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Global non-quadratic \mathcal{D} -stabilization of Takagi-Sugeno systems with piecewise continuous membership functions

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Abstract

This paper deals with the non-quadratic stabilization of Takagi-Sugeno (T-S) models with \mathcal{D} -stability constraints. Based on a recently proposed Non-Quadratic Lyapunov Function (NQLF), which involves the mean values of the membership functions (MFs) over a given time interval, three theorems are proposed for the design of non-Parallel Distributed Compensation (non-PDC) controllers satisfying closed-loop \mathcal{D} -stability specifications. Despite previous non-quadratic approaches and thanks to the nature of the considered NQLF, it is highlighted that the proposed LMI-based procedures not only apply for the global non-quadratic \mathcal{D} -stabilization of T-S models, but also for a larger class of T-S models with piecewise membership functions (i.e. a class of switching nonlinear systems), since no requirement is needed regarding to the bounds of the MFs derivatives. The effectiveness of the proposed LMI-based conditions and their relative degrees of conservatism, compared with previous quadratic \mathcal{D} -stabilization results, are illustrated through an academic example involving piecewise membership functions.

Keywords: Takagi-Sugeno models, \mathcal{D} -stability, Non-PDC controllers, Global non-quadratic design, Linear Matrix Inequalities.

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1. Introduction

Since the pioneer work of Takagi and Sugeno [1], dealing with the identification of nonlinear systems through the fuzzy formalism, Takagi-Sugeno (T-S) fuzzy model-based approaches have caught the attention of the control community. Indeed, thanks to their convex characters, these approaches may facilitate the analysis and control design for some classes of nonlinear systems by extending some linear results to the nonlinear framework [2].

A T-S model is a set of linear models aggregated by convex Membership Functions (MFs). It may accurately match a nonlinear system in a compact set of its state space by using sector nonlinearity transformations [2]. Their stability and stabilization are usually investigated via the well known direct Lyapunov methodology, aiming to obtain Linear Matrix Inequality (LMI) conditions [3, 4, 5, 6, 7]. Indeed, LMI conditions may be solved efficiently via convex optimization tools [8].

Controller design conditions for the stabilization of T-S models were firstly considered with common Quadratic Lyapunov Functions (QLF) and a Parallel Distributed Compensation (PDC) control scheme [2, 3]. However, they require to find common decision variables, solution of a set of LMI, which may lead to conservatism [9]. To reduce the conservatism, piecewise, switched or non-quadratic Lyapunov functions (NQLF) have been proposed with limitations [10, 4, 11, 12, 13, 14, 15]. Indeed, because of the MFs overlapping, piecewise or switched Lyapunov functions are inadequate for T-S models obtained from sector nonlinearity approaches. Moreover, when considering usual fuzzy NQLF in the continuous time case [4], the appearance of the MFs time derivatives in the stability conditions leads to local-based approaches, which require an estimation of the designed closed-loop domain of attraction [13], and makes harder the obtention of LMIs or their practical application. In addition, since these local non-quadratic approaches require the finite bounds of the time derivatives of the MFs, they can't be applied to T-S models involving piecewise MFs. Note

30 that in the discrete time case [11, 14, 15], these derivatives are no longer occur-
ring but these results are left out from the present study since it only focusses on
continuous time Takagi-Sugeno models. Thus, to circumvent these drawbacks
in the continuous time framework, another non-quadratic approach has been
proposed with the consideration of Line-Integral Lyapunov Functions (LILF)
35 [5]. This approach provides global stability conditions but is restricted in sta-
bilization to second order systems to obtain convex conditions [16, 17, 18]. In
[19], a new NQLF, which involves the mean values of the MFs on the interval
 $[t - \alpha, t]$ with $\alpha > 0$, has been proposed. The major milestone of such NQLF is
that the obtained closed-loop stability conditions are free of the time derivative
40 of the MFs and also hold globally when there exist a solution.

In parallel to the previous mentioned studies, the improvement of the closed-
loop transient responses of complex nonlinear systems represented by T-S mod-
els remain an important challenge. In this context, the concept of \mathcal{D} -stability
can be considered. It is borrowed from linear systems to provide closed-loop per-
45 formance while keeping the control signal suitable for real world applications.
 \mathcal{D} -stability consists on placing the distribution of the eigenvalues of the closed-
loop system in a prescribed region of the complex plane. This can be achieved
by adding constraints to the Lyapunov function [20, 21], which lead to obtain
LMI conditions that ensure both the closed-loop stability and some desired
50 performances. It was firstly proposed for linear systems [22], then extended to
polytopic uncertain linear systems [23, 24]. Based on these previous works, some
quadratic \mathcal{D} -stabilizing controller design conditions have been recently proposed
for T-S systems [25, 26, 27, 28]. Furthermore, it is to be pointed out that the less
the conservatism is, better transient performances can be achieved. Thus, to
55 reduce the conservatism, \mathcal{D} -stable LMI-based conditions for non-PDC controller
design have been proposed in the non-quadratic framework [29]. Nevertheless,
these non-quadratic conditions are only locally suitable since they suffer from
the occurrence of the time derivatives of the MFs, where the optimization of the
closed-loop domain of attraction remains a tricky challenge due to the presence
60 of the \mathcal{D} -stability constraints. Alternatively, some attempts have been done to

propose global closed-loop \mathcal{D} -stability conditions in the non-quadratic framework via LILF [30, 31], but these results are unfortunately unsuitable as it is detailed in Appendix A.

According to the above mentioned concerns, it appears that, from previous literature, suitable global non-quadratic approach for the \mathcal{D} -stabilization of T-S models are not available. The present study aims at filling this gap by considering a NQLF involving the mean values of the MFs instead of the LILF considered in [30, 31]. Moreover, in this work we ought to show that, unlike previous local non-quadratic approaches [4, 13, 29], considering a NQLF which involves the mean values of the MFs applies globally, not only for T-S models with smooth (continuous) MFs, but also for T-S models involving piecewise continuous MFs. This will make the proposed LMI conditions relevant for the global non-quadratic \mathcal{D} -stabilization of a larger class of nonlinear systems with piecewise continuous nonlinearities (e.g. a class of switching nonlinear systems) which is, until now and from the authors' knowledge, an open problem that hasn't been addressed in previous literature.

The outline of this paper is given as follows: In Section 2, some preliminaries on T-S fuzzy models with useful definitions and lemmas for their \mathcal{D} -stabilization are recalled. Then, the main results are presented in Section 3, where three theorems, summarizing the proposed non-quadratic LMI conditions for the design of \mathcal{D} -stabilizing non-PDC controllers for T-S models involving piecewise MFs, are given. Finally, the effectiveness and the relative degree of conservatism of the different results are illustrated and compared to suitable quadratic results through an academic example.

Notations. Along this paper, one denotes the subset of integers $\mathcal{I}_r = \{1, \dots, r\}$, $r \geq 1$. Moreover, the symbol $(*)$ in a matrix denotes a transpose quantity. In mathematical expressions, I denotes identity matrices with appropriate dimensions. $M > 0$ (resp. < 0) means that M is a positive definite matrix (resp. negative definite). \otimes denotes the Kronecker product. For any matrices M with appropriate dimensions, one denotes $\mathcal{H}(M) = M + M^T$. Finally, the time t is

omitted in the proofs of the proposed theorems when there is no ambiguity.

