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The teacher's action, the researcher's conception in mathematics

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[Abstract] Our point is to contrast both epistemological positions of teachers and researchers, by the means of their respective actions in a research process. Based on a threefold descriptive model of the teacher's action, our analyses examine the nature of the teaching techniques enacted about a given mathematical situation, the "Race to 20" (pupils aged 9-11 years) and the discourses of two teachers about this lesson. Our findings indicate that the teachers are primarily concerned with the educational coherence of the teaching process, using non specifically knowledge-related teaching techniques that researchers explain by some generic overdeterminations, from the didactical point of view. This gap has to be taken into account in further collaborative research, in order to make teacher's developing specific teaching techniques to foster the mathematical sense-making of the students.

This paper investigates the relationship between two experienced teachers(T1 and T2) and a researchers' team, both involved in a particular research. This relationship is to be considered through the epistemological conceptions of each part, that we need to compare in order to understand the respective behaviors and discourses.

In order to understand the teacher's action while he carries out mathematics lessons, we design a threefold model based upon didactical categories (Brousseau 1997 ; Chevallard, 1992). The first level of this theoretical frame comprises a set of *micro* teaching techniques, that were spotted several times among the interactions patterns concerning a given mathematical knowledge. These techniques should enable the researcher to describe in depth and precisely how a given piece of knowledge is handled by both the teacher and the students. A second level is also defined to gather the *macro* shifts that can be observed in the didactical contract, and therefore it sums up the main teacher's intentions about the knowledge. Then, a third level is to take into account the teachers' general practices and educational beliefs through behaviours observed during the lesson, and/or comments we may get from the teachers during interviews. Moving up the levels in the model shows a "gradient" in the teaching techniques : at the bottom, are the most knowledge-linked techniques

and at the top, are the less specific techniques. Actually, the two first levels should be the core of our theoretical point of view, as we defend that the knowledge specificities and the corresponding teaching situations in mathematics are very likely to constraint the didactical interactions. This model should be an analyzing tool for the researchers, and our comparative research project in didactics should contribute to sort out the generic or specific nature of these teaching techniques.

Therefore, the main purpose of this article is the confrontation of the two specific logics enacted in the teacher's action and researcher's action, through analyses using the teacher's action model. To organize empirically this confrontation, we focus the analysis on a gap between different interpretations (the researchers' one and the teachers' one) of two techniques used by one of the two teachers (T1). Thus, the third level of our model, which is the widest to gather different types of teaching techniques, seems to be the more convenient to start with, in this study.

In the first part, we make a brief comment on the activity suggested to both the teachers. In the second part, we present the context of the research. Part 3 is devoted to a description of the techniques used by the teacher from the researchers' point of view. In part 4, we present the teacher's (T1) point of view on the same techniques. The part 5 is dedicated to the other teacher's (T2) analysis of these techniques. The last part of this paper is an attempt to understand the two different systems of meaning and the two different epistemologies enacted in the different actions and discourses.

1. A didactical situation : the “Race to 20”

In this paper, we shall consider the teacher's action while he has to carry out a “Race to 20” lesson, proposed by the research team, and adapted from Brousseau's situation (Brousseau, 1997, pp. 3-17). This situation played an important role in the development of the Theory of didactic situations. This didactic setup is for us a paradigm for the studying of teacher's work (Sensevy *et al.*, 2004).

2. The research organisation

The research therefore involved two teachers (T1 and T2) of grade five classes in primary school. We first introduced the teachers to the “Race to 20” situation, presenting the main mathematical aspects of the game in a two hours training. Brousseau's complete text on this subject was handed out at the end of the training session, but the didactic engineering itself was not a training topic during this session. It was given as an opportunity that the teachers could take into account, or not. In the second phase, we asked each teacher to teach one or more lessons on the situation

“Race to 20”. The teachers were free to plan the lessons as they wished. Both decided to devote two lessons to the situation. The third phase consisted in the teachers’ self-analyzing their first lesson, based on a video recording of the class. The fourth phase consisted in the teacher’s cross-analyzing their first lesson: T1 was analyzing the T2’s lesson on the video recording (in presence of T2) and reciprocally.

