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# A Correlation for the Discontinuity of the Temperature Variance Dissipation Rate at the Fluid-Solid Interface in Turbulent Channel Flows

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1 **A correlation for the discontinuity of the temperature variance**  
2 **dissipation rate at the fluid-solid interface in turbulent channel**  
3 **flows**

4 **Cédric Flageul · Iztok Tiselj · Sofiane**  
5 **Benhamadouche · Martin Ferrand**

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8 **Abstract** Discontinuity of the dissipation rate associated with the temperature variance at  
9 the fluid-solid interface is analyzed in a turbulent channel flow at a Reynolds number, based  
10 on the friction velocity of 395 and a Prandtl number of 0.71. The analysis is performed with  
11 a wall-resolved Large Eddy Simulation and the results are used to derive a regression for the  
12 dissipation rate discontinuity, which depends only on the fluid-solid thermal diffusivity and  
13 conductivity ratios. Wall-resolved Large Eddy Simulations at a higher Reynolds number and  
14 a higher Prandtl number are used to investigate the validity of two correlations derived from  
15 the regression for the selected thermal properties ratios. The present results are obtained  
16 with the open-source Computational Fluid Dynamics solver *Code\_Saturne*, and use the fully  
17 conservative fluid-solid thermal coupling capability introduced by the authors in version  
18 5.0.

19 **1 Introduction**

20 Conjugate heat transfer refers to the thermal coupling between a fluid and a surrounding  
21 solid. It is of prime importance in industrial applications where thermal fatigue is a concern.  
22 In the nuclear field, thermal fatigue and fluctuating thermal stresses are particularly  
23 important in case of a severe emergency core cooling or long-term ageing of materials. Such  
24 complex applications are often studied experimentally and numerically. However, numerical  
25 investigations of turbulent flows at moderate to high Reynolds numbers remain challenging.  
26 CFD (Computational Fluid Dynamics) analysis of industrial applications usually relies  
27 on high or low Reynolds RANS (Reynolds-Averaged Navier-Stokes) and occasionally on  
28 wall-modelled LES (Large Eddy Simulation) for the low to moderate Reynolds numbers  
29 (Benhamadouche [4], Hassan [15]).

30 The turbulent flow applies on the solid domain a thermal load characterized by a broad  
31 spectrum. Qualitatively, the heat diffusion in the solid domain provides a strong damping

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at high frequencies. Therefore, smaller scales tend to apply a thermal stress in the vicinity of the fluid-solid interface while larger scales tend to penetrate deeper in the solid. Refined analysis actually shows that high stress amplitude events are generally associated with low probability ones (Costa Garrido et al. [5]), thus making accurate estimation of thermal fatigue in industrial applications even more challenging as the CFD simulations should provide at least a few minutes of operation in realistic conditions.

Analytical studies on conjugate heat transfer in turbulent flows were pioneered by Polyakov [24] and Geshev [13]. The fundamental solutions of the heat equation in the solid domain have a non-compact support. For instance, semi-infinite solids with a flat fluid-solid interface subjected to a statistically steady forcing can be characterized by a compatibility condition at the fluid-solid interface expressed as a spatio-temporal convolution (Flageul et al. [9]). Such non-local effects are specific to conjugate heat transfer and tend to become negligible only when the thermal properties of the fluid and of the solid differ by orders of magnitude.

Numerical studies on conjugate heat transfer in turbulent flows were pioneered by Kasagi et al. [17] and their 2D synthetic turbulence model. The first DNS (Direct Numerical Simulation) with conjugate heat transfer was a turbulent channel flow, performed by Tiselj et al. [27]. Following those studies, some of the present authors and co-workers performed additional DNS of the turbulent channel flow [9] to extract the budgets of the second moments (i.e. the turbulent heat fluxes and the temperature variance). This work was motivated by the global lack of validation data for second-order RANS turbulence models (Dehoux et al. [7], [8]) in case of an imposed heat flux, in case of a heat exchange coefficient, or in case of conjugate heat transfer, most of the previous DNS of the turbulent channel flow using an imposed temperature at the wall (Kasagi et al. [18], Kawamura et al. [19], Abe et al. [2]).

To the best of the authors knowledge, the only RANS turbulence model designed to take into account conjugate heat transfer — i.e. able to solve the temperature fluctuations both in the fluid and in the solid — was published by Craft et al. [6]. It is also the only RANS turbulence model designed to correctly handle cases with an imposed heat flux (Mangeon et al. [22]). However, it was recently shown that the dissipation rate ( $\epsilon_\theta$ ) associated with the halved temperature variance ( $\overline{T'^2}/2$ ) is discontinuous at the fluid-solid interface in case of conjugate heat transfer (Flageul et al. [10]). Although there is currently no coupled RANS model taking this discontinuity in account there is a global agreement that one is needed (Shams et al. [26]).

The background on such turbulence models is given in the paper of Craft et al. [6] and in classic textbooks on turbulence modelling (Hinze [16], Pope [25], Leschziner [21]). A simple — and thus approximate — sketch would be to say that RANS models use the turbulent kinetic energy  $k$  and the associated dissipation rate  $\epsilon$  to model velocity fluctuations. Regarding temperature fluctuations, the halved temperature variance  $\overline{T'^2}/2$  and the associated dissipation rate  $\epsilon_\theta$  are very similar to  $k$  and  $\epsilon$ , respectively. However, velocity fluctuations remain inside the fluid domain while temperature fluctuations may penetrate inside solid domains adjacent to the fluid one. In case of conjugate heat transfer, this fundamental difference must be taken into account, and the turbulence model should be able to evaluate  $\overline{T'^2}$  and  $\epsilon_\theta$  in the solid domain adjacent to the fluid one.

The objectives of this paper are twofold. First, we present the new stand-alone fluid-solid thermal coupling capability implemented in the open-source CFD code *Code\_Saturne*<sup>1</sup>, we use it to perform wall-resolved LES of the turbulent channel flow with conjugate heat transfer at moderate Reynolds numbers and we assess the results using DNS. The objective of this first step is to expose and validate our developments. Second, we perform and analyse

<sup>1</sup> See <https://www.code-saturne.org>

79 various wall-resolved LES with conjugate heat transfer at a higher Reynolds and Prandtl  
 80 numbers, and with various ratios of fluid-solid thermal properties. The objective of this  
 81 second step is to investigate their respective influence on the discontinuity of  $\varepsilon_\theta$  at the fluid-  
 82 solid interface. The correlations we propose for the discontinuity of  $\varepsilon_\theta$  are the outcome of  
 83 this second step. Indeed, the correlations proposed are expected to be valid only for the  
 84 range of fluid-solid thermal properties ratio investigated, and for turbulent flows similar to  
 85 the one simulated. However, the methodology used being generic, it could potentially be  
 86 applied to any configuration of interest, provided one can perform wall-resolved LES of this  
 87 configuration.

88 The present paper is divided into five sections. Section 2 presents the governing equa-  
 89 tions and describes the turbulent channel flows investigated. Section 3 presents the discreti-  
 90 sation used to solve the governing equations alongside with the extrapolation of statistics  
 91 performed at the fluid-solid interface. Section 4 presents the validation of our wall-resolved  
 92 LES against DNS. Section 5 presents some new LES results and our correlations for the dis-  
 93 continuity of  $\varepsilon_\theta$ . Ultimately, in Section 6, our results are further discussed, alongside with  
 94 some concluding remarks and some perspectives.

## 95 2 Governing equations and cases description

### 96 2.1 Governing equations

97 We consider the turbulent flow of a Newtonian fluid with constant physical properties. Fur-  
 98 thermore, we assume that the flow is incompressible and that the physical properties in the  
 99 solid domains are also constant. The subscripts  $f$  and  $s$  are used for the fluid and for the  
 100 solid, respectively.

101 In this subsection, we omit boundary conditions for the sake of simplicity. Firstly, the  
 102 conservation of mass in a fluid with a constant density reduces to the incompressibility  
 103 condition

$$\partial_i u_i = 0 \quad (1)$$

104 where  $u_i$  and  $\partial_i$  are the velocity and the spatial derivative in direction  $i$ , respectively.

105 Secondly, the conservation of momentum in the fluid domain is

$$\partial_t u_i + \partial_j (u_i u_j) = -\frac{1}{\rho_f} \partial_i P + \partial_j \sigma_{ij} + f_i \quad (2)$$

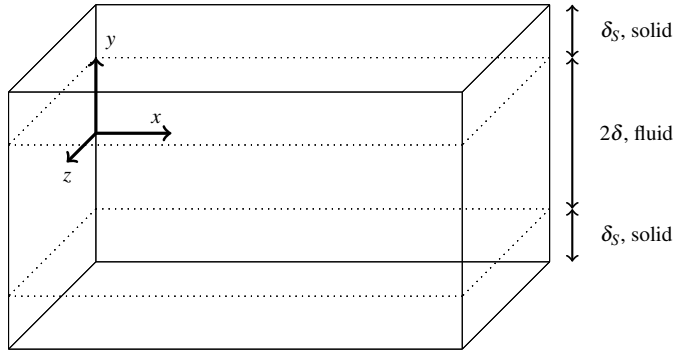
106 where  $\partial_t$  is the time derivative,  $\rho$  the density,  $P$  the dynamic pressure,  $\sigma_{ij}$  the deviatoric part  
 107 of the stress tensor and  $f_i$  a source term. We use the WALE subgrid-scale model (Nicoud  
 108 and Ducros [23]) as implemented in the version 5.0 of *Code\_Saturne*:  $\sigma_{ij} = (\nu + \nu_t) S_{ij}$  with

$$\nu_t = 4\sqrt{2} V_{cell}^{2/3} C_w^2 \frac{(S_{kl}^d S_{kl}^d)^{5/4}}{(S_{kl} S_{kl})^{5/2} + (S_{kl}^d S_{kl}^d)^{5/4}} \quad (3)$$

110 where  $\nu$  is the kinematic viscosity,  $V_{cell}$  the volume of the computational cell,  $C_w$  the model  
 111 constant,  $S_{ij}^d$  the deviatoric part of the square of  $\partial_j u_i$  and  $S_{ij}$  the mean strain rate tensor. The  
 112 constant  $C_w$  is set to 0.25, the default value in *Code\_Saturne*.

113 Lastly, the energy conservation equation is

$$\partial_t T_f + \partial_i (T_f u_i) = \partial_j \left( \left( \alpha_f + \frac{\nu_t}{Pr_t} \right) \partial_j T_f \right) + f^{T_f} \quad (4)$$



**Fig. 1** Channel flow with fluid-solid thermal coupling. The solid domains are on top ( $y > \delta$ ) and on bottom ( $y < -\delta$ ) of the fluid one ( $-\delta < y < \delta$ ). Dotted lines: fluid-solid interfaces at  $y = \pm\delta$ .

114 in the fluid domain and

$$\partial_t T_s = \alpha_s \partial_{jj} T_s + f^T \quad (5)$$

115 in the solid domain with

$$T_f = T_s \text{ and } \lambda_f \partial_n T_f = \lambda_s \partial_n T_s \quad (6)$$

116 at the fluid-solid interface. Here,  $T$  is the temperature,  $\alpha$  the thermal diffusivity,  $Pr_t$  the  
 117 turbulent Prandtl number,  $f^T$  a source term and  $\lambda$  the thermal conductivity. The turbulent  
 118 Prandtl number is set to 0.5 in the present study (Grötzbach [14]). Additional simulations  
 119 were performed to investigate the effect of the SGS model and the impact of  $Pr_t$ , as exposed  
 120 in the appendix B. Overall, the impact of the modelling parameter  $Pr_t$  is limited to the fluid  
 121 domain but is null while approaching the fluid solid interface. Thus, the turbulent Prandtl  
 122 number has little effect on the target values of the present study.

