HAL
open science

Electro-magneto-mechanical characterizations of the vibration of magnetic origin of electrical machines

H. Javadi, Yvan Lefèvre, Stéphane Clénet, Michel Lajoie-Mazenc

To cite this version:

H. Javadi, Yvan Lefèvre, Stéphane Clénet, Michel Lajoie-Mazenc. Electro-magneto-mechanical characterizations of the vibration of magnetic origin of electrical machines. IEEE Transactions on Magnetics, 1995, 31 (3), pp.1892-1895. hal-02002135

HAL Id: hal-02002135

https://hal.science/hal-02002135v1

Submitted on 31 Jan 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Open Archive Toulouse Archive Ouverte

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in: http://oatao.univ-toulouse.fr/21414

Official URL:

https://doi.org/10.1109/20.376408

To cite this version:

Javadi, H. and Lefevre, Yan and Clénet, Stéphane and Lajoie-Mazenc, Michel Electro-magneto-mechanical characterizations of the vibration of magnetic origin of electrical machines. (1995) IEEE Transactions on Magnetics, 31 (3). 1892-1895. ISSN 0018-9464

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

ELECTRO-MAGNETO-MECHANICAL CHARACTERIZATIONS OF THE VIBRATION OF MAGNETIC ORIGIN OF ELECTRICAL MACHINES

JAVADI H.* , LEFEVRE Y. , CLENET S. and LAJOIE MAZENC M.
Laboratoire d'Electrotechnique et d'Electronique Industrielle (LEEI)
U.R.A C.N.R.S n° 847
2 rue Charles Camichel 31071 Toulouse cedex , FRANCE
* PWIT, Minster of Energy, Tehran - currently with the LEEI

Abstract **– This paper contributes to the modeling of vibration of magnetic origin of electrical machines. It is generally difficult to calculate the damping matrix of actual mechanical structure of electrical machines. Mechanical model that we present here is known as the modal superposition method. It allows us to take into account the damping coefficients of the mechanical structure which can be inferred from experience. Associated with a numerical code for the computation of magnetic force it can be of a great help in the predetermination of the vibration of an electrical machine.**

I. INTRODUCTION

Nowadays the vast applications of permanent magnet machines in industrial area oblige engineers to improve their efficiency and to reduce their acoustic noise emission.

Design of noiseless permanent magnet machines takes several steps. A preliminary design using simple analytical expressions of the main electromechanical quantities [1] such as torque and flux gives roughly the main dimensions of the machine (airgap value, thickness of magnets,...). Then other parameters like the number of teeth, the number of poles, the type of winding, the shape of magnets must be determined. These parameters can be very important for the vibratory behavior of the machine. To make the optimum choice, designers must study for each set of these parameters the vibrations produced by the machine. Among the different type of vibrations produced by an electrical machine the one of magnetic origin is very important. Many searchers have proposed procedures based on the coupling of numerical codes for eletromagnetic field and mechanical structure analysis. In spite of improvment in computation speed of hardware, simulations still take a long time.

This paper proposes method which can simplify designer tasks. In section 1 the general procedure [3-5] is recalled and applied to predict the vibratory behavior of a variable speed synchronous motor. Section 2 shows how modal superposition method can reduce simulation time. Finally this method is used to characterize the magneto-mechanical behavior of an electrical machine.

II. COMPUTATION OF VIBRATION OF MAGNETIC ORIGIN

Vibrations of magnetic origin are computed by means of two numerical codes. The first code is devoted to calculate the evolution of magnetic forces applied on the stator. The second one is devoted to calculate the mechanical response of the stator.

A. Magnetic Force Calculation

In order to have a good evaluation of the distribution of magnetic forces along stators, forces exerted on teeth and on conductors are evaluated. A previous paper [3] has shown experimentally that the integration of the surface force density given by Maxwell's stress tensor over a surface covering partially a tooth leads to magnetic force applied on it.