3. A description of two techniques used by the teacher

After the analysis of the first lesson of T1, the researchers agree on the following point: two techniques used by T1 seem rather rare and unexpected. Indeed, two original ways of acting are used by T1 :

- in the beginning of the lesson, he asks students to question him questions about the topic of this lesson,
- in the pair work that he scheduled for this lesson, a third student has to watch the game played by the two others as a referee.

3.1 The “questions” technique

T1 begins the lesson by questioning the students about the meaning of the words “Race to 20”.

T1 Omin	To day we are going to work on the race to twenty. It’s a mathematical game. From the expression “race to twenty, what can you already tell me?
Student	(...) we jump from 20 by 20
T1	I forgot to tell you something: speak loud enough so as to be heard. Repeat it louder please. <i>The teacher walks across the classroom</i>
Student	Maybe we are going to count from twenty to the next twenty more.
T1	Counting from twenty to the next twenty more. Yes.
Student	Its a race. We have to be quick...
T1	Yes. The race. The idea is to be quick. So there is velocity, since we are in a race. Or else we would have called that the walk to twenty. Maybe. Then twenty, so you say “counting twenty by twenty”. Can you see another idea?
Student	Its a race involving twenty children, with twenty children who play, who are racing..
T1	Twenty is the number of children taking part to this. Can you see any other thing? Race to twenty. Its true that “to” may be...
Quentin	For example we are going to count three by three up to twenty. And the winner is the first to reach...
T1	And then in that case, what <u>is</u> twenty ? What does it represent? Yes, go on Quentin.
Quentin	Its the number up to which we must go.
T1	The number up to which we must go, the number we must reach. And why are you thinking of counting thee by three?
Quentin	Well, because at the moment we are working a little bit on mental counting, so that counting three by three, we learnt through going backwards.

Camille	Yes, but counting three by tree, if you start to Zero up to twenty, we reach thirty but not twenty. It won't be the exact number.
T1	You think we can reach twenty when we start from zero ?
Camille	No, with three...
T1	Jumping three by three...
Camille	Yes, and starting from zero. .
T1	And last week, what happened Romuald ?
Romua	There were some who... (...).
Student	There is Alice who had come and given us tests, and we had to count three by three then, four by four...
T1	(...) Well, in the race to twenty you suggested several things. The race, effectively, there is an idea of velocity and then, twenty, as Quentin said a few moments ago, you must reach twenty. You must go up to twenty. Another game, we can change it. So, in order to play that game, do you have enough information if I say to you "we are going to play the race to twenty"?
Student	No.
T1	Well, in that case ask me questions!
Student	We need to know the rules of the game.
T1	Michaël ?
Student	(...)
Student	Whether its a game with draughts or an oral game
30. T1	Ask me questions.
Student	Will it be an individual game or a collective one ?
T1	Well, so, at the beginning... we play one b y one, one against one, one child against one child. well
Student	What is the game purpose ?
T1	Ha ! Well Kevin, can you answer to this question ?
Kevin	Reach twenty.
T1	And as fast as possible. So, there we are, the game purpose, we have talk about it. And here we are dealing with the rules of the game.
Mélanie	How are we going to reach twenty ? (...)?
T1	Ask your question again, Melanie.
Mélanie	How are we going to reach twenty ?

In contrast with the other teachers previously studied (and with T2), T1 institutes a "Question-game" (ST 1). Is it an habit in this class? The students have to determine the meaning of the "Race to 20" by considering the meaning of the words. T1 refuses gently the "wrong" answers (e.g. on ST 9). Then he summarizes the students' answers, to emphasize the fact that the students have not "enough information" to play the game. On ST 25, the teacher produces an utterance: "So, ask me questions !", which is emblematic of this "teaching technique". It seems that the division of the activity between the teacher and the students is upside-down: the students ask the "relevant" questions, and the teacher give answers.

3.2 *The "referee" technique*

Ten minutes after the beginning of the lesson, the students are correctly playing the game. T1 organises the group work.