123 As we investigate forced convection, the transported scalars — like the temperature —  
 124 do not impact the velocity field or the pressure. Thus, we can transport simultaneously an  
 125 arbitrary number of passive scalars, and each scalar can use different thermal properties in  
 126 the fluid and in the solid. This allows significant savings in terms of CPU time: each passive  
 127 scalar is transported by the same velocity field. This peculiarity has eased our investigation  
 128 as it reduced the impact of the statistical uncertainty (Flageul and Tiselj [11]).

## 129 2.2 Geometry

130 Figure 1 is a sketch of the domain.  $x$ ,  $y$  and  $z$  are the streamwise, wall-normal and spanwise  
 131 directions, respectively. The fluid domain is bounded ( $-\delta < y < \delta$ ) and  $\delta$  is the channel  
 132 half-height. The solid domains on top and on bottom of the fluid one are located at  $y > \delta$  and  
 133  $y < -\delta$ , respectively. Both have the same total height,  $\delta_s$ . The exact extension of the channel  
 134 in the streamwise and spanwise directions and the height of the solid domains depend on  
 135 the case considered, as described in Subsection 2.6. Based on previous studies, the present  
 136 authors estimate that for all the cases considered here and summarized in Tables 1 and 2,  
 137 the solid domains extension in the wall-normal direction is sufficient so that the boundary  
 138 condition used at the outer wall has no significant impact on the statistics at the fluid-solid  
 139 interface.

140 For all the LES considered here, the size of the cells in wall-units in the streamwise and  
 141 spanwise directions is  $\delta x^+ = 30$  and  $\delta z^+ = 15$ , respectively. On both sides of the fluid-solid

142 interfaces, the first cell has an extension in the wall-normal direction of 1 wall-unit. Further  
 143 away from the wall, the wall-normal extension of the cell is bigger, according to a geometric  
 144 law with a progression factor of 1.09. In the fluid, as soon as the wall-normal extension  
 145 of the cell reaches 15 wall-units, this progression factor is set to 1 so that the wall-normal  
 146 extension of the cells does not exceed their spanwise extension. All the LES use a constant  
 147 time step with a maximal instantaneous CFL number around 0.5.

### 148 2.3 Source terms

149 The source term in the momentum equation in the fluid is an imposed pressure gradient,  
 150 which is constant in space and in time. It compensates the viscous friction at the wall and  
 151 keeps the flow in a statistically steady state. Using the averaged momentum equation, the  
 152 no-slip boundary condition and the vanishing  $v_t$  at the wall, one can derive  $\delta f_x = u_\tau^2$  where  
 153  $u_\tau$  is the friction velocity, derived from the wall shear stress. The Reynolds number based  
 154 on the friction velocity and the channel half-height is  $Re_\tau = \frac{u_\tau \delta}{\nu}$ . This definition, combined  
 155 with the previous relation, allows us to choose the source term  $f_x$  so that the target  $Re_\tau$   
 156 is exactly reached:

$$f_x = \frac{\nu^2 Re_\tau^2}{\delta^3} \text{ and } f_y = f_z = 0. \quad (7)$$

157 The source term in the energy equation in the fluid domain depends on the instantaneous  
 158 streamwise velocity and on the instantaneous bulk velocity, as defined by Kasagi et al. [18].  
 159 It is a volumetric source term which compensates the heat output at the boundaries, so that  
 160 the case remains statistically steady. Using the previous notations and writing  $q_w$  the heat  
 161 flux per unit surface outputted at the top and bottom boundaries, the source term in the  
 162 energy equation in the fluid domain is:

$$f^{T_f} = \frac{2\alpha_f q_w L_x L_z}{\lambda_f} \frac{u_x(x, y, z, t)}{\int_{x=0}^{L_x} \int_{y=-\delta}^{\delta} \int_{z=0}^{L_z} u_x dz dy dx} \quad (8)$$

163 Subsection 2.5 further describes the boundary conditions used. There is no source term in the  
 164 energy equation in the solid domain ( $f^{T_s} = 0$ ) since the heating of the solid wall is provided  
 165 through the constant heat flux imposed at the outer side of the wall.

166 The source terms used allow one to derive a theoretical friction velocity and friction tem-  
 167 peratures which are not plagued by any statistical uncertainty. The friction velocity verifies  
 168  $u_\tau = \frac{\nu Re_\tau}{\delta}$ . The friction temperature verifies  $T_\tau = \frac{\alpha_f q_w}{\lambda_f u_\tau}$ . In the present paper, those theoretical  
 169 friction values are used to express raw statistics in wall-units.

### 170 2.4 Dimensionless numbers and equations

171 At this stage, in addition to the friction Reynolds number, we can introduce additional di-  
 172 mensionless numbers. The Prandtl number is  $Pr = \frac{\nu}{\alpha_f}$ . The fluid-to-solid thermal diffusivity  
 173 ratio is  $G = \frac{\alpha_f}{\alpha_s}$ . The solid-to-fluid thermal conductivity ratio is  $G_2 = \frac{\lambda_s}{\lambda_f}$ . The thermal activ-  
 174 ity ratio — which is also the fluid-to-solid thermal effusivity ratio — is  $K = \frac{1}{G_2 \sqrt{G}}$ .

175 Although we have 3 dimensionless numbers describing the fluid and solid thermal prop-  
 176 erties ratios, only 2 are independent (Tiselj and Cizelj [28]). The resulting dimensionless

177 equations for the temperature are:

$$\partial_t T_f^+ + \partial_i (T_f^+ u_i^+) = \partial_j \left( \left( \frac{1}{Re_\tau Pr} + \frac{v_i^+}{Pr_i} \right) \partial_j T_f^+ \right) + f^{T_f^+} \quad (9)$$

178 in the fluid domain and

$$\partial_t T_s^+ = \frac{1}{G Re_\tau Pr} \partial_{jj} T_s^+ \quad (10)$$

179 in the solid domain with

$$T_f^+ = T_s^+ \text{ and } \partial_n T_f^+ = G_2 \partial_n T_s^+ \quad (11)$$

180 at the fluid-solid interface. The superscript + indicates a conversion into wall-units. Here-  
181 after,  $u_i$ ,  $u_i'$ ,  $v_i$ ,  $T$ ,  $T'$  and their gradients are expressed in wall-units. Thus, for the sake of  
182 clarity, the superscript + will be omitted.

183 The thermal activity ratio  $K$  does not appear naturally. However, it was the only param-  
184 eter in the analytical studies on conjugate heat transfer by Polyakov [24] and Geshev [13].  
185 The following short digression explains the origin of the thermal activity ratio, and briefly  
186 discusses the limiting cases of an imposed temperature and an imposed heat flux.

187 We consider a 1D case, and we rescale the space coordinate in the solid domain with  
188 a factor  $\sqrt{G}$  so that the thermal diffusivity in the fluid and in the solid are the same. The  
189 resulting differential equation for the temperature in the solid domain is

$$\partial_t T_s = \frac{1}{Re_\tau Pr} \partial_{yy} T_s \quad (12)$$

190 with

$$T_f = T_s \text{ and } \partial_y T_f = G_2 \sqrt{G} \partial_y T_s \iff \partial_y T_f = \frac{1}{K} \partial_y T_s \quad (13)$$

191 at the fluid-solid interface. However, if one were to extend this to a 2D or a 3D case, only  
192 the wall-normal coordinate in the solid could be rescaled as the continuity of temperature  
193 at the fluid-solid interface prevents any rescaling in the wall-parallel directions. Thus, the  
194 rescaling would introduce an anisotropic diffusion in the solid domain, and 2 dimensionless  
195 numbers would remain:  $K$  and  $G$ .

196 Regarding limiting cases, the following is based on the spatio-temporal convolution ex-  
197 hibited in [9]. We consider a flat fluid-solid interface subjected to a statistically steady forc-  
198 ing and a semi-infinite solid. Combining Fourier and Laplace transforms, one can show that  
199 temperature fluctuations are decaying exponentially in the solid. In addition, at the fluid-  
200 solid interface, there is a compatibility condition of the form:

$$G_2 R \hat{T}_f = \pm \partial_y \hat{T}_f \quad (14)$$

201 with

$$R^2 = k_x^2 + k_z^2 + \sqrt{-1} \omega G Re_\tau Pr \quad (15)$$

202 where  $\hat{T}_f$  is the temperature in the spectral space and  $k_x$  and  $k_z$  ( $\omega$ ) are the wavenumbers  
203 (angular frequency) associated with the Fourier transform in the wall-parallel directions  
204 (time). An integral form of equation (14) is

$$G_2 = \frac{\left\| \partial_y \hat{T}_f \right\|}{\left\| R \hat{T}_f \right\|} \quad (16)$$

205 Thus, as  $G_2$  tends to 0 or  $\infty$ , the case degenerates towards an imposed flux or value, re-  
 206 spectively. The impact of the number  $G$  — which is hidden inside  $R$  in the denominator —  
 207 cannot be easily exhibited as it is entangled inside the spatio-temporal convolution. Still,  
 208 one can say that it is connected with the non-local effects characteristics of conjugate heat  
 209 transfer.

210 Looking back at equation (10), the impact of  $G$  is more easily exhibited. When  $G$  tends  
 211 to  $\infty$ , the time derivative vanishes. It corresponds to solid domains with a high thermal inertia  
 212 and should be well represented by cases with an imposed temperature. The other limiting  
 213 case is when  $G$  tends to 0. The energy equation in the solid domain degenerates into an  
 214 homogeneous Laplace equation. It corresponds to solid domains with a negligible thermal  
 215 inertia. This situation can not be represented with an imposed temperature or heat flux.  $G$   
 216 can also be seen as the ratio of time scales for thermal diffusion in the solid and in the fluid.  
 217 Thus, the aforementioned limiting cases actually correspond to solid domains with very slow  
 218 ( $G \gg 1$ ) or very fast ( $G \ll 1$ ) thermal diffusion.

219 Overall, an imposed value of  $T_f$  at the fluid boundary (Dirichlet boundary condition)  
 220 is usually a good approximation when simulating the temperature inside a fluid in contact  
 221 with a conductive solid ( $G_2 \gg 1$ , or  $\lambda_s \gg \lambda_f$ , as in a metal). An imposed flux (Neumann  
 222 boundary condition) would rather correspond to the simulation of the temperature inside a  
 223 fluid in contact with a solid which has a low thermal conductivity ( $G_2 \ll 1$ , or  $\lambda_s \ll \lambda_f$ , as  
 224 in a glass). One might notice that the present analysis does not handle cases combining both  
 225 a high  $G$  (imposed temperature) and a low  $G_2$  (imposed heat flux). This is not an issue since  
 226 such parameters would represent very exotic pair of fluid and solid materials.

## 227 2.5 Boundary conditions

228 For the streamwise and spanwise directions, periodicity is used. For the wall-normal direc-  
 229 tion, the 3 components of the velocity vanish at the fluid-solid interfaces. Regarding the  
 230 transported scalars, 2 kinds of boundary conditions are envisaged: coupled or not.

231 If the scalar is not coupled, either its value (Dirichlet boundary condition) or its flux  
 232 (Neumann boundary condition) is imposed at the fluid boundaries, located at  $y = \pm\delta$ . In case  
 233 of an imposed value, the temperature is arbitrarily set to zero at the boundary. Such cases will  
 234 be referred to as *isoT*. In case of an imposed flux, its value must match the volumetric heat  
 235 source term imposed in the fluid domain. Such cases will be referred to as *isoQ*. Obviously,  
 236 if a scalar is not coupled, it does not correspond to conjugate heat transfer.

237 If the scalar is said to be coupled, it corresponds to the coupling of the fluid and solid  
 238 values. For such scalars, the flux is imposed at the outer walls, located at  $y = \delta + \delta_s$  and  
 239  $y = -\delta - \delta_s$ , respectively. Here again, the value of the flux must match the volumetric heat  
 240 source term imposed in the fluid domain. For coupled scalars, there is continuity of the  
 241 scalar and of its diffusive flux at the fluid-solid interface. Regardless of the case, the same  
 242 boundary condition is imposed on both sides, so symmetry is preserved.