To obtain evolution of force in time domain the evolution of the electromagnetic field inside motors must be calculated. In synchronous motors with no damper windings, induced currents are neglected. Under steady state conditions their dynamic operation can be assimilated to a succession of magnetic states governed by magnetostatic equation. For each state, which is characterized by a position of the rotor relatively to the stator, the current density is calculated by means of the characteristics of the external power supply while magnetic field is computed by means of numerical method.

For the general case, a field computation code called EFCAD [9] based on the finite element method is used. The rotor motion is taken into account by means of a moving band technique.

For permanent magnet synchronous motors without polar pieces a specific procedure [2] has been developed. In this case the rotor motion is taken into account by means of the superposition principle. This method can be employed because the laminated core is not generally saturated (constant permeability) for this kind of motor. Further more the rotor has no polar pieces and the permeability of magnets are practically equal to the one of air. Thus the reluctance of the motor doesn't change during rotor motion. Only one inversion of the matrix of reluctivity of the discretized magnetic equation is needed. An elementary source is defined for each phase supplied by a unit current and for each position of magnets. The actual potential vector solution is obtained by the superposition of each elementary solution multiplied by an adequate coefficient. This method combined with symmetry considerations reduce drastically simulation time.

B. Structural Analysis

When finite element is used, the general equation of motion of a discretized structure can be expressed in the following form:

$$[M]\frac{d^2}{dt^2}\{X\} + [C]\frac{d}{dt}\{X\} + [K]\{X\} = \{F(t)\} \qquad (1)$$

where $\{X\}$ is the generalized displacement vector, $\{F\}$ is the generalized equivalent force vector and $[K]$, $[M]$, $[C]$ are respectively the stiffness matrix, the mass matrix and the damping matrix. If the forces applied on the structure are periodic and if mechanical equations are linear then (1) can be solved by first splitting up $F(t)$ in its harmonics then by solving complex algebraic system of equations:

$$[\,[K] + j\,\omega_h\,[C] - \omega^2_h\,[M]\,]\,\{X_h\} = \{F_h\} \qquad (2)$$

In our problem $\{F_h\}$ is the complex vector force corresponding to harmonic of rank h of magnetic forces, $\{X_h\}$ is the complex displacement vector, ω_h is the pulsation of the harmonic h. $[K]$, $[M]$ and $[C]$ are real matrix. For an actual structure, matrix $[C]$ is generally difficult to calculate. When ω_h is far from the resonance pulsation of the structure, damping matrix can be neglected.

C. Application: Prediction of the Vibratory Behavior of Varaible Speed Permanent Magnet Motor

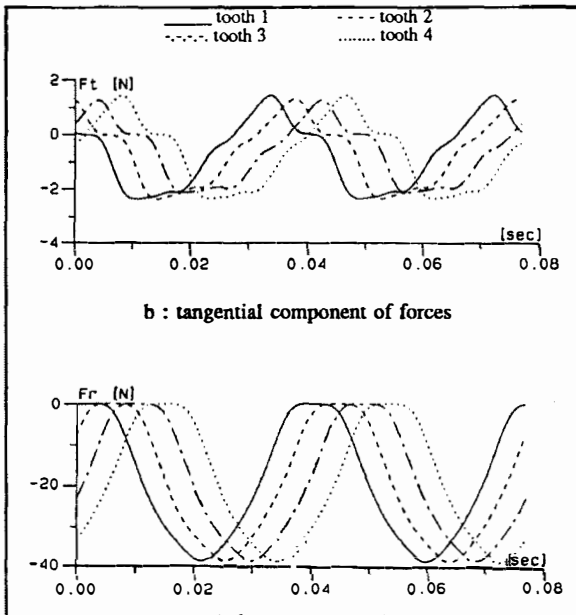
In the case of variable speed control, synchronous motors are generally self-piloted and supplied by current controlled voltage inverter. When the speed of the motor changes while the torque is constant, theoretically the current waveshape versus rotor position remains the same. For a synchronous motor without damper windings this means also that all magnetic quantities like flux, torque and force waveshapes remain the same. Only their frequency changes. Therefore to simulate the vibratory behavior of motor during speed variation only one magnetic forces calculation during one steady operation is needed. For each harmonic of magnetic forces the frequency response of the mechanical structure is computed by means of equation (2), ω_h varying from ω_{hmin} to ω_{hmax} corresponding to the minimum speed and the maximum speed of the motor.

