{Spr} \quad \textit{and} \quad \pi^C_{Spr} < \pi^{NC}_{Spr}$$
$$s^C_{Spu} > s^{NC}_{Spu} \quad \textit{and} \quad \pi^C_{Spu} > \pi^{NC}_{Spu}$$

Proof: Appendix D. □

Proposition 2 explains the firms' decisions at $t = 0$. Under both research funding policies, an increase in the R&D optimal subsidy s for each unit of R&D output encourages firms to cooperate in R&D. When spillovers are low ($\beta < 0.5$), R&D cooperation only emerges as part of the equilibrium of the whole game under the Spr policy. In order to encourage firms to invest substantially in R&D, the optimal subsidy is higher when firms decide to cooperate. Thus, the optimal subsidy has a positive effect on the firms' R&D investments and boosts firms' profits. By contrast, the government needs to provide more support for non-cooperative R&D investments, through higher subsidies s, when spillovers are high. Since non-cooperating firms benefit more from this R&D subsidy, their profits are higher under non-cooperative than cooperative R&D when spillovers are high. Under the Spu approach, as described above, the positive effect of the subsidy on the firms' R&D activity is stronger when firms cooperate in R&D provided spillovers are high enough ($\beta > 0.5$). Whatever the level of public externalities α, it therefore becomes more efficient socially to increase the R&D subsidy when firms coordinate their R&D activities and spillovers are high. Since cooperating firms benefit indirectly more from the subsidy under Spu for high spillovers, the profit in the cooperative R&D scenario is higher than in the non-cooperative one and firms prefer to cooperate at equilibrium. From Proposition 1, cooperative strategies outperform non-cooperation in terms of R&D investments, consumer surplus and welfare provided the spillover level is high. We conclude that R&D cooperation under the Spu policy needs to be encouraged to stimulate R&D activity with a low level of appropriation (high spillovers).

4 Comparing the two funding policies

We now look to compare the two public policies, Spr and Spu, whose objective is to encourage innovation and improve social welfare. First, we compare social welfare levels under the two policies before comparing the improvements in social welfare for the same public expenditure.

4.1 Comparison between the effectiveness of the two research policies

The result presented in Proposition 3 shows that subsidizing private sector innovation is more effective than subsiding public research is in terms of social welfare level (sum of producer and consumer surplus, net of subsidy payment).

Proposition 3 $\forall \alpha \in (0, 1]$, *at equilibrium,*

(i) $\forall \beta \in (0, 0.5)$, *firms only cooperate in R&D under Spr:*

$$SW_{Spr} > SW_{Spu}^{NC} > SW_{NS}^{NC}$$

(ii) for $\beta = 0.5$, *firms may cooperate or not in R&D:*

$$SW_{Spr} > SW_{Spu}^{NC} = SW_{Spu}^{C} > SW_{NS}^{NC} = SW_{NS}^{C}$$

(iii) $\forall \beta \in (0.5, 1]$, *firms only cooperate in R&D under Spu (and NS):*