PROF 10 min 35	Its Romuald. Who has not understood? I am waiting in case you have questions to ask. We go through the following steps, now you can pay, since you seem to have understood. Therefore you are going to play one against one, and one child will be the referee. That is to say, a few minutes ago I thought there was a mistake and eventually there was none. I had not heard correctly. So, do be careful. It might be a god thing if you wont go too fast and pronounce correctly for sometimes t actually looks like table tennis. It goes very fast. So, do be careful about this, so as to be well understood. So, you play one against one and the third child is the referee. That is to say, what part does referee Arnaud's play?
Arnaud	Do the counting up (...).
PROF	So this amounts a little bit to strike the match balance. Who won? Who lost? That's the game, isn't it, Jacques?
Jacques	Does the referee writes on a sheet ? Est-ce que l'arbitre marque sur une feuille ?
PROF	So, for the first game, we are going to watch very accurately what is happening. The referee plays another part as well. For instance if a child adds three. Can he do this?
Elève	No, he cannot.
PROF	If a child gives several times, he says one number then he says another one... in fact one does not know any longer what he said. So, in that case, the referee supervises a little bit the respect... sees to it that the rules be respected. Yes Jacques?

The teacher institutes the refereeing function of a third student in the group (ST 39). He defines the main features of his role. The referee will be maintained during both lessons. Many times, his task will be discussed in the whole class activity. The studying of the two lessons transcript make us conjecture that this way of acting could be a classroom habit, not specific to mathematics.

3.3 *A first analysis of these techniques*

The two techniques are analyzed by the team research in the same way. Our hypothesis was that these two techniques might be counter-productive from a didactic viewpoint. Indeed, the “questions-game” could slow down the student’s activity. The students’ attention could be taken off the mathematical aspects of the situation. We conjectured that the “question-game” could work as a metacognitive shift. In a similar way, the refereeing could affect the involvement of the students in the mathematical tasks. It could also draw their attention to the superficial features of the game (the “basic rules”), and be detrimental to the production of mathematical strategies.

4. **The teacher’s point of view on these techniques (T1)**

4.1 *About the “questions” technique*

When watching the video recording of his performance, T1 comments the

“questions” technique in the following way :

1.P1 So starting from a word or an expression or a sentence, there, the starting point was to try and find which... which ideas this could give us, regarding the game, I think I had said it but there I did not hear it and then I did : I said « mathematical game ».

2. C This is something you do rather frequently?

3. P1 So this is part of my habits, trying to link as much as possible activities which apparently could not be connected. So, try to find some coherence in specific domains. Hence the idea to work on a project as well. True in mathematics it's a bit more difficult. But here there actually is a link between them; for some pupils they made the link between the pretest and then what is going to be asked in the game.(...)

What is difficult as result this is something I tried to do rather frequently because, as regards my childhood experiences. I felt coherence very very late within what I was able to learn and we worked a lot on various unconnected topics; and so, I suppose I discovered many links, thank to my profession. But what is difficult in fact is to see that many pupils here are a little bit out of the game, all the same.

4. C Who are out of the play, in what sense do you mean?

5. P1 Well, a certain number of them don't look interested for the moment. They don't look motivated. Well, the idea to ask questions is in fact to motivate a little bit.

(...)

9. P1 It is something that I do, play, er... play the ninny well, or make believe I don't understand. I try to do that. Maybe sometimes I overdo it. I don't know. I enjoy playing that too. So, I try to get out of this... I have some problems with that, sometimes, because...

10. C For example ?

11. P1 Some of my pupils are waiting for me around the corner, shall we say. Boys for example, who don't necessarily understand the play and who will think finally “here again he made a mistake”. So that is a part rather difficult to play. It's tricky, playing with them and saying stupid things. Or making believe I have not understood, or have another child saying something which looks obvious.

14. C Em... And do you do it often?

15. P1 Yea, rather often. (...) I must admit that there is planted in my head, if you want to know, the fact... teachers are taking the power, shall we say. I want by all means avoid this, certainly because of my past.

We can infer from these comments that the “questions-game” is really important in this teacher's practice. It allows the students “to ...” (ST 3). It is obvious that the teaching intentions are far beyond the didactic goal of this lesson. The “questions-game” could be considered as a taken-as-shared way of acting, that seems normative in this classroom community. It is not specifically related to the mathematical knowledge but mostly an educational technique, that could apply to any subject matter area. This technique is produced in order to fulfill some constraints of the didactic process *i.e.* make sure the didactic experience remains coherent for the students, and develop inquiry procedures in the classroom. It is worth noticing the teacher's remarks on ST 17. This argument seems to corroborate our analysis of the different “division of the activity” that this technique entails. In the “question” game, the teacher's role is not so easy (ST11) and the students have to interpret the teacher's behaviors in the right way.

