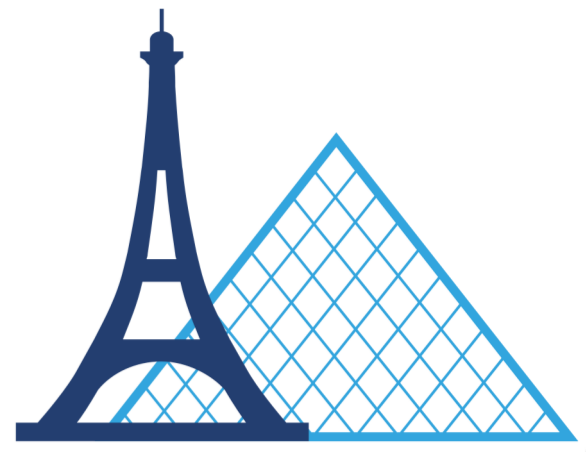


Large Surface LC-resonant Metamaterials: from Circuit Model to Modal Theory and Efficient Numerical Methods

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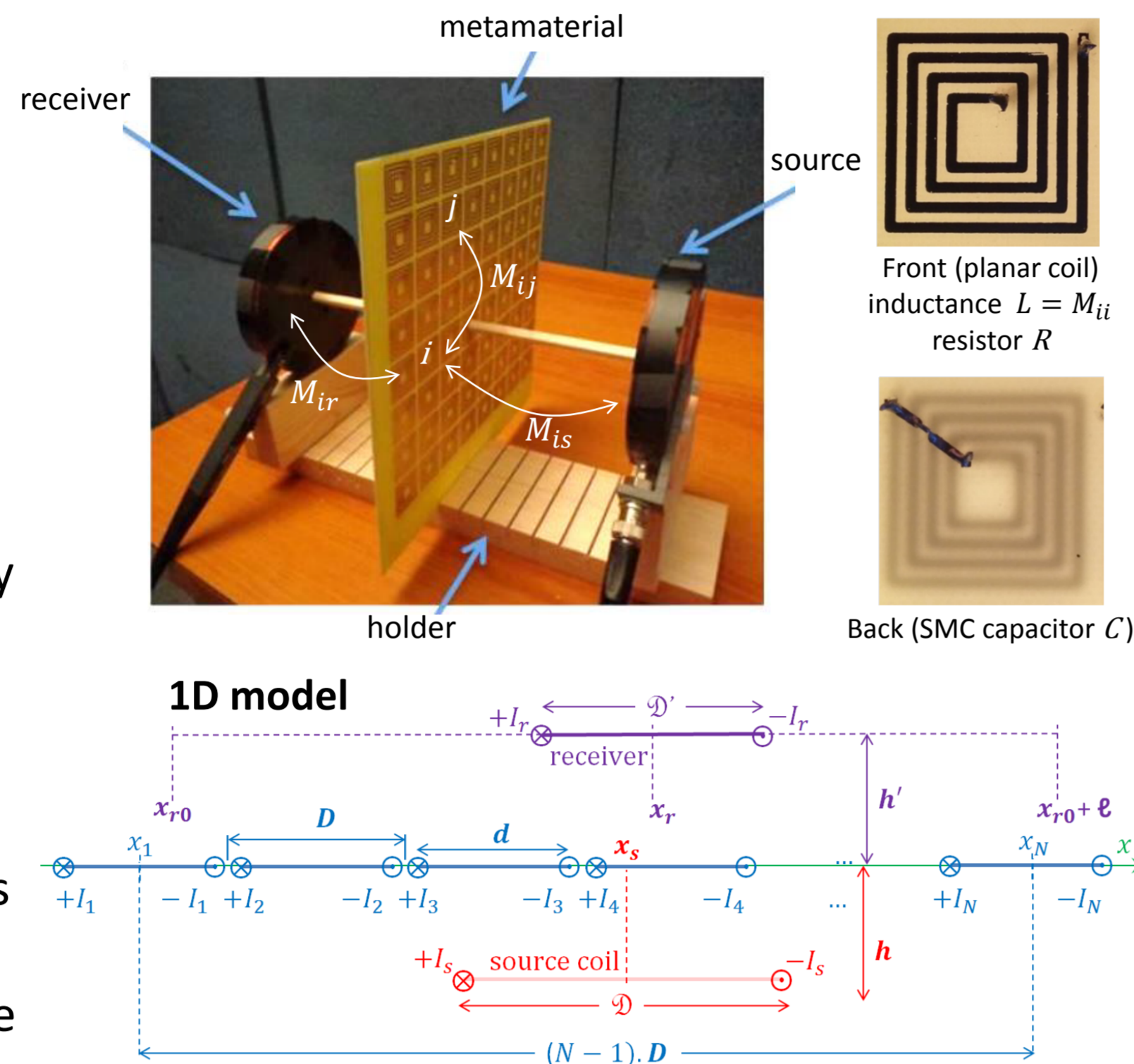
The MaSuRe project objectives

