A comparative study of different NMPC schemes for control of semi-active suspension system

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A COMPARATIVE STUDY OF DIFFERENT NMPC SCHEMES FOR CONTROL OF SEMI-ACTIVE SUSPENSION SYSTEM

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ABSTRACT

Control of semi-active suspension system for vertical dynamics of automobiles plays a vital role in guaranteeing comfort and safety for the on-board passengers. The seemingly simple task of control poses to be daunting under the presence of multiple nonlinearities, physical constraints and specification over objective satisfaction of the system and thereby, it is of paramount importance to account for these issues during control system design for better efficiency and prolonged endurance of the suspension system. Amongst the existing several control methodologies, predictive control techniques serves as a promising approach in dealing with the aforementioned issues. In this paper, we present several nonlinear model predictive control (NMPC) schemes and a detailed performance analysis of the methods. The incorporated NMPC schemes are the direct methods – single shooting, multiple shooting and collocation methods. The different methods were tested in simulation under MATLAB environment.

Keywords: Vehicle dynamics, Non-linear model predictive control, Suspension systems, Optimal control

1. INTRODUCTION

In the recent years, study on advanced control methods for automotive systems has gained a huge momentum and most of the automotive industries has embarked in research and development and implementation of better control algorithms for improved passenger comfort and safety and as well as vehicle's performance in terms of energy efficiency, pollution reduction etc. This sudden surge in interest is partly in response to the advent of autonomous vehicles and it has also been the key driving factor for automotive companies to strive for optimal performance. Conditioned upon the aforementioned requirements both objectively and subjectively, optimal control methods fares much better than other control methods due to the systematic approach embodied in its design to tackle the above issues. In order to implement optimal control in practice, model predictive control (MPC) formulation of the optimal control problem (OCP) provides the necessary feedback control framework to work seamlessly in real-time and real-world. The crux of the MPC scheme is the receding horizon method, where an OCP is solved online at every sampling instant. Despite the enormous benefits of the method, one of the major downside is the computational requirements for solving the underlying optimization problem of the OCP at every sampling period. However, given the exponential growth of computational power and easy/cheap availability of computing resources, the chasm between the two is closing in and the method seems promising in the near future.

The main ingredients of the MPC problem are a) the objective function, b) system constraints and c) the system dynamics [1] and when any of the ingredients induce any nonlinearity, the problem is known as nonlinear MPC (NMPC). In general, the NMPC problem can be solved in three prongs which are a) Direct methods (discretize then optimize), b) Indirect methods (optimize then discretize) and c) Dynamic programming [2]. This paper utilizes the direct methods approach, however irrespective of the method adopted the problem requires a nonlinear programming (NLP) solver to deduce the optimal solution for the NMPC problem. The direct methods can be broadly classified as single shooting, multiple shooting and collocation methods. Despite the fact that all the three methods solve the same problem, the solutions sought varies due to the difference in the problem formulation. The NLP solver that yields the solution might return different local minima solution for different cases. Under special case of convex problems, the solution of all the methods tends to be the same. In this paper, the subject of focus is on a comparative study on NMPC direct methods for control of semi-active suspension system for a quarter car model. The nonlinearity is induced due to the inherent dissipativity characteristics of the semi-active damper system [3],[4].

The paper is organized as follows. Section 2 describes the mathematical model of the quarter car model equipped with semi-active suspension system. Section 3 and Section 4 details the NMPC design requirements and direct methods formulation. Section 5 expounds the simulation results obtained and finally, the paper is concluded with conclusions and future works in Section 6.

2. MATHEMATICAL MODELING

The quarter-car model system equipped with semi-active suspension system shown in Fig.1 are given by the following set of equations

$$\begin{cases} \mathrm{m_s}\ddot{\mathrm{z}}_\mathrm{s} = -\mathrm{k_s}(\mathrm{z_s}-\mathrm{z_{us}}) - \mathrm{F} \\ \mathrm{m_{us}}\ddot{\mathrm{z}}_\mathrm{us} = \mathrm{k_s}(\mathrm{z_s}-\mathrm{z_{us}}) + \mathrm{F} - \mathrm{k_t}(\mathrm{z_{us}}-\mathrm{z_r}) \\ \mathrm{F} = \mathrm{f_c}\tanh\left(a_1\dot{z}_{def} + a_2 z_{def}\right)u + c_{nom}\dot{z}_{def} + k_{nom}z_{def} \end{cases} \tag{1}$$