$$SW_{Spr} > SW_{Spu}^{C} > SW_{NS}^{C}$$

with $SW_{Spr} = SW_{Spr}^{NC} = SW_{Spr}^{C}$

Proof: Appendix E. □

Subsidizing public research (Spu) leads to better performance in terms of overall net surplus than providing no research support does (NS policy, with or without R&D cooperation). But, we also show that social welfare is always higher under the Spr than under the Spu policy, whatever the levels of externality. However, even if the Spr policy seems preferable in terms of total surplus, the success of this policy does not rely on the firms cooperating. Indeed, the optimal R&D subsidy leads to the same levels of R&D investment, consumer surplus and social welfare regardless of the firms' behavior in the R&D stage. Hinloopen (1997, 2000) reached the same result considering a subsidy financed by a tax on profits. This implies that financing by taxation does not alter the results obtained here in terms of R&D investments and equilibrium outputs. However, our results differ in terms of the equilibrium profit: while with a tax-financed subsidy, the firms are indifferent to the presence (or absence) of cooperation, this is no longer the case in our model. Proposition 2 shows that cooperative strategies will not be adopted by subsidized firms (because of their low profitability), when the spillover level is high (> 0.5). By contrast, firms choose to cooperate in R&D to take advantage of higher levels of R&D subsidy when spillovers are low. The cost of the subsidy to the government ($s^C_{Spr}.x_{Spr}$ paid for each firm) is higher than when the firms do not cooperate in R&D ($s^{NC}_{Spr}.x_{Spr}$ paid for each firm) (see Lemma 2), despite the fact that it leads to the same levels of innovation and welfare as non-cooperative R&D. Consequently, the government could increase innovation and welfare just as much with a lower subsidy. This result suggests that sustaining R&D cooperation with an optimal subsidy to private sector research is costly, particularly when spillovers are low. Moreover, the success of this policy does not rely on R&D cooperation between firms because cooperation and non-cooperation during the R&D stage lead to the same results.

4.2 Comparison between the cost-effectiveness of the two research policies

Thus far we have assumed that the public budget is unlimited. Because the impacts per unit of money spent on the two alternative research funding policies differ in nature and intensity, it is useful to examine and compare the two policies for the same level of public investment.[15] By how much is social welfare increased for each additional unit of public money invested? Is it more effective to give this money to the private or to public sector research? We investigate these issues by measuring the improvement in social welfare under the two alternative policies (with respect to the benchmark case without subsidies) in relation to the R&D cost to the government (public spending, SC, see Lemma 2). Proposition 4 enables us to compare the cost-effectiveness of the two research funding policies when firms choose to cooperate in R&D or not at equilibrium:

[15] We would like to thank the referee for this suggestion.

Proposition 4 $\forall \alpha \in (0, 1]$,

(i) $\forall \beta \in (0, 0.5]$,

$$Eff_{Spr} = \frac{SW_{Spr} - SW_{NS}^{NC}}{SC_{Spr}^{C}} \geq \frac{SW_{Spu}^{NC} - SW_{NS}^{NC}}{SC_{Spu}^{NC}} = Eff_{Spu}$$

$$\text{iff} \quad h(\gamma, \beta) = 4.5\gamma 3\beta X - (2-\beta)(1+\beta)^2 Y \geq 0$$

(ii) $\forall \beta \in [0.5, 1]$,

$$Eff_{Spr} = \frac{SW_{Spr} - SW_{NS}^{C}}{SC_{Spr}^{NC}} \geq \frac{SW_{Spu}^{C} - SW_{NS}^{C}}{SC_{Spu}^{C}} = Eff_{Spu}$$

$$\text{iff} \quad f(\gamma, \beta) = 4.5\gamma X - 3\beta(1+\beta) W \geq 0$$

with $X = 4.5\gamma - 2(1+\beta)^2 > 0$, $Y = 4.5\gamma - (2-\beta)(1+2.5\beta) > 0$ *and* $W = 4.5\gamma - 1.5(1+\beta)^2 > 0$.

Proof: Appendix F. □

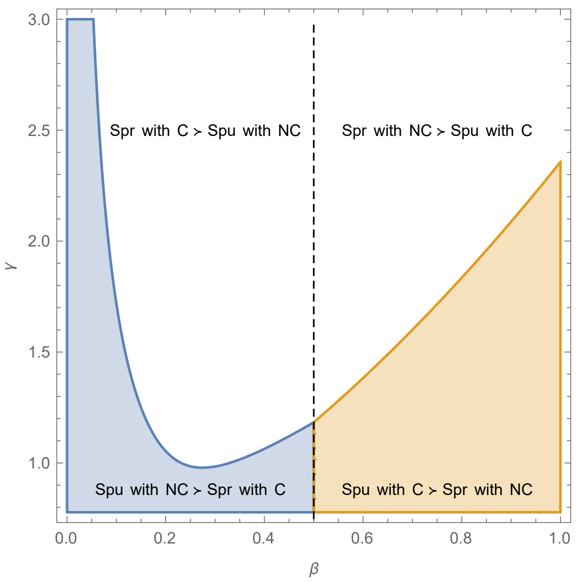

Figure 2: Cost-effectiveness of Spr vs Spu policies for different β and γ.