4.2 About the “referee” technique

Let us consider T1’s comments about the “referee” technique:

1. C Why do you have... chosen here... a situation with a referee?

2. P1 Well, it was to have... in fact, the p... the referee there observe the game, regarding the respect of the rules. So in fact, check that well... they were really adding 1 or 2, because well, you know, there may have been moments... checking that there was a proper enunciation too, clear enough, checking that everybody understood everybody. Seeing to it that nothing broke loose or that there was no communication because the game went too fast, for example. About all... It was in fact... In that case the word “referee” fitted perfectly. Because he was more a “respect of the rules” referee than an observer.

3. C All right. And you are used to putting referees that way?

4. P1 Yea. Well I do this very often during an assessment session for example there are very often associate in either phases where we are researching... well, here I come again on gym: so in some way there were pupils referees. (...)

6. P1 Yea, observers. Then we specify the success criteria. We try, because unfortunately, I cant take it with all the activities... So...

The teacher’s comments make us understand that the “referee technique”, as it is enacted in the “Race to 20” lesson, is a frequently used technique (also used, for instance, in Physical Education) that the teacher applied to the mathematical pair-work designed in this situation. In other cases, the referee is said to be useful for the evaluation tasks of the knowledge, but the teacher admits that criteria for assessment are not easy to define. This is interesting because the teacher reveals himself that these technique may not fit with all the class activities.

4.3 A first interpretation

In order to understand the teacher’s action in this lesson, and particularly in the management of the two techniques that we showed, one has to consider the function of these techniques in the teaching process. T1 is anxious to create an inquiry-based classroom, and possibly to delegate the assessment task to the students themselves, and this, not only in mathematics or in science, but in a general way, in all the classroom activities. In order to create such a self-directed learning, the teacher calls in some general techniques that can be *replicated* from one situation to another, which bring some coherence in the learning experiences. A didactic analysis make us conjecture that these techniques are not very efficient from a mathematical viewpoint. Nevertheless, the role of the teacher, in primary school, is not only to foster the mathematical thinking and sense-making of the students: it is also to educate them, to give them “cognitive values” (Putnam, 1992) embedded, for instance, in self-questioning or inquiry process.

5. The other teacher's (T2) analysis of these techniques

5.1 On the techniques

When watching the videotape of T1's lesson, T2 shows interest in the "questions-games" episode.

1.P2	Well, in that case it is... I don't separate the same way exactly. That's the point I wanted to ask you there. You... you... ask the pupils o speak?
2.P1	I wanted to.
3.P2	So this means... yes.
4.P1	I wanted to start from the race to twenty since... showing people... so there it was a questioning to see which meaning they could give when there was a new discovery.
5.P2	Yes I found that it was interesting exactly because they were taking part, I would not have thought at the beginning at first, but I found that it was worth doing it.
	(...)
6.P2	When you were having this questioning there, in fact, you felt that it should create links, or was it because you intended simply to explicit the vocabulary?
7.P1	Er... no, it was in fact, looking for meaning. Starting from an idea, well, from a proposition, an expression, different meanings, to be able to rebound afterwards a little bit later on. Er... Now we can say that it was rather that way. But that might have been something else.
8.P2	There it is interesting. Well, I am an observer so...
9.P1	There when we look at the children, some of them are working and then some others, finally...
10.P2	Yes but. Your asking a question. Er... On the meaning, and the children. Er... giving an interpretation linking it to something else, I find that, for me, it's interesting.

In his comment, T2 grasps T1's intentions. Notably in ST 6 and 10, T2 stresses that "making links" is important.. T2 seems to recognises some "valuable" features in these techniques, that may corroborate our hypothesis of a generic constraint about connections between tasks, that lies upon teachers.

5.2 T2 synthetic commentary

11.P1	Do you have something to add, to end the session?
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11.P2	About the session, in fact, I notice that we did not at all take the same beginning. There are things which I would never have thought about because I don't practice them in my class... In fact it gives me ideas, you know, I will try some things. I really enjoy the part of the referee coming from outside, because I do it as well among the groups but it is always within the groups. Each one has his part in the group. Each one takes turns, but I must admit I never bring it from outside, now, I find this is really worth doing it. (...)
16.P2	And then there is one thing which I will keep in mind as well: the way you get in the activity with your insisting on the language, the meaning of words ; that I find maybe a way to start. That can be done. That I can do with other activities. But I would never have thought about it, there, for example, and I find it is quite right when starting an activity, to make a link or to avoid disconnections with previous sessions.

