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Impact on a RC rock-shed slab : FEM Modelling

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ABSTRACT: Recent experiments were performed in France on a new type of protection system against rock fall characterized by a roof slab without damping medium and simply supported on vertical elements by a set of "fuse" steel supports. The direct study of the experimental measurements manages to approach the main mechanical mechanism during the impact phases. In parallel, this study proposes a rigorous three-dimensional modelling of the roof slab and its supporting elements. This numerical analysis introduces the impact load in a way similar to that of the real experiment, and is based on a finite elements approach with a stress-strain relationship that allows a realistic representation of the concrete behaviour under dynamic loads and its corresponding damages. Mesh dependency is prevented by the Hillerborg technique and comparative tests confirm its efficiency and mesh refinement limits. Finally, interesting results are obtained for successive impacts and slab repair, using in particular a pre-damage state to represent the previous history of the slab. The possible yielding of the supports are also taken into account, especially for border impacts.

1 INTRODUCTION

The continuous expansion of urban zones in mountainous areas increases the need of protection systems to protect civil structures and infrastructures from natural hazards such as falling rocks. As example, one rockslide of over 100 m3 occurs each year along cliffs surrounding Grenoble in French Alps.

In particular for roads, protection can consist (passive approach) in rock-shed. However, the dimensioning and design of reinforced concrete protection systems is based, in the current codes of practice, on the "limit states" method that consider a combination of static forces. In case of dynamic loadings such as rock impacts, this approach is limited by its own hypothesis. Consequently, usual technical solutions are conventionally composed of reinforced concrete sub-structural elements (walls, columns, and foundations) and a roof slab covered by a thick backfilling layer. The roof slab is rigidly connected to sub-structural elements, and the backfilling layer constitutes a damping medium that allows therefore designing the system only with static dead loads (own weight, backfilling and rock weights). The structure is not designed any more to resist to the impact of blocks but especially to support the backfilling layer.

This solution has the main disadvantage of producing over dimensioned reinforced concrete elements and needs to maintain the damping layer properties. The foundations, which must be dimensioned consequently, cause often some construction problems due to site difficult situations.

The optimal dimensioning of protection systems (resistance + limited cost + probability) should be based on the following conditions:

- Absolutely considering the dynamic feature of the loading.
- Using behaviour properties for the structural materials that allow an accurate description of the stress-strain relationship variation under the applied dynamic loads, including possible irreversible phenomena and limited damages.
- Performing numerical structural analysis that includes realistic problem data to take into account such situation complexity (three dimensional geometry, generation of dynamic effects due to impacts, non-linear behaviour...).

The present study focuses on a new type of rockshed galleries: considering that the request for this type of equipment will be increasing, an investigation was carried out to improve the design and limit costs. The basic idea was to eliminate the backfilling layer and to use a semi-probabilistic approach with the notion of "acceptable damage" to the structure.

For the purpose of finding an optimal solution, a new system was proposed in France by the engineering company TONELLO IC, which consists of a roof slab only supported with no continuity on the sub-structural elements. The roof slab is subjected to the direct impact of falling rocks and slab reactions are transmitted to the sub-structures throughout ductile steel supports that act as dissipating energy fuses and protect the sub-structural elements. The slab is then designed in order to resist directly to a falling rock impact that causes a local damage limited to the shock zone in case of field impact. In case of border impact, energy is dissipated in these steel fuses that have then to be replaced. The first example of this protection system was built in 1999 at ''les Essariaux'' between Albertville and Chamonix in French Alps.

1998 (3.9 t. à B6.00m)

Figure 1. Les Essariaux Rock-shed

2 EXPERIMENTAL RESULTS

The experiments consisted of a reinforced concrete slab ($12 \times 4.8 \times 0.28m$) set on two lines of 11 elastic plastic steel fuses (12.6 cm high, spaced of 1.14 m, buckling load of about 260 kN) and impacted by a 450 kg reinforced concrete cubic block falling from different heights to various locations on the slab. Block and slab were built with a concrete of 35 MPa compressive strength, strongly reinforced by a high-density bars network (about 270 kg of steel per cubic meter of concrete).