Figure 2 compares the effectiveness of the two policies for the same level of public expenditure for a set of feasible (γ, β) combinations. The feasible range consists of four regions, whose boundaries are defined by the conditions in Proposition 4.[16] We show that the effectiveness of the research policy is sensitive to changes in the parameter of the firms' R&D cost function (γ) and the spillover level (β). When β is neither too high nor too low, the Spr approach is more effective than the Spu approach in terms of improving social welfare for the same level of public investment. When the R&D efficiency is high (low γ), the Spu policy is more effective than the Spr policy. High spillovers increase the effectiveness of the Spu policy with R&D cooperation. Symmetrically, R&D non-cooperation makes the Spu policy more effective than the Spr one when spillovers are low. The reason for this is simple: when spillovers are low, the welfare improvement from the Spr policy does not offset the high cost of the subsidy to the government when firms cooperate. On the other hand, when spillovers increase (with $\beta > 0.5$), the rising cost SC in β to stimulate non-cooperative R&D investments under the Spr policy counteracts the gains in social welfare. Consequently, for high spillovers and a sufficiently low γ, the Spu policy with R&D cooperation can be more effective.

5 Conclusion

This paper investigates and compares the effects of two widely employed R&D stimulation policies within the same framework: providing R&D subsidies directly to firms (Spr), and stimulating private innovation indirectly by funding public research (Spu). The Spr policy consists in providing optimal R&D subsidies in a competitive market. The Spu policy involves a public research sector body whose aim is to maximise social welfare by generating public knowledge externalities for the private research sector (for both competing firms). The only motivation for the public research body is to maximise social welfare in the knowledge that its research is treated as a public good and will thereby stimulate private research.

Our theoretical approach allows us to compare the effectiveness of a policy mix that combines the Spr or the Spu approach with R&D cooperation. Martin (2016) highlights the importance of investigating how different R&D policy instruments interact to optimise the policy mix: "There seems to be little literature on policy choice and policy mix with regard to R&D policy". Our work opens the way for simultaneous investigations of R&D funding (Spr or Spu) and cooperation policies and compares the two research funding policies when the firms' choice to cooperate in R&D is endogenous.

Our first contribution is that combining the Spu policy with R&D cooperation is strictly more effective than R&D cooperation alone is in a context of high spillovers. The presence of a public research body in a cooperative setting leads to better results in terms of all criteria compared with just supporting R&D cooperation. But Proposition 3 shows also that this is not necessarily the case for the Spr policy: the success of the Spr policy does not hinge on the presence of cooperation during the R&D stage. Cooperation and non-cooperation perform equally well and lead to the same level of R&D investment,

[16] We can trace the functions $h(\gamma, \beta) = 0$ and $f(\gamma, \beta) = 0$ as a function of the two parameters γ and β.

the same consumer surplus and the same level of social welfare. Under the Spr policy, subsidizing R&D cooperation thus appears pointless in terms of stimulating innovation. According to Hinloopen (1997, 2000, 2001), the presence of a tax on profits does not alter this result. However, the implementation of other tax/subsidy schemes (a sales tax for example) may well modify the conditions under which the firms operate. In summary, it is unclear whether sustaining R&D cooperation is worthwhile or not in this context (Spr) and any policy recommendations in this regard should therefore be considered with care.