Where, $\mathrm{m_s}$, $\mathrm{m_{us}}$, $\mathrm{k_s}$, $\mathrm{k_t}$ are the sprung mass, unsprung mass, stiffness coefficient of the suspension system and stiffness coefficient of the tyre respectively. $\mathrm{z_s}$, $\mathrm{z_{us}}$, $\mathrm{z_r}$, $\dot{z}_{def} = \dot{z}_s - \dot{z}_{us}$, $z_{def} = \mathrm{z_s} - \mathrm{z_{us}}$ are the chassis mass displacement, unsprung mass displacement, road profile displacement, deflection velocity and displacement between the chassis and the tyre respectively. The force exerted due to the suspension system is given by the Guo's [5] nonlinear equation F with $\mathrm{f_c}, a_1$, a_2, c_{nom}, k_{nom} the appropriate parameters for the force model. The input for the system is u, which is the PWM duty cycle (DC) signal that drives the damper system. The nonlinear dynamics of the system is compactly expressed with $\dot{x} = f(x,u,w)$, where $x = [\mathrm{z_s}, \mathrm{z_{us}}, \dot{z}_s, \dot{z}_{us}]^T$, $w = \mathrm{z_r}$ the road disturbance acting on the system and u is the PWM-DC signal.

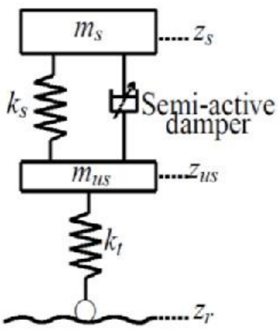

Fig.1 The quarter-car model

The parameters for the mathematical model are utilized from the INOVE test platform. The INVOVE test platform [6], shown in Fig.2 is a 1:5-scaled baja style racing car which consists of 4 controllable Electro-Rheological (ER) dampers and 4 DC motors to generate different road profiles for each wheel corner. The numerical values of the parameter are listed in the reference herein [3].

Fig.2 INOVE test platform

3. NMPC DESIGN REQUIREMENTS

3.1 Objective requirements

In this paper, the chosen objective design for the semi-active suspension pertains to comfort objective. The prime goal of the comfort based objective design is to minimize the vertical acceleration of the chassis [7] i.e. $\ddot{z}_s$. For a given look ahead period T_l, the objective is defined with

$$J_{com}\big(x(t), u(t)\big) = \int_o^{T_l} l(x(t), u(t), t)dt \quad (2)$$

Where, $l(x(t), u(t), t) = (\ddot{z}_s(t))^2$ which is obtained from equation (1).

3.2 Constraint requirements

The constraints incorporated into NMPC problem are:

- Semi-active damper input constraint: Minimum and maximum saturation force of the semi-active damper system i.e. $|F| \leq F_{max}$.
- State constraint: Minimum and maximum stroke displacement of the suspension system i.e. $|z_{def}| \leq z_{max}$.
- PWM-DC input constraint: The PWM-DC input u is constrained in the set $u \in [u_{min}, u_{max}]$.
- Road disturbance: The road profile is treated constant over the horizon i.e. $\dot{z}_r = 0$. This can be appended as an additional state variable and the augmented dynamics of the system can be expressed with $\dot{\tilde{x}} = f(\tilde{x}, u)$, where $\tilde{x} = [z_s, z_{us}, \dot{z}_s, \dot{z}_{us}, z_r]^T$.

The list of constraints can be compactly expressed with $h(\tilde{x}, u) \leq 0$, which is the mixed nonlinear input and state constraints of the system. Thus, given the objective and the constraints of the system, the OCP to be solved at every time instant with initial condition $\tilde{x}(0)$ for the NMPC problem is defined with

$$\begin{cases} \min_{\tilde{x}(.),u(.)} J_{com}\left(\tilde{x}(t), u(t)\right) \\ \text{subject to :} \\ \tilde{x}(0) = \tilde{x}_0 \\ \dot{\tilde{x}}(t) = f\left(\tilde{x}(t), u(t)\right), t \in [0, T_l] \\ h(\tilde{x}(t), u(t)) \leq 0,\ t \in [0, T_l] \end{cases} \quad (3)$$

