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Modeling and developing a conflict-aware scheduling in urban transportation networks

Yassine Idel Mahjoub^{a,b}, El houcine Chakir El-Alaoui^a, Ahmed Nait-Sidi-Moh^b

^a*Cadi Ayyad University-FSTG, LSET laboratory, 40000, Marrakesh, Morocco*

^b*University of Picardie Jules Verne-INSSET, LTI laboratory, 02100 Saint-Quentin, France*

Abstract

In this paper, we propose a Timed Coloured Petri Nets (TCPN) and $(\max, +)$ algebra based approach to model and evaluate the performances of a bus network characterized by complex phenomena such as conflicts, synchronization and concurrency. First of all, the modeling of the studied network by TCPN model is presented. Afterwards, $(\max, +)$ models representing the behavior of the network are developed. Both proposed models are considered for the evaluation of buses timetables and also passengers boarding, disembarking and waiting times. Furthermore, through the developed models, we show how the capacity of buses impact the passengers waiting times. An illustrative example is given to show the applicability of the proposed approach and the obtained results are promising. This example enables the verification and the validation of the developed models through a concrete scenario. It is worth noting that the proposed approach in this paper is highly adaptable to a variety of public transportation networks.

Keywords: Transportation systems; Modeling; Performance evaluation; Coloured Petri Nets; Dioid Algebra; Conflicts management.

1. Introduction

Nowadays, the importance of public transport systems continues to evolve [5] [12] [20]. For that reason, the study and analysis of such networks have become one of the major concerns of researchers in the field of public transport in order to maintain or improve the quality of the services offered to users [5].

Transport networks, especially bus networks, can be perceived taking into account their operating mode as Discrete Event System (DES) [2]. In fact, the behavior of such systems is mainly characterized by complex phenomena such as parallelism, synchronization, concurrency and conflicts [10]. The diversity and complexity of these phenomena make the study of these networks more difficult and

Email addresses: yassine.idelmahjoub@edu.uca.ma (Yassine Idel Mahjoub), e.chakirelalaoui@uca.ma (El houcine Chakir El-Alaoui), ahmed.nait@u-picardie.fr (Ahmed Nait-Sidi-Moh)

require the use of adequate and often complementary tools. Among these tools we underline the power and efficiency of Petri nets [7] namely TCPN [14] [13], multi-agent systems [6] [1], multicriteria analysis [22], dioid algebra [10] [12] etc, allowing to describe the behavior of DES in a formal way. For instance, the model developed, in [21], is based on the combination of the new computational paradigm - the Bee System and fuzzy logic with the aim to make real time decisions regarding route shapes for situations in which locations of the depot, nodes to be served and vehicle capacity are known, and the demand at the nodes is only approximated. Also, [22] presented an effective fuzzy multicriteria analysis approach to performance evaluation for urban public transport systems involving multiple criteria of multilevel hierarchies and subjective assessments of decision alternatives. A study of 10 bus companies in Taipei has been carried out to illustrate the effectiveness of the approach. Urban mobility via public transport is currently one of the major challenges facing policy makers and residents. Besides, [3] developed a multi-criteria model that provides support to managers and directors of transport systems, offering a comprehensive view that supports the implementation of public policies for the improvement of transport services. Also, [17] provided a framework which can be utilized as a tool for evaluating public transit performances. One of the important features of this framework is flexibility and involvement of each stakeholder in the public transportation system assessment. As a case study, the proposed framework was used to evaluate the newly established public bus system in the city of Abu Dhabi. Moreover, [1] proposed a solution approach based on cooperative multi-agent-based in order to achieve a more optimal vehicle routing and scheduling by fostering interaction and cooperation between Network operators and drivers. Furthermore, the approach is highly adaptable to a variety of networks and user populations.

Thanks to the conceptual simplicity of the model and its intuitive graphical presentation, PN has been proven to be a powerful modeling formalism for various kinds of DES. For instance, the issue addressed in [19] relates to the management of the correspondences in a bus transportation system. The objective is to evaluate the bus timetables as well as passengers waiting times at the correspondence stations. Furthermore, [4] has used stochastic PN to evaluate the performances of self-service bicycles. This study focused on rebalancing the distribution of bicycles in various network stations in order to satisfy demands of users. Also, [9] presented a modular framework based on TCPN in order to model the dynamics of signalized traffic network systems. The proposed model is applied and validated on a concrete intersection. In [8], a timed PN model representing an intermodal freight transport terminal (IFTT) was proposed, with the objective to simulate and evaluate the performance of the intermodal transportation network. This model is used to identify the IFTT bottlenecks, as well as to test different solutions to improve the IFTT dynamics.

On the other hand, $(\max, +)$ algebra is a powerful mathematical formalism that allows a linear analytic description and analysis of some DES like transportation networks. In the railway field, [15]

has shown how to model railway traffic and dispatching actions like changing tracks and breaking or joining trains using the max-plus algebra. Also, [16] has shown that the behavior of urban bus networks can be described by a min-max recursive equation which can be used for simulation issue in real time. Moreover, in [20], spectral theory in $(\max, +)$ algebra was used in order to evaluate passengers waiting times at the connection stations within a bus network. Furthermore, [11] dealt with the control of transportation systems. Actually, the authors proposed an approach based on dioid theory to compute timetables of a transportation network.

In this paper, we present a modeling approach based on linear and non-stationary $(\max, +)$ equations obtained from a graphical model describing the behavior of a bus transportation system and based on timed coloured Petri nets. The originality of this work lies firstly in the use of $(\max, +)$ algebra to describe the dynamic behavior of the TCPN model, and secondly in the combination of the advantages of these formal tools and their efficiency for the modeling and evaluation of the performances of such DES. In fact, the developed models present three main advantages. First, these models consider additional constraints related namely to buses finite capacity, random passengers arrival and the choice of their destination. Second, the models are modular and easily extendable for networks of huge size. Finally, the particularity and the strength of the developed models lie in their ability to be applied to any public transport network (such as railway networks). Comparing the presented approach with other models based on PN (e.g., [12]) or $(\max, +)$ algebra (e.g, [11], [20]), it may be enlightened that our contribution does not lie only in the modeling of the dynamics associated with the buses circuits and their timetables, but also in the evaluation of passengers random arrivals, their boarding and disembarking times as well as their differentiated itineraries and destinations. In addition, the developed models are validated on several scenarios and configurations of the system, and the obtained results are satisfying and promising.

This paper extends and generalizes the work presented in [13] while considering any number of stations for a line bus instead of just three. In fact, all the equations and algorithms obtained in the initial work are extended and more generalized.

To the best of our knowledge, we think that it is the first time that TCPN and $(\max,+)$ algebra are combined to model, analyze and evaluate the performance of a DES. Our main objective is to express TCPN with $(\max, +)$ equations, in order to evaluate departure/arrival of buses from/to the various network stations and also boarding, disembarking and waiting times of each passenger at every network station. Afterwards, given that the buses could become saturated in peak periods, the influence of the limited capacity of these buses on passengers waiting time is studied. In fact, the peak period is defined as a moment of great influx where the arrival rate of the passengers is too large compared to the capacities of buses.

This paper is organized as follows. The considered public transport system is introduced in section

2. In Section 3, TCPN model of the considered network is proposed. A mathematical modeling using (max, +) algebra is given in section 4. The results of a case study are presented and analyzed in section 5. Finally, in section 6, conclusion and indications of future work are reported.