2. Preliminaries and problem statement

Consider the class of Takagi-Sugeno systems described by the following polytopic state space representation [1]:

$$\dot{x}(t) = \sum_{i=1}^r h_i(z(t))(A_i x(t) + B_i u(t)) \quad (1)$$

where $x(t) = [x_1(t) \dots x_n(t)]^T \in \mathbb{R}^n$ is the state vector, $u(t) = [u_1(t) \dots u_m(t)]^T \in \mathbb{R}^m$ is the input vector, $z(t) = [z_1(t) \dots z_p(t)] \in \mathbb{R}^p$ is the premise vector, which is assumed to nonlinearly depend only on the state variables for stabilization purpose (i.e. $z(t) \equiv x(t)$), r is the number of polytopes, $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ are constant matrices describing the dynamics of each polytope, $h_i(z(t)) \geq 0$ are the Membership Functions (MFs) with the convex sum property $\sum_{i=1}^r h_i(z(t)) = 1$.

Inspired by the work of Marquez et al. [19], a non-quadratic Lyapunov Function (NQLF) candidate, which involves the mean value of the MFs $h_i(z(t))$ on the time interval $[t - \alpha, t]$ with $\alpha > 0$, will be considered to develop the main results proposed in the next Section. Hence, the following assumptions are considered in the present study.

Assumption 1. For a given scalar $\alpha > 0$ and $\forall t \in [-\alpha, 0)$, $h_i(z(t)) = h_i(z(0))$.

Assumption 2. For all $t \in [-\alpha, +\infty)$, each $h_i(t)$ is (at least) piecewise continuous.

According to assumptions 1 and 2, $\forall t > 0$ the piecewise MFs $h_i(z(t))$ are integrable on intervals $[t - \alpha, t]$. Hence, we can define their mean values on these intervals as:

$$v_i(z(t)) = \alpha^{-1} \int_{t-\alpha}^t h_i(z(\tau)) d\tau, \quad \forall i \in \mathcal{I}_r \quad (2)$$

As shown in [19], $\forall t > 0$ and $\forall i \in \mathcal{I}_r$, the mean values (2) exhibit the convex sum properties. Indeed, since $h_i(z(t)) \geq 0$ and $\sum_{i=1}^r h_i(z(t)) = 1$, it is obvious

that $v_i(z(t)) \geq 0$ and:

$$\begin{aligned}
\sum_{i=1}^m v_i(z(t)) &= \sum_{i=1}^m \alpha^{-1} \int_{t-\alpha}^t h_i(z(\tau)) d\tau \\
&= \alpha^{-1} \int_{t-\alpha}^t \sum_{i=1}^m h_i(z(\tau)) d\tau \\
&= \alpha^{-1} \int_{t-\alpha}^t 1 d\tau \\
&= 1
\end{aligned} \tag{3}$$

Note that, if the membership functions $h_i(z(t))$ are piecewise continuous, their derivatives $\dot{h}_i(z(t))$ are unbounded and previous classical and local non-quadratic approaches (see e.g. [4, 13, 29]) can't apply, whereas:

$$\dot{v}_i(z(t)) = \alpha^{-1} (h_i(z(t)) - h_i(z(t - \alpha))), \quad \forall i \in \mathcal{I}_r \tag{4}$$

are always finite and bounded for all $t \in [-\alpha, +\infty)$.

Moreover, it is to be noticed that if $\alpha \rightarrow 0$, then $\dot{v}_i(z(t)) \rightarrow \dot{h}_i(z(t))$. As a consequence, for small finite and strictly positive values of α , $v_i(z(t))$ is continuous and can be view as a smoothed approximation of $h_i(z(t))$ (see for example Figure 4 in section 4).

Summarizing, the mean values $v_i(z(t))$ being continuous with bounded derivatives make them relevant (instead of $h_i(z(t))$) as weighting functions for a NQLF candidate, presented in the next section, for the global stabilization of T-S systems (1) with piecewise MFs.

Remark 1. In [19], the focus was only put on the stabilization of T-S models with strictly continuous MFs and no mention was made on the applicability of the results with piecewise continuous MFs. If the authors of [19] miss this fact, the present study enable this oversight to be highlighted.

In the sequel, to lighten the mathematical expressions, for any sets of real matrix M_i , $i \in \mathcal{I}_r$, we will denote:

$$M_h = \sum_{i=1}^r h_i(z(t)) M_i, \quad M_{\bar{h}} = \sum_{i=1}^r h_i(z(t - \alpha)) M_i, \quad M_v = \sum_{i=1}^r v_i(z(t)) M_i \tag{5}$$

or for any other convex combinations (multiple fuzzy sums):

$$M_{h\bar{h}\bar{v}} = \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r \sum_{l=1}^r h_i(z(t))h_j(z(t))h_k(z(t-\alpha))v_l(z(t))M_{ijkl} \quad (6)$$

Moreover, from (4) and the above defined notations, we have:

$$\dot{M}_v = \sum_{i=1}^r \dot{v}_i(z(t))M_i = \alpha^{-1} (M_h - M_{\bar{h}}) \quad (7)$$

To stabilize (1), let us now consider the following non-PDC control law adapted from [32] and [19]:

$$u(t) = K_{h\bar{h}\bar{v}}H_{(\cdot)}^{-1}x(t) \quad (8)$$

where $K_{h\bar{h}\bar{v}} \in \mathbb{R}^{m \times n}$ and $H_{(\cdot)} \in \mathbb{R}^{n \times n}$ (to be further detailed) are gain scheduled matrices to be synthesized.

Substituting (8) in (1), the closed-loop dynamics is expressed by:

$$\dot{x}(t) = \left(A_h + B_h K_{h\bar{h}\bar{v}} H_{(\cdot)}^{-1} \right) x(t) \quad (9)$$

125 **Problem statement.** Providing that only few local non-quadratic approaches
for the \mathcal{D} -stabilization of T-S models involving continuous MFs are available
from previous literature (see e.g. [27, 29]), the goal of this paper is to propose
new non-quadratic LMI conditions for the design of the gain scheduled matrices
 $K_{h\bar{h}\bar{v}}$ and $H_{(\cdot)}$ such that (9) is globally non-quadratically \mathcal{D} -stable; i.e such
130 that the non-PDC controller (8) globally non-quadratically \mathcal{D} -stabilizes the T-S
fuzzy model (1) with piecewise MFs.

To deal with the \mathcal{D} -stability concept, let us now recall the following definition
of a LMI region [21].

Definition 1. [21]. A LMI region is a subset \mathcal{D} of the complex plane, defined
by the matrices $R_1 = R_1^T \in \mathbb{R}^{\delta \times \delta}$ and $R_2 \in \mathbb{R}^{\delta \times \delta}$ such that:

$$\mathcal{D} = \{ \lambda \in \mathbb{C} : R_1 + \lambda R_2 + \bar{\lambda} R_2^T < 0 \} \quad (10)$$

where δ is the LMI region order.