In these comments, we can find arguments that expose very clearly the epistemological gap between the teachers and the researchers. Indeed, T2 appreciates the referee as an outsider. In contrast, researchers analysis show how this referee could be a *mathematical outsider*, who does not mathematically benefit from the situation. Similarly, T2 emphasizes, in a very direct way, two fundamental functions of the "questions" technique : to link different activities *a priori* separated ; to avoid the temporal break between the different lessons. There is no consideration for the mathematical content at stake. Finally, the gap is obvious, between researchers who are primarily concerned with the specific mathematical meaning of the situations, and teachers who are primarily concerned with the coherence of the classroom activities and the educational relevance of replicated forms of teaching actions.

6. Two different systems of meaning, two different epistemologies

These two systems can be described as following. The "researchers system" is oriented towards the mathematical content, enacted in specific mathematical practices. The Race to 20 is a mathematical situation that includes several prominent features: the "alternative" (if I play 17, either my opponent plays 18, or he plays 19); "the backwards recurrence" (to play 20, I have to play 17, therefore the race to 20 is a race to 17; to play 17, I have to play 14, therefore the race to 20 is a race to 14... the race to 20 is a race to 2) ; the "methodological" triplet *proof-conjecture-refutation*. For the researchers, the appropriate didactic contract (Brousseau, 1997) contains these objects, but the teaching practices of T1 and T2 do not include them. Following the distinction coined by Cobb & al (2001), we could say that the researchers put into focus the "mathematical practices" in the classroom. On the contrary, the "teachers system" is based on the generic relevance of some teaching techniques. The appropriate categories are "the development of the student's autonomy", and "the necessity, for the students, of assuming their learning responsibilities". The generic teaching techniques bring the coherence in the didactic experience of the students. The priority lies, therefore, in fostering "social norms" (Cob& al; *ibid*) in the

classroom.

As we find important to explore the ways in which the gap might be bridged, we chose, at least in a first study, to put on hold our theoretical stance. In doing so, we took the opportunity to understand the practical logic of the action. Now, in order to organise the discussion of analyses, we shall come back to the levels of the model of the teacher's action, that we introduced at the beginning.

We hypothesize an *overdetermination* of the third level, in which cognitive values and teaching practices are embedded, upon the two *infra* levels, at least at primary school. If we want to take into account the teacher's action, we have to document how these constraints can shape the didactic transactions. First, let's start with sorting the nature of these constraints. The techniques used by the teachers appear to be answers to educational constraints (e.g., "try to establish a link between a maximum of activities that, on the face of it, don't have anything to do with each other" or "the assessment tasks must not rely only on me, the teacher"). However, a technique that the teacher replicates in a general way in different activities, could be the answer to a *generic* constraint, from the didactical point of view. Indeed, the didactical theory (Brousseau 1997) shows that pieces of knowledge have intrinsic links between each other. The mathematical situations are designed to help the student in building bridges between different pieces of knowledge. However, the links between situations, or class activities are not obvious and often depends on an *a priori* knowledge organisation made by institutions (curricula, textbooks). Therefore the teacher has to cope with the situations, trying to replicate some teaching techniques, but not very specifically related to the mathematical knowledge, in order to reach an educational achievement. Meanwhile some *generic* didactical constraints exist about the knowledge organization and could be playing in the background. Therefore conflicts may emerge between the teaching techniques and the didactical goal of a mathematical situation, as we saw in T1's lesson. This explanation induces that the teacher may need some specifically knowledge-related techniques, to meet both didactical and educational achievements.

To conclude, we think that the researchers have to understand the very nature of the teacher's action. That means to identify the different constraints that the teachers have to cope with, in particular the necessity of *educational coherence* for the teachers which that can be explained by some *generic overdeterminations, in the didactical point of view*, that we conjecture in this paper.

Against such a background, a collaborative research could allow the researchers to acknowledge the multi-determination of the practical logic, and the teachers to analyse the mathematical content in a more efficient way, the collaborative research attempting to answer the following questions : what could be the *specific teaching technique* that the teacher has to produce to foster the mathematical thinking and sense-making of the students ? What could be the effective conditions of their

productions ?

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