The second contribution of this study is showing that subsidizing private sector innovation is more effective than subsidizing public research is in terms of social welfare (Proposition 3). This result should however be nuanced by comparing the effectiveness of the two funding research policies for the same level of public spending. The policy implications of the comparison between the performance of the two policies are then not so clear-cut. For the same expense, mixing Spu and R&D-cooperation-encouraging policies produces better results provided spillovers are high and γ is sufficiently low. The Spr policy becomes more effective when the level of spillovers is neither too high nor too low. Higher spillovers do indeed lead to greater improvements in social welfare under the Spr policy than under the Spu policy, but also increase the cost of the subsidy for the government. An important result for policy-makers is that the cost for the government of the Spu policy is always lower than that of the Spr policy. In conclusion, policy-makers need to choose the most effective research funding policy given their budgetary constraints.

Our analysis can be seen as a first step towards understanding how R&D policy instruments interact with one another and what effects they have on the incentives that drive R&D investment. The theoretical basis of this article is the assumption that public sector research is a means of supporting private R&D efforts. Further development of our research includes the possibility for research public bodies to sell their R&D output. It could also be worthwhile to consider that the public support is split between the public and private sector research. New theoretical insights are still needed to better assess the performance of public policies to support research efforts.

Appendix A The Spr policy: the equilibrium outcomes

We first calculate the equilibrium outcomes under the Spr policy in the non-cooperative subgame before considering the cooperative subgame.
In the last stage, each firm i chooses the output that maximises its objective function (Equation 2), including the output of its rival. The first and second order conditions yield Equation (Appendix A).

$$q_i(x_i, x_j) = \frac{a - c + (2 - \beta)x_i + (2\beta - 1)x_j}{3}$$

1) *The non-cooperative subgame*
In the third stage, the firms choose the R&D level that maximises their profit:

$$\pi_i(x_i, x_j) = (q_i(x_i, x_j))^2 - \frac{\gamma}{2}x_i^2 + s.x_i, \quad i \neq j, i, j \in \{1, 2\}$$

The solution of this equation is unique and symmetric, such that the two firms choose the same amount in R&D, i.e. $x_i = x_j = x_{Spr}^{NC}$, and have the same equilibrium level of output, q_{Spr}^{NC}: [17]

$$x_{Spr}^{NC} = \frac{(2 - \beta)(a - c) + 4.5s}{4.5\gamma - (2 - \beta)(1 + \beta)}$$

$$q_{Spr}^{NC} = \frac{1.5(\gamma(a - c) + (1 + \beta)s)}{4.5\gamma - (2 - \beta)(1 + \beta)}$$

2) *The cooperative subgame*
In the third stage, the firms coordinate their R&D levels to maximise the sum of profits:

$$\Pi_{Spr}^{C} = \sum_{i,j=1}^{2} \left\{(q_i(x_i, x_j))^2 - \frac{\gamma}{2}x_i^2 + s.x_i\right\}, \quad i \neq j$$

The symmetric solution, $x_{Spr}^{C} = x_i = x_j$, gives a unique equilibrium solution for the coordination of R&D investments as a function of s, [18]

$$x_{Spr}^{C} = \frac{(1 + \beta)(a - c) + 4.5s}{4.5\gamma - (1 + \beta)^2}$$

from which the two firms' output as a function of s can be deduced:

$$q_{Spr}^{C} = \frac{1.5(\gamma(a - c) + (1 + \beta)s)}{4.5\gamma - (1 + \beta)^2}$$

[17] Second-order conditions require that $4.5\gamma > (2 - \beta)^2$.
[18] The second-order condition is $4.5\gamma > (1 + \beta)^2$.