To illustrate this modelling approach, it is applied to calculate the vibrations produced by a permanent magnet synchronous motor without polar pieces. This motor has 3 phases, 4 poles and 36 teeth. Magnetic forces are calculated under steady conditions. The motor is supplied by nominal currents at frequency equal to 13 Hz. Figure 1 shows the radial and tangential forces applied on 4 successive teeth under one pole. Analysis of these curves shows that the forces exerted over stator teeth are related by relation:

$$F(t)\,\big|_{tooth\ n} = F(t - N_{dpp}\,\Delta t)\,\big|_{tooth\ n+N_{dpp}} \qquad (3)$$

where Δt is the time for travelling from tooth n to tooth $n+1$ and N_{dpp} is the number of teeth per pole and per phase.

For this motor N_{dpp} is equal to 3. These results show also that the frequency of force are equal to twice the electrical frequency. Figure 2 shows for each harmonic the picture of distribution of forces along the stator. When picture of each distribution is animated, it looks like a swing progressive wave rotating at a speed proportional to the rang of harmonic. Results show that the distribution shape of harmonic of rank h, where h is an integer inferior to the number of teeth per pole N_d, is closely similar to the one of harmonic of rank $(N_d\text{-}h)$ and also to the one of harmonic of rank $(m.N_d \pm h)$ where m is an integer. For the distributed motor there are only five types of ditribution of the harmonics of magnetic forces. All of them are displayed on figure 2.



b : tangential component of forces

a : radial component of forces

Fig. 1. Evolution of magnetic force in the synchronous machine WPP ($N_p = 4$ and $N_d = 9$)

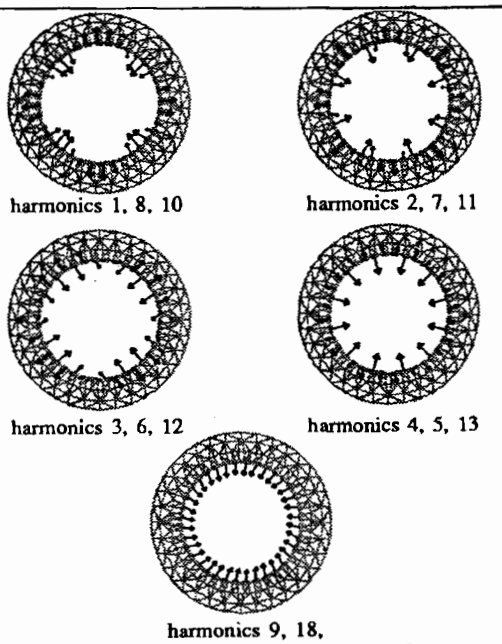


Fig. 2. Distribution of the magnetic forces over the stator of a permanent magnet machine

The frequency response of the mechanical structure to each harmonic of magnetic forces is calculated by means of the software EFMEC* which solved the inhomogenous equations (2). As mentionned above and according to many authors [7],[8], damping matrix $[C]$ is difficult to calculate and is commonly neglected. Frequency responses to harmonic of rank 1 and 9 are shown on figure 3. Each response curve presents resonance frequency. The resonance frequencies correspond to the natural frequencies of the mechanical structure calculated by means of another software EFFMP*, which computes eigenvalues and eigenvectors of the undamped homogenous equation (2). Figure 4 shows the five first mode shapes and the natural frequencies of the stator of the studied motor. According to figure 3 and 4, resonance frequency appears at the natural frequency of mode 4 if the first harmonic of magnetic forces is applied, and at the natural frequency of mode 0 if the 9th harmonic is applied. Next section is devoted to make clear relations between distribution of forces and frequency response characteristics.