Steel fuses consisted of a tube welded between two plates. The concrete block was released to fall freely from 15 m (to simulate a common impact loading – 68 kJ) and 30 m high (exceptional impact loading – 135 kJ), and to impact the slab. The impact velocity varied from 17.2 to 24.2 m/s. Different impacts were carried out in the inner part or on the edge of the slab to test the support line (table 1). After test $n^{\circ}2$ (T2) and before test $n^{\circ}4$ (T4), the corresponding zone was repaired using high-pressure water jet capacity to eliminate damaged concrete: the impact zone was then replaced with new concrete and some bars reinforcement.

Among different tests, test n°4 was one of the most monitored and is particularly interesting because of the slab repair.

Test	State of the impact zone	Impact en- ergy (kJ)	Impact location	
T1	Initial	68	Middle part	
T2	Initial	135	Extremity of the inner part	
T3	Initial	135	On the edge	
T4	Repaired after T2	138	Extremity of the inner part	
T5	Damaged by T1	134	Middle part	

Table 1. Impact history on the slab.

3 THE NUMERICAL MODEL

3.1 Objectives

Different approaches were possible to model such impacts: (Hentz 2003) proposes a distinct element modelling, very powerful to analyse phenomena in the impact zone but costly with complex procedure to evaluate local material parameters. Secondly, this approach is not able to model correctly the total structure and its temporal evolution for a long time.

In order to develop tools able to perform numerical experiments (as a "reality simulator" of the main phenomena that can occur in such situation) useful for civil-engineers, we propose here an exhaustive finite elements approach based on the Abaqus code with a damage model for concrete behaviour and including effectively the block impact on the slab supported by its steel fuses.

3.2 The finite elements model

A realistic prediction of the structural response through a finite elements numerical analysis requires a rigorous three-dimensional modelling of the different structural components of the structures. Abaqus offers also the possibility of managing several interactive entities (the slab and the block in this case). The analysis can, therefore, introduce the impact in a way similar to that of the experiment, with the block impacting effectively the slab and managing only the impact characteristics.

The slab and the block are modelled separately and completely with volumetric finite elements including different degree of mesh refinement (essentially at the impact zones of the slab). A particular attention is paid for the respect of the real shape of the block, including its edges chamfers. Reinforcements are represented independently by linear elements including longitudinal, transversal and even shear bars. Considering the structural damages level after impact, bars are considered embedded in concrete elements and the bond between reinforcement and concrete is assumed perfect: this solution does not introduce directly a particular bond behaviour but to not prevent either corresponding damage developments around one bar inside concrete elements.

To be able to model the complete behaviour of the slab even after the first impact and as equivalent string elements would introduce a perturbing connection to the ground, the steel supports are modelled completely with volumetric elements including the pipe between its metallic plates (for efficiency, the inferior plate can be eliminated for supports far from the impact: the neoprene layer is then placed at the interface with the slab). Welding points are supposed to be equivalent to numerical stuck points with forbidden rotations. With this solution, the contact between the slab and metallic supports introduces a possibility of friction and even separation as in reality. This separation is prevented by six vertical springs equivalent to vertical metallic bars used for real experiments and connecting the ground to the upper side of the slab to limit only its vertical up displacements.

3.3 Calculation procedure

In the present study, the finite elements code Abaqus is used. The explicit module of this code allows highly non-linear transient dynamic analysis of phenomena like impacts. However, this explicit approach needs to limit the modelled simulation duration that is directly linked with calculation costs. In the same time, it is necessary to impact the slab only after it stands under its own weight: this first phase is performed in the same calculation with a preliminary and progressive gravity application during 0.15 s (optimal time to prevent dynamic effects appearance). Then, the concrete block impacts the slab with the right speed and with the real impact conditions (angle, position).