Appendix B Proof of Proposition 1

1) Comparisons between Spr and Spu policy:

$\forall\beta \in (0,1]$ and $4.5\gamma > 2(1+\beta)^2$ (the second-order condition), we remark that

$$\begin{cases} Y = 4.5\gamma - (2-\beta)(1+2.5\beta) > 4.5\gamma - 2(1+\beta)^2 = X \\ \Leftrightarrow (2-\beta)(1+2.5\beta) < 2(1+\beta)^2 \Leftrightarrow 3.5\beta^2 > 0 \\ W = 4.5\gamma - 1.5(1+\beta)^2 > 4.5\gamma - 2(1+\beta)^2 = X > 0 \end{cases}$$

Hence, from Table (1),

$$x_{Spr}^{NC} = x_{Spr}^{C} = \frac{2(1+\beta)}{X}(a-c) > \frac{2-\beta}{Y}(a-c) = x_{Spu}^{NC}$$

$$x_{Spr}^{NC} = x_{Spr}^{C} = \frac{2(1+\beta)}{X}(a-c) > \frac{(1+\beta)}{W}(a-c) = x_{Spu}^{C}$$

$$Q_{Spr}^{NC} = Q_{Spr}^{C} = \frac{3\gamma}{X}(a-c) > \frac{3\gamma}{Y}(a-c) = Q_{Spu}^{NC}$$

$$Q_{Spr}^{NC} = Q_{Spr}^{C} = \frac{3\gamma}{X}(a-c) > \frac{3\gamma}{W}(a-c) = Q_{Spu}^{C}$$

Since $4(1+\beta)(1-2\beta) < (2-\beta)^2$, (or $9\beta^2 > 0$) and $X < Y$ we deduce

$$\pi_{Spr}^{NC} = \frac{\gamma(4.5\gamma - 4(1+\beta)(1-2\beta))}{2X^2}A^2 > \frac{\gamma(4.5\gamma - (2-\beta)^2)}{2Y^2}A^2 = \pi_{Spu}^{NC}$$

Since $X < Y$ and $X < W$ we deduce

$$\pi_{Spr}^{C} = \frac{4.5\gamma^2}{2X^2}A^2 > \frac{\gamma(4.5\gamma - (1+\beta)^2)}{2W^2}A^2 = \pi_{Spu}^{C}$$

and

$$SW_{Spr}^{NC} = SW_{Spr}^{C} = \frac{2\gamma}{X}(a-c)^2 > \frac{2\gamma}{Y}(a-c)^2 = SW_{Spu}^{NC}$$

$$SW_{Spr}^{NC} = SW_{Spr}^{C} = \frac{2\gamma}{X}(a-c)^2 > \frac{2\gamma}{W}(a-c)^2 = SW_{Spu}^{C}$$

2) Comparisons with NS policy:

From Table 1, $\forall\beta \in (0,1]$, since $\Omega = 4.5\gamma - (2-\beta)(1+\beta) > Y > 0$ and $\Gamma = 4.5\gamma - 2(1+\beta)^2 > W > 0$, we remark that:

$$x_{Spu}^{NC} > x_{NS}^{NC}, \quad x_{Spu}^{C} > x_{NS}^{C}$$

$$Q_{Spu}^{NC} > Q_{NS}^{NC}, \quad Q_{Spu}^{C} > Q_{NS}^{C}$$

$$\pi_{Spu}^{NC} > \pi_{NS}^{NC}, \quad \pi_{Spu}^{C} > \pi_{NS}^{C}$$

Now, we calculate the differences

$$SW_{Spu}^{NC} - SW_{NS}^{NC} = \frac{2\gamma A^2}{4.5\gamma - (2-\beta)(1+2.5\beta)} - \frac{\gamma(9\gamma - (2-\beta)^2)A^2}{[4.5\gamma - (2-\beta)(1+\beta))^2}$$

$$SW_{NS}^{C} - SW_{SPU}^{C} = \frac{\gamma(9\gamma - (1+\beta)^2)A^2}{(4.5\gamma - (1+\beta)^2)^2} - \frac{2\gamma A^2}{4.5\gamma - 1.5(1+\beta)^2}$$