4. NMPC DIRECT METHODS FORMULATION

The OCP of the NMPC scheme (3) leads to an infinite dimensional problem and it is cumbersome and in most cases it is impractical to implement. The direct methods, transcribes the problem into a finite dimensional problem, which yields an approximate solution to (3) by virtue of NLP solvers. In order to set the transcription procedure, the following assumptions are taken into consideration [8]:

4.1 Assumptions

- The input $u(t), \forall t \in [0, T_l]$ is finitely parameterized by piecewise constant vector $\mathcal{U}$ at an integer multiple of the sampling period T_s over the horizon. With this representation, the input can be expressed with $u(t) = \mu(t, \mathcal{U})$, which is a piecewise continuous input signal.
- The dynamics (defined by the ODE in (3)) of the system is numerically simulated for the given input signal $u(t) = \mu(t, \mathcal{U})$, which is compactly expressed with $\tilde{x}(t) = \phi(t; \mathcal{U}, \tilde{x}(0))$, which is evaluated at N discrete time instants $T_d = \{t_1, t_2, \dots, t_N\} \subset [0, T_l]$. The time stamps T_d typically corresponds to the integer multiple of the sampling period and it is utilized to discretize the dynamics, objective function and the constraint functions listed in (3). The ODE solver utilized in this paper is a 4th order Runge-Kutta (RK) solver. With the aforementioned assumptions, the generic NLP framework for the direct method is expressed with

$$\begin{cases} \min_w F(w) \\ \text{subject to :} \\ G(\tilde{x}_0, w) = 0 \\ H(w) \leq 0 \end{cases} \quad (4)$$

Where, w is the optimization variable which depends upon the direct method formulation utilized.

Remark 1. The ODE solver mentioned in the assumption is to be considered as a computer code and not in terms of algebraic equations. The ODE solver is a simulator which takes the numerical input trajectory $u(t) = \mu(t, \mathcal{U})$ and outputs the numerical state trajectory which is utilized in the objective and constraint functions for the NLP problem mentioned in (4). Thus, the objective and the constraint functions is embedded with the ODE solver code and is ought to be deemed as computer codes (function code). Under conditions of twice differentiability of all the functions (codes) listed in (3), the Jacobians and Hessians for the NLP solver are numerically obtained by methods such as finite differences, algorithmic differentiation etc. [9] (also known as oracles in optimization parlance) and this information aids the optimization procedure. Thanks to MATLAB's "fmincon" routine which automatically computes the derivatives from the objective and constraint functions.

4.2 Direct single shooting

The direct single shooting method also known as sequential method eliminates the dynamics equality constraint in (3) by forward simulation and thus, removing the state variables from the OCP NMPC problem. This reduces the optimization problem only to the input variables $\mathcal{U}$, which is obtained from the following NLP problem.

$$\begin{cases} \min_{\mathcal{U}} \sum_{k=1}^{N} l(\phi(t_k; \mathcal{U}, \tilde{x}(0)), \mu(t_k, \mathcal{U}))(t_k - t_{k-1}) \\ \text{subject to :} \\ h(\phi(t_k; \mathcal{U}, \tilde{x}(0)), \mu(t_k, \mathcal{U})) \leq 0, \; t_k \in T_d \end{cases} \quad (5)$$

The objective is discretized by means of Riemann sum at time stamps T_d. The states are replaced with the ODE simulator evaluated at these time stamps in the objective as well as the constraints. The optimal solution $\mathcal{U}^*$ obtained from (5) and the first input $\mathcal{U}^*(0)$ is injected into the system and the process is repeated in receding horizon manner.

4.3 Direct multiple shooting

The direct multiple shooting method also known as simultaneous method retains the state variables as optimization variables and this increases the number of decision variables in the optimization formulation in (3). The ODE solver simulates the system over multiple time intervals i.e. $[t_{k-1}, t_k], \forall\, k = \{1,2, \dots, N\}$ simultaneously and the final state value $\tilde{x}(t_k)$ for the simulation in the each interval $[t_{k-1}, t_k]$ is stipulated to obey the dynamics of the system, which is enforced by equality constraints. The NLP optimization problem is formulated as

$$\begin{cases} \min_{\mathcal{U}, \{\tilde{x}(t_1), \tilde{x}(t_2), \dots \tilde{x}(t_N)\}} \sum_{k=1}^{N} l(\tilde{x}(t_k), \mu(t_k, \mathcal{U}))(t_k - t_{k-1}) \\ \text{subject to :} \\ h(\tilde{x}(t_k), \mu(t_k, \mathcal{U})) \leq 0, \; t_k \in T_d \\ \tilde{x}(t_{k+1}) - \phi(t_k; \mathcal{U}, \tilde{x}(t_k)) = 0, t_k \in T_d \\ \tilde{x}(t_0) = \tilde{x}(0) \end{cases} \quad (6)$$