243 Although the solver is fully conservative, tiny sources of error remain. They can accu-  
 244 mulate and introduce a small temporal drift for coupled and non-coupled scalars with an  
 245 imposed heat flux at the boundaries. This can perturb the collection of statistics, especially  
 246 deep in the solid domain where thermal fluctuations have a small amplitude. This can be-  
 247 come prominent for long averaging times. To avoid such a pitfall, cases with an imposed  
 248 heat flux at the boundaries are corrected at each time step by a term constant in space so that  
 249 the instantaneous bulk temperature remains zero. The amplitude of this correction is around  
 250  $10^{-10}$ . It is thus expected to be relatively harmless.



**Table 1** Validation cases in Section 4

	DNS	LES	DNS	LES
$Re_\tau$		150		395
$Pr$		0.71		0.71
$\delta_S/\delta$		1		1
$L_x/\delta$	25.6	12.566	$2\pi$	6.283
$L_z/\delta$	8.52	6.284	$\pi$	3.142
Scalars		11		3

**Table 2** LES computations in Section 5

$Re_\tau$	395		1020	
$\delta_S/\delta$	0.375		0.147	
$L_x/\delta$	6.283		6.283	
$L_z/\delta$	3.142		3.142	
$Pr$	0.71	1	0.71	1
Scalars	51	5	5	5

## 251 2.6 Simulations summary

252 All the LES and DNS presented include 2 non-coupled scalars. Those non-coupled scalars  
 253 actually represent the limit of coupled cases when the solid thermal conductivity is very  
 254 high, or very low, compared to the fluid one. All the simulations also include at least one coupled  
 255 scalar, with the same thermal properties in the fluid and in the solid ( $G = G_2 = K = 1$ ).

256 Section 4, our LES are compared with some DNS. The friction Reynolds numbers investigated  
 257 are 150 and 395. The Prandtl number investigated is 0.71. Table 1 summarizes  
 258 the performed simulations. The DNS at the lower Reynolds number was already presented  
 259 in [10] and includes 9 coupled scalars: 3 values of the ratios  $G$  and  $G_2$  were simulated. The  
 260 DNS at the higher Reynolds number is novel and includes only one coupled scalar with unit  
 261 fluid-solid thermal properties ratios. The corresponding LES use the same thermal properties  
 262 ratios.

263 In Section 5, four LES are presented. The friction Reynolds numbers investigated are  
 264 395 and 1020. The Prandtl numbers investigated are 0.71 and 1. Table 2 summarizes the  
 265 performed simulations. The case  $Re_\tau = 395$  and  $Pr = 0.71$  includes 49 coupled scalars: 7  
 266 values of the ratios  $G$  and  $K$  are simultaneously simulated. The other LES of this section in-  
 267 clude 3 coupled scalars: one with unit fluid-solid thermal properties ratios, one with  $G = 1.3$   
 268 and  $K = 2.8$  and one with  $G = 0.1$  and  $K = 0.23$ . The second case is an approximation of  
 269 the thermal coupling of air and plexiglas. The last one is an approximation of the thermal  
 270 coupling of pressurized water and steel. This is further discussed in Section 5.

## 271 3 Discretisation and extrapolation of statistics

### 272 3.1 Discretization

273 *Code\_Saturne* is an open-source CFD solver for incompressible or weakly dilatible flows. It  
 274 is based on the FVM library and can handle unstructured meshes. The finite volume solver is  
 275 collocated. The predictor / corrector algorithm used for pressure-velocity coupling is com-

276 bined with a Rhie and Chow filter to avoid odd-even oscillations. Further details can be  
277 found in Archambeau et al. [3].

278 For the velocity and the scalars, the convection scheme used in this study is fully centered:  
279 the existing slope-test by default for the scalars is deactivated. Regarding time advancement,  
280 a Crank-Nicolson scheme is used, except for the convective term which uses an Adams-Bashforth  
281 scheme for the transporting velocity. As our meshes are made of orthogonal hexahedra, no gradient  
282 reconstruction sweeps are needed.

### 283 3.2 Extrapolation of statistics at the fluid-solid interface

284 First, we recall the definition of the dissipation rate, which is the quantity pursued in the  
285 present paper:

$$\varepsilon_\theta = \overline{\alpha \partial_i T' \partial_i T'} \quad (17)$$

286 The definition can be applied directly in the solid domain, but not in the fluid one. There,  
287 the scalar dissipation rate depends on the subgrid-scale model as follows:

$$\varepsilon_{\theta,f} = \overline{\left( \alpha_f + \frac{v_t}{Pr_t} \right) \partial_i T'_f \partial_i T'_f} \quad (18)$$

288 with a vanishing  $v_t$  at the wall. The scalar dissipation rate in the fluid is actually obtained  
289 with the following combination of terms

$$\varepsilon_{\theta,f} = \alpha_f \overline{\partial_i T'_f \partial_i T'_f} + \frac{\overline{v_t}}{Pr_t} \overline{\partial_i T'_f \partial_i T'_f} + \frac{\overline{v_t'}}{Pr_t} \overline{\partial_i T'_f \partial_i T'_f} \quad (19)$$

$$\text{with } \frac{\overline{v_t'}}{Pr_t} \overline{\partial_i T'_f \partial_i T'_f} = \frac{\overline{v_t}}{Pr_t} \overline{\partial_i T_f \partial_i T_f} - 2 \frac{\overline{v_t}}{Pr_t} \overline{\partial_i T_f} \overline{\partial_i T_f} - \frac{\overline{v_t}}{Pr_t} \overline{\partial_i T'_f \partial_i T'_f} + \frac{\overline{v_t}}{Pr_t} \overline{\partial_i T_f} \overline{\partial_i T_f}$$

290 As shown in [10], this quantity is discontinuous at the fluid-solid interface, where the  
291 ratio of the solid and fluid dissipation rates verify

$$\begin{aligned} \frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}} &= \frac{\overline{\partial_y T'_f \partial_y T'_f}}{\overline{\partial_i T'_f \partial_i T'_f}} K^2 + \left( 1 - \frac{\overline{\partial_y T'_f \partial_y T'_f}}{\overline{\partial_i T'_f \partial_i T'_f}} \right) \frac{1}{G} \\ &= \frac{1}{G} + \left( K^2 - \frac{1}{G} \right) \frac{\overline{\partial_y T'_f \partial_y T'_f}}{\overline{\partial_i T'_f \partial_i T'_f}} \end{aligned} \quad (20)$$

292 At this stage, it seems important to stress that the discontinuity of  $\varepsilon_\theta$  at the fluid-solid in-  
293 terface is a direct consequence of the continuity of the temperature and normal heat flux at  
294 this interface, combined with distinct thermal properties on both sides of the interface. In  
295 *Code\_Saturne*, most of the relevant physical quantities are defined at the center of the cells.  
296 However, the fluid-solid interface is located at the faces of the cells and not at their center.  
297 In this subsection, we describe the strategy used to extrapolate statistical quantities at the  
298 fluid-solid interface for coupled fields.

299 This important post-processing step is performed after the simulation and uses quantities  
300 averaged in time and over homogeneous directions. In the following,  $y_f$  and  $y_s$  are the coor-  
301 dinate of the center of the first fluid and solid cells, respectively, and  $y_{fs}$  is the coordinate of  
302 the fluid-solid interface.

303 The proposed strategy was designed with a modeller's perspective. Thus, from the LES,  
304 we extract quantities a RANS model adapted to conjugate heat transfer could have provided.

305 As those quantities are defined at the cell center, their evaluation at the fluid-solid interface  
 306 implies an extrapolation. To remain within the framework of unstructured meshes, only cells  
 307 adjacent to the fluid-solid interface are considered.

308 For the streamwise contribution to  $\varepsilon_\theta$ , one can use a first-order Taylor expansion. Com-  
 309 bined with the continuity of temperature and heat flux at the fluid-solid interface, one gets

$$\begin{aligned}\overline{\partial_x T'_f \partial_x T'_f}(y) &= \overline{\partial_x T'_f \partial_x T'_f}(y_f) + b_f (y - y_f) \\ \overline{\partial_x T'_s \partial_x T'_s}(y) &= \overline{\partial_x T'_s \partial_x T'_s}(y_s) + b_s (y - y_s) \\ \overline{\partial_x T'_f \partial_x T'_f}(y_{fs}) &= \overline{\partial_x T'_s \partial_x T'_s}(y_{fs}) \\ b_f &= G_2 b_s\end{aligned}\quad (21)$$

310 The two unknowns are  $b_f$  and  $b_s$ . The last two equations allow to close and solve the linear  
 311 system. The situation is exactly the same for the spanwise contribution to  $\varepsilon_\theta$ .

312 For the wall-normal contribution to  $\varepsilon_\theta$ , the situation is different as there is no compat-  
 313 ibility condition for its derivative. Thus, there is no simple way to close and solve a linear  
 314 system similar to the previous one. As a workaround, we use the continuity of the one-  
 315 point correlation between the temperature and its wall-normal derivative at the fluid-solid  
 316 interface. We define the angle  $\phi$  with

$$\cos(\phi) = \frac{\overline{T' \partial_y T'}}{\sqrt{\overline{T'^2}} \sqrt{\overline{\partial_y T' \partial_y T'}}}\quad (22)$$

317 The numerator and the first term in the denominator can be estimated with  $\overline{T'^2}$ .

318 For the variance of the temperature in the cells adjacent to the fluid-solid interface,  
 319 one can use a second-order Taylor expansion. As the LES is wall-resolved and the flow  
 320 statistically steady, we assume equilibrium between dissipation and viscous diffusion in the  
 321 budget equation of  $\overline{T'^2}$ . Combined with the continuity of temperature and heat flux at the  
 322 fluid-solid interface, one gets:

$$\begin{aligned}\overline{T'^2_f}(y) &= \overline{T'^2_f}(y_f) + a_f (y - y_f) + Pr (y - y_f)^2 \varepsilon_{\theta,f}(y_f) \\ \overline{T'^2_s}(y) &= \overline{T'^2_s}(y_s) + a_s (y - y_s) + GPr (y - y_s)^2 \varepsilon_{\theta,s}(y_s) \\ \overline{T'^2_f}(y_{fs}) &= \overline{T'^2_s}(y_{fs}) \\ a_f + 2Pr (y_{fs} - y_f) \varepsilon_{\theta,f}(y_f) &= G_2 (a_s + 2GPr (y_{fs} - y_s) \varepsilon_{\theta,s}(y_s))\end{aligned}\quad (23)$$

323 The two unknowns are  $a_f$  and  $a_s$ . The last two equations allow to close and solve the linear  
 324 system.

325 In this study,  $\cos(\phi)$  at the fluid-solid interface is approximated with its value at  $y_s$ .  
 326 Although one can estimate it with a combination of the values at  $y_f$  and  $y_s$ , the authors  
 327 have observed that such combinations tend to produce non-physical  $\varepsilon_\theta$  at the fluid-solid  
 328 interface. For instance, estimating  $\cos(\phi)$  at  $y_{fs}$  with the halved sum of the values at  $y_f$  and  
 329  $y_s$  can produce a value of  $\varepsilon_{\theta,s}$  lower at  $y_{fs}$  than at  $y_s$ . Such a result is not admissible: for a  
 330 statistically steady case, without source term in the solid domain, the diffusion of  $\varepsilon_{\theta,s}$  must  
 331 remain positive in the solid domain. For the present 1D case, it implies a convex profile and  
 332 a decaying  $\varepsilon_{\theta,s}$  with increasing distance to the fluid-solid interface.

333 Once  $\cos(\phi)$  at the fluid-solid interface is obtained, it is straightforward to estimate  
 334  $\overline{\partial_y T' \partial_y T'}$  on both sides of the fluid-solid interface. For instance, at the fluid-solid interface  
 335 and on the fluid side, one gets

$$\overline{\partial_y T'_f \partial_y T'_f} = \left[ \frac{\partial_y (\overline{T_f'^2}) / 2}{\sqrt{\overline{T_f'^2}} \cos(\phi)} \right]^2 \quad (24)$$

336 where all quantities are taken at the fluid-solid interface.