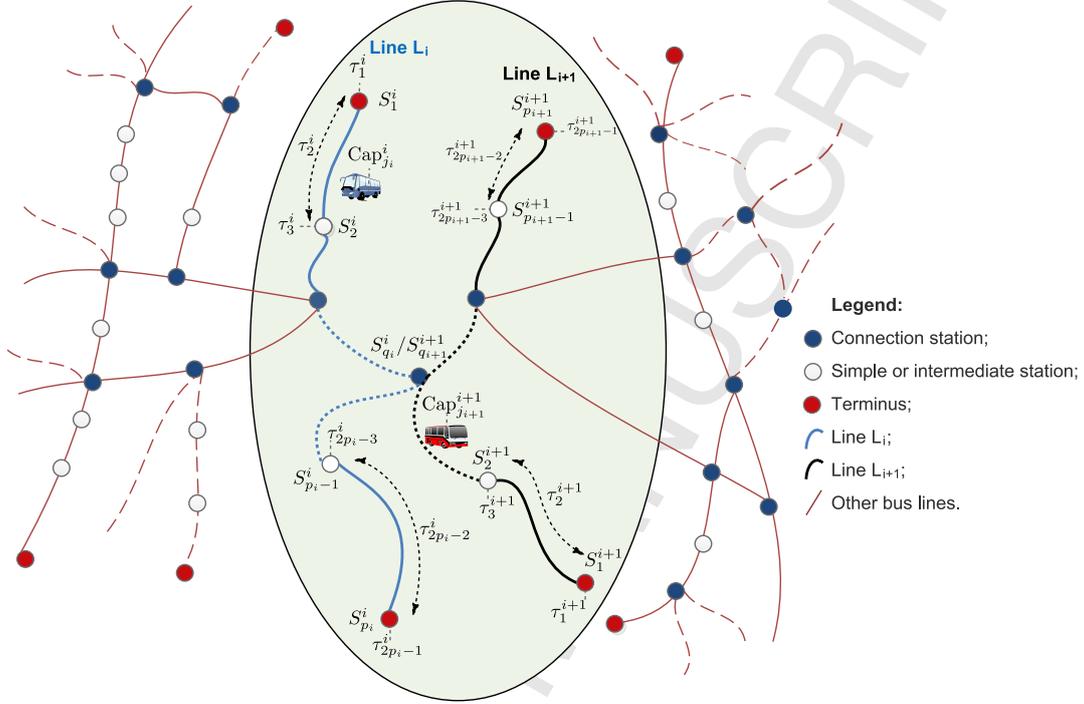


Figure 1: The studied network

2. Studied Network

In this section, we consider a bus network assumed to be a generic structure for any bus transportation network. The studied part is composed of two connected lines L_i and L_{i+1} with a single connection station (Figure 1). Each line L_l ($l \in \{i, i+1\}$) is represented by p_l stations: departure and arrival terminus (denoted respectively S_1^l and $S_{p_l}^l$), connection station (denoted $S_{q_l}^l$) and intermediate stations (denoted $S_2^l, \dots, S_{q_l-1}^l, S_{q_l+1}^l, \dots, S_{p_l-1}^l$). Also, each line L_l is supposed to be served by a finite number n_l of buses. Every bus j_l ($1 \leq j_l \leq n_l$) is characterized by its limited capacity $Cap_{j_l}^l$ that refers to the maximum number of passengers to be transported at the same time. The arrival of passengers to their departure stations is supposed to be given by a random distribution law. Moreover, the buses are sent from their starting station one after the other (the passage sequence of the buses is $1, 2, \dots, n_l, 1, 2, \dots$). For each line L_l , the following data are supposed to be known a priori :

- The number n_l of buses circulating on the line L_l ;

3. Modeling using timed coloured Petri nets

Petri nets, particularly TCPN, constitute a formalism well adapted to the modeling of DES. Their understandable graphical representation makes it possible to model easily several phenomena namely parallelism, synchronization and conflicts. In the following, TCPN (Figure 2) is used to model (i) buses circuit, (ii) passengers arrival, boarding and finally (iii) passengers destination choice and disembarking. In the remainder of this study, we will only develop models of the line L_i . For the other line (L_{i+1}), we obtain the same models while replacing the index i by $i+1$. Basic concepts and further details on TCPN can be found for example in [18], [14].

Definition 1: We recall that a token of color x is denoted $\langle x \rangle$. So, every bus j_i (resp. station s_i) has its own color denoted $\langle j_i \rangle$ (resp. $\langle s_i \rangle$). The various color domains used in the TCPN model are presented in Table 1.

Table 1: Color domains of the TCPN model

Variables and color domains
j_i in C_1^i with C_1^i is $1, \dots, n_i$
s_i, dest_i in C_2^i with C_2^i is $S_1^i, \dots, S_{p_i}^i$;
d in C_4^i with C_4^i is Crsp, Dst;
C_3^i is $\langle C_1^i, C_2^i \rangle$;
C_5^i is $\langle C_1^i, C_4^i \rangle$;
C^i is $1^{Cap_1^i} \cup \dots \cup n_i^{Cap_{n_i}^i}$;

Sub-model 1: Buses circuit

All the buses are initially at their departure station S_1^i (presence of n_i tokens of different colors in Ps_1^i). The transition u^i models departure permissions of buses. The firing of the transition $Dpt^{i,1}$ by $\langle j_i \rangle$ means that the bus j_i begins its circuit. This bus will wait τ_1^i time units before leaving its departure station (by firing $xd^{i,1}$), so that the passengers waiting at this station can get on the bus (by firing M^i). Afterwards, the bus is in its way to the next station (by adding a token in Pd_1^i). τ_2^i units of time later, the bus j_i arrives to the next station. The firing of $xa^{i,2}$ by $\langle j_i \rangle$ puts simultaneously a token $\langle j_i \rangle$ in the place P_2^i (which models bus waiting), and another token $\langle j_i, S_2^i \rangle$ in the place OM^i (which models the authorization order for passengers to get on the bus) and finally another token $\langle j_i, S_2^i \rangle$ in the place OD^i (which models the authorization order for passengers to get off the bus). The parameter δ , associated to the places OM^i and OD^i , refers to the average time that a passenger can take to get on/off the bus. The same operating mode is performed in the next stations ($S_3^i, \dots, S_{p_i}^i$). The place $Pd_{p_i}^i$ models the return path of the buses. A token in this place means that the bus j_i is going back to its departure station S_1^i .

Sub-model 2: Passengers arrival

Passengers arrival at their departure stations are given randomly, so they can be modeled by a stochastic transitions (PA^i) endowed for example with an exponential distribution of rate denoted λ . The firing of PA^i by $\langle s_i \rangle$ means that a passenger is waiting at the station s_i . So, the passengers waiting at station s_i get on the bus j_i (by firing the transition M^i) when this bus arrives at this station with empty seats inside (existence of tokens $\langle j_i \rangle$ in Pv^i). Moreover, in a connection station, passengers can come from the line L_{i+1} by firing the transition $Crp^{i+1,i}$ (Figure 2) and adding $\langle S_{q_i}^i \rangle$ to the place At^i . Finally, passengers boarding is supposed made by FIFO (First In First Out) rule. This means that the first passenger arriving at a given station is the first one who gets on the bus.

Sub-model 3: Passengers destination choice

Every passenger has his own destination ($dest_i$ is the color which indicates the desired destination). When a passenger gets on a bus j_i , the place Pc^i contains a token $\langle j_i, dest_i \rangle$. A transition $x^{i,m}$ ($m \in \{1, \dots, p_i\}$) will be fired when a passenger has chosen station S_m^i of line L_i as his destination. The transition $D^{i,m}$ models passengers disembarking at station S_m^i . However, a passenger, who has a station of the line L_{i+1} as destination, will choose the connection station to make the correspondence and change the bus. This passenger disembarks from the bus (by firing $Crp^{i,i+1}$) and waits for the bus of line L_{i+1} (by adding $\langle S_{q_{i+1}}^{i+1} \rangle$ in At^{i+1}). Finally, passengers release their seats (by adding $\langle j_i \rangle$ in Pv^i) after getting off the bus j_i (by firing $D^{i,m}$ or $Crp^{i,i+1}$).

The advantage of this model, compared to other models in the literature, is that it represents the state of all buses (on/off the road, number of passengers on board etc.) and all stations (empty, full, number of waiting passengers, etc.). In addition, the arrival, boarding, choice of destination, disembarking and correspondence of all passengers are modeled. This model is simulated and verified by the software CPN tools.

In the next section, the (max, +) models representing the dynamic behavior of the previous TCPN model are given.