The optimal solution for the above optimization problem yields both the optimal state trajectory and optimal input sequence. As per the standard receding horizon policy, the first input $\mathcal{U}^*(0)$ is applied to the system and repeated in the future. The benefits of utilizing direct multiple shooting methods include a) Better simulator stability with unstable system, b) Parallelizability of ODE simulation, c) Structural properties of the Hessian matrices aid the optimization routine. However the flip side is that the optimization is carried out over an increased number of variables and a good initialization for the NLP solver is required for faster convergence to the optimal/sub-optimal solution (this can be ameliorated by warm start procedure).

4.4 Direct collocation

Direct collocation methods are extension to the simultaneous methods, where the ODE simulator is expunged from the multiple shooting formulation (6) and is replaced with algebraic equality constraints enforced at the collocation points. The optimization problem is expressed with

$$\begin{cases} \min_{\mathcal{U},\{\tilde{x}(t_1),\tilde{x}(t_2),\ldots\tilde{x}(t_N)\}} \sum_{k=1}^{N} l(\tilde{x}(t_k),\mu(t_k,\mathcal{U}))(t_k - t_{k-1}) \\ \text{subject to :} \\ h\big(\tilde{x}(t_k),\mu(t_k,\mathcal{U})\big) \leq 0,\ t_k \in T_d \\ \Psi\big(\tilde{x}(t_{k+1}),\tilde{x}(t_k),\mu(t_k,\mathcal{U})\big) = 0,\ k = 0,\ldots,N-1 \\ \tilde{x}(t_0) = \tilde{x}(0) \end{cases} \tag{7}$$

The fundamental difference between (6) and (7) is Ψ (Implicit solver), which satisfies the dynamics of the system at the collocation points. Direct collocation methods are typically suited for stiff systems and implicit RK methods are popularly adopted in optimal control literature. This introduces additional algebraic variables which are casted as equality constraints in (7). In this paper, vis-à-vis to the system considered, trapezoid collocation method is utilized to enforce the dynamics constraints at the collocation points.

5. SIMULATION RESULTS

The NMPC methods were implemented in MATLAB environment and NLP solver utilized was the “fmincon” routine. The sampling period T_s was chosen to be 5ms and the look ahead period T_l as 15ms. The input parameterization $\mathcal{U}$ was chosen to be a constant signal over the horizon. The “fmincon” solver was set to sequential quadratic programming (SQP) mode with maximum number of Newton iteration as 3. The road profile utilized was a chirp signal with an amplitude of 2.5mm and a frequency sweep from 1Hz to 8Hz for a duration of 25s.

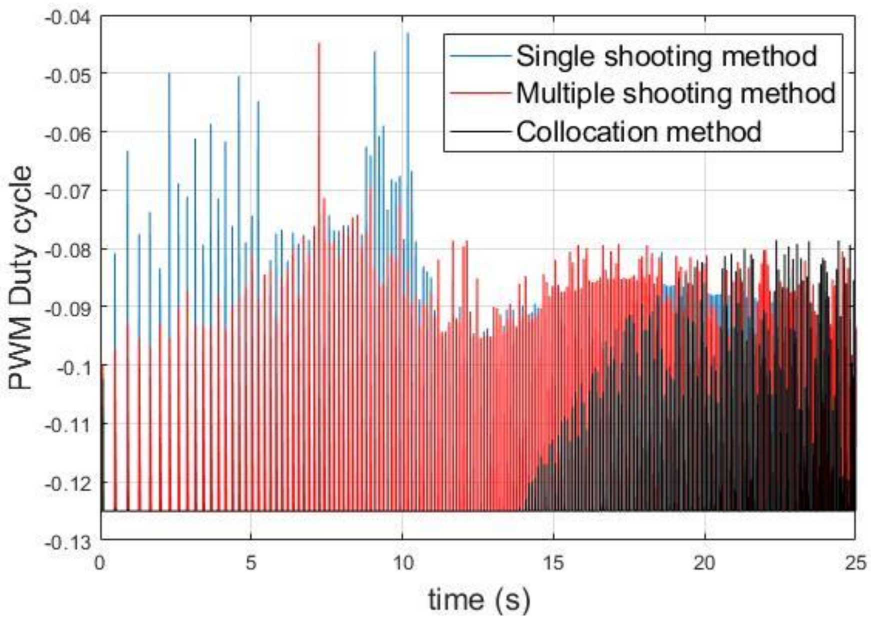

Fig.3 PWM duty cycle input

Fig.3 displays the PWM-DC computed from the NMPC methods. Clearly, it is evident that the input profiles are not the same for the methods due to the difference in the problem formulation.