It can be easily checked that,

$$\text{sign}(SW_{Spu}^{NC} - SW_{NS}^{NC}) = \text{sign}((2-\beta)^2(4.5\beta^2)) > 0$$
$$\text{sign}(SW_{Spu}^{C} - SW_{NS}^{C}) = \text{sign}(0.5(1+\beta)^4) > 0$$

which holds for any $\beta, \alpha \in (0,1]$. We deduce that

$$SW_{Spu}^{NC} > SW_{NS}^{NC}$$

and

$$SW_{Spu}^{C} > SW_{NS}^{C}$$

As a result from 1) et 2)

$$x_{Spr}^{NC} > x_{Spu}^{NC} > x_{NS}^{NC} \quad \text{and} \quad x_{Spr}^{C} > x_{Spu}^{C} > x_{NS}^{C}$$

$$Q_{Spr}^{NC} > Q_{Spu}^{NC} > Q_{NS}^{NC} \quad \text{and} \quad Q_{Spr}^{C} > Q_{Spu}^{C} > Q_{NS}^{C}$$

$$\pi_{Spr}^{NC} > \pi_{Spu}^{NC} > \pi_{NS}^{NC} \quad \text{and} \quad \pi_{Spr}^{C} > x_{Spu}^{C} > \pi_{NS}^{C}$$

$$SW_{Spr}^{NC} > SW_{Spu}^{NC} > SW_{NS}^{NC} \quad \text{and} \quad SW_{Spr}^{C} > SW_{Spu}^{C} > SW_{NS}^{C}$$

Appendix C Proof of Lemma 2

From Table (1), $\forall\beta \in (0,1]$, we compare the cost of the R&D subsidy to government. Since $X < W$ and $X < Y$:

$$SC_{Spr}^{NC} = 2x_{Spr}^{NC}.s_{Spr}^{NC} = 6\gamma\beta\frac{2(1+\beta)}{X^2}(a-c)^2 > 6\gamma\beta\frac{(2-\beta)}{Y^2}(a-c)^2 = x_{PU}^{NC}.s_{Spu}^{NC} = SC_{Spu}^{NC}$$

$$SC_{Spr}^{C} = 2x_{Spr}^{C}.s_{Spr}^{C} = 4\gamma\frac{(1+\beta)^2}{X^2}(a-c)^2 > 2\gamma\frac{(1+\beta)^2}{W^2}(a-c)^2 = x_{PU}^{C}.s_{Spu}^{C} = SC_{Spu}^{C}$$

Appendix D Proof of Proposition 2

From Table (1), $\forall \alpha \in (0,1]$,

(i) Under the Spr policy,

$$\pi^{C}_{Spr} > \pi^{NC}_{Spr}, \quad \forall \beta \in (0, 0.5) \quad \text{and} \quad \pi^{C}_{Spr} < \pi^{NC}_{Spr}, \quad \forall \beta \in (0.5, 1]$$

(ii) Under the Spu policy,

$$\pi^{C}_{Spu} < \pi^{NC}_{Spu}, \quad \forall \beta \in (0, 0.5) \quad \text{and} \quad \pi^{C}_{Spu} > \pi^{NC}_{Spu}, \quad \forall \beta \in (0.5, 1]$$

We deduce that firms cooperate in R&D under Spr Policy for $\beta < 0.5$ and under Spu policy for $\beta > 0.5$.

Appendix E Proof of Proposition 3

From Proposition 1, we verify that $SW^{NC}_{Spu} > SW^{NC}_{NS}$ and $SW^{C}_{Spu} > SW^{C}_{NS}$, $\forall \beta, \alpha \in (0,1]$.

From Appendix B, we have
$\forall \beta \in (0,1]$, $X < Y$ and $X < W$ and for $\beta = 0.5$, $Y = W$.
From Table (1), $SW^{NC}_{Spr} = SW^{NC}_{Spr} = SW_{Spr} = \frac{2\gamma}{X} A^2$.

