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A MODEL FOR EXAMINING TEACHERS' DIDACTIC ACTION IN MATHEMATICS, THE CASE OF THE GAME "RACE TO 20"¹

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Abstract

This paper is based on an exploratory study aimed at providing a better understanding of the teaching process. By analyzing interactions between teachers and students in a particular situation (Race to 20), we developed a model for examining teachers' didactic action. This model is rooted in a number of theoretical concepts in French didactics, i.e. topogenesis and chronogenesis management, monitoring the didactic contract, the devoluting and instituting processes. The teaching process is considered from a theoretical standpoint involving three levels of description: the fundamental structures of the didactical relationship, the types of tasks teachers have to meet the demands of that relationship, and the classes of techniques they have to produce in order to carry out those tasks. Through the use of this model, we emphasize the necessity of considering these different types of techniques as mixed techniques that are shaped by the different constraints of the teacher's action inside the didactic relationship.

Introduction

1. Teachers' action

1a. General purpose

This paper is based on an exploratory study, aimed at providing a better understanding of the teaching process. The research published on this topic in the past few years is plentiful, and has been conducted from different points of view. In this paper, attention must be paid to two distinctive features of our work. First, this exploratory study is based on an empirical observation of student-teacher interactions. These interactions are scrutinized using a fined-grained analysis method, and they are characterized both in "natural" language, and in the theoretical language of mathematics didactics. Second - and this is a crucial point - we do not consider the teachers' action from a *prescriptive* viewpoint. We try to account for teachers' specific rationality. This means we do not evaluate and deplore the gap between their actual practices and some "right way of teaching" pronounced by the "authorities". Our point of view is a descriptive, comprehensive, explanatory one.

1b. Human action, the teachers action

We cannot mention all of the general features of our theoretical framework for analyzing the human action. Only, an outline of such a frame can be given. We consider human action, and more specifically a teacher's action, to be as follows. Action is expressed in terms of *dispositions*, which are elaborated by adaptation to particular *situations*. *Didactic institutions* are settings for these situations.

In order to understand the teacher's action, let us first state some features of its general structure as it is modeled in this paper. In paragraph 4b, this structure will be described in greater detail. In a general way, we shall say that the teacher's work consists of initiating, establishing, and monitoring the didactic relationship, which is a ternary relationship between the teacher, the students, and knowledge. In doing so, the teacher has to produce and apply numerous techniques, which are usually communicative techniques. We call these techniques *didactic techniques*. So, if we want account for the teacher's action, we have to describe, understand, and explain the techniques used to teach, i.e., to enable students to appropriate knowledge.

2. The theoretical concepts of the model

¹ For a description of a didactical use of this game, see Brousseau (1998), chap. 1.

In order to describe the teacher's work, we will use the theoretical framework in French didactics, developed by Brousseau (1998) and Chevallard (1999). As Brousseau (1998, p. 19) says: "[...] what is at issue... is to describe certain kind of human relationships in such a way the concepts of *didactique* are made to appear in order to serve as useful means to description [...]". These concepts will be presented as Brousseau or Chevallard defined them.

2a. *The didactic contract*

In the didactic system (built from three subsystems: the teacher sub-system, the knowledge content subsystem, and the student subsystem), "[...] a relation is formed which determines - explicitly to some extent, by mainly implicitly - what each partner, the teacher and the student, will have the responsibility for managing, and, in some way or other, be responsible to the other person for. This system of reciprocal obligation resembles a contract. What interest us here is the didactical contract, that is to say, the part of the contract which is specific to the 'content', the target mathematical knowledge." (Brousseau, 1998, p. 31). Brousseau suggests that "[...] this interplay of obligations is not exactly a contract, [because] it cannot be made completely explicit. There are no known, recognized, sufficient ways of allowing the construction of new knowledge of or ensuring, against all resistance, the student's appropriation of the target knowledge." (Brousseau, 1998, p. 32). So Brousseau emphasizes the following point: "The theoretical concept in *didactique* is therefore not the contract (the good, the bad, the true, or the false contract), but the hypothetical *process of finding a contract*. It is this process which represents the observations and must model and explain them" (Ibid.). In our modelling of the teacher action, we have to use the dual concept of *adidactic situation* and *setting* (named "milieu", in Brousseau, 1998) which is related to the the didactic contract.