135 *Remark 2.* By extension of the pioneer work [21], where the \mathcal{D} -stability concept
has been defined for linear systems with constant uncertainties, it is used in
the present study for nonlinear systems (i.e. the considered class of T-S fuzzy
models) in the sense that, for each instant t , the distribution of the eigenvalues
of (9) lies in a subset \mathcal{D} of the complex plane. Moreover, because the proposed
140 conditions are in the LMI form (see the main results in the next section), when
the particular subset \mathcal{D} recovers the left half-plane, the asymptotical stability
of the nonlinear system can be concluded following similar steps as those used
in Theorem 1 in [33].

For control purpose, the LMI region depicted in Figure 1 is usually consid-
145 ered. It is the intersection of three elementary regions of the complex plane:
the left half-plane defined by $\mathcal{R}e(\lambda) < -\beta$, a conic sector defined by its apex at
 $(\gamma, 0)$ with an inner angle $\pi/2 - \theta$ and a circle centered at $(q, 0)$ with a radius s .

- Place Figure 1 here -

According to Definition 1, this LMI region is defined by the following ma-
trices [21]:

$$R_1 = \begin{bmatrix} 2\beta & 0 & 0 & 0 & 0 \\ 0 & -2\gamma \cos \theta & 0 & 0 & 0 \\ 0 & 0 & -2\gamma \cos \theta & 0 & 0 \\ 0 & 0 & 0 & -s & -q \\ 0 & 0 & 0 & -q & -s \end{bmatrix} \quad (11)$$

and

$$R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 & 0 \\ 0 & -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

See e.g. [20, 21] for several LMI region examples illustrating how to set the
150 matrices R_1 and R_2 .

With regards to the Lyapunov theory, the following definition is adapted from [21] as basic \mathcal{D} -stability conditions for nonlinear systems [34].

Definition 2. [34]. Given an LMI region defined by (10), a nonlinear system $\dot{x} = f(x)x$ is said to be \mathcal{D} -stable if there exists a Lyapunov function $\mathcal{V}(x(t))$ satisfying $\frac{1}{2} \frac{\dot{\mathcal{V}}(x(t))}{\mathcal{V}(x(t))} \in \mathcal{D}$, i.e.:

$$R_1 \otimes \mathcal{V}(x(t)) + R_2 \otimes \frac{1}{2} \dot{\mathcal{V}}(x(t)) + R_2^T \otimes \frac{1}{2} \dot{\mathcal{V}}(x(t)) < 0 \quad (13)$$

where \otimes denotes the Kronecker product.

Remark 3. When $R_1 = 0$ and $R_2 = 1$, this definition reduces to the standard asymptotical stability conditions of the considered nonlinear system (i.e. the LMI region \mathcal{D} reduces to the left half-plane, see also Remark 2).

Let us now recall some useful properties of the Kronecker product.

Property 1. [35] For any scalars η and matrices X , Y and Z with appropriate dimensions, the following properties of the Kronecker product holds:

$$(X \otimes Y)^T = X^T \otimes Y^T$$

$$(X + \eta Y) \otimes Z = (X \otimes Z) + \eta(Y \otimes Z)$$

$$X \otimes (Y + \eta Z) = (X \otimes Y) + \eta(X \otimes Z)$$

In addition, to further relax the LMI conditions proposed in the next section, the following lemma [36], usual in the context of T-S models, is adopted in the sequel since it is wellknown as a good compromise between computational complexity and conservatism improvement [9].

Lemma 1. (Tuan's lemma [36]): Let Γ_{ij}^{kl} , for $(i, j, k, l) \in \mathcal{I}_r^4$, be matrices of appropriate dimensions. $\Gamma_{hh}^{\tilde{h}v} < 0$ is satisfied if both the following conditions hold:

$$\Gamma_{ii}^{kl} < 0, \quad \forall (i, k, l) \in \mathcal{I}_r^3 \quad (14)$$

$$\frac{2}{r-1} \Gamma_{ii}^{kl} + \Gamma_{ij}^{kl} + \Gamma_{ji}^{kl} < 0, \quad \forall (i, j, k, l) \in \mathcal{I}_r^4 / i \neq j \quad (15)$$

To conclude this preliminary section, let us recall that from Definition 2, LMI-based quadratic \mathcal{D} -stability conditions and relaxed quadratic ones have been proposed in [34, 25, 26, 27, 28]. To further relax these LMI conditions, local non-quadratic LMI-based conditions have been proposed in [29]. Nevertheless, these previous local non-quadratic approach suffers from the occurrence of the time derivatives of the MFs and are not suitable when T-S systems involve piecewise continuous MFs (since their time derivatives are unbounded). Moreover, some attempts to provide global non-quadratic \mathcal{D} -stabilization approaches, based on the consideration of a LILF, have been proposed in [30, 31]. Unfortunately, as detailed in Appendix A, these previous LILF-based results are not correct. Therefore, in the next section, we aim at proposing new relaxed LMI-based non-quadratic conditions for the design of non-PDC controllers (8), which globally non-quadratically \mathcal{D} -stabilizes (1), even when they involve piecewise continuous MFs.

3. Main results

In this section, three theorems are proposed with new non-quadratic LMI-based conditions for the global non-quadratic \mathcal{D} -stabilization of T-S model (1), which may have piecewise MFs, under the non-PDC control law (8). In this context, let us consider the following NQLF candidate [19], which involves the mean values of the MFs (2), under assumptions 1 and 2, as scheduling variables:

$$\mathcal{V}(x(t)) = x^T(t)P_v^{-1}x(t) \tag{16}$$

Remark 4. Note that Assumption 1 is similarly to what is done for time-delay systems to ensure the existence of a valued function for initial conditions, see e.g. [37]). Moreover, Assumption 2 is the only requirement on the MFs h_i for (16) to be continuous, making it relevant for the stability analysis of T-S models involving piecewise MFs without needing any dwell time or switching instants assumptions.

The following theorem summarizes the first proposed global non-quadratic conditions.

Theorem 1. *Let R_1 and R_2 be two matrices defining a prescribed LMI region \mathcal{D} in the left half-plane of the complex plane. The T-S model (1) is globally and non-quadratically \mathcal{D} -stabilized by the non-PDC control law (8) if there exists a scalar $\alpha > 0$ and, for $(j, k, l) \in \mathcal{I}_r^3$, the matrices $P_k = P_k^T > 0$, K_{jkl} , such that the LMI conditions (14) and (15) are verified with:*

$$\Gamma_{ij}^{kl} = R_1 \otimes P_l + \mathcal{H} \left(R_2 \otimes \left(A_i P_l + B_i K_{jkl} - \frac{1}{2\alpha} (P_j - P_k) \right) \right) \quad (17)$$

185 *In that case, $H_{(\cdot)} = P_v$ in the non-PDC controller (8).*

Proof. Consider the NQLF (16). From the convex properties of the MFs (2), (16) is obviously positive if $P_k = P_k^T > 0$, $\forall k \in \mathcal{I}_r$. Moreover, from the closed-loop dynamics (9), the time derivative of (16) can be expressed as:

$$\begin{aligned} \dot{V}(x) &= \dot{x}^T P_v^{-1} x + x^T P_v^{-1} \dot{x} + x^T \dot{P}_v^{-1} x \\ &= 2x^T \left(P_v^{-1} \left(A_h + B_h K_{h\bar{h}v} H_{(\cdot)}^{-1} \right) + \frac{1}{2} \dot{P}_v^{-1} \right) x \end{aligned} \quad (18)$$