We study the harmonic magnetodynamic behavior (low frequency) of a resonant surface metamaterial, made up of many identical, regularly arranged LC resonant cells.

- How does it work?
- Why is power transfer possible also with large excentricity of the receiver?
- How can we explain the antiresonance
- Find an efficient numerical method for very large numbers of cells (e.g. 1000 × 1000)

Perspectives:

- Effective numerical homogenization
- Review of possible manufacturing methods
- Development of a specific SMPS
- Experimental measurements of a very large structure



Equations

Reference solution (circuit equations for N cells, full $N \times N$ matrix):

$$\left(R + \frac{1}{j\omega C}\right) I_i + j\omega \sum_{j=1}^N M_{ij} I_j = -j\omega (M_{is} I_s + M_{ir} I_r) \quad i = 1 \dots N$$

Space modal development of cell currents and fluxes (discrete values i):

$$X_{is} = \sum_{l=-n_y}^{n_y} \sum_{k=-n_x}^{n_x} X_{k,l,s} e^{-j2\pi k \frac{x_i}{L_x}} e^{-j2\pi l \frac{y_i}{L_y}}$$

Exact modal equation:

Note: for a 2D $N_x \times N_y$ metamaterial, $N = N_x \times N_y \Rightarrow$ matrix dimension = $(N_x \times N_y)^2$

Example: 1000 cells × 1000 cells \Rightarrow full $10^6 \times 10^6$ matrix.

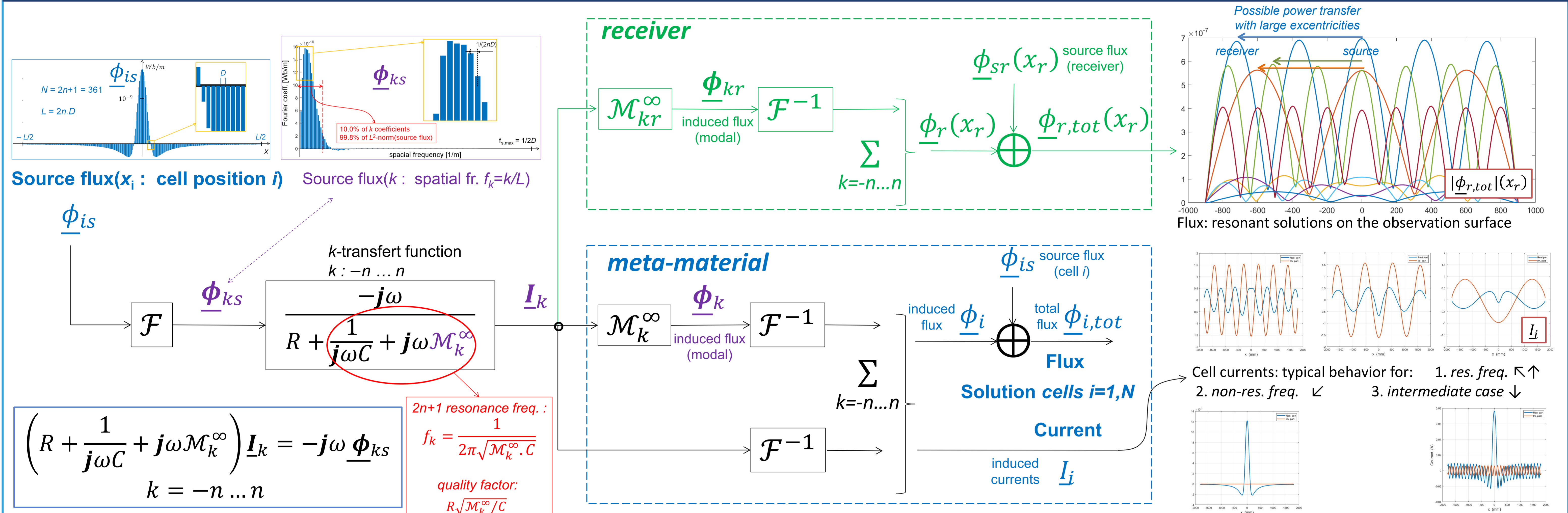
For a 1D problem ($N \times N$ matrix, $N = 2n + 1$):