2b. *The adidactic situation and the setting*

An adidactic situation is a learning environment designed by a teacher. Three criteria for this. First, the student must not be conscious of the teacher's intentions about the knowledge. Second, the student is engaged in a game "bringing together a 'milieu' and a 'player', with this game being such that a given piece of knowledge will appear as *the* means of producing winning strategies" (Brousseau, 1998, p.57). Brousseau thus presents the milieu (the setting) as "the system opposing the taught system" (Ibid.). We emphasize these point: the setting in a non didactical situation is incapable of provoking learning at all, most of didactical virtue being contained in the didactic contract; the student interaction with a setting can be expected to provoke the expected adaptations and learning when imbedded in a didactical contract i.e. in an adidactic situation. Thus, and this is the third criterium, the students must be aware of this: the setting is designed by a teacher, and knowledge acquisition is the expected effect of playing the game. Such an adidactical situation (the Race to 20), and its setting are presented later in this paper, so that these definitions will no longer be abstract.

We'll use these concepts in a description of the teacher's action. In fact, the teacher can be considered as someone who, at all times in the didactical relationship, is trying to build a setting for the student's action and to ensure that the student has elaborated the right relationship to that setting. We will see that a very large set of didactic techniques are ones we can call the *setting establishment* techniques.

3. The teaching tasks

3a. *Monitoring adidactic situations: the devoluting and the instituting processes*

In order to learn, the student must become engaged in the learning situation and build a relationship with the setting that defines this adidactic situation. "Devolution is the act by the teacher makes the student accept the responsibility for an (adidactical) learning situation or for a problem, and accepts the consequences of this transfert of this responsibility." (Brousseau, 1998, p.230). In our model, the devolution process is one of the fundamental structures of the teacher's action. Another fundamental structure of this action, symmetrical to devolution, is the instituting process, when "[the teacher] defines the relationships that can be allowed between the student's 'free' behavior or production and the cultural or scientific knowledge and the didactical project; she provides a way of reading these activities and gives them a status." (Brousseau, 1998, p.56).

Devoluting and instituting processes are two sorts of teachers' tasks in monitoring adidactic situations. They are subtle processes in the didactical relationship, running through the whole learning-teaching process.

3b. Managing the didactical contract: topogenesis and chronogenesis

The core of a learning-teaching process can be viewed as follows:

a) At any time in this process, the teacher and the student have a specific set of tasks to carry out their mathematical works. This division of didactical labor has been called *topogenesis* (Chevallard, 1991). Each participant in the didactical relationship has a *topos*, i.e. a specific set of tasks to accomplish, which define his position in the didactic system. These reciprocal positions develop throughout every learning-teaching processes. *Topogenesis monitoring* is synonymous with producing and managing teacher's and student's topos.

b) In order to describe the development of the teacher and the student mathematical works, Chevallard (1991) proposed the concept of *didactic time*. That is the time of the teaching progression through the study of knowledge. In fact, the teacher's action is constrained by the necessity of presenting to his students a body of knowledge, part by part, shaped for teaching. So, in their action, teachers have to give a certain amount of time to pieces of knowledge, in order to cover its content. When teachers give up some item of knowledge, replacing it by a new one, they produce an unit of didactic time. This type of monitoring implies an efficient pacing (Mercier, 1992, 1995; Sensevy, 1996). *Chronogenesis monitoring* is synonymous with producing and managing didactic time.

We can now consider the didactic contract as rooted in two dimensions of teachers' action. First teachers have to ensure the running of didactic time, i.e. monitor the chronogenesis; second, at every moment in this chronogenesis, they have to make the reciprocal positions of the student and the teacher clear, i.e. monitor the topogenesis.