III. Modal Superposition Method

In the space of generalized displacement vector $\{X\}$ of the discretized structure, the undamped homogenous equation of (1) is a standard eigenvalue problem with the eigenvalues being the square of natural pulsations and eigenvectors the corresponding mode shapes. The eigenvectors are orthogonal with respect to both matrix $[M]$ and $[K]$. And if they are normalized the generalized displacement vector $\{X\}$ and the generalized force vector $\{F\}$ can be expressed in the modal space [5]. For the harmonic of rank h we have:

$$\{X_h\} = \sum_{i=1}^{N_{mode}} y_{h,i}\,\{x^{(i)}\} = \sum_{i=1}^{N_{mode}} \left[\left\{x^{(i)}\right\}^T \{X_h\}\right] \{x^{(i)}\} \qquad (5)$$

$$\{F_h\} = \sum_{i=1}^{N_{mode}} f_{h,i}\,\{x^{(i)}\} = \sum_{i=1}^{N_{mode}} \left[\left\{x^{(i)}\right\}^T \{F_h\}\right] \{x^{(i)}\} \qquad (6)$$

where $y_{h,i}, f_{h,i}$ are the modal component of vector of displacements and the vector of forces and $\{x^{(i)}\}$ is the mode vector i. Relations (5) and (6) define the transformation matrix transferring physical coordinates to modal coordinates. Application of this transformation to equation (1) gives equation:

$$[m]\{y''\} + [c]\{y'\} + [k]\{y\} = \{f\} \qquad (7)$$

where $[m]$, $[k]$ and $[c]$ are matrix defined by relations:

$$\begin{aligned} [m] &= [\{x^{(1)}\},\ldots,\{x^{(Nmode)}\}]^T [M] [\{x^{(1)}\},\ldots,\{x^{(Nmode)}\}] \\ [k] &= [\{x^{(1)}\},\ldots,\{x^{(Nmode)}\}]^T [K] [\{x^{(1)}\},\ldots,\{x^{(Nmode)}\}] \\ [c] &= [\{x^{(1)}\},\ldots,\{x^{(Nmode)}\}]^T [C] [\{x^{(1)}\},\ldots,\{x^{(Nmode)}\}] \end{aligned} \qquad (8)$$

$[m]$ and $[k]$ are diagonal matrix but $[c]$ is generally not. This means that equations of motion in the modal space are coupled only by the viscous damping. However, in common assumption $[c]$ is often considered as a diagonal array the components of which are inferred from experience. Thus the equation of motion is decoupled into a set of equations in single degree of freedom. Thus for the hth harmonic of magnetic force, the equation of motion along the ith mode shape is:

$$m_i \frac{d^2}{dt^2} y_{h,i} + c_i \frac{d}{dt} y_{h,i} + k_i\, y_{h,i} = f_{h,i} \qquad (9)$$

From relations (5) and (6) and the complex form of equation (9) the following equation is deduced:

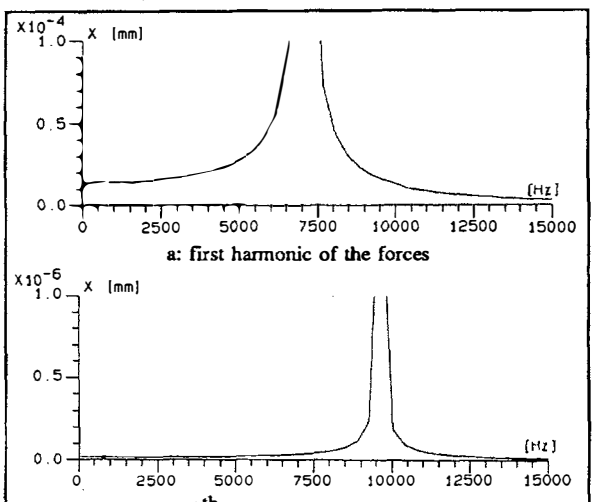
$$\{X_h\} = [G(\omega_h)]\{F_h\} \qquad (10)$$

where $[G(\omega_h)]$ is the transfer matrix of the mechanical structure which can be expressed by;

$$[G(\omega_h)] = \sum_{i=1}^{N_{mode}} \left\{ \frac{1}{m_i\,\omega_i^2\left(1 - \frac{\omega_h^2}{\omega_i^2}\right) + j\,c_i\,\omega_h} \left\{x^{(i)}\right\}\left\{x^{(i)}\right\}^T \right\} \qquad (11)$$

In this expression m_i, ω_i, and c_i are respectively the generalized mass, natural pulsation and the damping coefficient of mode i.