337 Once the quantities  $\overline{\partial_i T' \partial_i T'}$  have been extrapolated on both sides of the fluid-solid in-  
 338 terface, it is straightforward to estimate the dissipation rates. As the extrapolation procedure  
 339 proposed here relies deeply on the continuity of the temperature and heat flux, equation (20)  
 340 is automatically satisfied by the reconstructed quantities.

#### 341 4 Validation against DNS

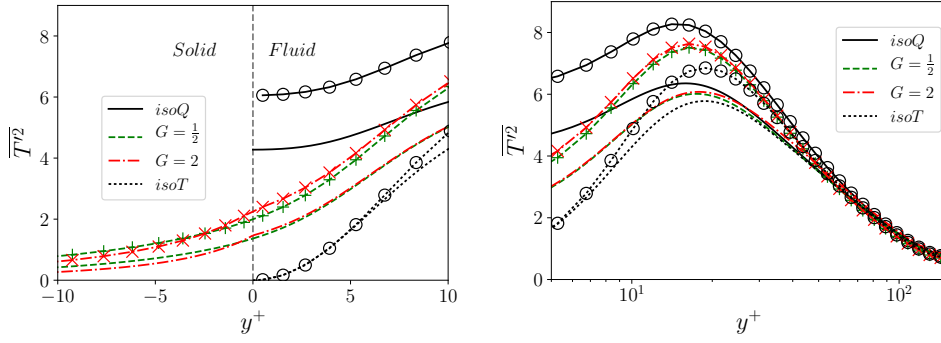
342 In this section, our wall-resolved LES are validated against existing and new DNS data.  
 343 Indeed, the coupling strategy has already been verified against analytical solutions. The  
 344 validation performed here is twofold: both the fluid-solid thermal coupling strategy and the  
 345 extrapolation of statistics at the fluid-solid interface are assessed.

##### 346 4.1 $Re_\tau = 150$ and $Pr = 0.71$

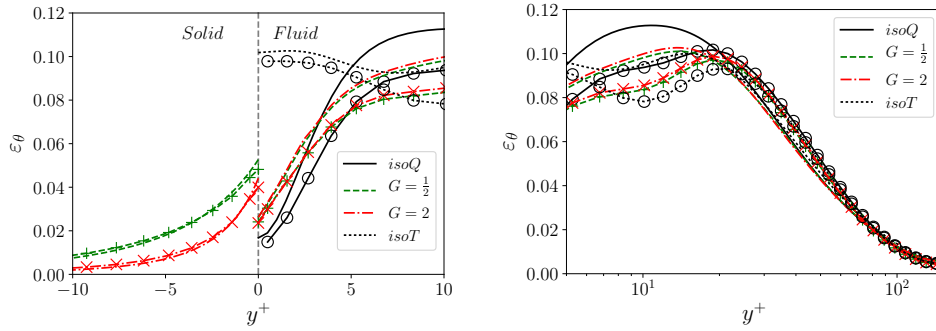
347 In this subsection, the LES is assessed using the DNS from [10]. The friction Reynolds  
 348 number  $Re_\tau$  is 150 and the Prandtl number  $Pr$  is 0.71. Figures 2 and 3 allow to compare  $\overline{T'^2}$   
 349 and  $\varepsilon_\theta$ , respectively, for the cases with  $K = \sqrt{2}$ .

350 In the middle of the channel ( $y^+ > 30$ ), the LES compares favourably with the DNS.  
 351 Closer to the wall ( $0 < y^+ < 30$ ), the LES overestimates  $\overline{T'^2}$ . This trend is less visible for  
 352 the *isoT* case thanks to the boundary condition which enforces  $\overline{T'^2} = 0$  at  $y^+ = 0$ . Near the  
 353 wall, the LES also underestimates  $\varepsilon_\theta$  for non-coupled cases. For coupled cases, the LES  
 354 provides a good estimation of  $\varepsilon_\theta$ , especially in the viscous sublayer ( $0 < y^+ < 5$ ) and in the  
 355 solid domain.

356 As shown in Table 3, the LES provides a reasonably good estimation of  $\varepsilon_\theta$  at the fluid-  
 357 solid interface. The maximal error on  $\varepsilon_\theta$  is around 11% and is reached when  $G = 1/2$  and  
 358  $G_2 = 1/2$ . Thanks to some compensations, the maximal error on the ratio  $\varepsilon_{\theta,s}/\varepsilon_{\theta,f}$  is lower,  
 359 with a maximal value around 4%, reached when  $G = 1/2$  and  $G_2 = 1/2$ . Regarding the  
 360 case with  $G = G_2 = 1$ , the ratio of the dissipation rates at the fluid-solid interface should  
 361 be exactly one: there is no discontinuity of  $\varepsilon_\theta$  for unit fluid-solid thermal properties ratios.  
 362 This is the case for the extrapolated LES statistics, but not for the DNS results due to a non-  
 363 conservative coupling strategy and a statistical uncertainty of 1%. Overall, the agreement  
 364 between the extrapolated LES results and the DNS is quite good. Especially considering the  
 365 overestimation of  $\overline{T'^2}$  observed in Figure 2 and the relatively low Reynolds number of the  
 366 case. As the Reynolds number gets higher, the LES should perform better. This is shown in  
 367 the next subsections.



**Fig. 2** Temperature variance for  $Re_\tau = 150$ ,  $Pr = 0.71$  and  $K = \sqrt{2}$  (case  $G = 1/2$  and  $G_2 = 1$  or case  $G = 2$  and  $G_2 = 1/2$ ). Lines: DNS. Lines + symbols: LES. Continuous lines for  $isoQ$ , dashed lines for  $G = 1/2$ , dash-dotted lines for  $G = 2$ , dotted lines for  $isoT$ . Fluid domain:  $y^+ > 0$ . Solid domain:  $y^+ < 0$ .



**Fig. 3** Scalar dissipation rate for  $Re_\tau = 150$ ,  $Pr = 0.71$  and  $K = \sqrt{2}$  (case  $G = 1/2$  and  $G_2 = 1$  or case  $G = 2$  and  $G_2 = 1/2$ ). Lines: DNS. Lines + symbols: LES. Continuous lines for  $isoQ$ , dashed lines for  $G = 1/2$ , dash-dotted lines for  $G = 2$ , dotted lines for  $isoT$ . Fluid domain:  $y^+ > 0$ . Solid domain:  $y^+ < 0$ .

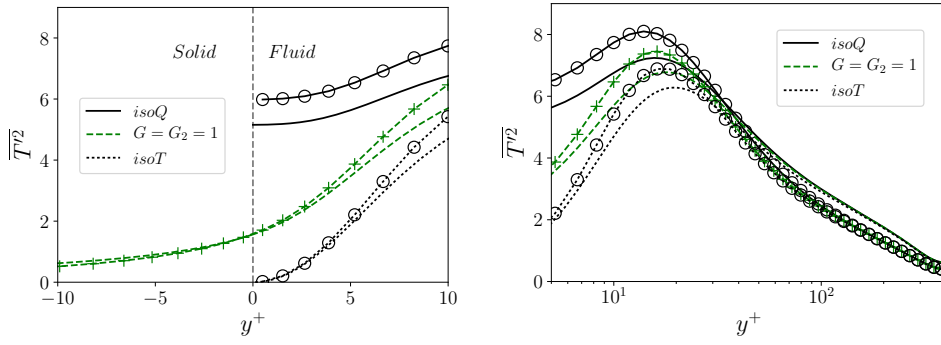
**Table 3** Coupled cases with  $Re_\tau = 150$  and  $Pr = 0.71$ . Relative error on  $\varepsilon_{\theta,f}$ ,  $\varepsilon_{\theta,s}$  and  $\varepsilon_{\theta,s}/\varepsilon_{\theta,f}$  at the fluid-solid interface, in %. Highest values in bold.

		$G_2$			
		$1/2$	$1$	$2$	
G	$1/2$	<b>11.5</b> , 8.03, <b>3.94</b>	8.47, 8.57, 0.101	7.60, 8.74, 1.24	
	1	10.4, 8.87, 1.68	9.06, 9.17, 0.120	8.71, 9.01, 0.327	
	2	9.91, <b>9.47</b> , 0.492	9.29, 9.38, 0.101	8.47, 8.39, 0.0884	

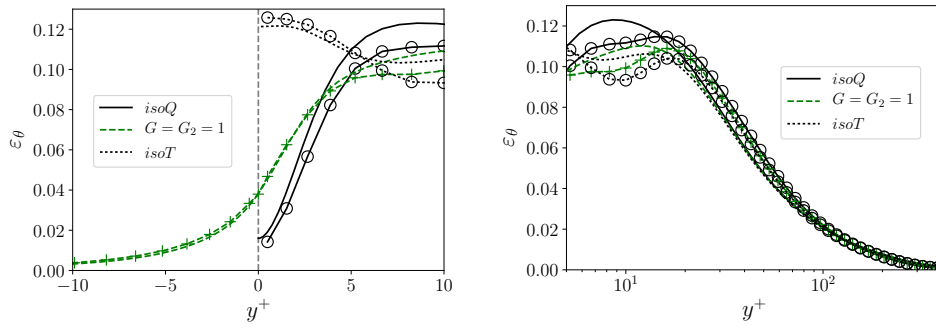
#### 368 4.2 $Re_\tau = 395$ and $Pr = 0.71$

369 In this subsection, the LES is assessed using a new DNS. It is performed with the pseudo-  
 370 spectral code previously used in Tiselj et al. ([27], [28]). The friction Reynolds number  $Re_\tau$   
 371 is 395 and the Prandtl number  $Pr$  is 0.71. Figures 4 and 5 allow to compare  $\overline{T'^2}$  and  $\varepsilon_\theta$ ,  
 372 respectively.

373 In the middle of the channel ( $y^+ > 30$ ), the LES correctly estimates  $\varepsilon_\theta$  and tends to  
 374 underestimate  $\overline{T'^2}$ . Closer to the wall ( $0 < y^+ < 30$ ), the LES overestimates  $\overline{T'^2}$ , as observed  
 375 at  $Re_\tau = 150$ , although the error is about two times lower at  $Re_\tau = 395$ . The LES also



**Fig. 4** Temperature variance for  $Re_\tau = 395$ ,  $Pr = 0.71$  and  $K = G = G_2 = 1$ . Lines: DNS. Lines + symbols: LES. Continuous lines for  $isoQ$ , dashed lines for  $K = G = G_2 = 1$ , dotted lines for  $isoT$ . Fluid domain:  $y^+ > 0$ . Solid domain:  $y^+ < 0$ .



**Fig. 5** Scalar dissipation rate for  $Re_\tau = 395$ ,  $Pr = 0.71$  and  $K = G = G_2 = 1$ . Lines: DNS. Lines + symbols: LES. Continuous lines for  $isoQ$ , dashed lines for  $K = G = G_2 = 1$ , dotted lines for  $isoT$ .

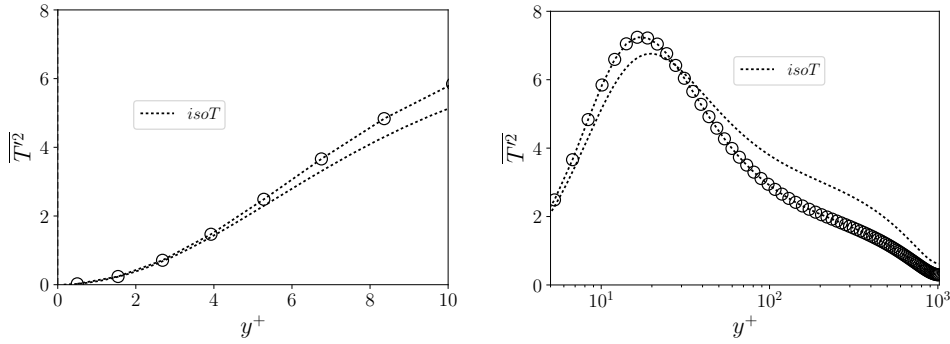
376 underestimates  $\varepsilon_\theta$  for non-coupled cases, although this trend is more disputable for the  $isoT$   
 377 one in the viscous sublayer ( $0 < y^+ < 5$ ). For the coupled case, the LES provides a good  
 378 estimation of  $\varepsilon_\theta$ , especially in the viscous sublayer and in the solid domain.