Thus, from (16), (18) and according to Definition 2, the closed-loop dynamics (9) is \mathcal{D} -stable if:

$$R_1 \otimes x^T P_v^{-1} x + \mathcal{H} \left(R_2 \otimes x^T \left(P_v^{-1} \left(A_h + B_h K_{h\bar{h}v} H_{(\cdot)}^{-1} \right) + \frac{1}{2} \dot{P}_v^{-1} \right) x \right) < 0 \quad (19)$$

Now, according to Property 1, with $\tilde{x} = (I \otimes x)$ we can rewrite (19) as:

$$\tilde{x}^T \left(R_1 \otimes P_v^{-1} + \mathcal{H} \left(R_2 \otimes \left(P_v^{-1} \left(A_h + B_h K_{h\bar{h}v} H_{(\cdot)}^{-1} \right) + \frac{1}{2} \dot{P}_v^{-1} \right) \right) \right) \tilde{x} < 0 \quad (20)$$

which holds $\forall \tilde{x}$ if:

$$R_1 \otimes P_v^{-1} + \mathcal{H} \left(R_2 \otimes \left(P_v^{-1} \left(A_h + B_h K_{h\bar{h}v} H_{(\cdot)}^{-1} \right) + \frac{1}{2} \dot{P}_v^{-1} \right) \right) < 0 \quad (21)$$

Now, multiplying (21) left and right by $(I \otimes P_v)$ and since $P_v \dot{P}_v^{-1} P_v = -\dot{P}_v$, (21) is equivalent to:

$$R_1 \otimes P_v + \mathcal{H} \left(R_2 \otimes \left(A_h P_v + B_h K_{h\bar{h}v} H_{(\cdot)}^{-1} P_v - \frac{1}{2} \dot{P}_v \right) \right) < 0 \quad (22)$$

Moreover, from (4) yields:

$$R_1 \otimes P_v^{-1} + \mathcal{H} \left(R_2 \otimes \left(A_h P_v + B_h K_{h\bar{h}v} H_{(\cdot)}^{-1} P_v - \frac{1}{2\alpha} (P_h - P_{\bar{h}}) \right) \right) < 0 \quad (23)$$

Finally, by choosing $H_{(\cdot)} = P_v$, then applying Lemma 1, (23) is satisfied if the LMI conditions expressed in Theorem 1 hold. \square

Remark 5. From Theorem 1, by taking $P_j = P$ common and $K_{jkl} = K_j$, the basic quadratic \mathcal{D} -stabilizing controller design conditions proposed as Theorem 7 in [27] are recovered. Thus, the non-quadratic conditions of Theorem 1 are obviously less conservative than the quadratic ones (including those presented in [26]).

To further relax the global non-quadratic conditions proposed in Theorem 1, we propose two alternatives. The first one is inspired by the way borrowed in [23] for linear systems with polytopic uncertainties and the second one can be considered, in the global non-quadratic \mathcal{D} -stabilization context of this study, as an extension of recent stabilization results (see e.g. [38, 32, 18, 28]), obtained via the application of the Finsler's lemma [39].

The following theorem summarizes the first alternative.

Theorem 2. *Let R_1 and R_2 be two matrices defining a prescribed LMI region \mathcal{D} in the left half-plane of the complex plane. The T-S model (1) is globally and non-quadratically \mathcal{D} -stabilized by the non-PDC control law (8) if there exists a scalar $\alpha > 0$ and, for $(j, k, l) \in \mathcal{I}_r^3$, the matrices $P_k = P_k^T > 0$, K_{jkl} , G_{jkl} and Q_{jkl} , such that the LMI conditions (14) and (15) are verified with:*

$$\Gamma_{ij}^{kl} = \begin{bmatrix} \Phi_{ij}^{kl} & (*) \\ R_2 \otimes (P_l - Q_{jkl}) + I \otimes G_{jkl}^T A_i^T & -I \otimes \mathcal{H}(G_{jkl}) \end{bmatrix} \quad (24)$$

and:

$$\Phi_{ij}^{kl} = R_1 \otimes P_l + \mathcal{H} \left(R_2 \otimes \left(A_i Q_{jkl} + B_i K_{jkl} - \frac{1}{2\alpha} (P_j - P_k) \right) \right) \quad (25)$$

In that case, $H_{(\cdot)} = P_v$ in the non-PDC controller (8).

Proof. If the conditions of Theorem 2 hold, from Lemma 1, they provide that:

$$\begin{bmatrix} \Phi_{hh}^{\tilde{h}v} & (*) \\ R_2 \otimes (P_l - Q_{jkl}) + I \otimes G_{jkl}^T A_i^T & -I \otimes \mathcal{H}(G_{jkl}) \end{bmatrix} < 0 \quad (26)$$

with $\Phi_{hh}^{\tilde{h}v} = R_1 \otimes P_v + \mathcal{H}(R_2 \otimes (A_h Q_{h\tilde{h}v} + B_h K_{h\tilde{h}v} - \frac{1}{2\alpha}(P_h - P_{\tilde{h}})))$.

Pre- and post-multiplying (26) respectively by $\begin{bmatrix} I & I \otimes A_i \end{bmatrix}$ and its transpose, after expanding and simplifying the obtained mathematical expression, yields the inequality (23) with $H_{(\cdot)} = P_v$. This guarantees that the closed-loop dynamics (9) is globally non-quadratically \mathcal{D} -stable. \square

The second alternative to relax the basic global non-quadratic conditions of Theorem 1 is proposed by the following theorem.

Theorem 3. *Let R_1 and R_2 be two matrices defining a prescribed LMI region \mathcal{D} in the left half-plane of the complex plane. The T-S model (1) is globally and non-quadratically \mathcal{D} -stabilized by the non-PDC control law (8) if there exists the scalars $\alpha > 0$ and $\varepsilon > 0$, and, for $(j, k, l) \in \mathcal{I}_r^3$, the matrices $P_k = P_k^T > 0$, K_{jkl} and H_{jkl} , such that the conditions (14) and (15) are verified with:*

$$\Gamma_{ij}^{kl} = \begin{bmatrix} \Psi_{ij}^{kl} & (*) \\ R_2 \otimes (P_l - H_{jkl}) + \varepsilon I \otimes (H_{jkl}^T A_i^T + K_{jkl}^T B_i^T) & -\varepsilon I \otimes \mathcal{H}(H_{jkl}) \end{bmatrix} \quad (27)$$

and:

$$\Psi_{ij}^{kl} = R_1 \otimes P_l + \mathcal{H}\left(R_2 \otimes \left(A_i H_{jkl} + B_i K_{jkl} - \frac{1}{2\alpha}(P_j - P_k)\right)\right) \quad (28)$$

In that case, $H_{(\cdot)} = H_{h\tilde{h}v}$ in the non-PDC controller (8).

Proof. If the conditions of Theorem 3 hold, from Lemma 1, they provide that:

$$\begin{bmatrix} \Psi_{hh}^{\tilde{h}v} & (*) \\ R_2 \otimes (P_v - H_{h\tilde{h}v}) + \varepsilon I \otimes (H_{h\tilde{h}v}^T A_h^T + K_{h\tilde{h}v}^T B_h^T) & -\varepsilon I \otimes \mathcal{H}(H_{h\tilde{h}v}) \end{bmatrix} < 0 \quad (29)$$

with $\Psi_{hh}^{\tilde{h}v} = R_1 \otimes P_v + \mathcal{H}(R_2 \otimes (A_h H_{h\tilde{h}v} + B_h K_{h\tilde{h}v} - \frac{1}{2\alpha}(P_h - P_{\tilde{h}})))$.