$$\sum_{k=-n}^n \left(R + \frac{1}{j\omega C} + j\omega \mathcal{M}_{k,i}\right) I_k e^{-j2\pi k \frac{x_i}{L}} = -j\omega \sum_{k=-n}^n \phi_{ks} e^{-j2\pi k \frac{x_i}{L}}$$

Asymptotic solutions for infinite dimensions (N equations with one unknown):

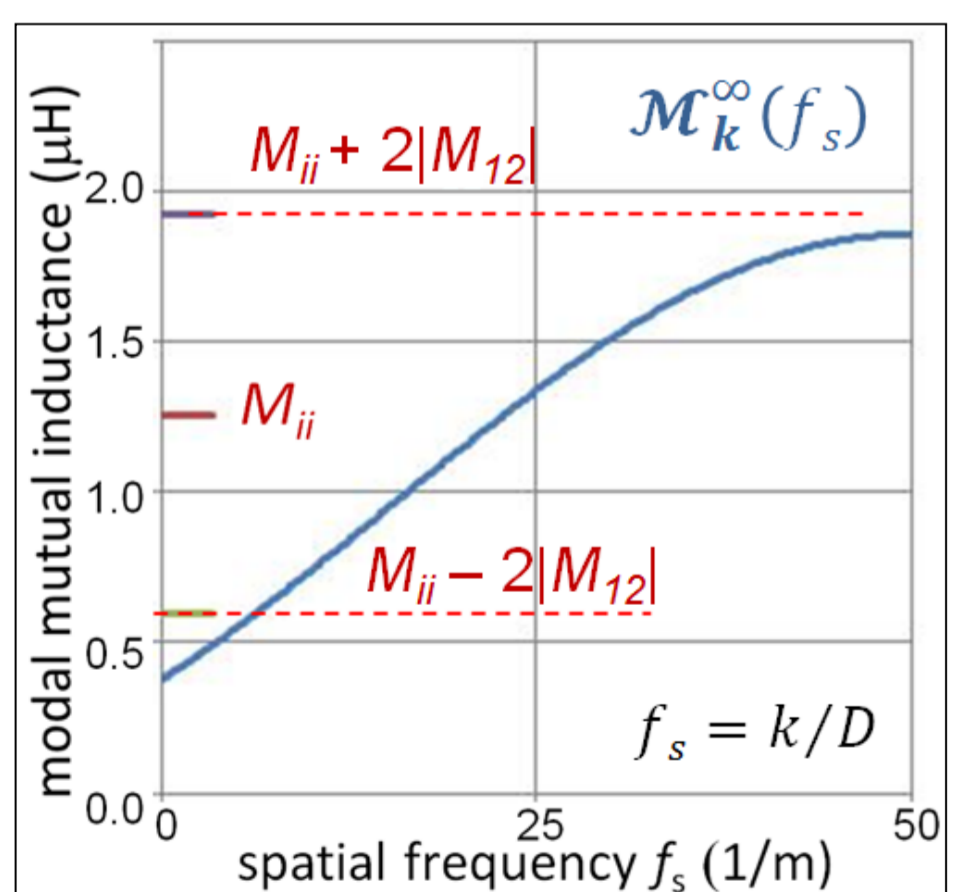
$$\left(R + \frac{1}{j\omega C} + j\omega \mathcal{M}_k^\infty\right) I_k = -j\omega \phi_{ks} \quad k = -n \dots n, n \rightarrow \infty$$

Modal approach (very large number of cells)