When $\beta < 0.5$, firms cooperate in R&D under Spr but doesn't cooperate under Spu.
Hence

$$SW_{Spr} = \frac{2\gamma}{X} A^2 > \frac{2\gamma}{Y} A^2 = SW^{NC}_{Spu} > SW^{NC}_{NS}$$

When $\beta = 0.5$, firms are indifferent between cooperation and non-cooperation in R&D whatever the research policy.
Hence

$$SW_{Spr} > SW^{NC}_{Spu} = SW^{C}_{Spu} > SW^{NC}_{NS} = SW^{C}_{NS}$$

When $\beta > 0.5$, firms cooperate in R&D under Spu but doesn't cooperate under Spr.
Hence

$$SW_{Spr} > \frac{2\gamma}{W} A^2 = SW^{C}_{Spu} > SW^{C}_{NS}$$

Appendix F Proof of Proposition 4

From Table (1), we calculate the differences between the ratios of effectiveness under the two approach (Spr and Spu) at the SPNE:
1) $\forall \beta \leq 0.5$,

$$\text{Eff}_{Spr} = \frac{SW^{C}_{Spr} - SW^{NC}_{NS}}{2 s^{C}_{Spr} x^{C}_{Spr}} = \frac{\frac{2\gamma A^2}{4.5\gamma - 2(1+\beta)^2} - \frac{\gamma(9\gamma - (2-\beta)^2)A^2}{(4.5\gamma - (2-\beta)(1+\beta))^2}}{2\left(\frac{\gamma(1+\beta)A}{4.5\gamma - 2(1+\beta)^2}\right)\left(\frac{2(1+\beta)A}{4.5\gamma - 2(1+\beta)^2}\right)}$$

and

$$\mathrm{Eff}_{Spu} = \frac{SW_{Spu}^{NC} - SW_{NS}^{NC}}{s_{Spu}^{NC} x_{PU}^{NC}} = \frac{\frac{2\gamma A^2}{4.5\gamma-(2-\beta)(1+2.5\beta)} - \frac{\gamma(9\gamma-(2-\beta)^2)A^2}{(4.5\gamma-(2-\beta)(1+\beta))^2}}{\left(\frac{4\alpha\gamma A}{4.5\gamma-(2-\beta)(1+2.5\beta)}\right)\left(\frac{1.5\beta(2-\beta)A}{\alpha(4.5\gamma-(2-\beta)(1+2.5\beta))}\right)}$$

Hence,

$$\mathrm{Eff}_{Spr} - \mathrm{Eff}_{Spu} = \frac{40.5\gamma\beta^2 X}{4(1+\beta)^2(4.5\gamma-(2-\beta)(1+\beta))^2} - \frac{3\beta(2-\beta)Y}{4(4.5\gamma-(2-\beta)(1+\beta))^2}$$

and the difference has the same sign as

$$h(\gamma,\beta) = \frac{3\beta 4.5\gamma}{(1+\beta)^2}X - (2-\beta)Y$$

with $X = 4.5\gamma - 2(1+\beta)^2 > 0$ and $Y = 4.5\gamma - (2-\beta)(1+2.5\beta) > 0$.

We can trace the graph $h(\gamma,\beta) = 0$ as a function of the two parameters γ and $\beta \in (0, 0.5]$ (see Fig.2).

2) $\forall \beta \geq 0.5$,

$$\mathrm{Eff}_{Spr} = \frac{SW_{Spr}^{NC} - SW_{NS}^{C}}{2 s_{Spr}^{NC} x_{Spr}^{NC}} = \frac{\frac{2\gamma A^2}{4.5\gamma-2(1+\beta)^2} - \frac{\gamma(9\gamma-(1+\beta)^2)A^2}{(4.5\gamma-(1+\beta)^2)^2}}{2\left(\frac{\gamma(1+\beta)A}{4.5\gamma-2(1+\beta)^2}\right)\left(\frac{2(1+\beta)A}{4.5\gamma-2(1+\beta)^2}\right)}$$