4. Empirical study

4a. Race to 20

Brousseau designed and used it as a paradigm to introduce most of the main features of the Theory of Didactical Situations. The game is played by two players. The first player says "1" or "2" (for example, "1"); the other continues by adding 1 or 2 to this number ("2" for example) and saying the result (which would be "3" in this example); the first person then continues by adding 1 or 2 to this number ("1" for example) and saying the result (which would be "4" in this example); and so on. Each player tries to reach "20". The winning series for one player (2, 5, 8, 11, 14, 17, 20) is $20 \div 3$ i.e. the numbers $p=20-3*n$. The quotient of the division $20 \div 3$ gives the number of stages which is necessary to win ($20 \div 3 = 6$ means there are 6 steps to reach 20); the rest of this division gives the "starting" number (for the race to 20, 2). The race to 20 (by adding 1 or 2) is a peculiar case of the general race to n (by adding $(p, \text{ or } p-1, \text{ or } p-2, \dots, \text{ or } 1)$). Brousseau organized several phases in the teaching process based on this game, referring to different types of didactical situations (action, formulation, and validation).

4b. Methodology

This paper is based on a study of 5 lessons in the Race to 20, given in the 5th grade by 5 teachers (4 regular teachers, 1 student teacher). The methodological device and procedure was as follows. The researchers gave the teachers a text that described the Race to 20. It was a simple description of a few lines, without the "solution" and without any theoretical or mathematical terms, similar to the description we presented above at the beginning of paragraph 3a. In order to make sure the teacher understood the meaning of this text, the researcher played the game with the teacher, without giving any information. It was the first time the teachers came in contact with this situation.

Then the researcher asked teachers to organize a mathematics lesson based on this game in their classroom. Teachers were free to design the lesson as desired. The methodology of this study, mainly designed by Schubauer-Leoni and Leutenegger (1997), and Leutenegger (1999), can be summarized as follows: a) Each teacher engaged in a "preactive" interview called the "anteinterview" (before the lesson), in which he/she was asked about his intentions regarding the lesson. b) The lesson was videotaped, and transcriptions were made of the audio part of the videotapes. c) Each teacher engaged in a "postactive" interview (immediately after the lesson). This interview was conducted "blindfold" i.e., by a researcher who had not seen the lesson. d) A final interview was conducted several weeks after the lesson. It was used a self-confrontation interview based on videotape (or the transcription) of the lesson, for the teacher confrontation with his/her own teaching.

In the present paper, we describe this methodology to situate the general context of this study, but we do not utilize all its possibilities. Rather, we focus our analysis on a single lesson, produced by a particular teacher.

4c. Three classes of techniques

In this paragraph, we present three fundamental classes of techniques. They are grounded on what we considered the three fundamental functions of the teacher's didactical action: elaborating the *topogenesis*; managing the *chronogenesis*; establishing the *setting*.

The description of the teaching techniques is organized as follows: the left part of the table is devoted to the transcription of classroom interactions and the right part is devoted to the simple description of this action in "natural semantics" (i.e. in the words of someone "familiar enough" with the action). After this, the same action is analyzed from our theoretical viewpoint. The teacher is a experienced teacher who has elaborated the lesson in alternating phases of pair work, group work, and whole-class debate.

A topogenetic technique: a move of devolution

In a general way, the devolution produced in this lesson (and in the other "Race to 20" lessons studied) is a devolution of the *correct action to play the game*.

	So you have to play two rounds and then, w'll try to figure out, uh, can we try to figure out who will be the winner... and when we know that?	Indication of the number of rounds to play (re-using of a relevant feature in a student's sentence). Incentive to predict (anticipate the winner) and identify (when we know that).
68	Well I don't know	Affirmation of ignorance.
	W'll try to find the rule... Ok, it works, we try to go...	Indication: use of the term "rule".

150	Ah! You think it comes from the starting number.	Monitoring a dialogue (you think) and indication: introduction of a meaning specific to the game, "starting number".
152	Therefore, you think it's this way?	Incentive to develop a logic argumentation (therefore). Asking for a confirmation.
154	Well, I don't know, we have to see.	Affirmation of ignorance.