Pre- and post-multiplying (29) respectively by $\begin{bmatrix} I & I \otimes (A_z + B_z K_z H_z^{-1}) \end{bmatrix}$

and its transpose, after expanding and simplifying the obtained mathematical expression, yields the inequality (23) with $H_{(\cdot)} = H_{h\tilde{h}v}$. This guarantees that the closed-loop dynamics (9) is globally non-quadratically \mathcal{D} -stable. \square

Remark 6. Note that with Theorem 3, the non-PDC scheduled gain matrix $H_{h\tilde{h}v}$ is not required to be symmetric. Nevertheless, its invertibility is guaranteed if the conditions of Theorem 3 hold since it yields from blocks (2,2) of (27) that $H_{h\tilde{h}v} + H_{h\tilde{h}v}^T > 0$.

Remark 7. : Similarly to previous **stabilization results without \mathcal{D} -stability constraints** based on the Finsler's lemma (see e.g. [38, 32, 18]), the conditions of Theorem 3 involve a prefixed scalar parameter $\varepsilon > 0$ and so are not strictly LMIs. **Note that this scalar is an additional slack decision variable as it can be easily shown that it results from the introduction of a null term by taking the converse way to prove Theorem 3 from Theorem 1.** As quote in [32], this parameters is usually prefixed or optimized **in practice** by linear programming inside a logarithmically spaced family of values **such like** $\varepsilon \in \{10^{-6}, 10^{-5}, \dots, 10^6\} \times \rho$, **with** $\rho \in \{1, 2, \dots, 9\}$, in order to avoid an exhaustive linear search. Moreover, in [38], it has been shown that the introduction of such additional parameter was **outperforming previous results in a large way for thousands of LPV models and comparing with numerous results (classical Q approach, Finslers application, and several other variations).**

Remark 8. **Note that, from their respective proofs, it is obvious that the conditions of theorems 2 and 3 are less conservative than the conditions of Theorem 1. Indeed, both these proofs show that from the LMI-based conditions of theorems 2 and 3, the LMI conditions of Theorem 1 can be recovered by simple mathematical manipulation (congruences). Moreover, theorems 2 and 3 involve slack decisions variables, which give more degree of freedom to their respective LMI problems than the one of Theorem 1. Therefore, Theorem 1 is a special case of both theorems 2 and 3, which in turns are obviously less conservative than Theorem 1. Moreover,** due to the absence of the terms $K_{jkl}^T B_i^T$ in the blocks (2,1) of (24) and their presence in the blocks (2,1) of (27), we cannot to prove

that the Theorem 2 is a special case of Theorem 3. Nevertheless, we will show through a numerical example given in the next section that we may expect less conservative results from Theorem 3 than Theorem 2, despite Theorem 3 is not strictly LMI (see Remark 7). We assume that the conservatism improvement
 245 of Theorem 3 comes from the more flexibility given by the terms including the control gains as decision variables to relax the anti-diagonal blocks of (27).

Remark 9. Note that $P_h = \lim_{\alpha \rightarrow 0} P_v$ and $\dot{P}_h = \lim_{\alpha \rightarrow 0} \dot{P}_v$. Thus, taking to that limit and assuming $|\dot{h}_k| < \phi_k$, theorems 1 and 3 lead respectively to the local non-quadratic results proposed as theorems 1 and 2 in [27]. In addition
 250 to being local (see also [29]), these non-quadratic \mathcal{D} -stabilization results are not suitable for T-S models with piecewise continuous. Consequently, these local non-quadratic results are left out in this paper.

4. Numerical example

In this section, an academic example is propose to show the effectiveness
 255 of theorems 1, 2 and 3, as well as to compare their conservatism regarding to recently proposed quadratic results [26, 27].

Let us consider a T-S model with two vertices involving piecewise continuous MFs and given by:

$$\dot{x}(t) = \sum_{i=1}^2 h_i(z(t))(A_i x(t) + B_i u(t)) \quad (30)$$

with :

$$A_1 = \begin{bmatrix} -1 & -10 & 5 & a \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -2 & 3 \\ 3 & 2 & 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & -10 & -5 & 4 \\ 0 & -1 & 0 & 1 \\ -1 & 2 & -4 & 0 \\ 3 & 1 & 2 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ b \\ 2 \\ 1 \end{bmatrix},$$

and the piecewise continuous MFs :

$$h_1(x) = \frac{1 - \text{sign}(\cos(x_4))\sin(x_1)}{2}, h_2(x) = \frac{\text{sign}(\cos(x_4))\sin(x_1) + 1}{2}$$

Remark 10. The considered MFs are piecewise continuous (i.e. discontinuous for all $x_4 = \pi/2 + \kappa\pi$, $\kappa \in \mathbb{Z}$). Assuming that the discontinuities are left limits and right continuous, the MFs are integrable $\forall x \in \mathbb{R}^2$. Thus, the recent local non-quadratic \mathcal{D} -stabilizing controller design conditions, proposed in [29], are unsuitable since they are dependent of the bounds ϕ_k of $|\dot{h}_i|$, which cannot be globally defined in this example (see also Remark 9).

In (30), the parameters a and b are dedicated to compare the conservatism of the considered LMI-based conditions. Thus, Figure 2 shows the obtained feasibility fields, computed using YALMIP [40] and SeDuMi [41], of theorems 1, 2, and 3, with $\alpha = 0.1$ and an LMI region defined by (11) and (12) (see also Figure 1) with $s = 21$, $q = -17$, $\beta = 0.5$, $\theta = 3\pi/10$, $\gamma = 9$. Figure 2(a) shows a comparison between Theorem 1 and the quadratic conditions of Theorem 7 in [27] and Theorem 2 in [26]. Figure 2(b) shows a comparison between Theorem 2 and the quadratic conditions of Theorem 1 in [28] (without parametric uncertainties). Figure 2(c) shows a comparison between Theorem 3 and the quadratic conditions of Theorem 10 in [27]. As expected, we observe that the proposed non-quadratic results are less conservative than their respective quadratic particular cases (see Remark 3) as well as outperforming another recent related quadratic \mathcal{D} -stabilization result [26]. Moreover, Figure 2(d) summarizes the feasibility fields obtained from theorems 1, 2 and 3. We observe that Theorem 2 provide slightly less conservative results than Theorem 1. Moreover, Theorem 3 provide significantly less conservative results than Theorem 2. This shows the conservatism improvements of the global non-quadratic stabilization conditions proposed in this paper.