Modal mutual inductance

$$\mathcal{M}_{k,l}^\infty = \mathcal{M}(f_{sx}, f_{sy}) = \lim_{n \rightarrow \infty} \sum_{j=-n}^n M_{ij} e^{-j2\pi k \frac{x_j - x_i}{L_x}} e^{-j2\pi l \frac{y_j - y_i}{L_y}}$$



1D: analytic values for $\mathcal{M}_{k,i}^\infty$

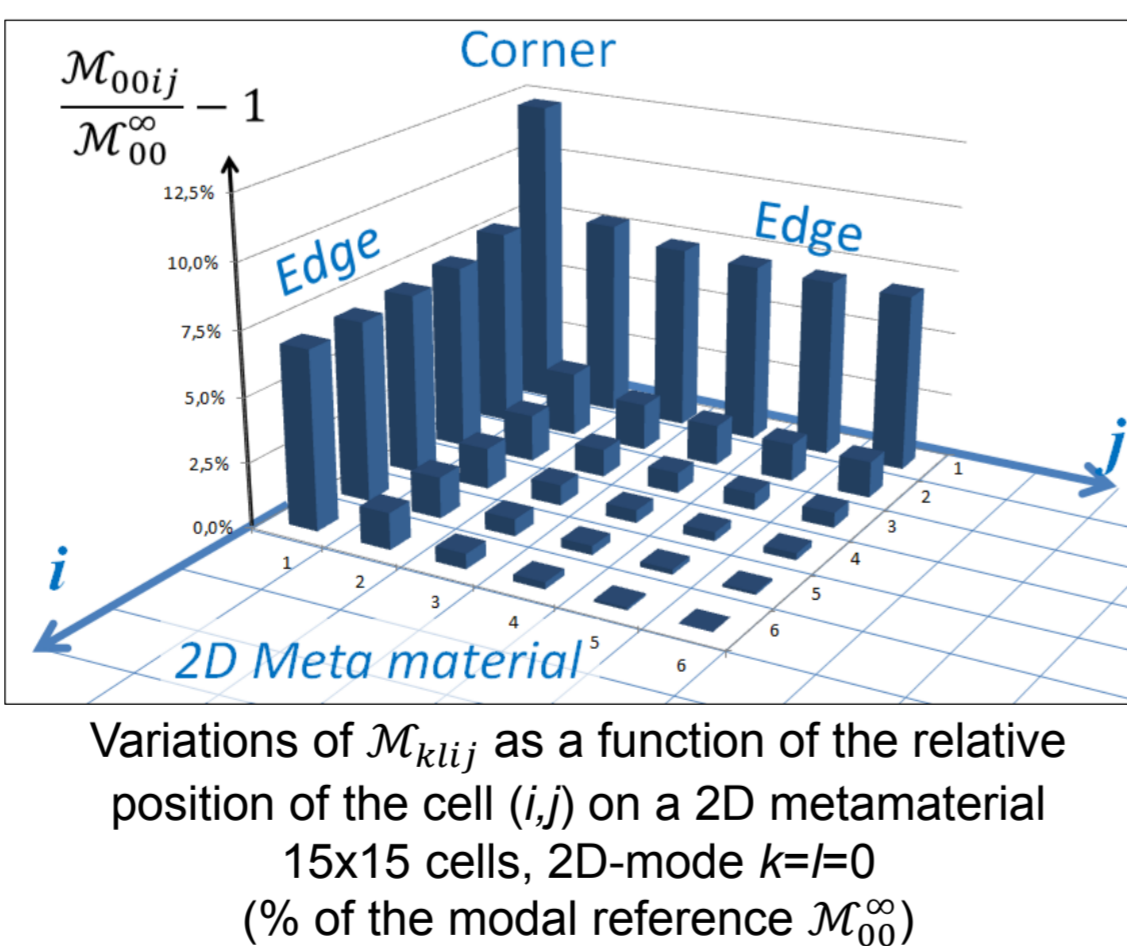
Data used for this example:

Metamaterial:
 $D = 0.010$
 $d = 0.009$
 wire $\phi = 0.001$
 $N = 361$
 $R = 1\text{m}\Omega$ $C = 1.95\mu\text{F}$

Source and receiver:
 $h = h' = 0.10$
 $\mathcal{D} = 0.15$
 $\mathcal{D}' = 0.10$

Why is the modal solution not exact?