Our model does not include a specific numerical damping. In the same time, many damping solutions are available with numerical codes but can constitute an easy non-physical way to exactly fit results with experiments without being finally representative of the model capacities. We keep here a clear position: we do not force and modify internal numerical parameters that manage the resolution algorithm and use only the original "bulk viscosity" advised automatically by Abaqus for dynamic explicit calculation. This general viscosity introduces a damping linked to voluminal strains (HKS 6.4).

3.4 Materials behaviours

For an accurate simulation of the system structural response, it is necessary to use a realistic representa-

tion of the materials behaviour under dynamic loads. For concrete, the behaviour properties must include some phenomena that are related to the damage under dynamic loads such as decrease in material stiffness due to cracking, stiffness recovery related to closure of cracks and inelastic strains concomitant to damage.

In this numerical analysis, the stress-strain relationship is represented by the PRM (Pontiroli-Rouquand-Mazars) (Pontiroli 1995; Rouquand and Pontiroli 1995) damage model that uses a scalar damage variable (see (Berthet-Rambaud & al 2003) for more details).

The strain threshold that release damage initiation depends on the strain rate in order to model the strain rate effect (Elices & Planas 1996) under dynamical loading:

$$\varepsilon_o = \varepsilon_o^s \left(1 + a. \dot{\varepsilon}^b \right) \tag{1}$$

where ε_o^s corresponds to the static threshold value. a and b are material parameters: assuming (Pontiroli 1995) that the strain effect has the same influence on the dynamic strength / static strength ratio and on the dynamic damage threshold / static damage threshold ratio, their values were identified from references like (Bischoff & Perry 1991) or (Malvar & Crawford 1998) (table 2). It takes into account a different influence in tension and compression but concrete physical properties like Young modulus are not modified.

The stress-strain relationship for steel (reinforcing bars and fuses supports) is considered as simply elastic plastic. Neoprene is considered as elastic. Material physical properties values corresponding to the experiments are obtained from classical identification tests.

3.5 Contact between block and slab

Abaqus Explicit [HKS 6.4] uses an efficient cinematic prediction – correction algorithm to manage interactions between block nodes (slave) and slab upper surface elements (master).

Then, it remains necessary to choose correctly the contact properties at interface: many model exist between hard and soft contact to manage locally normal and tangential interactions. Soft contact allows interpenetration between the two entities whereas this possibility is forbidden by hard contact. In our case and as the two interacting entities (the slab and the block) are themselves deformable the contact is chosen hard.

The only missing parameter is also the friction between the concrete of the block and the concrete surface of the slab with tangential effects at the interface. This aspect has quite a great influence because the block did not fall exactly horizontally on one of its faces. After a parametric study (Berthet-Rambaud 2004), the best value proved to be 0.15. It seems to be rather small for a concrete-concrete contact but this value appeared also to be the best one for impact in a different situation with an inclined slab. Some arguments can be proposed: firstly, this is not a static situation. Secondly, impact phase includes concrete erosion from block, which is not directly taken into account in the model: it can modified interface contact properties by eroded material presence (ball bearing effect) and friction coefficient can include indirectly fracture energy and other dissipative phenomena in the model.

3.6 Mesh dependency and refinement

Many authors showed a mesh dependency for softening behaviour models like PRM: without any correction, most of the strain is no more distributed but can be localised in one single element that has became weaker than its neighbours. This will then generate a localised damage that is unrealistic and dependant of the corresponding element size.

The (Hillerborg 1976) regularization technique is introduced in the PRM model in order to avoid such mesh dependency: this technique modifies the material behaviour depending on the finite element size in comparison with a length parameter Lh. Dissipated energy in softening post-peak part becomes independent from element sizes.

In the PRM model, this is obtained introducing a correction factor w for the damage evolution (Pontiroli 1995). We use a value Lh = 0.3 m, coherent with literature data and PRM model experience.