- Place Figure 2 here -

Let us now consider the particular point $(a, b) = (-4, 17)$, where a solution has only been found with Theorem 3. This solution is summarized by the

following non-PDC controller (8) gain matrices (the other decision variables, completing the solution of Theorem 3, are left out for space reasons):

$$F_{111} = \begin{bmatrix} -0.0319 & -0.0292 & -0.0236 & -0.0096 \end{bmatrix},$$

$$F_{112} = \begin{bmatrix} -0.0259 & -0.0263 & -0.0218 & -0.0133 \end{bmatrix},$$

$$F_{121} = \begin{bmatrix} -0.0184 & -0.0231 & -0.0241 & -0.0098 \end{bmatrix},$$

$$F_{122} = \begin{bmatrix} -0.0163 & -0.0217 & -0.0231 & -0.0113 \end{bmatrix},$$

$$F_{211} = \begin{bmatrix} -0.0272 & -0.0189 & -0.0127 & -0.0012 \end{bmatrix},$$

$$F_{212} = \begin{bmatrix} -0.0256 & -0.0180 & -0.0118 & -0.0015 \end{bmatrix},$$

$$F_{221} = \begin{bmatrix} -0.0180 & -0.0170 & -0.0184 & -0.0040 \end{bmatrix},$$

$$F_{222} = \begin{bmatrix} -0.0139 & -0.0152 & -0.0183 & -0.0061 \end{bmatrix},$$

$$H_{111} = \begin{bmatrix} 0.0702 & 0.0116 & -0.0340 & -0.0510 \\ 0.0223 & 0.0110 & 0.0079 & -0.0102 \\ -0.0126 & 0.0163 & 0.0567 & 0.0240 \\ -0.0456 & -0.0011 & 0.0398 & 0.0439 \end{bmatrix},$$

$$H_{112} = \begin{bmatrix} 0.0694 & 0.0110 & -0.0352 & -0.0517 \\ 0.0186 & 0.0090 & 0.0062 & -0.0084 \\ -0.0219 & 0.0114 & 0.0533 & 0.0295 \\ -0.0476 & -0.0017 & 0.0401 & 0.0455 \end{bmatrix},$$

$$H_{121} = \begin{bmatrix} 0.0751 & 0.0148 & -0.0350 & -0.0601 \\ 0.0149 & 0.0081 & 0.0058 & -0.0082 \\ -0.0364 & 0.0056 & 0.0517 & 0.0382 \\ -0.0565 & -0.0061 & 0.0396 & 0.0533 \end{bmatrix},$$

$$H_{122} = \begin{bmatrix} 0.0755 & 0.0146 & -0.0359 & -0.0618 \\ 0.0143 & 0.0075 & 0.0050 & -0.0084 \\ -0.0384 & 0.0041 & 0.0502 & 0.0393 \\ -0.0583 & -0.0067 & 0.0395 & 0.0554 \end{bmatrix},$$

$$H_{211} = \begin{bmatrix} 0.1506 & 0.0617 & -0.0003 & -0.0672 \\ 0.0916 & 0.0586 & 0.0300 & -0.0028 \\ 0.0386 & 0.0360 & 0.0399 & 0.0183 \\ -0.0104 & 0.0336 & 0.0536 & 0.0768 \end{bmatrix},$$

$$\begin{aligned}
H_{212} &= \begin{bmatrix} 0.1476 & 0.0598 & -0.0035 & -0.0671 \\ 0.0860 & 0.0558 & 0.0270 & -0.0006 \\ 0.0328 & 0.0334 & 0.0384 & 0.0202 \\ -0.0169 & 0.0330 & 0.0530 & 0.0847 \end{bmatrix}, \\
H_{221} &= \begin{bmatrix} 0.1268 & 0.0472 & -0.0102 & -0.0688 \\ 0.0661 & 0.0495 & 0.0386 & -0.0004 \\ 0.0148 & 0.0367 & 0.0662 & 0.0347 \\ -0.0323 & 0.0254 & 0.0724 & 0.0762 \end{bmatrix}, \\
H_{222} &= \begin{bmatrix} 0.1255 & 0.0450 & -0.0118 & -0.0707 \\ 0.0579 & 0.0454 & 0.0369 & 0.0032 \\ 0.0002 & 0.0317 & 0.0674 & 0.0438 \\ -0.0410 & 0.0248 & 0.0724 & 0.0857 \end{bmatrix}.
\end{aligned}$$

Remark 11. Note that, as quote in Remark 7, the parameter ε has been searched by linear programming in an exponential family space. To save computational time, our LMI algorithm is stopped as soon as a solution is obtained and, for the particular point $(a, b) = (-4, 17)$ we obtained $\varepsilon = 0.0006$. Nevertheless, let us notice that other values of this parameter can also lead to feasible LMIs. Hence, in the case of a more stringent \mathcal{D} region specification, such linear programming search on this parameter can be helpful to find a solution.

Figure 3 shows the obtained closed-loop state trajectories and the migration of the closed-loop eigenvalues for the initial state $x(0) = \pi \begin{bmatrix} 2 & -2 & -1 & 1 \end{bmatrix}^T$.

- Place Figure 3 here -

The time evolution of the MFs and their mean values during this simulation are shown in Figure 4. It is clearly illustrated the piecewise character of the MFs h_1 and h_2 , with the red marks \times (left limits) and \circ (right continuous) pointing-out the discontinuities, while v_1 and v_2 are continuous.

- Place Figure 4 here -

Remark 12. To show the flexibility of the \mathcal{D} -stability concept, a simulation has been done by considering only a decay rate $\beta = 0.5$ (exponential stabilization)

300 instead of the whole LMI region depicted in Figure 3(b) (with the same initial state). As shown in Figure 5(a), when considering only a decay rate $\beta = 0.5$, the control signal exhibits a peak of 498.4 while it is significantly reduced to 76.8 when considering the whole LMI region (including the circle constraint). Figure 5(b) shows the distribution of the eigenvalues when only the decay rate
 305 is considered. Compared to Figure 3(b), we can observe that the distribution of the eigenvalues goes far on the left and have larger imaginary parts, increasing the transient control cost when only a decay rate is considered.

- Place Figure 5 here -

Remark 13. Note that in [19], \mathcal{D} -stability constraints have not been considered. Hence, it can be easily shown that theorems 1 and 2 in [19] are respectively
 310 special cases of theorems 1 and 2 by setting the left half-plane as \mathcal{D} region (see remarks 2 and 3). Thus, in this case, checking their respective conservatism leads to obtain the same feasibility regions (plots are left out since it is straightforward). However, there is no theorem in [19] which is a special case of Theorem
 315 3 and, since it has been shown below that it improves the results obtained with theorems 2 and 3, we can expect less conservative results regarding to the results from both theorems 1 and 2 in [19].

To conclude this numerical example, let us recall that, for this example, no quadratic conditions exist with $(a, b) = (-4, 17)$ from the previous results [27,
 320 28]. Moreover, due to the piecewise character of the membership functions, the previous non-quadratic \mathcal{D} -stabilization approaches are unsuitable [29]. Moreover, when only the standard stabilization is considered (left half-plane set as LMI region), Theorem 3 has been shown less conservative than the results in [19]. Hence, this clearly shows the interest of the theoretical procedure proposed
 325 in this paper since it is not only improving the conservatism but also enlarging the considered class of T-S systems, which may now involve piecewise MFs.