4. (Max, +) description of TCPN behavior

The originality of this contribution is to propose a mathematical formulation of the TCPN model in order to evaluate the firings dates of the various transitions in a formal way. So, in order to obtain a linear analytic description of the studied network characterized by synchronization and conflicts, we adapt (max, +) algebra and then we obtain linear and non stationary equations representing the behavior of the studied system.

In this section, the (max, +) models describing all components of the TCPN (i) the buses circuit, (ii) passengers arrival and boarding, and finally (iii) passengers destination choice and disembarking

are presented. It's well known that $(\max, +)$ algebra is dedicated to the management and calculus of the occurrence dates of events (called "daters"). For more details about dioids algebra, especially $(\max, +)$ algebra, the reader can refer to [2]. Moreover, for uncoloured PN, for each transition x we associate a dater $x(k)$ which corresponds to the date of the k^{th} firing of x . However, in our study, the concept of **color** must be integrated into the $(\max, +)$ equations. So, we consider the following definition.

Definition 2: the associated dater with a transition x fired by a token with color y is denoted $x_{\langle y \rangle}(k)$ and represents the date of the k^{th} firing of x by $\langle y \rangle$.

To our knowledge, it should be noted that the description of the TCPN by $(\max, +)$ equations has never been addressed in the literature.

4.1. (Max, +) model of buses circuit

In the following section, we describe the dynamic behavior of the buses (sub-model 1 in Figure 2), in terms of daters, by mathematical equations in the usual algebra and then by $(\max, +)$ equations. So, the k^{th} firing of each transition of our TCPN model, describing the bus circuit, is given by the following system. $\forall k > 1$

$$\left\{ \begin{array}{l} Dpt_{\langle j_i \rangle}^i(k) = \max(u_{\langle j_i \rangle}^i(k), xa_{\langle j_i \rangle}^{i,1}(k-1)) \\ xa_{\langle j_i \rangle}^{i,1}(k) = \max(Dpt_{\langle j_i \rangle}^i(k) + \tau_1^i, M_{\langle j_i, S_1^i \rangle}^i(k_{1,j_i}^i) + \delta, D_{\langle j_i \rangle}^{i,1}(o_{1,j_i}^i) + \delta) \\ \dots\dots\dots \\ xa_{\langle j_i \rangle}^{i,q_i}(k) = xa_{\langle j_i \rangle}^{i,q_i-1}(k) + \tau_{2(q_i-1)}^i \\ xa_{\langle j_i \rangle}^{i,q_i}(k) = \max(xa_{\langle j_i \rangle}^{i,q_i}(k) + \tau_{2q_i-1}^i, M_{\langle j_i, S_{q_i}^i \rangle}^i(k_{q_i,j_i}^i) + \delta, D_{\langle j_i \rangle}^{i,q_i}(o_{q_i,j_i}^i) + \delta, Crp^{i,i+1}(o_{p_i+1,j_i}^i) + \delta) \\ \dots\dots\dots \\ xa_{\langle j_i \rangle}^{i,p_i}(k) = xa_{\langle j_i \rangle}^{i,p_i-1}(k) + \tau_{2(p_i-1)}^i \\ xa_{\langle j_i \rangle}^{i,p_i}(k) = \max(xa_{\langle j_i \rangle}^{i,p_i}(k) + \tau_{2p_i-1}^i, M_{\langle j_i, S_{p_i}^i \rangle}^i(k_{p_i,j_i}^i) + \delta, D_{\langle j_i \rangle}^{i,p_i}(o_{p_i,j_i}^i) + \delta) \\ xa_{\langle j_i \rangle}^{i,1}(k) = xa_{\langle j_i \rangle}^{i,p_i}(k) + \tau_{2p_i}^i \end{array} \right. \quad (1)$$

- For the first equation of (1): the bus j_i starts its k^{th} turn (k^{th} firing of Dpt^i by $\langle j_i \rangle$) after both completing his $(k-1)^{th}$ turn ($(k-1)^{th}$ firing of $xa^{i,1}$ by $\langle j_i \rangle$), and a departure permission is given (k^{th} firing of u^i by $\langle j_i \rangle$).

- For the bus departure daters (equations $xd^{i,1}(k), \dots, xd^{i,p_i}(k)$ of the system (1)): A bus j_i leaves a station S_m^i ($m \in \{1, \dots, p_i\}$) for the k^{th} time (k^{th} firing of $xd^{i,m}$ by $\langle j_i \rangle$) after waiting τ_{2m-1}^i time units in this station so that passengers can get on (firing of M^i by $\langle j_i, S_m^i \rangle$) and off (firing of $D^{i,m}$ by $\langle j_i \rangle$) this bus. The parameter $k_{m,j_i}^i(k)$ (resp. $o_{m,j_i}^i(k)$) are counters that represents the number of firings of M^i (resp. $D^{i,m}$) by $\langle j_i, S_m^i \rangle$ (resp. $\langle j_i \rangle$) before the k^{th} departure of the bus j_i from the station S_m^i (see next section). So, the expression $M_{\langle j_i, S_m^i \rangle}^i(k_{m,j_i}^i)$ (resp. $D_{\langle j_i, S_m^i \rangle}^{i,m}(o_{m,j_i}^i)$) represents the last passenger boarding (resp. disembarking) time to/from the bus j_i , in its k^{th} turn, at station

S_m^i .

- For the bus arrival dates (equations $xa_1^i(k), \dots, xa_{p_i}^i(k)$ of the system (1)): the bus j_i arrives at a station S_m^i for the k^{th} time after $\tau_{2(m-1)}^i$ time units after leaving the previous station.

In order to make the system 1 in a matrix form, we define the buses state vector $X_{<j_i>}^i(k)$ and the input vector $U_{<j_i>}^i(k)$:

$$\begin{cases} X_{<j_i>}^i(k) = [Dpt_{<j_i>}^i(k), xd_{<j_i>}^{i,1}(k), \dots, xa_{<j_i>}^{i,p_i}(k), xd_{<j_i>}^{i,p_i}(k), xa_{<j_i>}^{i,1}(k)]^T; \\ U_{<j_i>}^i(k) = u_{<j_i>}^i(k). \end{cases}$$

We assume that every bus j_i leaves a station after having boarded and disembarked all the concerned passengers. Considering this hypothesis and using (max, +) notations, the system (1) can be written as the following:

$$X_{<j_i>}^i(k) = A_1 \otimes X_{<j_i>}^i(k) \oplus A_2 \otimes X_{<j_i>}^i(k-1) \oplus B_1 \otimes U_{<j_i>}^i(k) \quad (2)$$

$$\text{With: } A_1 = \begin{bmatrix} \varepsilon & \varepsilon & \dots & \varepsilon & \varepsilon \\ \tau_1^i & \varepsilon & \dots & \varepsilon & \varepsilon \\ \varepsilon & \tau_2^i & \dots & \varepsilon & \varepsilon \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varepsilon & \varepsilon & \varepsilon & \tau_{2p_i}^i & \varepsilon \end{bmatrix}, A_2 = \begin{bmatrix} \varepsilon & \dots & \varepsilon & \varepsilon \\ \varepsilon & \dots & \varepsilon & \varepsilon \\ \varepsilon & \dots & \varepsilon & \varepsilon \\ \vdots & \ddots & \vdots & \vdots \\ \varepsilon & \dots & \varepsilon & \varepsilon \end{bmatrix} \text{ and } B_1 = \begin{bmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ \vdots \\ \varepsilon \end{bmatrix}$$

The matrix A_1 is nilpotent, so the implicit equation (2) can then be rewritten as follows:

$$X_{<j_i>}^i(k) = A_i \otimes X_{<j_i>}^i(k-1) \oplus B \otimes U_{<j_i>}^i(k) \quad (3)$$

With: $A_i = A_1^* \otimes A_2$, $A_1^* = \bigoplus_{k \geq 0} A_1^k$ and $B = A_1^* \otimes B_1$ where A_1^* is the Kleene star of A_1 .

In the next section, the (max, +) model that describes passengers boarding is presented.