379 At  $Re_\tau = 395$ , the relative error on  $\varepsilon_\theta$  at the fluid-solid interface for the coupled case  
 380 is 0.860 %. Compared with the validation case at  $Re_\tau = 150$ , the accuracy is improved by  
 381 a factor of 10. Thus, the higher the Reynolds number, the more accurate our wall-resolved  
 382 LES, which is an expected trend.

### 383 4.3 $Re_\tau = 1020$ and $Pr = 0.71$

384 In this subsection, the LES is assessed using the DNS data from Abe et al. ([2]). The friction  
 385 Reynolds number  $Re_\tau$  is 1020 and the Prandtl number  $Pr$  is 0.71. The validation exposed  
 386 here is less extensive as the only case available is an imposed temperature at the wall, and  
 387 the scalar dissipation rate is not available. Figure 6 allows to compare  $\overline{T'^2}$ .

388 Qualitatively, the agreement is similar to the one observed at  $Re_\tau = 395$ . Overall, the  
 389 outcome of this validation is satisfactory. Indeed, the comparison of the  $isoT$  and  $isoQ$  cases  
 390 with DNS shows rather moderate agreement, especially for the temperature variance at very



**Fig. 6** Temperature variance for  $Re_\tau = 1020$ ,  $Pr = 0.71$  and an imposed temperature at the wall. Lines: DNS (Abe et al. [2]). Lines + symbols: LES.

low Reynolds number  $Re_\tau = 150$ . It seems important to stress that despite moderate predictions for non-coupled cases, predictions for coupled scalars are quite satisfying, especially for  $\varepsilon_\theta$  at the fluid-solid interface. This, in our opinion, comes from the refined mesh used and from the fact that the solid domain tends to filter out higher turbulent frequencies. We believe that this combination allows our LES with conjugate heat-transfer to capture most of the temperature fluctuations generating  $\varepsilon_\theta$  at the wall.

In addition, one should keep in mind that the friction Reynolds numbers considered are not very high, especially at  $Re_\tau = 150$ . The quality of the LES results is better for higher Reynolds numbers. The new coupling strategy implemented in *Code\_Saturne* described in Appendix A, combined with the extrapolation of statistics at the fluid-solid interface described in Subsection 3.2 provide a good estimation of the scalar dissipation rates at the fluid-solid interface for wall-resolved LES.

## 5 New LES results and proposed correlations

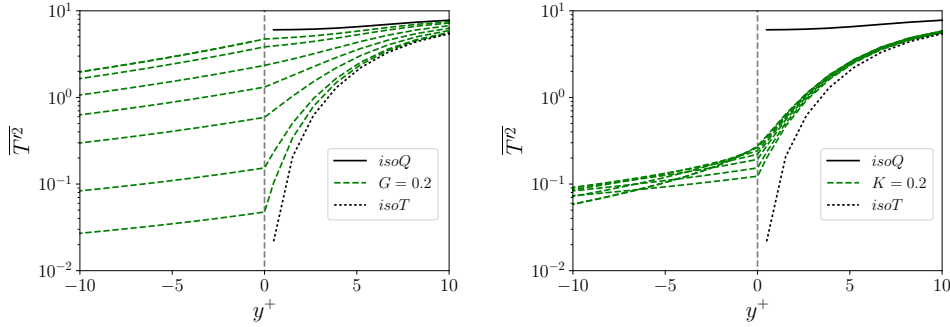
In this section, the new LES results are presented and discussed, alongside with the regression and the correlations. The first subsection presents the regression, and starts with a description of the LES which was used to derive it. The second subsection presents additional LES at higher  $Re_\tau$  and  $Pr$ , which are used to assess the correlations.

### 5.1 Case $Re_\tau = 395$ , $Pr = 0.71$

In this subsection, the main LES performed is presented, alongside with some key results and a regression for the discontinuity of  $\varepsilon_\theta$  at the fluid-solid interface. As indicated Table 2, the simulation contains 51 passive scalars, 49 being coupled at the fluid-solid interface while the remaining two have either an imposed value or an imposed flux at the fluid boundary. Regarding the 49 coupled scalars, the fluid-solid thermal properties ratios investigated and cases label are described in the Table 4. For both the fluid-to-solid thermal diffusivity ratio  $G_i$  and the thermal activity ratio  $K_j$ , 7 values are simultaneously investigated. They vary over a range of two decades, centered around unity. The coupled passive scalar with fluid-solid thermal properties ratios  $G_i$  and  $K_j$  is denoted as  $CHT_{ij}$ .

**Table 4** Thermal properties ratios investigated in the subsection 5.1. Solid-to-fluid thermal conductivity ratio  $G_2$ .

$G_2 = \frac{1}{K\sqrt{G}}$	$K_j$							
	0.1	0.2	0.5	1	2	5	10	
$G_i$	0.1	32	16	6.3	3.2	1.6	0.63	0.32
	0.2	22	11	4.5	2.2	1.1	0.45	0.22
	0.5	14	7.1	2.8	1.4	0.71	0.28	0.14
	1	10	5	2	1	0.5	0.2	0.1
	2	7.1	3.5	1.4	0.71	0.35	0.14	0.071
	5	4.5	2.2	0.89	0.45	0.22	0.089	0.045
	10	3.2	1.6	0.63	0.32	0.16	0.063	0.032

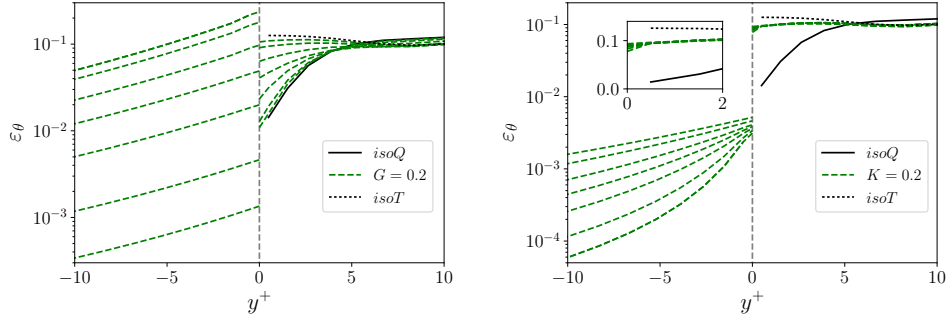
**Fig. 7** Temperature variance for the wall-resolved LES at  $Re_\tau = 395$  and  $Pr = 0.71$ . Left frame: Cases  $G = 0.2$ . Right frame: Cases  $K = 0.2$ .

418 Regarding the *isoT* case, monitoring the bulk temperature, we observed a transient of  
 419 about  $44.10^3 v/u_\tau^2$  before reaching a statistically steady state. However, for coupled cases,  
 420 monitoring the bulk temperature in the solid domain, we observed transients up to a decade  
 421 longer ( $440.10^3 v/u_\tau^2$ ). Thus, the simulation was started with a maximal instantaneous CFL  
 422 around 1. After  $2.10^6$  time steps ( $dt^+ \approx 0.44v/u_\tau^2$ ), and about  $880.10^3 v/u_\tau^2$  simulated, the  
 423 time step was halved. Then, after a million and a half time steps ( $331.10^3$  additional  $v/u_\tau^2$ ),  
 424 all the coupled scalars were fully developed and statistics could be gathered. They were  
 425 gathered for a million time steps (around  $220.10^3 v/u_\tau^2$ ). Similar precautions were taken for  
 426 all the LES in the present paper.

427 The left frame of Figure 7 illustrates the impact of the thermal activity ratio  $K$  on the  
 428 temperature variance. Lower values of  $K$  correspond to conjugate cases closer to the *isoT*  
 429 one. Oppositely, higher values correspond to conjugate cases closer to the *isoQ* one. The  
 430 right frame of Figure 7 shows that statistics, in the fluid domain, are mostly driven by the  
 431 thermal activity ratio  $K$ , the impact of the thermal diffusivity ratio  $G$  being much weaker.  
 432 However, deep in the viscous sublayer ( $y^+ < 2$ ) and inside the solid, the situation is more  
 433 complex and the impact of  $G$  on  $\overline{T'^2}$  and  $\varepsilon_\theta$  is more visible.

434 Those observations based on the profiles of  $\overline{T'^2}$ , also apply on the profiles of  $\varepsilon_\theta$  in Figure  
 435 8. Overall, the present LES results with an extended range of fluid-solid thermal properties  
 436 ratios corroborate well the DNS results, obtained at a lower Reynolds number  $Re_\tau = 150$   
 437 and with thermal properties ratios closer to unity previously published in [10] : in the fluid  
 438 domain, the parameter  $G$  has a small impact only deep inside the viscous sublayer ( $y^+ < 2$ )  
 439 while the parameter  $K$  has a strong impact in the near-wall layer ( $y^+ < 15$ ). The strong





**Fig. 8** Scalar dissipation rate for the wall-resolved LES at  $Re_\tau = 395$  and  $Pr = 0.71$ . Left frame: Cases  $G = 0.2$ . Right frame: Cases  $K = 0.2$ , with a focus on the near-wall behaviour in the fluid.

440 impact of  $K$  on the turbulent statistics probably comes from the nature of the flow, which  
 441 is dominated by wall-normal fluctuations in a turbulent channel, and may not hold for more  
 442 complex flow configurations. The reconstructed statistics at the fluid-solid interface ( $\overline{T'^2}$  and  
 443  $\varepsilon_\theta$ ) have a realistic behaviour: both quantities monotonically decay in the first solid cell.

444 On top of those qualitative observations, the large number of combination of fluid-solid  
 445 thermal properties ratios investigated allow a more quantitative analysis. We look for power-  
 446 law regressions for statistics at the fluid-solid interface as follow:

$$\frac{\overline{T'^2}(CHT_{ij})}{\overline{T'^2}(isoQ)} = \frac{1}{1 + C_{T'} \cdot G_i^{a_{T'}} \cdot K_j^{b_{T'}}} \text{ with } C_{T'} > 0 \quad (25)$$

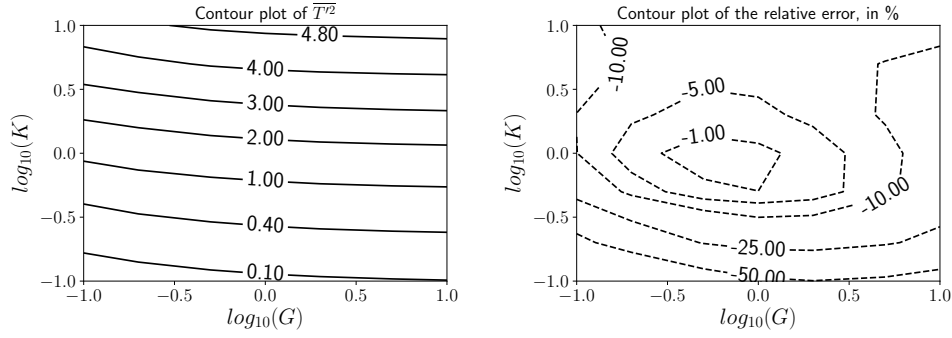
$$\frac{\frac{\partial_y T'_f \partial_y T'_f}{\partial_i T'_f \partial_i T'_f}(CHT_{ij})}{\frac{\partial_y T'_f \partial_y T'_f}{\partial_i T'_f \partial_i T'_f}(isoQ)} = \frac{1}{1 + C_{\varepsilon_\theta} \cdot G_i^{a_{\varepsilon}} \cdot K_j^{b_{\varepsilon}}} \text{ with } C_{\varepsilon_\theta} > 0 \quad (26)$$

447 The latter one, combined with equation (20), allows us to reconstruct the ratio of the scalar  
 448 dissipation rates at the fluid-solid interface.