5. Conclusion

In this paper, the problem of the global non-quadratic \mathcal{D} -stabilization of T-S systems with piecewise continuous membership functions has been considered. First, usual definitions of the \mathcal{D} -stability concept have been recalled. Then, from the choice of a non-quadratic Lyapunov function involving the mean values of the membership functions over a time interval $[t - \alpha, t]$ with $\alpha > 0$, three LMI-based theorems have been proposed for the design of non-PDC controllers for the \mathcal{D} -stabilization of T-S systems. It is highlighted that, thanks to the choice of such a Lyapunov function candidate, the non-quadratic design conditions are no longer depending on the time derivative of the membership functions, yielding to global asymptotical stabilization, also suitable for T-S models involving piecewise membership functions (i.e. for a class of switched nonlinear systems). This has been confirmed through a numerical example, where the conservatism improvements regarding to quadratic approaches have been illustrated.

The main advantages of the proposed global non-quadratic \mathcal{D} -stabilization approaches for T-S models are multiple regarding to previous tools. First, they allow to provide less conservative conditions than quadratic ones (see e.g. [25, 26, 27, 28]), which are also global. Secondly, most of the previous non-quadratic approaches were only local ones. Indeed, they require the estimation of the closed-loop Domain of Attraction (DA), which can be challenging when considering performance specifications such like \mathcal{D} -stability constraints (leading to small DA), but also can't guarantee the closed-loop stability in the whole state space (initial conditions outside the DA can diverge), e.g. [29]. Then, note that these local non-quadratic approaches are unsuitable for T-S models with piecewise MFs. Finally, let us also recall that some other previous attempts have been done to provide global non-quadratic \mathcal{D} -stabilization approaches with the use of LILF [30, 31]. But, as detailed in Appendix A, these results are incorrect and the use of a LILF candidate doesn't appear suitable to obtain convex conditions (LMI) with \mathcal{D} -stability conditions. Hence, the conditions proposed in this paper provide a convenient solution to the problem of global non-quadratic

\mathcal{D} -stabilization of T-S models, and so a suitable alternative to [30, 31].

Several perspectives of this study are foreseen. First, since the \mathcal{D} -stabilization allows to improve the closed-loop performances while keeping the energetic control cost to reasonable values (i.e. avoiding the distribution of the eigenvalues to goes far left in the complex plane), a study showing the links with optimal control design would be interesting. In the same way, analyzing the effects of associating input constraints such like anti-windup techniques with \mathcal{D} -stability constraints can be appealing for practical applications. Finally, since the considered mean values of the MFs hold filter properties, employing them as PDC gain scheduling variables may help to smoother the control signal and to handle external disturbances attenuation. Therefore, our future prospect will be to propose a rigorous robustness analysis of such control plant design with regards to H_∞ performances.

Appendix A.

In recent studies [30, 31], some attempts have been done to propose new global non-quadratic LMI conditions for the design of PDC controllers with \mathcal{D} -stability constraints for T-S models via LILF, following the pioneer work of Rhee and Won [5]. Such LILF is given by:

$$\mathcal{V}(x(t)) = 2 \int_{\Gamma(0, x(t))} f^T(\psi) \cdot d\psi \quad (\text{A.1})$$

where $\Gamma(0, x(t))$ is the path from the origin 0 to the current state $x(t)$, (\cdot) stands for the inner product of vectors, $\psi \in \mathbb{R}^n$ is a dummy vector for the integral and $d\psi$ is an infinitesimal displacement. Note that such approach requires the following structure of $f(x(t))$ to guarantee the path-independency of the LILF (A.1) [5, 16]:

$$f(x(t)) = \left(P_0 + \sum_{i=1}^r h_i(x(t)) P_i \right) x(t) \quad (\text{A.2})$$

$$\text{with } P_0 = \begin{bmatrix} 0 & p_{1,2} & \cdots & p_{1,n} \\ p_{1,2} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & p_{n-1,n} \\ p_{1,n} & \cdots & p_{n-1,n} & 0 \end{bmatrix}, P_i = \begin{bmatrix} p_{1,1}^{\alpha_{i1}} & 0 & \cdots & 0 \\ 0 & p_{2,2}^{\alpha_{i2}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & p_{n,n}^{\alpha_{in}} \end{bmatrix}$$

and where α_{il} specifies which θ_l -based fuzzy set is used in the i^{th} fuzzy rule, r_l is the number of θ_l -based fuzzy sets, i.e. $r = \prod_{l=1}^p r_l$. For example, if $\alpha_{il} = \kappa$ then it means that the κ^{th} fuzzy set among θ_l -based fuzzy sets is used in the i^{th} fuzzy rule, then $1 \leq \alpha_{il} \leq r_l$ for any i .

Note that from the LILF (A.1), the stability conditions are free of the time derivatives of the MFs. However in [30, 31], to apply the \mathcal{D} -stability conditions given as Definition 2, it has been assumed that the LILF (A.1) can be rewritten as:

$$\mathcal{V}(x(t)) = x^T(t) \left(P_0 + \sum_{i=1}^r h_i(x(t)) P_i \right) x(t) \quad (\text{A.3})$$

Unfortunately, such way of doing is not correct since it is obvious that (A.1) cannot be rewritten as (A.3) to cope with global asymptotical stabilization. Moreover, let us highlight that in the LILF framework, from Definition 2 and considering (A.1) instead of (A.3), the \mathcal{D} -stability condition (13) becomes:

$$R_1 \otimes 2 \int_{\Gamma(0,x(t))} f^T(\psi) \cdot d\psi + 2R_2 \otimes x^T(t) \left(P_0 + \sum_{i=1}^r h_i(x(t)) P_i \right) \dot{x}(t) < 0 \quad (\text{A.4})$$

which convexification remains an open and strong problem due to the presence of the line-integral term.

These concerns make unsuitable the LMI-based design for global non-quadratic \mathcal{D} -stabilization of T-S models proposed in [30, 31], while the procedures proposed above as Theorem 1, 2 and 3 provide alternative solutions.

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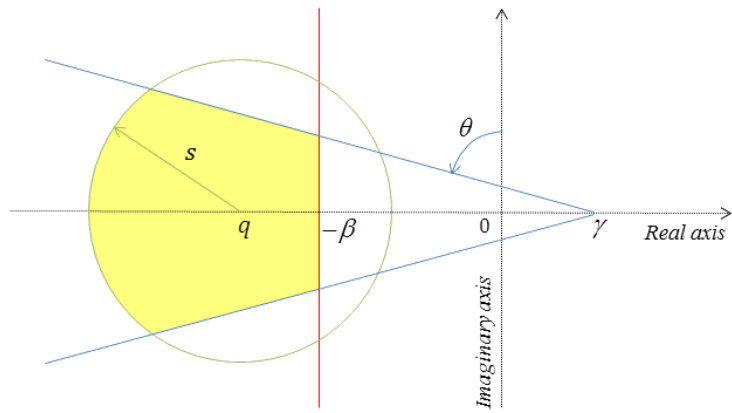
Figures:

Figure 1: Usual LMI region for control purpose.