4.2. (Max, +) model for passengers boarding

The difficulty in expressing passengers boarding times appears in finding the different relations between the firing of all transitions in conflict situation, namely M^i , PA^i and $\text{Crp}^{i+1,i}$ by the various colors (sub-model 2 of Figure 2). In order to remedy to this difficulty we introduce some functions called routing functions (namely α_{m,j_i}^i , β_{m,j_i}^i and $\delta_{j_i}^i$ with $m \in \{1, \dots, p_i\}$), in equation (4) below, to manage and solve the conflicts and express the date of the transition related to passengers boarding (M^i).

First, we describe the dynamic behavior of the sub-model 2 of Figure 2 by a system of equations expressed in (max, +) algebra. These equations make it possible to evaluate boarding time of each

passenger k to the bus j_i at the station S_m^i . So, for $m=1, \dots, p_i$ and $j_i=1, \dots, n_i$, $\forall k \geq 1$,

$$M_{\langle j_i, S_m^i \rangle}^i(k) = \begin{cases} PA_{\langle S_m^i \rangle}^i(\alpha_{m, j_i}^i) \oplus M_{\langle j_i, S_m^i \rangle}^i(k-1) \cdot \delta \oplus xa_{\langle j_i \rangle}^{i, m}(\beta_{m, j_i}^i) & \text{if } m \neq q_i \text{ and } m \neq 1, \\ PA_{\langle S_m^i \rangle}^i(\alpha_{m, j_i}^i) \oplus M_{\langle j_i, S_m^i \rangle}^i(k-1) \cdot \delta \oplus Dpt_{\langle j_i \rangle}^i(\beta_{m, j_i}^i) & \text{if } m = 1, \\ PA_{\langle S_{q_i}^i \rangle}^i(\alpha_{q_i, j_i}^i) \oplus Crp_{\langle S_{q_i}^i \rangle}^{i+1, i}(\delta_{j_i}^i) \oplus M_{\langle j_i, S_{q_i}^i \rangle}^i(k-1) \cdot \delta \oplus xa_{\langle j_i \rangle}^{i, q_i}(\beta_{q_i, j_i}^i) & \text{if } m = q_i. \end{cases} \quad (4)$$

Indeed, in case of terminus or intermediate stations ($m \neq q_i$), the k^{th} firing of the transition M^i by

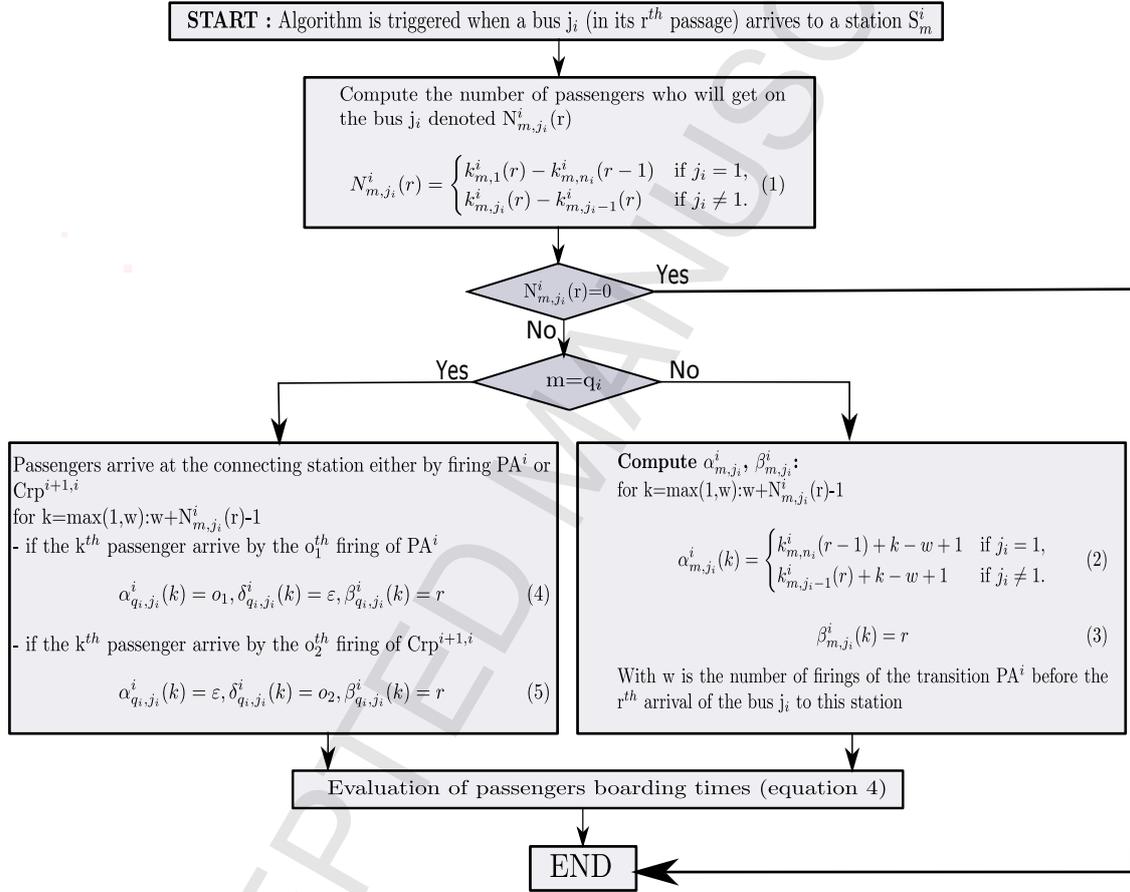


Figure 3: Algorithm for the calculation of routing functions

$\langle j_i, S_m^i \rangle$ occurs after the $\alpha_{m, j_i}^{i, th}(k)$ firing of the transition PA^i by $\langle S_m^i \rangle$ and $\beta_{m, j_i}^{i, th}(k)$ firing of the transition $xa^{i, m}$ by $\langle j_i \rangle$. Besides, in the connection station, passengers may come from the other line. For that reason, the k^{th} firing of the transition M^i by $\langle j_i, S_{q_i}^i \rangle$ occurs after $\beta_{q_i, j_i}^{i, th}(k)$ firing of the transition xa^{i, q_i} by $\langle j_i \rangle$ and either the $\alpha_{q_i, j_i}^{i, th}(k)$ firing of the transition PA^i by $\langle S_{q_i}^i \rangle$ or the $\delta_{j_i}^{i, th}(k)$ firing of the transition $Crp^{i+1, i}$ by $\langle S_{q_i}^i \rangle$ (equation (4)).

The routing functions depend essentially on the arrival dates of the passengers and the buses limited capacities ($Cap_{j_i}^i$). The total number (from the beginning of the day) of passengers arriving at the

station S_m^i before the k^{th} departure of the bus j_i is denoted by $a_{m,j_i}^i(k)$ and is given by equation (5). This parameter also represents the firing number of the transition PA^i by a token $\langle S_m^i \rangle$ (plus the firing number of the transition $Crp^{i+1,i}$ by $\langle j_{i+1}, Crsp \rangle$ in case of $m = q_i$) before the k^{th} departure of the bus j_i from the station S_m^i .

$$a_{m,j_i}^i(k) = \begin{cases} \sup_p \{PA_{\langle S_m^i \rangle}^i(p) < xd_{\langle j_i \rangle}^{i,m}(k)\} & \text{if } m \neq q_i, \\ \sup_p \{PA_{\langle S_{q_i}^i \rangle}^i(p) < xd_{\langle j_i \rangle}^{i,q_i}(k)\} + \sum_{j_{i+1}=1}^{n_{i+1}} \sup_p \{Crp_{\langle j_{i+1}, Crsp \rangle}^{i+1,i}(p) < xd_{\langle j_i \rangle}^{i,q_i}(k)\} & \text{if } m = q_i. \end{cases} \quad (5)$$

Thus, the firing number of the transition M^i by $\langle j_i, S_m^i \rangle$ before the k^{th} departure of the bus j_i from the station S_m^i is given by the following parameter $k_{m,j_i}^i(k)$.