Devolution is a process. This process runs through the whole learning-teaching process. Here, the technique of devolution was characterized by a "topogenetic boundary" change: the teacher said, "well I don't know", or "well I don't know, we have to see". Thus, by affirming his "ignorance" (true or false, it does not matter), he emphasized a certain symmetry with the students in this didactical work. We can consider this as an *up-down topogenetic move*: the distance between the students' topos and the teacher's topos is reduced.

On the other hand, the topogenetic move could be a *bottom-up* one, and increase the topogenetic distance. For example, when the teacher said "for me, I would like to listen to the Reds" (the "Reds" are one of the groups into which the teacher has split the class), he increased the distance between himself and the student: he acted as an expert who focused the student's attention on a particular point. In contrast when he said, "well I don't know", or "well I don't know, we must see", the teacher stood in a (fictitious) researcher's position. He acknowledged a certain kind of ignorance for himself, which legitimized the students' ignorance and urged them to take on a searcher's position. In saying that, the teacher meant "I, who most of the time knows very well what I am teaching, I move to a searcher's position, and I seek an answer which I don't possess *a priori*. Therefore we have to search." *We*, i.e. *you*. Of course, teacher's ignorance was fictitious, for he announced, at the beginning of the lesson, that he was able to win "all the time". This up-down topogenetic move is a technique for managing the didactic contract in a devolution process: the students have to search because the (winning) "rule" or the "starting number" are new pieces of knowledge; when the teacher indicated them his declaration of ignorance "gave some space" to the students.

We can analyze this technique in the following way: the teacher made a sign to the students, and this sign had to be interpreted inside the didactical contract specific to the didactical situations. It is a weak form of the Topaze Effect, which we can call Topaze Indication. The teacher did not suggest the right answer directly, but at least for the students able to understand the specific codes of this didactical contract, suggested the proper way to work: search for the "rule" and the "starting number". This point raises an interesting issue, the question of the epistemological habitus² the students need to communicate successfully with the teacher.

² On the notion of habitus, see Bourdieu (1990); on the notion of didactical habitus see Mercier (1986, 1992), Schubauer-Leoni (1988), Schubauer-Leoni and Perret-Clermont (1997), Sensevy (1998, 2000), Mercier, Sensevy, Schubauer-Leoni (1999).

We would like to emphasize a last point concerning this technique, a crucial point: it occurs when the students are confronted with a question (the "starting number") which is fundamental for mathematical understanding of the game. Thus, didactical techniques are not only linguistic and communicative, but are rooted in mathematical setting. We can conjecture that such topogenetic techniques are produced at mathematically critical times.

A chronogenetic technique

In the ante-interview, when we asked the teacher about the difficulties the students might encounter, he (and the other teachers interviewed in this study) stressed on the problem of a students' hypothesis about evenness of the winning series. Actually, the students did not have many means of acting on numbers.

When they were asked to characterize a number, they could think of evenness, because for these students, this is the only relevant property for speak about numbers (for example, they do not master the notions of multiple and factor). Therefore, before the lesson, the teacher was waiting for students to focus on this topic. His strategy for dealing with this point was elaborated in three steps.

First, at the beginning of the lesson, when certain students were searching in the direction of even numbers, and when some among them proposed this winning series (2, 4, 6...) they were ignored by the teacher, who seemed to listen to them with only one ear and did not take up their argument.

Second, after several minutes of the lesson, the teacher was working among the groups (the class was broken down in 4 student groups) and he reacted specifically to the problem: inside a group, a student thought that 11 was a winning number but an other student in the same group was an advocate of the rule "every even number is winning". Then the teacher brought out the contradiction, as follows:

185	You, you are thinking of even numbers, he, 11; it's not an even. I don't know	<i>focus on the contradiction</i>
	You, you use even numbers.	
189	To go to 14, you are playing only pair numbers?	
191	And he is playing 11, thus...	
193		

Third, later in the lesson, the problem was brought up again. Conducting the debate, the teacher gave more room to the even series proposal, as follows:

203	They have, wait, they have another theory. It's the even number theory, that's right.	<i>A competing theory is named. A virtual confrontation is organized, the term "theory" is enacted</i>
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The first time, the teacher did not react. The second time, the teacher drew the student's attention to the contradiction that involved evenness. The third time, the "conjecture" was named a theory. In order to understand this, we can propose the following interpretation.