A simple dynamic numerical application inspired from (Belytschko & Lasry 1989) is given figure 2: it consists in a strain wave generated at the free extremity of embedded numerical concrete bars. These bars are meshed differently. The wave is foreseen to traverses them without damage except at the embedded extremity where reflexion doubles the corresponding strain. This exercise confirms that the PRM model with the Hillerborg regularization technique is correctly mesh independent in that case.

In the same time, it is also important to verify that the mesh quality is sufficient to be representative of



Figure 2. Strain wave in different-meshed concrete bars

the main concerned phenomena. For the slab impacted by a block, the impact zone is the crucial one, subjected locally and dynamically to bending and shear. Preliminary calculations are performed to test the influence of the slab mesh refinement with threezones mesh:

- one impact zone (9 m² around the impact point)
- one transitional zone (60 cm wide around the impact zone)
- one external zone (the rest of the slab, supported by steel fuses).

The horizontal mesh of the external zone is rough, non-uniform but always the same. The size elements and the number of layers in the impact zone are given table 2: this impact mesh is uniform with regular elements (for mesh compatibility, the number of layer is the same for the complete slab). Finally, the transitional zone links these two zones.

Identical numerical impacts corresponding to T4 are performed (but without preliminary gravity application to the slab) and we compare the maximum deformed shape of the slab (table 2): Slabs n° 1, 2 and 3 give an equivalent deformed basin comparable with that obtained experimentally. For slab n° 5, the basin depth is clearly not sufficient: finite elements size prevents a correct representation of the high-curved part. The same effect begins to concern also slab n° 4.

It is also to note that these simulations need long calculations (0.15 s have already to be simulated only for gravity application!) and depending on the mesh refinement, calculation costs can become prohibitory: slab $n^{\circ}1$ would need more than two and half days to apply gravity, about eleven time more than slab $n^{\circ}5$. Finally, an average mesh refinement like slab $n^{\circ}3$ provides the best compromise in comparison with calculation costs.

4 NUMERICAL ANALYSIS RESULTS – TEST N°4

4.1 The impact phase

Firstly, it is important to verify that the impact phase is correctly modelled all the more it governs the energy transfer from the block to the slab and also the global quality of the simulation. Numerical impact conditions are corrected to be consistent with experimental ones: angle, position, speed and mass of the block correspond exactly to reality.

Then, figure 3 confirms that the block impact sequence includes the different phases observed experimentally with two successive shocks on opposite corners. This sequence is clearly influenced by tangential contact behaviour and friction value: high values can even allow the block to rebound directly from the first contact corner without touching the slab by the second corner.

Table 2.	Deformed	shapes	with	different	mesh	refineme	ent
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Slab n°	Horizontal size of elements in the impact zone	Number of layers	Total number of elements in the im- pact zone	CPU time to simulate 0.01s (with Pentium 4, 3 GHz, 512 Mo memory)	Slab maximum deformed shape (vertical scale factor of 200)
1	4 * 4 cm	7	39375	4 h 10' 9"	
2	6.66 * 6.66 cm	6	12150	1 h 30' 48"	
3	10 * 10 cm	5	4500	48' 11"	
4	20 * 20 cm	4	900	24' 06"	
5	30 * 30 cm	4	400	22' 14"	

The vertical speed of the block and of the slab impact point during this phase appear very closed to experimental results: in particular, the slab and the block move down with an equivalent speed between

the two successive corner shocks. Each of these two shocks lasts 5.2 ms with a total impact time of 16 ms that is consistent with reality. However, the maximum impact force for the first shock reaches 4.05



Figure 3. Block and slab vertical speeds during impact

MN and only 1.8 MN for the second one: this can go against simplified punching analysis that over evaluate the second shock (Delhomme & al 2005). In our case, it is also to note that the block finally rebounds numerically much more than in reality: this supplementary energy that remains can come from the block fracture and erosion that are not directly included in the model. But the energy transmitted to the slab seems correct and allows us to study the slab evolution.