$$k_{m,j_i}^i(k) = \begin{cases} \min \left[a_{m,j_i}^i(k), Cap_{j_i}^i + \sum_{r=1}^m k_{r,n_i}^i(k-1) - \sum_{r=1}^{m-1} k_{r,j_i}^i(k) + \sum_{r=1}^m Dt_{r,j_i}^i(k) \right] & \text{if } j_i = 1, \\ \min \left[a_{m,j_i}^i(k), Cap_{j_i}^i + \sum_{r=1}^m k_{r,j_i-1}^i(k) - \sum_{r=1}^{m-1} k_{r,j_i}^i(k) + \sum_{r=1}^m Dt_{r,j_i}^i(k) \right] & \text{if } j_i \neq 1. \end{cases} \quad (6)$$

Where:

- $Dt_{m,j_i}^i(k)$ represents the number of passengers who want to get off the bus j (in its k^{th} passage) at station S_m^i . This parameter is deduced from passengers destination choice.
- $\forall k \leq 0, k_{m,j_i}^i(k) = 0$

The algorithm, presented in Figure 3, is developed to manage the conflicts in the sub-model 2 of Figure 2, by calculating the above mentioned routing functions ($\alpha_{m,j_i}^i, \beta_{m,j_i}^i$ and $\delta_{j_i}^i$ with $m \in \{1, \dots, p_i\}$). In fact, this algorithm is triggered when a bus j_i arrives to a station S_m^i . Firstly, we compute the number of passenger who will get on the bus j_i (in its r^{th} passage) given by $N_{m,j_i}^i(r)$. Afterwards, according to the type of station S_m^i (correspondence, intermediate or terminus), we compute the considered routing functions. The expression of these routing functions are explicitly developed with mathematical equations (Figure 3).

4.3. (Max, +) model for passengers disembarking

In this section, the (max, +) equations describing passengers disembarking time are presented. To do so, we need to establish the relations between the firings of the transitions $M^i, x^{i,m}, D^{i,m}, xa^{i,m}$ ($m \in \{1, \dots, p_i\}$) and $Crp^{i,i+1}$. For that reason, another routing functions ξ_{h,j_i}^i ($h \in \{1, \dots, p_i(p_i + 1) + 5\}$) are used to handle conflicts, synchronization and resource sharing (sub-model 3 of Figure 2).

The (max, +) equations for passengers disembarking times are given by: $\forall k > 1$,

$$\left\{ \begin{array}{l} D_{\langle j_i \rangle}^{i,1}(k) = x_{\langle j_i, S_1^i \rangle}^{i,1}(k) \oplus x a_{\langle j_i \rangle}^{i,1}(\xi_{p_i^2+1, j_i}^i) \cdot \delta \oplus D_{\langle j_i \rangle}^{i,1}(k-1) \cdot \delta \\ \dots\dots\dots \\ D_{\langle j_i, Dst \rangle}^{i, q_i}(k) = x_{\langle j_i, S_{q_i}^i \rangle}^{i, q_i}(\xi_{p_i(p_i+1)+2, j_i}^i) \oplus x a_{\langle j_i \rangle}^{i, q_i}(\xi_{p_i^2+q_i, j_i}^i) \cdot \delta \oplus D_{\langle j_i, Dst \rangle}^{i, q_i}(k-1) \cdot \delta \oplus Crp_{\langle j_i, Crsp \rangle}^{i, i+1}(\xi_{p_i(p_i+1)+4, j_i}^i) \cdot \delta \\ \dots\dots\dots \\ D_{\langle j_i \rangle}^{i, p_i}(k) = x_{\langle j_i, S_{p_i}^i \rangle}^{i, p_i}(k) \oplus x a_{\langle j_i \rangle}^{i, p_i}(\xi_{p_i(p_i+1), j_i}^i) \cdot \delta \oplus D_{\langle j_i \rangle}^{i, p_i}(k-1) \cdot \delta \\ Crp_{\langle j_i, Crsp \rangle}^{i, i+1}(k) = x_{\langle j_i, S_{q_i}^i \rangle}^{i, q_i}(\xi_{p_i(p_i+1)+3, j_i}^i) \oplus x a_{\langle j_i \rangle}^{i, q_i}(\xi_{p_i(p_i+1)+1, j_i}^i) \cdot \delta \oplus Crp_{\langle j_i, Crsp \rangle}^{i, i+1}(k-1) \cdot \delta \\ \oplus D_{\langle j_i, Dst \rangle}^{i, q_i}(\xi_{p_i(p_i+1)+5, j_i}^i) \cdot \delta \end{array} \right. \quad (7)$$

Where $x^{i,m}(k)$ is given by the following equations: $\forall k > 1$,

$$\left\{ \begin{array}{l} x_{\langle j_i, S_1^i \rangle}^{i,1}(k) = M_{\langle j_i, S_1^i \rangle}^i(\xi_{1, j_i}^i) \oplus \dots \oplus M_{\langle j_i, S_{p_i}^i \rangle}^i(\xi_{p_i, j_i}^i) \\ \dots\dots\dots \\ x_{\langle j_i, S_{q_i}^i \rangle}^{i, q_i}(k) = M_{\langle j_i, S_1^i \rangle}^i(\xi_{p_i(q_i-1)+1, j_i}^i) \oplus \dots \oplus M_{\langle j_i, S_{p_i}^i \rangle}^i(\xi_{p_i q_i, j_i}^i) \\ \dots\dots\dots \\ x_{\langle j_i, S_{p_i}^i \rangle}^{i, p_i}(k) = M_{\langle j_i, S_1^i \rangle}^i(\xi_{p_i(p_i-1)+1, j_i}^i) \oplus \dots \oplus M_{\langle j_i, S_{p_i}^i \rangle}^i(\xi_{p_i^2, j_i}^i) \end{array} \right. \quad (8)$$

In fact, passengers get off the bus j_i at station S_m^i (by firing the transitions $D^{i,m}$ when these passengers have chosen station S_m^i as their destination (by firing of the transitions $x^{i,m}$). For example, in the first equation of (7), the k^{th} passenger, who has chosen departure station as his destination (firing of $x^{i,1}$ by $\langle j_i, S_1^i \rangle$), get off the bus j_i at the departure station (k^{th} firing of the transition $D^{i,1}$ by $\langle j_i \rangle$) when the bus j_i , in its $\xi_{p_i^2+1, j_i}^{i, th}$ passage, arrives to this station. Furthermore, if several passengers want to get off the FIFO rule is applied. The other equations are expressed in the same way.

To evaluate disembarking time of passengers at different stations, we have to solve the equations (7) and (8). To do so, the routing functions ξ_{h, j_i}^i ($h \in \{1, \dots, p_i^2\}$) are used to solve the free choice in the place Pc^i . Moreover, when the k^{th} passenger, who has boarded the bus j_i at S_m^i , wants to get off at station S_{sa}^i (by firing $x^{i, sa}$ with $sa \in \{1, \dots, p_i\}$), we have the following expression:

$$\text{For } o \in \{1, \dots, p_i\}, \quad \xi_{(sa-1)p_i+o, j_i}^i = k \otimes 1_{o=m}^{max} \quad (9)$$

With:

$$1_{c_1=c_2}^{max} = \begin{cases} e & \text{if } c_1 = c_2, \\ \varepsilon & \text{if } c_1 \neq c_2. \end{cases}$$

Furthermore, the functions $\xi_{p_i^2+h, j_i}^i$ ($h \in \{1, \dots, p_i\}$) represent the tour of the bus j_i . The other functions $\xi_{p_i(p_i+1)+h, j_i}^i$ ($h \in \{1, \dots, 5\}$) are used to solve the resource sharing and free choice related to the connection station (place $Pm_{q_i}^i$ and OD^i). These parameters depend essentially on passengers destinations. An

algorithm based on the equation (9) and similar to the algorithm of Figure 3 is developed to express these new routing functions.

In brief, the developed approach to evaluate the performances of the studied network consists of three related models: (i) buses circuit, (ii) passengers arrival and boarding and finally (iii) passengers destination choice and disembarking. In the first part, we developed a (max, +) equations to evaluate buses timetables given by the vectors $X_{<j_i>}^i$ ($\forall j_i \in \{1, \dots, n_i\}$). In the second part, given that passengers arrival are random, we introduce some routing functions to manage and solve the conflicts in Figure 2, in order to express passengers boarding by (max, +) equations. Finally, the last part is dedicated to manage passengers destination choice in order to evaluate their disembarking times.