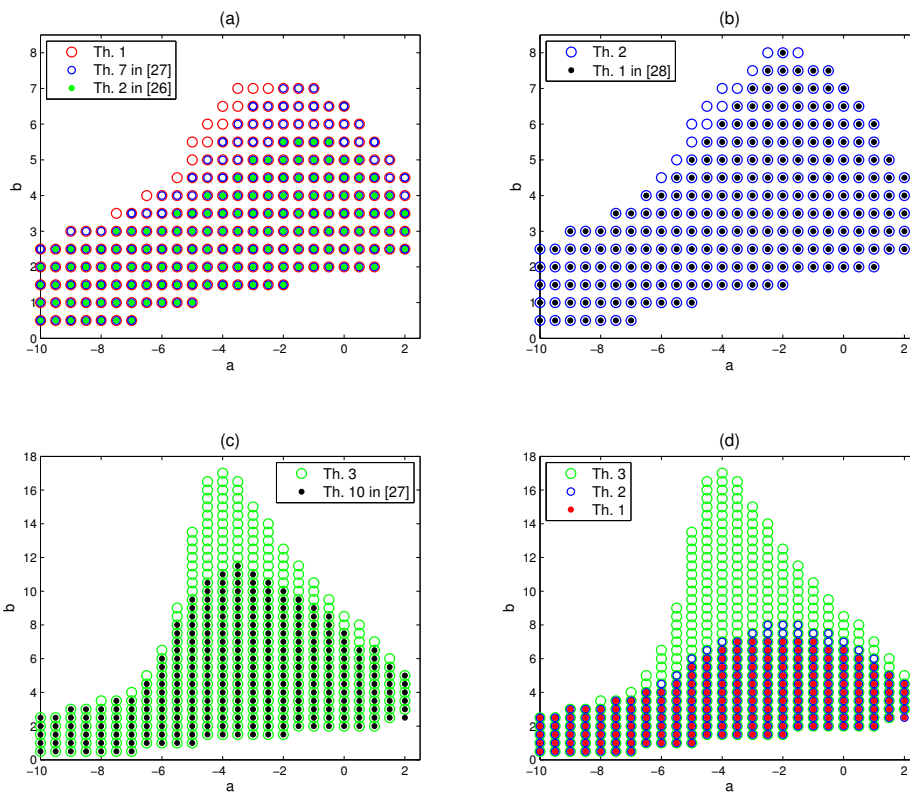


Figure 2: Comparison of the feasibility fields on parameter space $a \times b$.

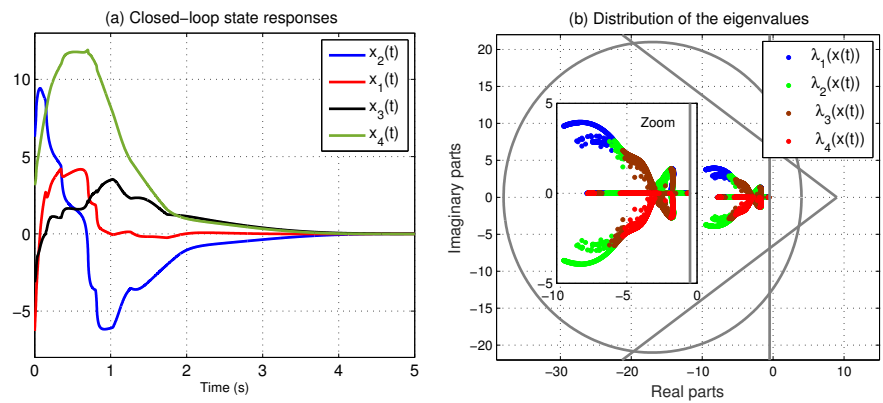


Figure 3: Closed-loop state responses and distribution of the closed-loop eigenvalues.

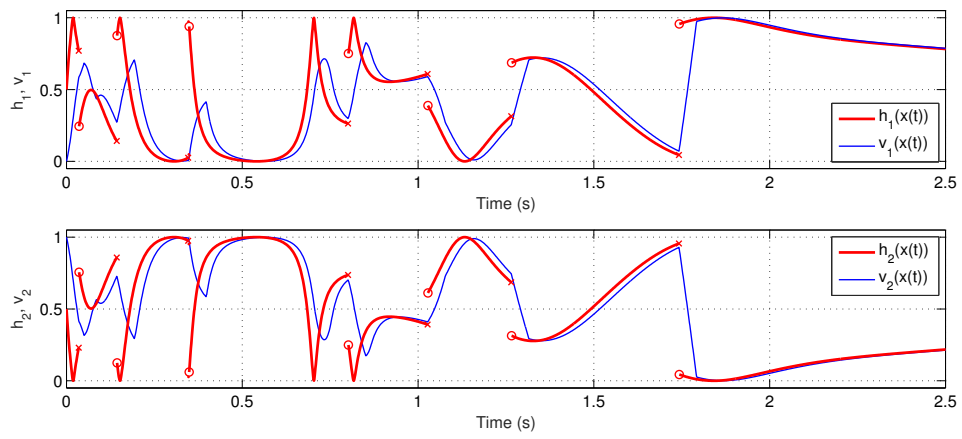


Figure 4: Time evolution of the membership functions.

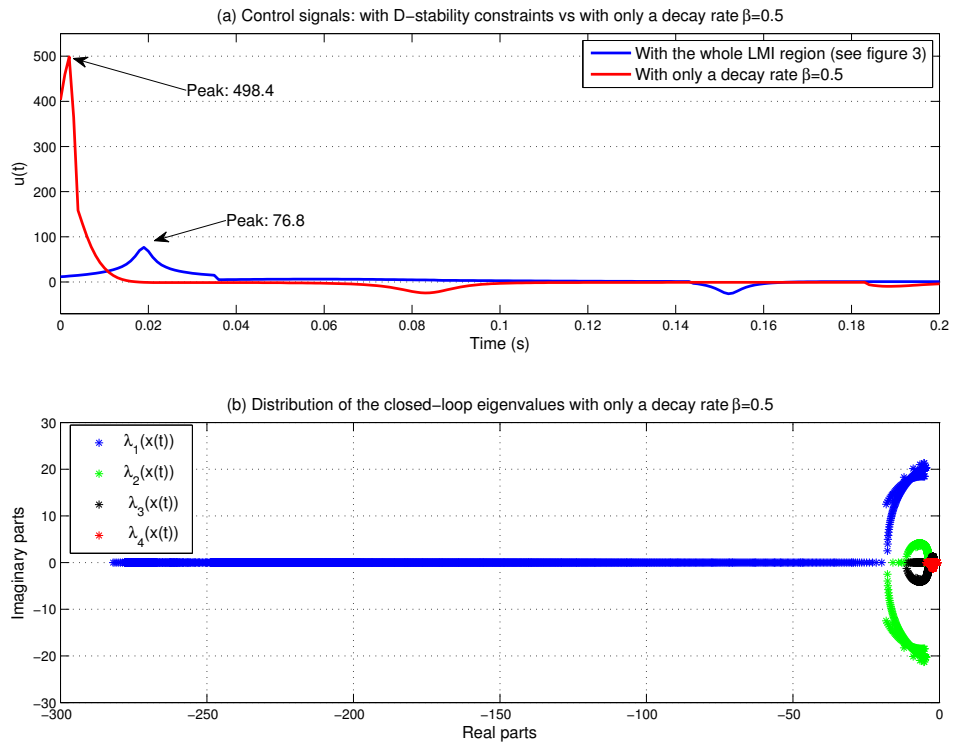


Figure 5: (a) Comparison of the control signals with the whole considered LMI region vs with only a decay rate $\beta = 0.5$; (b) distribution of the closed-loop eigenvalues with only a decay rate $\beta = 0.5$.