449 The main property of equation (25) is that the temperature variance of the conjugate  
 450 cases does not exceed the one obtained with an imposed heat flux. The main property of  
 451 equation (26) is that the relative wall-normal contribution remains bounded in  $[0, 1]$ . The re-  
 452 sulting ratio of dissipation rates at the fluid-solid interface will thus remain bounded between  
 453  $K^2$  and  $\frac{1}{G}$ , the theoretical bounds. Both equations are relatively simple. Optimal estimation  
 454 of  $a$  and  $b$  is thus easier.

455 In the present study, the parameters  $C_{T'}$  and  $C_{\varepsilon_\theta}$  are derived from the case  $CHT_{44}$ , which  
 456 has unit fluid-solid thermal properties ratios. For the temperature variance at the wall in the  
 457  $isoQ$  case, we simply use the value at the center of the first fluid cell. This approximation  
 458 should be relatively harmless as the LES is wall-resolved and in case of an imposed constant  
 459 heat flux, the derivative of  $\overline{T'^2}$  vanishes at the boundary. Regarding the coefficients  $a$  and  $b$ ,  
 460 they are simply determined with a linear regression using logarithms:

$$\begin{aligned} a_{T'} \log(G_i) + b_{T'} \log(K_j) &= \log \left( \frac{\overline{T'^2}(isoQ)}{\overline{T'^2}(CHT_{ij})} - 1 \right) - \log(C_{T'}) \\ a_{\varepsilon} \log(G_i) + b_{\varepsilon} \log(K_j) &= \log \left( \frac{\frac{\partial_y T'_f \partial_y T'_f}{\partial_i T'_f \partial_i T'_f}(CHT_{ij})}{\frac{\partial_y T'_f \partial_y T'_f}{\partial_i T'_f \partial_i T'_f}(isoQ)} - 1 \right) - \log(C_{\varepsilon_\theta}) \end{aligned} \quad (27)$$



**Fig. 9** Contour plots at the fluid-solid interface for the wall-resolved LES at  $Re_\tau = 395$  and  $Pr = 0.71$ . Left frame: Temperature variance. Right frame: Relative error for the regression (28), in %.

461 In the present study, the linear regressions are performed with the Analysis ToolPak available  
 462 in Microsoft Excel. The resulting regressions at the fluid-solid interface are

$$\frac{\overline{T'^2}}{\overline{T'^2}(isoQ)} = \frac{1}{1 + 2.78 \cdot G^{-0.140 \pm 0.062} \cdot K^{-1.28 \pm 0.062}} \text{ with } \overline{T'^2}(isoQ) = 6.04 \quad (28)$$

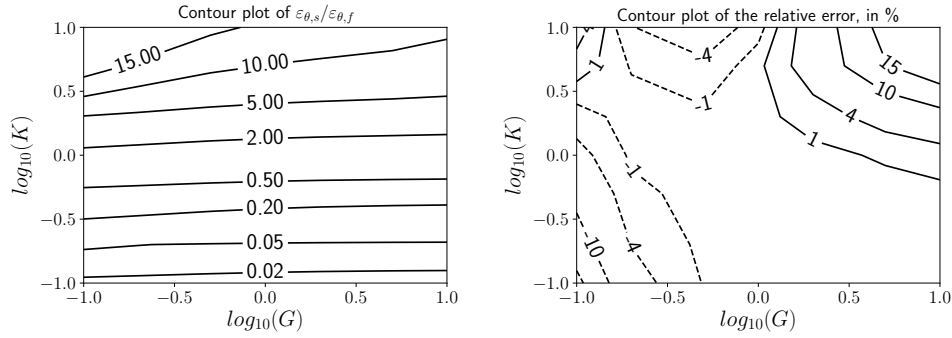
$$\frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}} = \frac{1}{G} + \left( K^2 - \frac{1}{G} \right) \frac{1}{1 + 0.0799 \cdot G^{0.225 \pm 0.029} \cdot K^{1.90 \pm 0.029}} \quad (29)$$

463 where the coefficients  $a$  and  $b$  are given with their 95 % confidence interval.

464 The right frame of Figure 9 shows that the regression for  $\overline{T'^2}$  produces a high relative  
 465 error for the lower values of  $K$ . The linear regression minimizes the global error. However,  
 466 lower values of  $K$  correspond to conjugate cases closer to the  $isoT$  one, which have a reduced  
 467 temperature variance at the fluid-solid interface compared with the conjugate cases at higher  
 468  $K$ . Therefore, the high relative error corresponds to a moderate absolute error, and is a direct  
 469 consequence of the regression used. Another remarkable trend for  $\overline{T'^2}$  is that the correlation  
 470 overestimates it for most of the conjugate cases, except the case  $CHT_{34}$ , which has  $G = 1/2$   
 471 and  $K = 1$ . This may be a direct consequence of the overestimation of  $\overline{T'^2}$  at the wall for the  
 472  $isoQ$  case observed Figure 4 when comparing wall-resolved LES and DNS.

473 The right frame of Figure 10 shows that the regression for the ratio of  $\varepsilon_\theta$  is fairly accurate  
 474 for the range of thermal properties investigated here: the relative error is below 4 % for most  
 475 of the cases considered. However, the regression tends to overestimate the ratio  $\frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}}$  when  
 476 both  $K$  and  $G$  are low (lower-left corner), and underestimates it when both  $K$  and  $G$   
 477 are high (upper-right corner). The latter one was expected to be problematic as high values of  $G$   
 478 should correspond to an imposed temperature boundary condition but high values for both  $G$   
 479 and  $K$  actually correspond to a low  $G_2$  (see Table 4), which should correspond to an imposed  
 480 heat flux boundary condition.

481 Indeed, the analysis proposed here remains qualitative as a better strategy to derive the  
 482 regressions would have been to randomly select combinations of fluid-solid thermal prop-  
 483 erties ratios inside the interval  $[0.1, 10]$ . It seems important to stress that the objective of  
 484 the present work is not to derive highly accurate regressions for turbulent channel flows as  
 485 it would probably require more complex formulations than the ones in equations (25) and  
 486 (26). The objective is rather to propose a methodology, which can be reproduced, adapted  
 487 and applied on any configuration of interest. The main idea of this methodology is that the



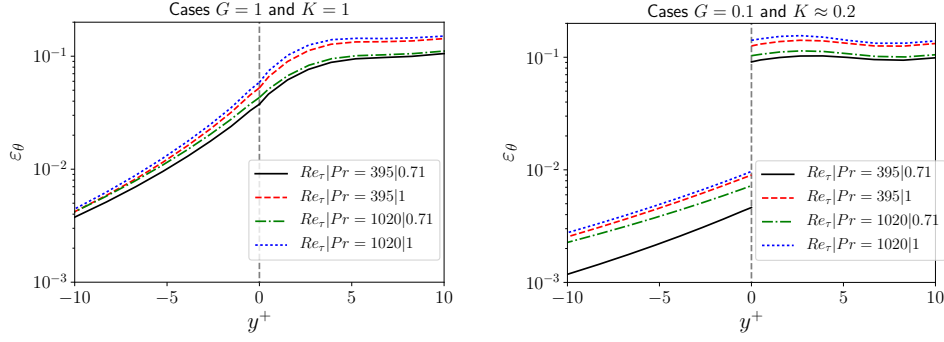
**Fig. 10** Contour plots at the fluid-solid interface for the wall-resolved LES at  $Re_\tau = 395$  and  $Pr = 0.71$ . Left frame: Ratio of the solid and fluid scalar dissipation rates. Right frame: Relative error for the regression (29), in %.

488 temperature variance should be maximal when the heat flux is imposed, and the ratio of the  
 489 scalar dissipation rates should remain within the theoretical bounds of  $K^2$  and  $\frac{1}{G}$ . Regarding  
 490 the discontinuity of  $\varepsilon_\theta$  at the fluid-solid interface, the key is definitively the anisotropy of the  
 491 fluctuating temperature gradient at the wall. The authors believe that the anisotropy obtained  
 492 with unit ratio of fluid and solid thermal properties reflects the natural — or unconstrained  
 493 — anisotropy of the heat transfer at the fluid-solid interface, and should be used as a refer-  
 494 ence. As the regression for  $\overline{T'^2}$  looks more qualitative, the remaining part of our study will  
 495 focus on correlations for  $\varepsilon_\theta$ .

## 496 5.2 Cases at a higher $Re_\tau$ or $Pr$

497 Regarding the simulations at a higher  $Re_\tau$  ( $395 \Rightarrow 1020$ ) or  $Pr$  ( $0.71 \Rightarrow 1$ ), only 3 coupled  
 498 passive scalars were transported. Among those, 2 have non-unit ratio of fluid and solid ther-  
 499 mal properties, as described hereafter. Although those properties are not fully representative  
 500 of the reality, they allow qualitative comparisons. The left frame of Figure 11 shows the  
 501 impact of the Reynolds and Prandtl numbers on  $\varepsilon_\theta$  for cases with unit ratio of fluid-solid  
 502 thermal properties ( $G = K = 1$ ). The Reynolds number has a very limited impact on the pro-  
 503 file around the fluid-solid interface while the Prandtl number has a more pronounced impact,  
 504 especially in the fluid domain. Overall, the impact of the Reynolds and Prandtl numbers ob-  
 505 served in our LES is inline with previous DNS of a turbulent channel flow (Abe and Antonia  
 506 [1], Kawamura et al. [19], Abe et al. [2], Kozuka et al. [20]). However, the comparison must  
 507 remain qualitative as those previous DNS are only considering an imposed temperature at  
 508 the wall, and we are looking at cases with fluid-solid thermal coupling.

509 The right frame of Figure 11 shows this impact on  $\varepsilon_\theta$  for cases with  $G = 0.1$  and  $K \approx 0.2$ .  
 510 The case  $Re_\tau = 395$  and  $Pr = 0.71$  has exactly  $K = 0.2$  while the others have  $K = 0.23$ . This  
 511 is representative of pressurized water and steel, except for the value of  $G$ , which should be a  
 512 decade lower. We limited it to 0.1 to remain within the bounds of validity of the correlation.  
 513 Overall, the impact of the Reynolds and Prandtl numbers is similar to what was observed at  
 514  $G = K = 1$ . Remarkably, the case  $Re_\tau = 395$  and  $Pr = 0.71$  exhibits a lower  $\varepsilon_\theta$  in the solid  
 515 domain. On the one hand, this might be the effect of the small difference on  $K$ . On the other,  
 516 it might be a distortion due to the logarithmic ordinate.



**Fig. 11** Scalar dissipation rate for the wall-resolved LES at various  $Re_\tau$  and  $Pr$ . Left frame: Cases  $G = 1$  and  $K = 1$ . Right frame: Cases  $G = 0.1$  and  $K \approx 0.2$ .

517 More quantitatively, we can measure the relative error produced by the regression (29).  
 518 The cases with  $G = K = 1$  can be discarded as they all lead to a continuity of  $\varepsilon_\theta$  at the fluid-  
 519 solid interface. The remaining cases are  $G = 1.3$  and  $K = 2.8$  — representative of air and  
 520 plexiglas — and  $G = 0.1$  and  $K = 0.23$ , representative of pressurized water and steel (except  
 521 for  $G$ , as already explained). For the LES at  $Re_\tau = 395$  and  $Pr = 0.71$ , we measure the  
 522 error of the regression at the approximate values of  $(G, K) = (1, 2)$  and  $(G, K) = (0.1, 0.2)$ ,  
 523 respectively. As a complement, we also measure the relative error at  $Re_\tau = 150$ ,  $Pr = 0.71$   
 524 and  $(G, K) = (1, 2)$ . This was one of our validation cases, see Table 3 (the case  $G = 1$  and  
 525  $K = 2$  has  $G_2 = \frac{1}{K\sqrt{G}} = 1/2$ ).