and

$$\mathrm{Eff}_{Spu} = \frac{SW_{Spu}^{C} - SW_{NS}^{C}}{s_{Spu}^{C} x_{PU}^{C}} = \frac{\frac{2\gamma A^2}{4.5\gamma-1.5(1+\beta)^2} - \frac{\gamma(9\gamma-(1+\beta)^2)A^2}{(4.5\gamma-(1+\beta)^2)^2}}{\left(\frac{4\alpha\gamma A}{4.5\gamma-1.5(1+\beta)^2}\right)\left(\frac{(1+\beta)^2 A}{2\alpha(4.5\gamma-1.5(1+\beta)^2)}\right)}$$

Hence,

$$\mathrm{Eff}_{Spr} - \mathrm{Eff}_{Spu} = \frac{4.5\gamma(1+\beta)X}{12\beta(4.5\gamma-(1+\beta))^2} - \frac{(1+\beta)^2 W}{4(4.5\gamma-(1+\beta)^2)^2}$$

and the difference has the same sign as

$$f(\gamma,\beta) = 4.5\gamma X - 3\beta(1+\beta)W$$

with $X = 4.5\gamma - 2(1+\beta)^2 > 0$ and $W = 4.5\gamma - 1.5(1+\beta)^2 > 0$.

We can trace the graph $f(\gamma,\beta) = 0$ as a function of the two parameters γ and $\beta \in [0.5, 1]$ (see Fig.2).

	NS policy (no subsidy)		Spr policy		Spu policy	
	NC	**C**	**NC**	**C**	**NC**	**C**
s	-		$\frac{3\gamma\beta}{X}A$	$\frac{\gamma(1+\beta)}{X}A$	$\frac{4\gamma\alpha}{Y}A$	$\frac{4\gamma\alpha}{W}A$
x_{PU}	-		-		$\frac{1.5\beta(2-\beta)}{\alpha Y}A$	$\frac{0.5(1+\beta)^2}{\alpha W}A$
x_{pr}	$\frac{(2-\beta)}{\Omega}A$	$\frac{(1+\beta)}{\Gamma}A$	$\frac{2(1+\beta)}{X}A$		$\frac{(2-\beta)}{Y}A$	$\frac{(1+\beta)}{W}A$
Q	$\frac{3\gamma}{\Omega}A$	$\frac{3\gamma}{\Gamma}A$	$\frac{3\gamma}{X}A$		$\frac{3\gamma}{Y}A$	$\frac{3\gamma}{W}A$
π	$\frac{\gamma(4.5\gamma-(2-\beta)^2)}{2\Omega^2}A^2$	$\frac{\gamma}{2\Gamma}A^2$	$\frac{\gamma(4.5\gamma-4(1+\beta)(1-2\beta))}{2X^2}A^2$	$\frac{4.5\gamma^2}{2X^2}A^2$	$\frac{\gamma(4.5\gamma-(2-\beta)^2)}{2Y^2}A^2$	$\frac{\gamma(4.5\gamma-(1+\beta)^2)}{2W^2}A^2$
SW	$\frac{\gamma(9\gamma-(2-\beta)^2)}{\Omega^2}A^2$	$\frac{\gamma(9\gamma-(1+\beta)^2)}{\Gamma^2}A^2$	$\frac{2\gamma}{X}A^2$		$\frac{2\gamma}{Y}A^2$	$\frac{2\gamma}{W}A^2$

Table 1: Welfare-optimal subsidies, Public R&D, Private R&D, Total outputs, Profits, Total surplus
$A = (a-c)$, $\Omega = 4.5\gamma - (2-\beta)(1+\beta)$, $\Gamma = 4.5\gamma - (1+\beta)^2$, $X = 4.5\gamma - 2(1+\beta)^2$,
$Y = 4.5\gamma - (2-\beta)(1+2.5\beta)$, $W = 4.5\gamma - 1.5(1+\beta)^2$

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