Table 1 gives for some harmonics of magnetic forces their modal components $f_{h,i}$. According to this table harmonic of rank 1 excites more probably the mode 4 and harmonic of rank 9 excites the mode zero. These results are consistent with the dynamic response frequency displayed on figure 3.



a: first harmonic of the forces

b: 9th harmonic of the forces

Fig. 3. Evolution of the dynamic response of the stator induced by two different harmonics of the magnetic forces

mode shapes	harmonics of the forces				
	1	2	6	9	12
2	1.9×10^{-3}	1.6×10^{-4}	2.5×10^{-4}	2.0×10^{-5}	9.1×10^{-6}
3	2.1×10^{-3}	2.5×10^{-3}	1.1×10^{-3}	3.0×10^{-4}	6.5×10^{-5}
4	14.951	0.1780	1.0×10^{-3}	1.5×10^{-3}	2.5×10^{-4}
5	3.8×10^{-2}	5.9×10^{-3}	3.4×10^{-4}	1.4×10^{-3}	3.2×10^{-4}
6	1.7×10^{-2}	1.3×10^{-3}	2.8×10^{-3}	6.1×10^{-4}	2.1×10^{-4}
0	1.2×10^{-2}	1.1×10^{-2}	2.1×10^{-2}	0.457	3.6×10^{-3}

Table 1: *amplitude of the harmonic of magnetic forces of an electrical machine in the space of its mode shapes*

From equation (10) the frequency response to each harmonic can be again calculated this time by taking into account the viscous damping of each mode. Figure 5 shows results for harmonics of rank 1 and 9 of magnetic forces for three values of the damping factor of mode 4 and 0, the other factors are fixed to a random constant between 0.01 and 0.2. These results are similar to results shown on figure (3) except that resonance peaks are not infinite and resonance frequencies are a bit different from natural frequencies.