The third time, in the teacher's mind, the discovery of the winning series was almost made: so he could bring up the even numbers idea. Contrary to the situation at the beginning of the lesson, there was no risk of weakening the "true theory". Instead of that, the teacher could conjecture that the discovery of the winning series would be strengthened by the discussion.

Obviously, the "resonance" of the teacher i.e., the way the teacher seizes or does not seize the students' answers (Comiti, Grenier, Margolinas, 1995; Comiti & Grenier, 1995) depends on the chronogenesis. In other words, a given student's statement is not taken into account at time t1, is considered in a certain way at time t2, and in an other way at time t3: these different reactions are grounded on the progress of the didactic time.

A setting establishment (mesogenetic) technique

In the learning-teaching process, the teacher has to establish the setting in which the student works. The following episode can be analyzed as a particular way of doing so.

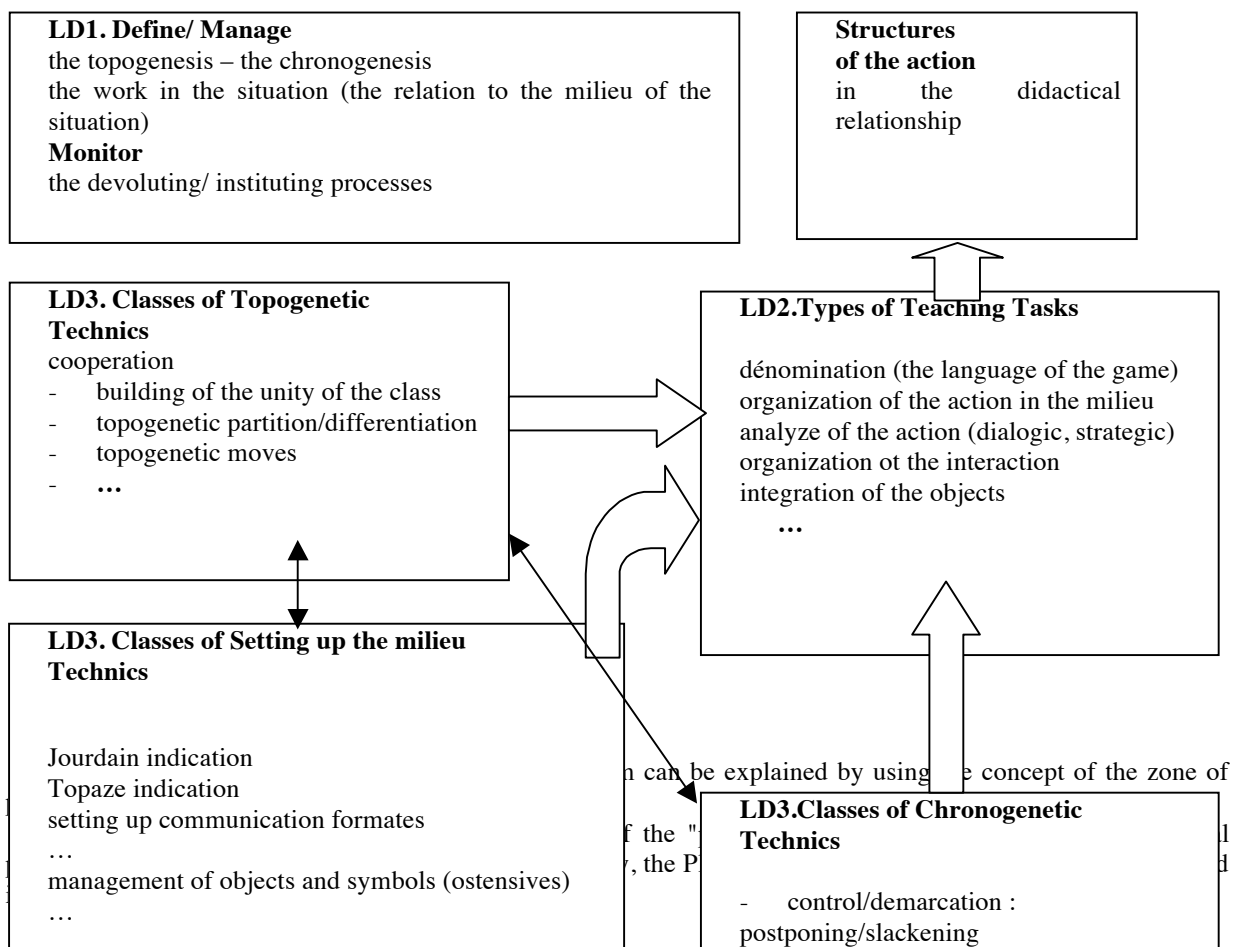
155	Student: Yes yes, look at this! This is the infallible numbers! When we were beginning ... The teacher: So, wait, wait, stop stop stop, you are engaged already... So, shh! Please...Thus you already have some series and for, of, you... You try to use the infallible strategy by comparing your, your series.	<i>A new task is designated</i>
156		<i>Seizing of the term "series"</i> <i>Proposing the term "infallible strategy"</i>