In order to illustrate the proposed approach, a numerical example is presented in the following section and the obtained results are reported and analyzed.

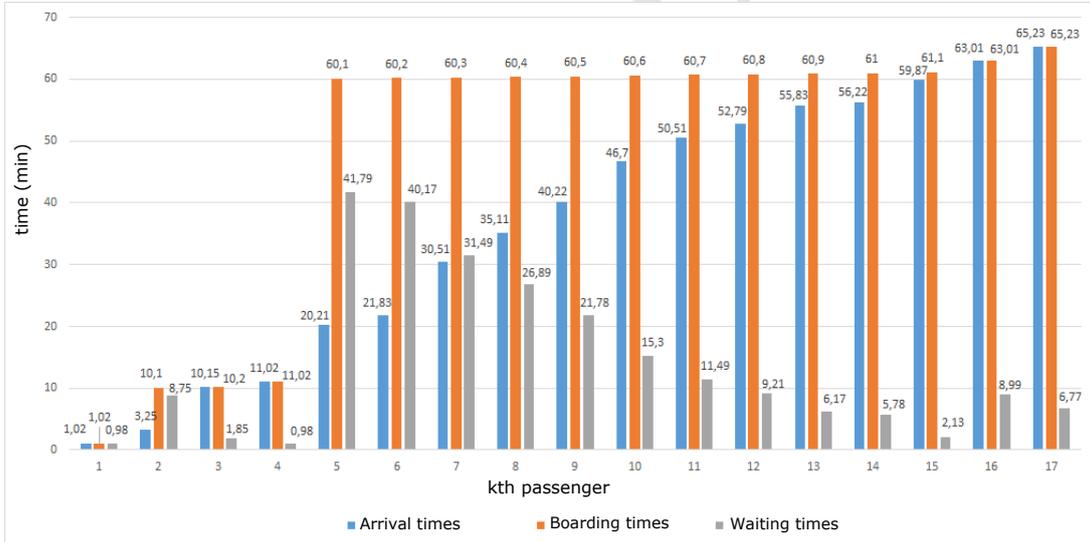


Figure 4: Passengers arrival, boarding and waiting times at the departure station

5. The case study

In this section, we present a case study to show and prove the effectiveness of the proposed models. To this aim, we present a simple bus network composed of two lines L_i and L_{i+1} . However, the proposed approach may be applied to a huge network with many lines and many intermediate and correspondence stations. The data of the considered network is reported in Table 2. In fact, the line L_i is composed of three stations (S_1^i, S_2^i, S_3^i) while the other one (L_{i+1}) is composed of four stations ($S_1^{i+1}, S_2^{i+1}, S_3^{i+1}, S_4^{i+1}$). Moreover, the firing dates of the input transitions u^i, u^{i+1}, PA^i and PA^{i+1} , for a given time interval, are presented in Table 3 and 4. The simulation results are reported in what follows.

Table 2: Data of the considered network

Bus network data	Line L_i	Line L_{i+1}
Number of stations	$p_i=3$ with $q_i=2$	$p_{i+1}=4$ with $q_{i+1}=2$
Bus stop time (min)	$\tau_1^i=2, \tau_3^i=2, \tau_5^i=3$	$\tau_1^{i+1}=3, \tau_3^{i+1}=3, \tau_5^{i+1}=4, \tau_7^{i+1}=5$
Bus traveling time (min)	$\tau_2^i=13, \tau_4^i=11, \tau_6^i=25$	$\tau_2^{i+1}=10, \tau_4^{i+1}=12, \tau_6^{i+1}=11, \tau_8^{i+1}=30$
Number of buses	$n_i=2$	$n_{i+1}=3$
Buses capacity	$Cap_1^i=20, Cap_2^i=20$	$Cap_1^{i+1}=30, Cap_2^{i+1}=25, Cap_3^{i+1}=30$
Parameter δ	$\delta=0.1\text{min}$	$\delta=0.1\text{min}$

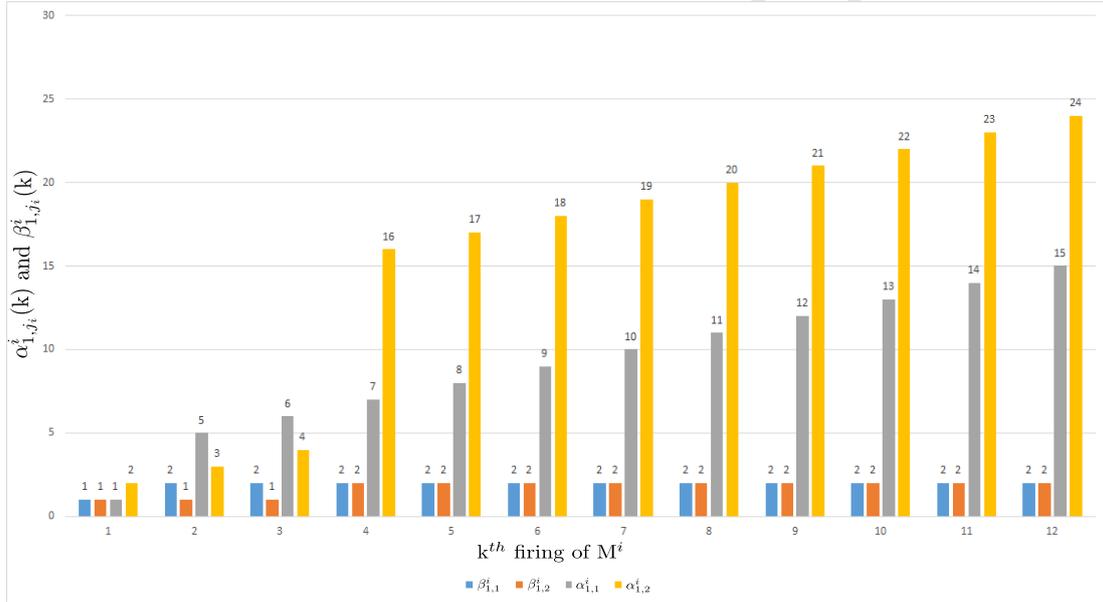


Figure 5: Routing functions at the departure station

5.1. Simulation results

The developed models are implemented and simulated using Scilab and CPN tools environments. Indeed, some properties (liveness, boundedness...) of graphical model are validated with the software CPN tools. Actually, the TCPN simulation, using CPN tools, could be used to evaluate the performances of the studied system in terms of dates of firing of various transitions. Furthermore, the proposed (max, +) equations and algorithms are implemented in Scilab environment and the obtained results are promising as we will show through the figures given in this section.

Considering the described network, simulations of two scenarios are performed with buses endowed with various capacities in a specific period where both peak and off-peak periods are observed. Our aim is to evaluate buses timetables, passengers waiting, boarding and disembarking times under these two scenarios in order to study, thereafter, the influence of the bus capacity on passengers waiting times. Initially, two buses ($n_i=2$) are in the starting station (S_1^i). The firing dates of the transition PA^i (in