526 At this stage, it is important to stress that the regression (29) has no dependence on  $Re_\tau$   
 527 and  $Pr$  and is thus not expected to perform well at higher  $Re_\tau$  and  $Pr$ . The amount of data  
 528 available so far (3 values for  $Re_\tau$  and 2 values for  $Pr$ ) is clearly insufficient to introduce such  
 529 a dependence. As a workaround, we derive the following correlation from (29):

$$\frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}} = \frac{1}{G} + \left(K^2 - \frac{1}{G}\right) \frac{1}{1 + C_\varepsilon \cdot G^{0.225} \cdot K^{1.90}} \quad (30)$$

$$\text{with } \frac{1}{1 + C_\varepsilon} = \frac{\overline{\partial_y T_f' \partial_y T_f'}}{\overline{\partial_i T_f' \partial_i T_f'}} (Re_\tau, Pr, G = 1, K = 1)$$

530 For the case  $Re_\tau = 395$  and  $Pr = 0.71$ , this is exactly the regression (29). For the other  
 531 cases, it uses the anisotropy of the fluctuating temperature gradient obtained at  $G = K = 1$   
 532 to improve the prediction, while keeping the exponents on  $G$  and  $K$  as in (29).

533 The relative wall-normal contribution in  $\varepsilon_\theta$  at the fluid-solid interface is usually not  
 534 directly available in a (U)RANS turbulence model, even when the fluid and solid thermal  
 535 properties are identical. Thus, (30) is not easily exploitable for future turbulence models.  
 536 Hereafter, we assume  $C_\varepsilon$  depends only on  $Re_\tau$ , on  $Pr$ , and on three quantities taken at the  
 537 fluid-solid interface for the conjugate case  $K = G = 1$ :  $T_f'^2$ ,  $\partial_y (T_f'^2)$  and  $Pr\varepsilon_{\theta,f} = \overline{\partial_i T_f' \partial_i T_f'}$ .  
 538 Furthermore, we assume a simple power-law relationship between dimensionless quantities.  
 539 Using the 5 LES performed with  $K = G = 1$  and a least-square regression, we derive the

540 following correlation:

$$\frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}} = \frac{1}{G} + \left( K^2 - \frac{1}{G} \right) \frac{1}{1 + C'_\varepsilon \cdot G^{0.225} \cdot K^{1.90}} \quad (31)$$

$$\text{with } C'_\varepsilon = 0.00432 \cdot Re_\tau^{0.0262} \cdot Pr^{-0.976} \cdot \left( \frac{\sqrt{\frac{T_f'^2}{Pr\varepsilon_{\theta,f}}}}{\frac{T_f'^2}{\partial_y(T_f'^2)}} \right)^{16.5}$$

541 It is potentially a more interesting correlation than (30) because it does not include the  
 542 relative wall-normal contribution in  $\varepsilon_\theta$ . However, the 4 coefficients in  $C'_\varepsilon$  were fitted on a set  
 543 of 5 values, and are thus plagued with a relatively large uncertainty. To be more specific, the  
 544 small coefficient found on  $Re_\tau$  suggests  $C'_\varepsilon$  is not depending on it. The near unity coefficient  
 545 on  $Pr$  is also interesting. However, the narrow range of  $Pr$  numbers investigated does not  
 546 allow any extrapolation and simply suggests that  $C'_\varepsilon$  increases when  $Pr$  decreases.

547 On this ground, we can build a correlation tailored to the available dataset. First, we  
 548 discard the dependence on  $Re_\tau$ . Second, we assume a unit coefficient on  $Pr$ . This leads to  
 549 the following correlation

$$\frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}} = \frac{1}{G} + \left( K^2 - \frac{1}{G} \right) \frac{1}{1 + C'_\varepsilon \cdot G^{0.225} \cdot K^{1.90}} \quad (32)$$

$$\text{with } C'_\varepsilon = 0.00612 \cdot Pr^{-1} \cdot \left( \frac{\sqrt{\frac{T_f'^2}{Pr\varepsilon_{\theta,f}}}}{\frac{T_f'^2}{\partial_y(T_f'^2)}} \right)^{15.3 \pm 2.80}$$

550 where the power-law exponent is given with its 95 % confidence interval. Regarding the  
 551 leading coefficient, the 95% confidence interval is [0.00401;0.00932]. As a side remark,  
 552 when  $\partial_y(T_f'^2)$  vanishes,  $C'_\varepsilon$  also vanishes and the discontinuity scales with  $K^2$ .

553 As shown Table 5, the correlation (30) performs much better compared to (29). Indeed,  
 554 this could have been expected for thermal properties ratios close to unity, as in the case  
 555 ( $G, K \approx (1.3, 2.8)$ ). It seems to hold quite well for ratio of thermal properties further away  
 556 from unity, as in the case ( $G, K \approx (0.1, 0.23)$ ). The correlations (31) and (32) also perform  
 557 well, even for ratios of thermal properties far away from unity.

**Table 5** Relative error in % for the regression (29) and for the correlations (30), (31) and (32)

		$Re_\tau$	150		395		1020	
		$Pr$	0.71	0.71	1	0.71	1	
Correlation (29)	$(G, K) \approx (1.3, 2.8)$		7.58	0.219	6.28	8.32	11.9	
Correlation (30)	$(G, K) \approx (1.3, 2.8)$		0.397	0.219	1.20	1.69	1.00	
Correlation (31)	$(G, K) \approx (1.3, 2.8)$		0.476	0.483	2.24	1.78	0.224	
Correlation (32)	$(G, K) \approx (1.3, 2.8)$		0.0705	0.843	2.27	2.25	0.183	
Correlation (29)	$(G, K) \approx (0.1, 0.23)$			12.4	14.8	16.7	19.4	
Correlation (30)	$(G, K) \approx (0.1, 0.23)$			12.4	8.35	8.11	5.13	
Correlation (31)	$(G, K) \approx (0.1, 0.23)$			13.1	9.60	8.21	3.76	
Correlation (32)	$(G, K) \approx (0.1, 0.23)$			14.0	9.64	8.78	3.79	

## 558 6 Discussion and conclusions

559 In the present paper, we have exposed a strategy to implement a fully conservative fluid-solid  
560 thermal coupling capability in a finite-volume solver, currently implemented in the open-  
561 source CFD solver *Code\_Saturne*. Comparisons with DNS have shown that the proposed  
562 implementation allows one to perform wall-resolved LES of conjugate heat transfer with a  
563 satisfying level of accuracy in case of forced convection at a Prandtl number close to unity.

564 We have also proposed a strategy to extrapolate statistics at the fluid-solid interface.  
565 As it is, the extrapolation strategy is applicable to wall-resolved LES of statistically steady  
566 flows. Extension to wall-modelled LES is not straightforward as the current strategy assumes  
567 the first fluid cell is located inside the viscous sublayer. Indeed, the proposed extrapolation  
568 strategy may locally fail on a complex case where  $\cos(\Phi) = 0$  in equation (24). However,  
569 without such a case at hand, it is dubious to propose solutions. One way to avoid the recon-  
570 struction would be to collect statistics at the fluid-solid interface during the simulation.

571 A regression for the discontinuity of  $\varepsilon_\theta$  at the fluid-solid interface for turbulent chan-  
572 nel flows at  $Re_\tau = 395$  and  $Pr = 0.71$  is also proposed, as expressed in equation (29). The  
573 regression is valid for fluid-solid thermal properties ratios  $G$  and  $K$  within  $[0.1, 10]$ . Exten-  
574 sion to thermal properties ratios further away from unity could be achieved but special care  
575 should be taken. First, transient might become excessively long and statistical convergence  
576 in the solid domain might be hard to reach when  $G$  becomes too large. In addition, as the  
577 coupled case degenerates towards an imposed temperature or an imposed heat flux, temper-  
578 ature fluctuations in the solid domain might get damped over a very short distance and a  
579 very short time scale, thus requiring a reduced time step and a very fine mesh in the solid  
580 domain to provide accurate statistics.

581 An original theoretical analysis also suggests that cases with very fast solid thermal  
582 diffusion ( $G \ll 1$ ) might correspond to an asymptotic state actually differing from the ex-  
583 pected limit of an imposed temperature or heat flux. This state is characterized by a Laplace  
584 equation for the solid temperature, which corresponds to a vanishing time derivative, or a  
585 steady-state limit. However, this is out of the scope of the present study and requires deeper  
586 investigations.

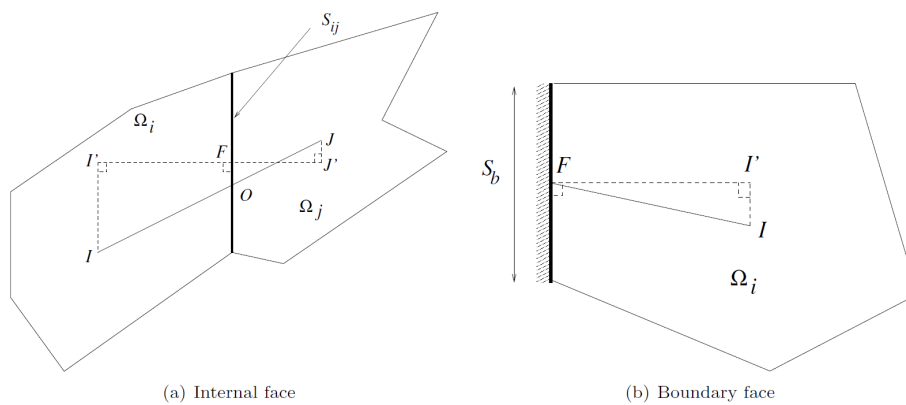
587 Correlations towards cases at a higher Reynolds number and different Prandtl number, as  
588 expressed in equations (30), (31) and (32), are proposed and assessed with additional LES.  
589 The strategy is promising but would require more simulations to be thoroughly validated.  
590 The authors would like to point out that DNS data at  $Pr = 0.01$  from [28] show encouraging  
591 trends. Regarding simulations at Prandtl numbers further away from unity, the LES model  
592 tested here remains to be thoroughly validated. Regarding conjugate heat transfer in turbu-  
593 lent flows, the anisotropy of the fluctuating temperature gradient at the fluid-solid interface  
594 clearly remains the key quantity, both for the proposed correlations and for future models.  
595 This work suggests that, given a flow regime, the impact of the fluid-solid thermal properties  
596 ratios can be clearly isolated. Indeed, much more work remains to be done to confirm this  
597 point.

598 Data and source code associated with the present paper are available online at <https://repo.ijs.si/CFLAG/CS-fluid-solid> under the GNU GPL v3 licence.

600 **Acknowledgements** This work was financially supported by the research project of the Slovenian Research  
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**Fig. 12** Quantities used in the finite volume discretization. Left: cell  $\Omega_i$ , cell  $\Omega_j$  and the internal face  $S_{ij}$  connecting them. Right: boundary cell  $\Omega_i$  and associated boundary face  $S_b$ . *Code\_Saturne* 5.0.0 theory guide [12].

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- 675

## 676 A Coupling strategy

677 Since version 5.0, *Code\_Saturne* can be used stand-alone to perform conjugate heat transfer with monolithically solving the fluid and solid temperatures in a fully conservative way, both in space and time, as described in this appendix. The coupling strategy developed contains two main entities. The geometric entity contains information related to the coupling interface and allows cells on both sides of it to exchange information. The physical entity contains information related to the coupled fields and their physical properties and allows to solve the coupled fields in a fully conservative way.

683 We start with a global mesh containing both the fluid and the solid. Using a given criterion, we split the volume into a solid part and a (complementary) fluid one: each computational cell is either inside the fluid or inside the solid. The internal faces separating fluid and solid cells are duplicated, alongside with the corresponding vertices and edges. Then, one of the duplicates is associated with the fluid cell while the other is associated with the solid cell. Both faces are then converted into boundary faces. This first step is performed by the preprocessor and changes the connectivity of the mesh. At the end of this step, the fluid-solid interface appears as an infinitely thin boundary which no variable can cross.