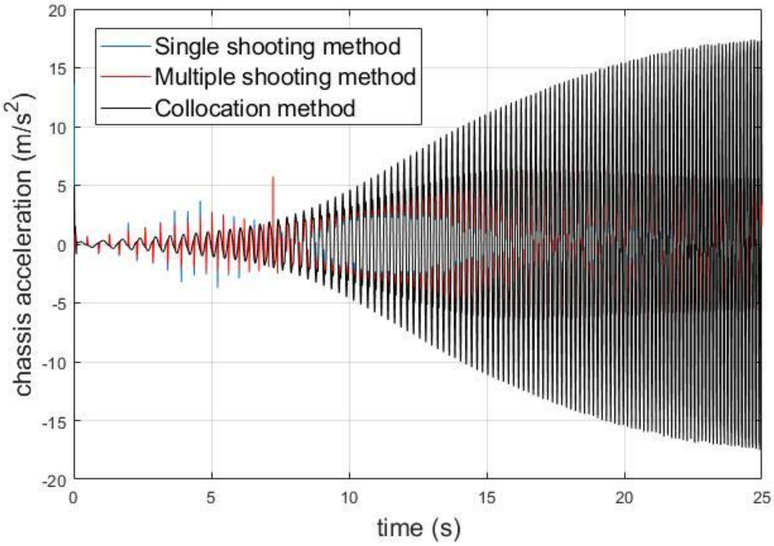

Fig.4 Chassis acceleration $\ddot{z}_s$

Fig.4 displays the chassis acceleration for the system. From the plot is evident that the solution for the single shooting, multiple shooting and collocation method are nearly equal in performance, however the collocation method tends to be have better greater RMS value in comparison with the other two methods. However, this is subjected to choice of the collocation method utilized in the problem formulation. The RMS values of the acceleration is listed in the table Table I. The dissipativity constraints are illustrated in Fig.5.

NMPC method	**RMS value (m/s^2)**
Single shooting	6.9020
Multiple shooting	6.8808
Collocation	6.9685

Table I Chassis acceleration RMS values

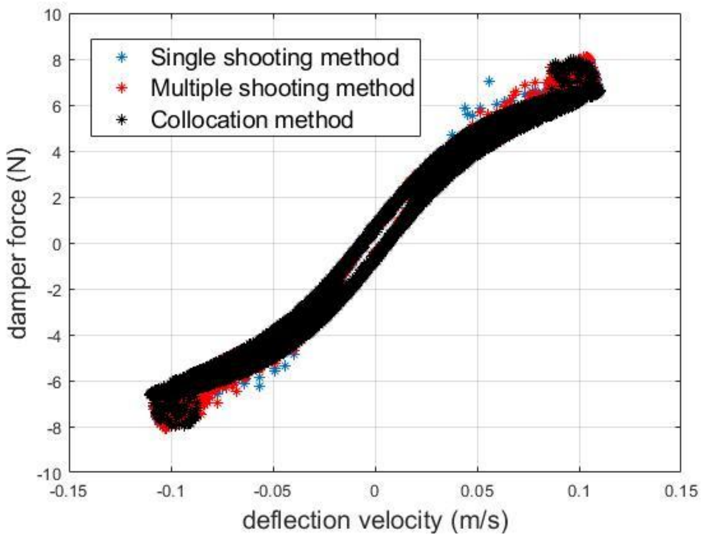

Fig.5 Dissipativity constraint

6. CONCLUSION AND FUTURE WORKS

In this work, a comparative simulation study is conducted on the different popular NMPC methods and its integration with a quarter car model equipped with semi-active suspension system. The work is conducted in the spirit of learning, exploring and investigating different NMPC methods and its suitability for automotive systems. In the future works, real-time implementation of the above methods are to be conducted to validate the real-time feasibility and viability in the INOVE test platform at GIPSA-lab, Grenoble.

7. ACKNOWLEDGEMENT

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