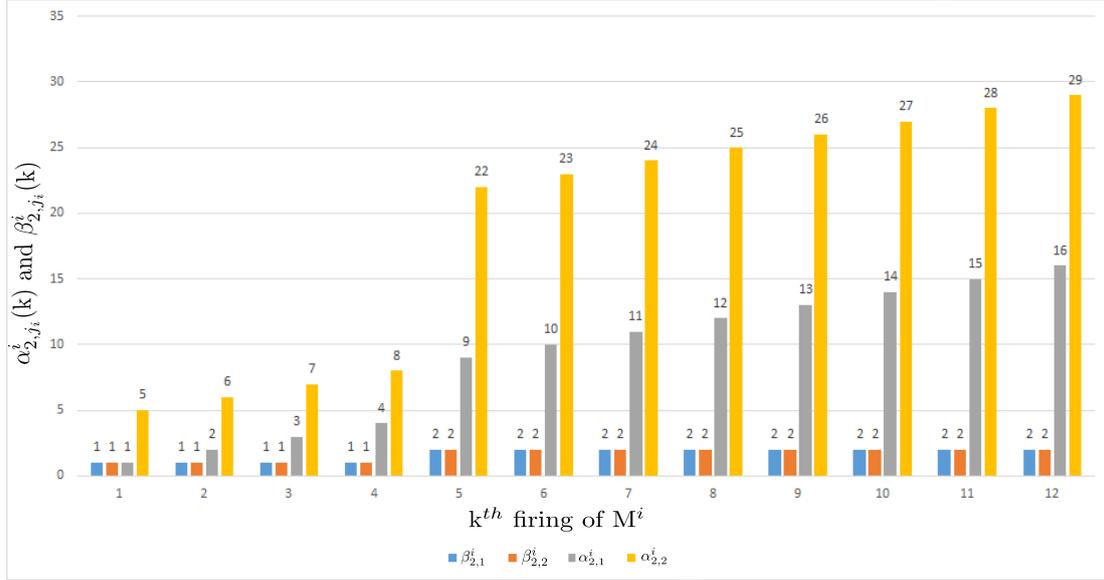


Figure 6: Routing functions at the connection station

Table 3: Firing times of the input transitions u^i and u^{i+1}

k	u^i		u^{i+1}		
	< 1 >	< 2 >	< 1 >	< 2 >	< 3 >
1	0	10	0	10	20
2	60	75	80	90	100
3	130	140	160	170	180
4	200	215	250	260	270
..

a specific period 6h00 - 6h53) by various colors ($\langle S_1^i \rangle$, $\langle S_2^i \rangle$ and $\langle S_3^i \rangle$) are given randomly by Scilab software using the command "grand" which generates random numbers according to a given exponential distribution law of parameter $\lambda = 0.7$. First of all, the firing dates of the transitions that models departure/arrival times of buses from/to different stations are given by the equation (3). For instance, the first four time passage of the first bus of both lines L_i and L_{i+1} are given by:

$$\left\{ \begin{array}{l} X_{\langle 1 \rangle}^i(1) = [e, 2, 15, 17, 28, 31, 56]^T \\ X_{\langle 1 \rangle}^i(2) = [60, 62, 75, 77, 88, 91, 116]^T \\ X_{\langle 1 \rangle}^i(3) = [130, 132, 145, 147, 158, 161, 186]^T \\ X_{\langle 1 \rangle}^i(4) = [200, 202, 215, 217, 228, 231, 256]^T \\ X_{\langle 1 \rangle}^{i+1}(1) = [e, 3, 13, 16, 28, 32, 43, 48, 78]^T \\ X_{\langle 1 \rangle}^{i+1}(2) = [80, 83, 93, 96, 108, 112, 123, 128, 158]^T \\ X_{\langle 1 \rangle}^{i+1}(3) = [160, 163, 173, 176, 188, 192, 203, 208, 238]^T \\ X_{\langle 1 \rangle}^{i+1}(4) = [250, 253, 263, 266, 278, 282, 293, 298, 328]^T \end{array} \right.$$

Table 4: Firing times of the input transitions PA^i and PA^{i+1} for a given period

k	PA^i			PA^{i+1}			
	$\langle S_1^i \rangle$	$\langle S_2^i \rangle$	$\langle S_3^i \rangle$	$\langle S_1^{i+1} \rangle$	$\langle S_2^{i+1} \rangle$	$\langle S_3^{i+1} \rangle$	$\langle S_4^{i+1} \rangle$
1	1,02	0,25	10,11	0,11	5,12	2,55	4,23
2	3,25	5,23	12,12	1,01	6,13	3,23	5,26
3	10,15	9,87	15,56	1,21	11,21	4,82	6,23
4	11,02	11,25	16,12	2,23	11,53	5,25	10,72
5	20,21	17,25	19,87	2,58	11,73	5,64	11,13
6	21,83	20,21	22,05	3,24	12,52	6,12	20,62
7	30,51	25,33	24,04	3,51	13,01	6,42	25,94
8	35,11	26,57	27,15	3,95	13,98	7,63	26,12
9	40,22	30,21	30,25	4,11	15,15	10,52	27,95
10	46,7	35,48	31,35	4,62	16,17	17,23	28,13
11	50,51	40,89	40,45	4,89	18,65	17,25	28,89
12	52,79	42,31	46,51	5,12	19,21	18,12	29,78
..

For the considered bus network, Figure 4 (resp. Figure 7) reports random arrival, boarding and waiting times of passengers at the departure station (resp. the connection station). For example, in Figure 4, the fifth passenger arrives at 20,21 min and waits 41,76 min to leave the departure station. The same approach is applied for the other stations. These parameters depend essentially on the routing functions computed using the algorithm of Figure 3. Actually, Figure 5 and Figure 6 reports the value of these routing functions, in a given period, in both departure and connection stations. In these figures, we deduce the relation between the firing of the transitions related to passengers boarding namely PA^i , $Crp^{i+1,i}$ and M^i . Thereafter, we evaluate passengers boarding and waiting times according to the equation (4). For instance, the first firing of the transition M^i by $\langle 1, S_1^i \rangle$ (resp. by $\langle 2, S_1^i \rangle$) occurs after the first (resp. second) firing of the transition PA^i by $\langle S_1^i \rangle$ and the first firing of the transition xa^{i+1} by $\langle 1 \rangle$ (resp. $\langle 2 \rangle$).

In the departure station (Figure 4), the arrival date of the 1st passenger is less than 2 min, so he can take the first bus who will leave this station after waiting 2 min. However, the 2nd (resp. 3rd and 4th) will wait 8,75 min (resp. 1,85 min and 0,98 min) to take the second bus and so on for the other passengers. In addition, we notice from this figure that passengers waiting time is very high (average of 20 min), because the number of buses used ($n_i=2$) is not enough for arrival rate of passengers specially in peak periods.

In order to show that the proposed models are able to describe the influence of the buses capacity on passengers waiting times, we consider more scenarios (Figure 8 and 9). More precisely, Figure 8 considers two scenarios; scenario 1 where the first bus capacity (Cap_1^i) equals to 20 (as mentioned in Table 2) and scenario 2 where the capacity is reduced to just 8. Based on developed (max, +) models, the obtained results show that the capacity of the buses affects passengers disembarking times. For

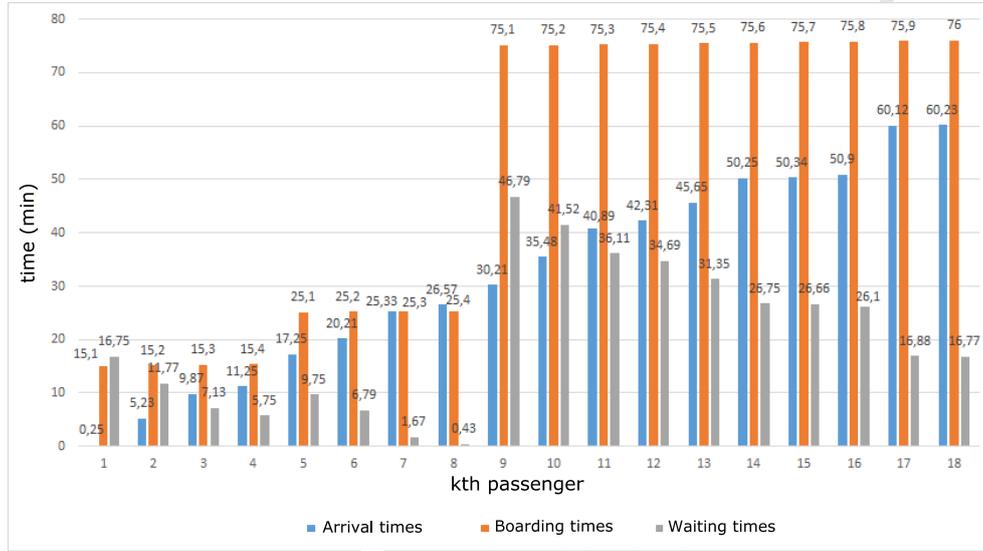


Figure 7: Passengers arrival, boarding and waiting times at the connection station

instance, the waiting times of the 13th, 14th and 15th passengers, under scenario 2, increased due to capacity limited to 8. In fact, these passengers find the bus saturated (no empty seats inside). Therefore, boarding times of these passengers increases over time. Furthermore, we can observe on the Figure 9 which considers peak periods scenarios, that the two buses could become saturated. So, there will be passengers who will not find empty places. Therefore, the average of passengers waiting time increases.