690 Then the solver starts. The fields specified by the user as coupled are flagged as such. Then, each time the convection-diffusion equation of a coupled field is solved, or when its gradient is computed, the explicit and implicit parts of the corresponding linear system are modified on the fly to take into account the coupling contributions. As the velocity vanishes on the fluid-solid interface, the coupling will be fully conservative if the diffusive term is correctly estimated. To further discuss this point, we must define a number of quantities specific to the finite volume discretization, as shown in Figure 12.



696 For an internal face — i.e.  $S_{ij}$  —, the thermal flux from cell  $\Omega_i$  to cell  $\Omega_j$  per unit surface, written  $D_{ij}$ ,  
697 is

$$D_{ij} = \lambda_{ij} \frac{T_{j'} - T_{i'}}{l'_{j'}} \quad (33)$$

698 where  $\lambda_{ij}$  is the thermal conductivity on the internal face  $S_{ij}$ . For a boundary face — i.e.  $S_b$  —, the thermal  
699 flux out of cell  $\Omega_i$  per unit surface, written  $D_{ib}$ , is

$$D_{ib} = \lambda_i \frac{T_F - T_{i'}}{l'_{iF}} \quad (34)$$

700 Thus, the thermal flux from a boundary face to a neighbouring cell  $\Omega_j$  per unit surface, written  $D_{bj}$ , is

$$D_{bj} = \lambda_j \frac{T_{j'} - T_F}{l'_{j'F}} \quad (35)$$

701 If the field is coupled, the fluxes per unit surface  $D_{ib}$  and  $D_{bj}$  are equal. This flux will be written  $D_{ibj}$ . The  
702 faces temperature  $T_F$  in equations (34) and (35) are also equal. This leads to

$$\left( \frac{F_{j'}}{\lambda_j} + \frac{l'_{iF}}{\lambda_i} \right) D_{ibj} = (T_{j'} - T_{i'}) \quad (36)$$

703 To conclude, the flux  $D_{ibj}$  of the coupled field should be equal to the flux  $D_{ij}$  we would have obtained on an  
704 internal face. Thus, using equation (33) and writing  $\eta_{ij} = \frac{l'_{iF}}{l'_{j'}}$ , one gets

$$\frac{1}{\lambda_{ij}} = \left( \frac{1 - \eta_{ij}}{\lambda_j} + \frac{\eta_{ij}}{\lambda_i} \right) \quad (37)$$

705 This demonstrates that an harmonic interpolation for the face thermal conductivity gives a fully conservative  
706 coupling strategy. Second order accuracy in space on non-orthogonal meshes is reached thanks to an iterative  
707 process.

708 The main advantage of the present coupling strategy is that the solver remains fully conservative in case  
709 of conjugate heat transfer. In addition, second-order accuracy in time is preserved. The main disadvantage is  
710 that the velocity and the pressure are also defined in the solid domains. As both remain exactly zero inside the  
711 solid, this is not an issue, but it does lead to higher memory usage compared with a dedicated solver for the  
712 solid thermal diffusion. This coupling strategy will provide a valuable framework for future RANS models  
713 able to tackle conjugate heat transfer.

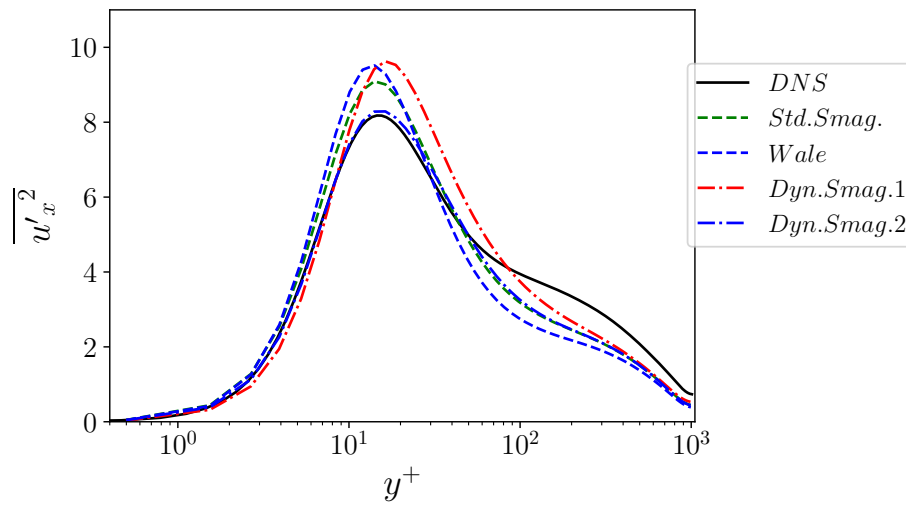
## 714 B Impact of the SGS model and $Pr_t$

715 Prior to the present study, several LES of a turbulent channel flow at  $Re_\tau = 1020$  were performed to se-  
716 lect a SGS model for the momentum equation. The mesh used corresponds exactly to the one described in  
717 the Subsection 2.2. SGS models tested already implemented in Code\_Saturne were the Standard Smagorin-  
718 sky (*Std.Smag*), the WALE model (*Wale*), and 2 variants of the Dynamic Smagorinsky (*Dyn.Smag.1* and  
719 *Dyn.Smag.2*). The two variants of the Dynamic Smagorinsky SGS model are using different clippings. The  
720 authors also tested some variants of the Dynamic Smagorinsky model with spatial averaging over homoge-  
721 neous directions. As they did not bring significant improvements, they are simply omitted here.

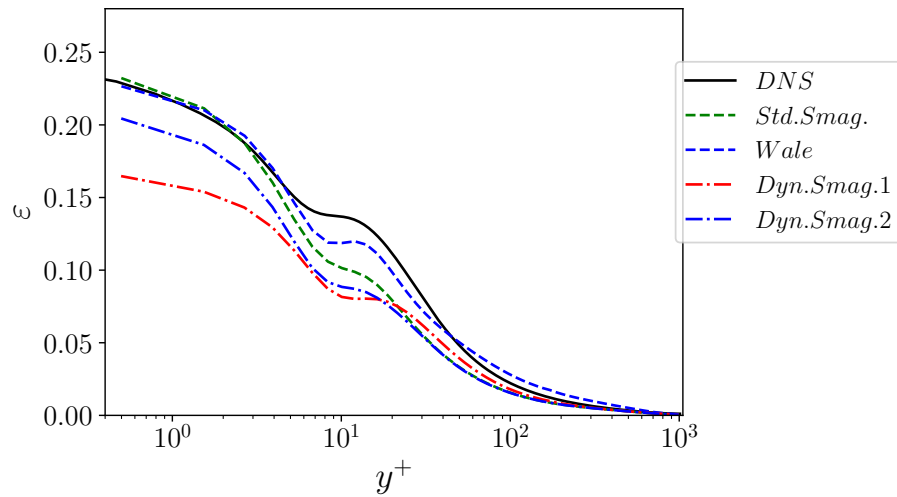
722 Overall, the SGS models tested only provide a qualitative agreement for the Reynolds stresses. This is  
723 illustrated in Figure 13 with the variance of the streamwise velocity ( $\overline{u_x'^2}$ ). Regarding the dissipation rate ( $\epsilon$ )  
724 associated with the turbulent kinetic energy, Figure 14 shows that the *Wale* SGS model provides the best  
725 estimation near the wall. The *Wale* SGS model was selected on this criterion.

726 The authors have also looked at the impact of the turbulent Prandtl number ( $Pr_t$ ). Two simulations were  
727 performed at  $Re_\tau = 395$  and  $Pr = 1$  using conjugate heat transfer with unit ratios of fluid-solid thermal  
728 properties ( $K = G = G_2 = 1$ ). The mesh used corresponds exactly to the one described in the Subsection 2.2.  
729 One simulation were performed with  $Pr_t = 0.5$  and the other with  $Pr_t = 1$ . As visible on Figures 15 and 16,  
730 the turbulent Prandtl number has no significant impact on the statistics in the near-wall layer ( $y^+ < 5$ ).

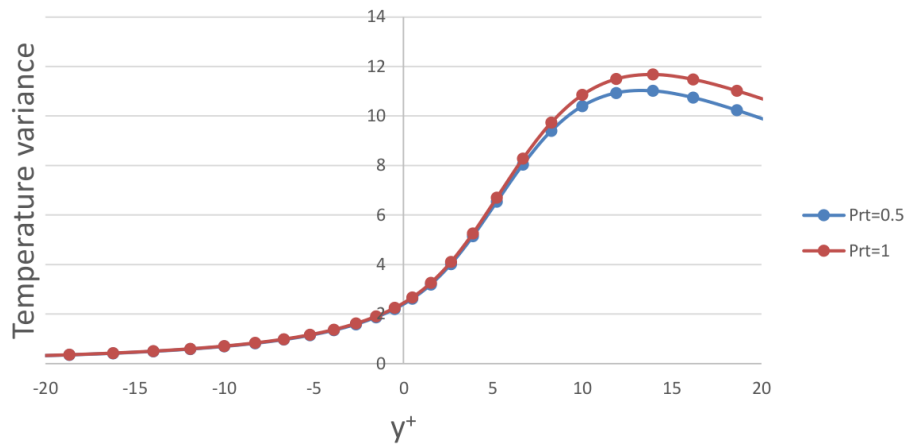
731 When performing a LES, the choice of the SGS models used in the momentum equation and in the  
732 energy equation is of paramount importance. However, development and testing of refined SGS models is  
733 out of the scope of the present study. Thus, the authors have decided to combine a fine mesh with relatively  
734 simple SGS models already implemented in Code\_Saturne, and to carefully assess the obtained results using  
735 DNS.



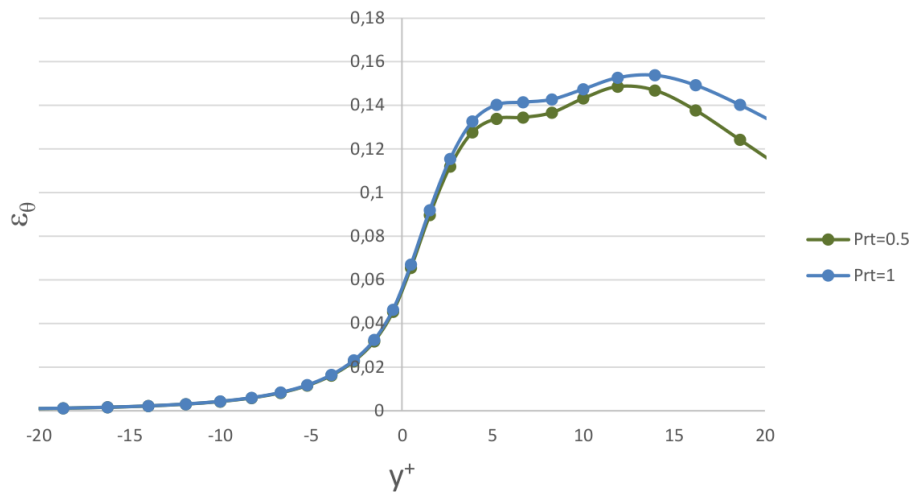
**Fig. 13** LES of a turbulent channel flow at  $Re_\tau = 1020$ . Impact of the SGS model on the variance of the streamwise velocity.



**Fig. 14** LES of a turbulent channel flow at  $Re_\tau = 1020$ . Impact of the SGS model on the dissipation rate associated with the turbulent kinetic energy.



**Fig. 15** LES of a turbulent channel flow at  $Re_\tau = 395$  and  $Pr = 1$ . Conjugate heat transfer with  $K = G = G_2 = 1$ . Impact of the turbulent Prandtl number ( $Pr_t$ ) on the variance of the temperature.



**Fig. 16** LES of a turbulent channel flow at  $Re_\tau = 395$  and  $Pr = 1$ . Conjugate heat transfer with  $K = G = G_2 = 1$ . Impact of the turbulent Prandtl number ( $Pr_t$ ) on  $\varepsilon_\theta$ .