We previously used the notion of "Topaze Indication", in this case we can use the notion of "Jourdain Indication". This notion was illustrated when the teacher seized and used a word produced by the students ("infallible" numbers), and used it - this is the actual Jourdain indication - together with a meaningful teacher's word ("strategy"), in order to create the new relevant meaning of "infallible strategy".

This episode is illustrating the concept of "mimetic postulation" (Sensevy, 2000). Communication between two persons is grounded in the necessity, for a person, of considering the interlocutor as the same person oneself. If the teacher allows himself to cover the student's meaning (infallible) by his own meaning (the infallible strategy), it is because he postulates that the conceptual distance between the two expressions is small enough to be travelled by the student.³ Thus, the didactical relationship seems to increase the fundamental constraint of all types of communication: if I want him to learn, I must consider the student I teach as a rational person, who shares already with me the rationality that I want him to build.⁴

From a didactical viewpoint, this technique allows the teacher to organize the confrontation of the students' statements and monitor it (Schubauer-Leoni, 1997). If this technique is powerful enough, the setting changes: the teacher brings in the milieu some meanings that the students need to consider, and that they have to evaluate. Therefore, the *establishing the setting* consists, among other features we cannot mention in this short paper, of diffusing the relevant statements of the students, if need by changing them!

4. The model





This model is produced from three levels of description of the teacher action. The first level of description (LD1) concern what we call the structures of the didactical relationship. The second level (LD2), according to Chevallard (1999), is the level of what we call the teaching tasks. The third level (LD3), which is the main topic of this paper, referred to the different classes of techniques. This model is grounded on the study of "race to 20" lessons, but it could be, and it aims to be, a more general model of the teacher's didactical action in mathematics.

It is not possible to describe neither the prominent feature, nor the wholeness of teaching techniques or monitoring tasks. However, we want to emphasize two points:

a) There is no bijection techniques/tasks. Several techniques of different natures (chronogenetic, topogenetic, setting-up the milieu) may converge towards a specific task. We have to remind that the tasks themselves are not isolated, but are included in a functional system of tasks, a way of teaching.

b) Perhaps the more crucial point of this model: the techniques specify themselves reciprocally. This feature is a consequence of the very nature of the didactical action: teaching involves the pacing of didactic time (chronogenesis), the monitoring of topogenesis, and managing the effective pupils' relation to the setting. Most of times, it is not possible to isolate one of these types of action: the relevance and the efficiency of didactical processes are grounded on "mixed techniques", oriented by the functional structures of the didactical relationship.

Conclusion

In conclusion, we would like to recall the prominent features of our way of modelling the teacher's didactical action.

In order to understand and to explain the teacher action, we think that we have to:

- clearly identify the teacher action as functionally structured by the necessities of the didactical relationship
- describe his activity in terms of types of tasks responding to the fact that the didactical relationship is grounded on communicative acting
- understand that these tasks are most of time accomplished by mixed techniques, in which a particular status of the chronogenesis and the topogenesis (status that the teacher produces in cooperation with the class) specify (and is specified by) the work in progress about the knowledge.

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