These observations allow concluding that the used bus are not enough to transport all the passengers at the same time specially in peak periods. So, in this case and in order to ensure or improve the quality of the service, the transportation company should either choose a bus with very high capacity or increase the number of buses circulating on the network. This will be addressed in the future work of this contribution as a control problem.

Finally, we mention that the change in passengers random arrival, buses waiting and traveling times or passengers destination simply result in modifying the routing functions and firing dates of different transitions, while the TCPN model structure is unaffected.

6. Conclusion and perspectives

In this study, we proposed a modeling approach combining TCPN and $(\max, +)$ algebra in order to evaluate the behavior of a public transportation system. The studied system is subject to complex phenomena such as conflicts, synchronization and concurrency which make the study of these networks more difficult. The addressed issue through this paper is to bring our contribution by proposing a $(\max,$

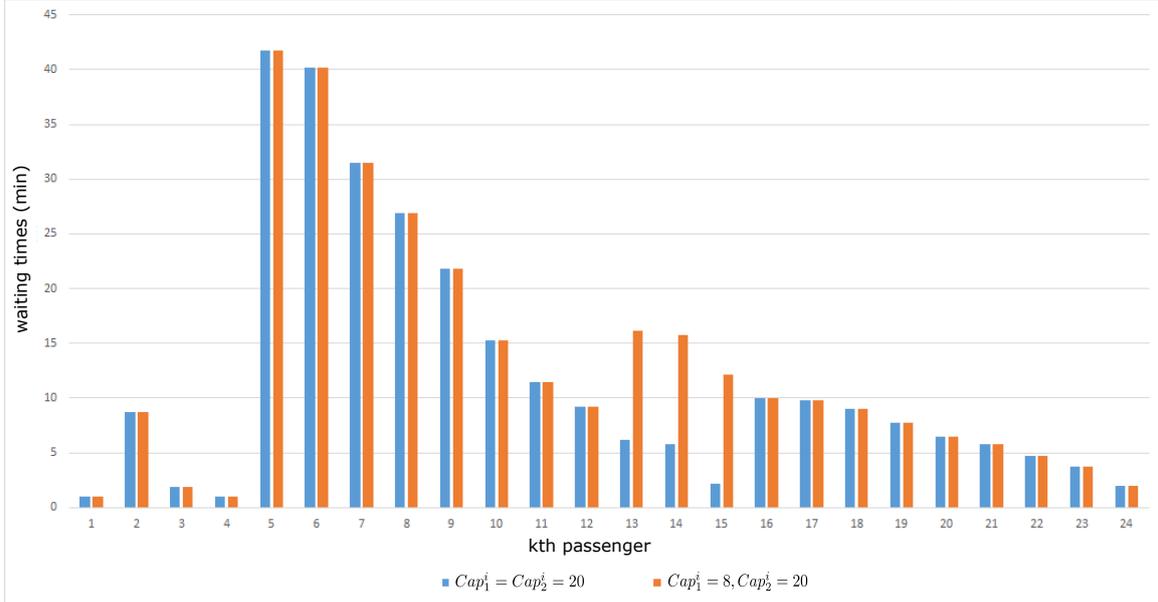


Figure 8: Influence of capacity on passengers waiting times

+) modeling approach for this type of DES. More precisely, the $(\max, +)$ equations describing the behavior of the TCPN model are proposed and routing functions enabling to manage all encountered conflicts are developed.

From these models, we have evaluated arrival and departure time of buses at different stations of the network and also waiting, boarding, disembarking times of each passenger. Furthermore, the influence of limited capacity of buses on passengers waiting time is studied. For instance, in peak periods scenario, the buses could become saturated and alternative solutions should be adapted to improve the provided transportation service to users. Finally, the proposed approach in this paper is highly adaptable to any public transportation networks with several lines and correspondence stations. Simulations are performed for two working scenarios and obtained results are reported and analyzed.

Our perspectives can include the use of TCPN and $(\max, +)$ algebra to evaluate and study more complex configurations of transportation networks with further operating constraints. Also, we intend to proceed with a control approach in order to determine a compromise between the capacity and number of buses to use on the network in order to minimize both passengers waiting times and increase performance indicators of transportation companies.

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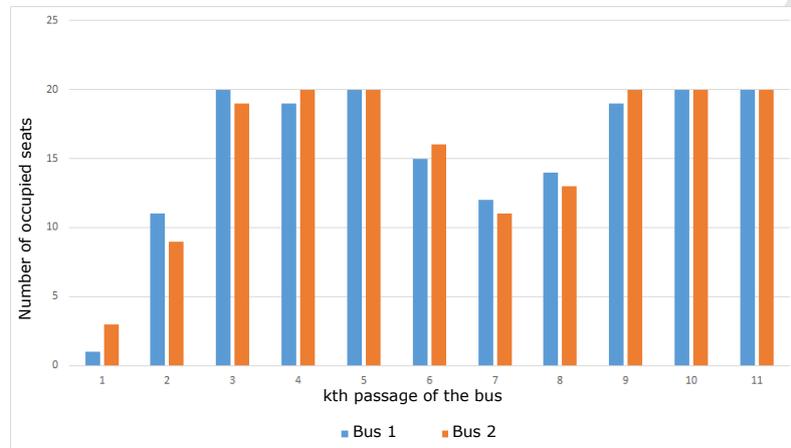


Figure 9: Number of buses occupied seats in a given period

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IDEL MAHJOUR Yassine

Idel Mahjoub Yassine received his engineering degree in Automation and Computer Engineering at the University of Cadi Ayyad in Morocco in 2015. Currently, he is a PhD student at the University of Cadi Ayyad (Morocco) and the University of Picardie Jules Verne (France). His research interests are discrete event dynamic systems (SDED), modelling, simulation, optimal control, performance evaluation and multimodal transportation systems.

Ahmed Nait-Sidi-Moh:

Ahmed Nait-Sidi-Moh received his PhD degree in 2003 in computer science and automatic control from the University of Technology of Belfort (UTBM). He is currently Associate Professor at the University of Picardie Jules Verne (UPJV), where he obtained his HDR (Habilitation à diriger des Recherches) in 2016 in the field of computer engineering. Nait-Sidi-Moh's research has been supported by many research and development projects including European grants, regional projects and EU EACEA Erasmus Mandus projects. His research interests include modelling, analysis and control of Discrete Event Systems and Hybrid Systems, performance evaluation, resolution and management of conflicts and resource sharing, routing, optimization, scheduling and interoperability for service composition, information and communication technologies. Nait-Sidi-Moh is the author of several papers published in international journals, international conferences and workshops. He is involved in several national and international events such as conferences and workshops organization, evaluation committee member of many international journals and conferences; participation to national research groups.

EL Houcine Chakir El Alaoui

El Houcine Chakir El Alaoui received his Ph.D thesis in 1996 in Automatic and Applied Mathematics from University of Rouen, France. Since 2001, he is a professor at the Faculty Sciences and Technologies, Marrakech. He is actually member of Electrical Systems and Telecommunication laboratory of Cadi Ayyad University. His research interests include optimal control, fuzzy control, AQM design and TCP network performances, Discrete Event Dynamic Systems (SDED), Petri Nets, Dioid algebra, Transportation system.

Highlights

- Modeling of a public transportation system characterized by conflicts, synchronization and concurrency.
- Use of CPN and $(\max, +)$ algebra for behavior study and analysis of considered system.
- Resolution of system conflicts with routing functions.
- Evaluation of system performances: waiting and traveling times of passengers.
- Study of the influence of bus capacity on passenger waiting times.