4.2 Damage distribution and accumulation in the slab – optimal pre-damaged slab

The advantage of this type of analysis, in addition to traditional output (strain, stress, internal forces, reactions...), is to allow obtaining additional results, in particular evaluating the damage states by mapping the values of D in the slab. Figure 4 shows a typical distribution of damage D in the impact zone due to a T4 impact sequence. Considering that numerical damages begin to represent effectively physical damages for important value of D (Berthet-Rambaud 2004), we represent only damage with D> 0.8.

It is firstly interesting to note that the experimental crack network visible on the inferior slab side is of an equivalent size than the numerical damage dis -tribution. Then, this numerical damage inside the slab thickness shows also a tendency to a conical distribution: this is to be linked with punching effect and punching fractures that could be located after other tests (Perrotin & al 2002).

Before to model quantitatively test n°4, it is necessary to take into account damages around the repaired zone due to previous tests and particularly test 2. Firstly, it is assumed that the different impact zones are independent: this hypothesis is acceptable considering typical crack distribution zone size. Then and keeping in mind the objective to develop engineering tools, it appears difficult and non optimal to simulate the complete history of the slab to study test n°4 especially because of very important numerical costs: damages due to test 2 have to be pre-introduced in the model for T4 impact using in the same time an efficient mesh.

A supplementary model of slab (called "average slab") is also compared to slab n°2: firstly, its mesh is simplified and does not include any particular zones but an average distributed mesh with a refinement between slab n°3 and 4. Secondly, this slab can be "pre-damaged" to represent damages due to a first impact (T2) and to perform directly test T4 without a preliminary test. This pre-damage is assumed regularly distributed around the repaired zone (Fig. 5): damage values that correspond only to tension, are obtained from an average of damage in these different parts after the simulation of a pre-liminary impact equivalent to T2.

Then, two scenarios are tested

- Scenario "T2+T4" (for slab n°2 and the "average slab"): the slab is impacted 2 times identically (with T4 conditions, considered as equivalent to T2 for the first impact) but is "repaired" between the two impacts the damage D is given to zero in the corresponding zone before the second impact.
- Scenario "direct T4" (for the "average and predamaged slab" only): the slab is impacted directly to perform test T4, without any preliminary impact



Figure 4. Damage distribution (between 0.8 and 1) in the impact zone (slab $n^{\circ}1$ mesh)



Figure 5. Numerical pre-damaging of the "average slab" for scenario "Direct T4"

It is verified that damage distributions and slab deformations are comparable for both scenarios. Of course, the damage distribution cannot be as precise as for a refined mesh, in particular near the impact point, but the "average pre-damaged slab" already provides interesting results and represents well the global behaviour of the slab.

4.3 Slab deformations and reactions

The quality of the simulation is confirmed by the comparison about the maximum displacement of different points under the slab (table 3). Numerical results are globally closed to experimental measurements even if some difference exist: A refined mesh like slab n°2 over estimates a little bit the impact zone deformations whereas a more rough mesh ("average slab") will rather increase the vertical displacements around the impact basin and decrease them inside. This is naturally due to the direct link between the mesh refinement and its capacity to correctly envelop and describe the curves of the basin due to impact.

It is also to note that the prediction of the maximum vertical displacement without using an efficient model like PRM but only with an elastic behaviour (using the Young modulus as rigidity) for concrete gives about a 60% lower result: it confirms the necessity to model the non-linear behaviour of

Table	3.	Maximum	vertical	displacements -	– comparison	"ex-
perime	ent	al-numerica	1"			

