Optimal switching instants for the Control of Hybrid Systems

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Problem Statement

In this talk, we will consider the problem of determining optimal switching instants for the control of hybrid systems under reachability constraints. First, we define an n-mode dynamical system

$$(\mathcal{S}_i)\left\{\dot{x} = f_i(x) \quad , \quad x(t_i) = x_i\right\}$$

in the time interval $[t_i, t_{i+1}]$ with $f_i : \mathbb{R}^m \to \mathbb{R}^m$ where $x_i \in \mathbb{R}^m$ is the initial condition for all modes $0 \leqslant i \leqslant n-1$. Apart from x_0 that is fixed, x_i is taken as the solution at time t_i of the previous dynamical system $(\mathcal{S}_{i-1})$. This sequence of dynamical systems corresponds to the switching of control law. Our problem can be modeled using the following optimization problem

$$\begin{bmatrix} \max_{t_1,\dots,t_{n-1}} & g(x(\tau)) & \text{(cost function)} \\ \text{s. t.} & \forall 0 \leqslant i \leqslant n-1, (\mathcal{S}_i) & \text{(dynamical constraint)} \\ & h(x(\tau)) > 0 & \text{(reachability constraint)} \\ & \tau \in [t_{n-1}, t_n] & \end{bmatrix} \tag{1}$$

with the decision variables $t_1, \dots, t_{n-1} \in \mathbb{R}^n_+$ the search space for the different times; $g : \mathbb{R}^m \to \mathbb{R}$ the cost function on the state variable at given time $\tau \in [t_{n-1}, t_n]$, some constraints defined by the dynamical systems $(\mathcal{S}_i)$ and the times t_i; a reachability constraint using $h : \mathbb{R}^m \to \mathbb{R}$.

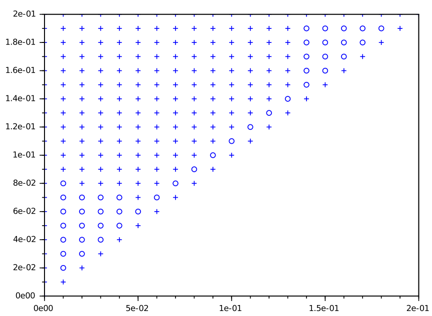
Approach

The optimization problem (1) is cast into a global optimization problem with differential constraints, where validated simulation techniques [1] and dynamic time meshing are used for its solution.

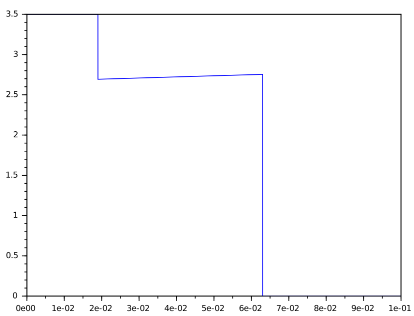
Example of the Goddard's Rocket

The Goddard problem [2] models the ascent of a rocket through the atmosphere. The rocket has to reach a given altitude while consuming the smallest amount of fuel. Physically, the optimal solution is a bang-singular arc-bang controller following the steps: 1) full power to break out 2) an increasing function to compensate for the drag effect and 3) turn off the engine and continue with the impulse. The question is when to switch from one dynamics to the following one.

Our approach provides the controller given in figures below, results agreed with [3] such as $t_1 = 0.019$, $t_2 = 0.063$ for a mass $m = 0.6273$.



Mesh for t_2 w.r.t. t_1



Optimal controller

References

[1] J. Alexandre dit Sandretto, and A. Chapoutot, Validated Explicit and Implicit Runge-Kutta Methods, *Reliable Computing*, 2016.

[2] R.H. Goddard, A Method of Reaching Extreme Altitudes, *Smithsonian Miscellaneous Collections*, 1919.

[3] K. Graichen and N. Petit, *Solving the Goddard problem with thrust and dynamic pressure constraints using saturation functions*, IFAC 2008.