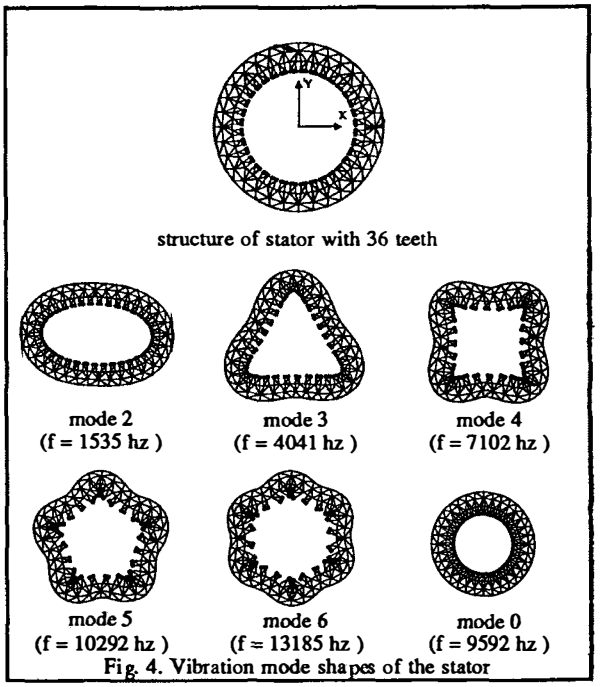


Fig. 4. Vibration mode shapes of the stator

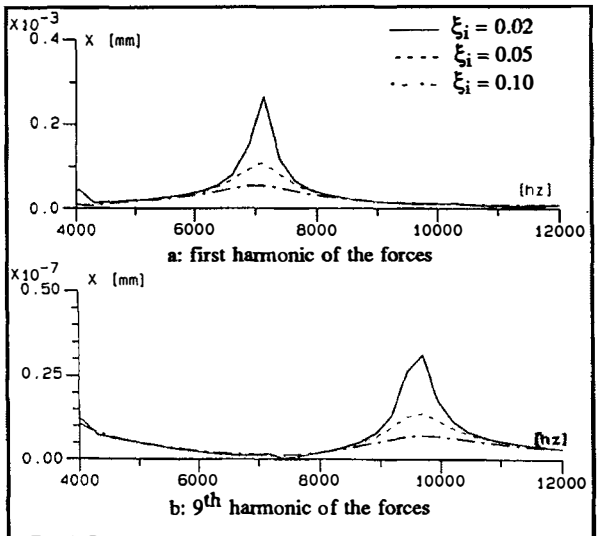


Fig.5. Dynamic response frequency determined by the modal superposition method

Using equation (10) and (11) can predict the vibration of the mechanical structure. Nevertheless, their utilisation must be made carefully because numerical problems may occur. These problems are due to the fact that to perform this computation only the first modes are taken into account, higher modes are neglected.

In fact in a design procedure, these last simulations are not necessary because table 1 synthesizes very well the studied mechanical behavior. This saves much computation time because only mode shapes, natural frequencies and the modal components of force have to be calculated.

CONCLUSION

Using the method of modal superposition for calculating the vibration level of an electrical machine structure, the designers are free from calculating its displacement by the mechanical motion equation. In this calculation they can also consider the coefficient damping of the structure. For this, we have to determine accurately the mode shapes of the stator that can be done at once and it allows us to reduce the total computation time. Moreover, the present paper gives a new conception of the distribution form of the magnetic forces over the stator. Knowing of these distribution forms leads to distinguish the excitation mode of the stator produced by the magnetic force harmonics.

REFERENCES

[1] Kone A.D., Nogarede B. et Lajoie-Mazenc M. "Le dimensionnement des actionneurs électriques: un problème de programmation non linéaire" J.Phys. III , February 1993, pp. 285-301

[2] Javadi H., Lefèvre Y. et Lajoie-Mazenc M. "Procédure spécifique pour modélisation et le calcul des vibrations d'origine magnétique dans une machine électrique" Le Centenaire ESIM, Marseille 1993, pp. 134-140

[3] Lefèvre Y., Davat B. and Lajoie-Mazenc M. " Determination of synchronous motor vibrations due to electromagnetic force harmonics" IEEE Trans. Magn. Vol 25 , No 4 , 1989, pp. 2974-2976

[4] Henneberger G., Sattler Ph. K., Hadrys W., Shen D. "Procedure for the numerical computation of mechanical vibrations in elctrical machine" IEEE Trans. Magn. Vol 28 , No 2 , 1992, pp. 1351-1354

[5] Benbouzid M.E.H., Reyne G., Dérou S. and Foggia A. "Finite element modeling of a synchronous machine: Electromagnetic forces and mode shapes" IEEE Trans. Magn. Vol 29 , No 2 , 1993, pp. 2014-2018

[6] Lefèvre Y. "Etude de la répartition des efforts électromagnétiques dans les machines synchrones à aimants permanents et commutation électrique. détermination des vibrations d'origine magnétiques" Thèse de Doctorat INP Toulouse 1988

[7] Imbert J.F."Analyse des structures par éléments finis" 2ème Edition , Cepadues Editions 1984

[8] Morel J. "Vibrations des machines et diagnostic de leur état mécanique" Edition Eyrolles, Paris, 1992

[9] Sadowski N."Modélisation des machines électriques à partir de la résolution des équations du champ en tenant compte du mouvement et du circuit d'alimentation (Logiciel EFCAD)" Thèse de docteur de l'I.N.P. Toulouse, 1993

* EFMEC *and* EFFMP *are the developed softwares at LEEI for computing the natural frequencies and the vibration mode shapes of a structure in 2D.*