	Maximum vertical depression T4 (mm)						
Point		Numerical					
i onn	experimental	Slab n°2 scenario "T2 + T4"	average pre-damaged slat - scenario "direct T4"				
Α	7.9	9.7	11.6				
В	12.0	13.0	11.6				
С	7.9	8.4	6.3				
D	19.7	21.2	17.8				
Е	16.2	18.2	15.1				
F	6.4	9.5	7.6 15.0 10.5				
G	15.2	18.7					
Η	10.3	10.5					
Ι	2.4	1.6	1.8				
J	11.0	13.8	10.6				
Κ	10.0	11.1	9.0 13.1 10.6				
L	14.0	16.2					
М	11.5	13.2					
×	Y	+1 +	F + + H +G +D +B +A M+ + K				
			+c				

materials in such impact cases. Secondly, calculations show also that for scenario "T2+T4", vertical displacements due to the second impact are only about 10% (depending on the location) superior to ones of the first impact: this limited difference confirms that after repairing, the slab recovers almost its initial resistance, providing a supplementary proof of its fitness for use.

Then, we look at the temporal evolution and oscillations of the slab after the T4 impact obtained with the "average pre-damaged slab" (Fig. 6): for the displacement sensor E, which is located just near the impact point, we can see that the first oscillation is very well computed. The post impact behaviour is much more difficult to foreseen but remains correctly evaluated by the simulation with a consistent damping: here, we must particularly recall that this slab is a very complex cinematic system just set on twenty-two supports and anti-elevation devices that have a large influence on its temporal evolution post-impact! This is shown by the results of sensor I located at about 4.7 m of the impact point, still correctly foreseen by the model, and that begins to move up during impact.

Concerning finally forces generated on the support, figure 7 shows a comparison between experimental measurements and numerical results. During experiments, five force sensors supported a massive beam that supported her-self one line of eleven steel fuses above the slab. Theses force sensors were distributed along the supporting beam but unfortunately



Figure 6. Vertical displacements of points E (closed to the impact point) and I (at about 4.7m)



Figure 7. Time (s) – Force (kN) diagrams from sensors under a support line – comparison experimental (thin line) and numerical results (thick line)

the middle one did not work and provided no results (whereas it supported also the beam). Assuming a perfect rigidity of the beam, it was possible to evaluate corresponding numerical forces from stress at fuses basis: the two first curves given figure 16 correspond to sensors closer to the impact point. Peak forces are correctly computed with a correct time evolution. In particular, phases when the slab edges are rebounding are clearly visible. For the two last curves for sensors that were far from the impact, it is more difficult to obtain good results but the average force is correctly evaluated with interesting peaks.

5 CONCLUSIONS:

Finally, the numerical results proved to be in agreement with the experimental measurements, as well as representing the damage states under the different cases of loadings. This confirms the capacity of used numerical tools to treat such situation: the PRM model appears very efficient and enough easy to use for reinforced concrete structures subjected to dynamic and cyclic loadings. Abaqus with its explicit time-integration scheme generates no difficulty to achieve all these calculations. This combination based on a finite element tool even allows us to study correctly impact different phases and main phenomena to correctly model the interactions between the block and the slab with a right energy transfer.

In the same time, used tools can provide a numerical experiments device: instead of performing expensive and difficult tests, they allow to test simply and rapidly new situations and are very useful for parametric studies. Moreover, the numerical analysis allows to obtain additional information that can not be measured experimentally: energy transfer, strain or stress everywhere...In that way, such a numerical approach completes the experimental results analysis and constitutes a major help to better understand phenomena, which occur during the different phases of the problem: impact and post impact.

Then, and as refined mesh can need impressive calculation time, it is also possible to obtain interesting results with an "average slab" and low costs. The pre-damaging procedure is also useful to simulate a particular impact without all the previous history of the structure: this can be particularly interesting for real structures subjected to many (often unknown) impacts to imagine different scenario. However, work is still necessary to better link the numerical D to physical damages.

Next step is then to continue to develop knowledge about such rock-sheds and their behaviour in reality: one aspect is to provide simplified way to dimension these structures, including efficient rules and numerical tools adapted to engineering. The second one is to study the real life of an on-site rockshed with questions about successive damages, repairing and residual protection capacity.

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