- 1) $\mathcal{M}_{k,l}^n \neq \mathcal{M}_{k,l}^\infty$ near edges (finite material)
- 2) numerical solution $n \neq \infty$:
 foldover distortion
 infinite anti-periodic excitation



Results

Meta-material global frequency behavior:

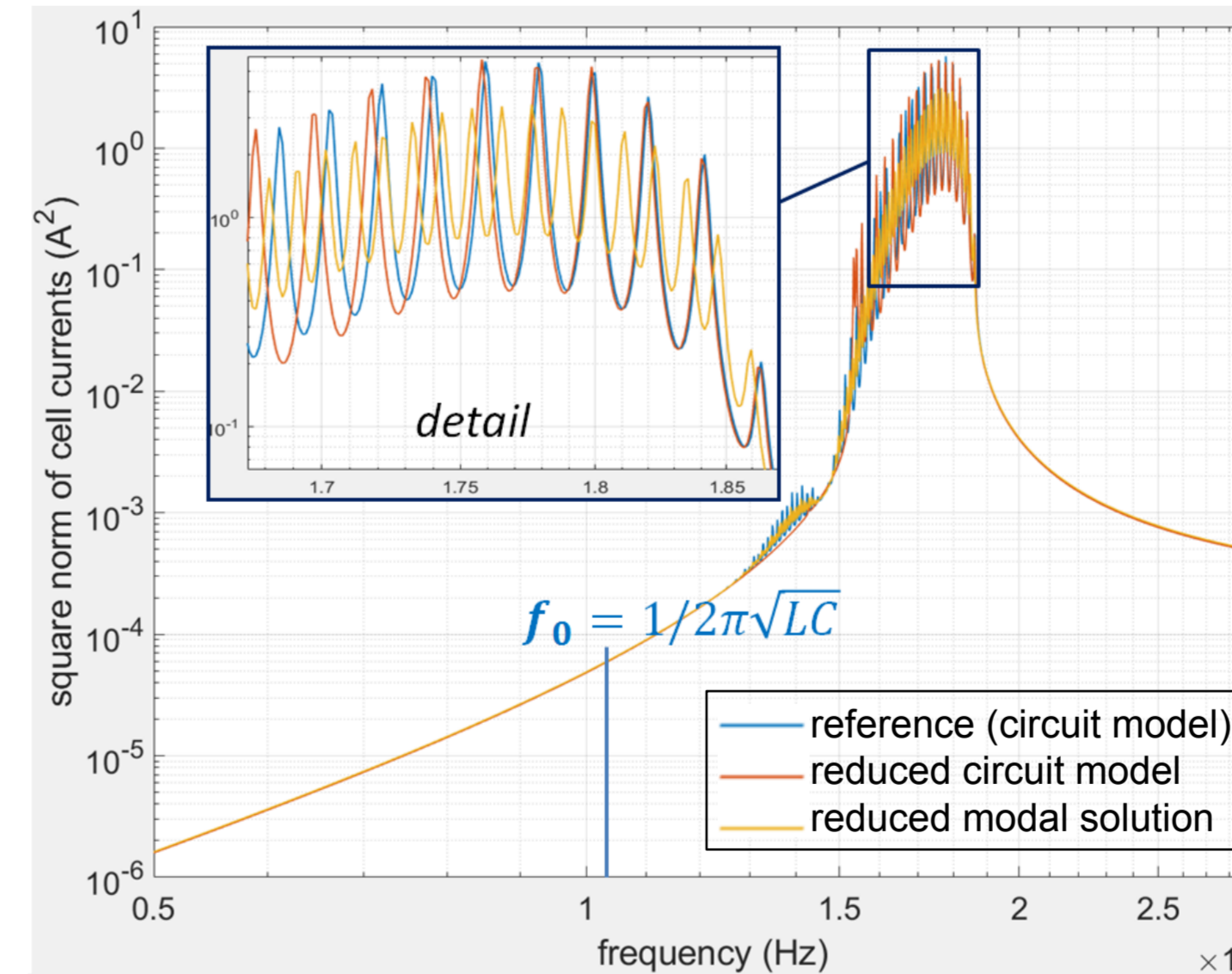
$$|I|^2(\omega) = \sum_N |I_i(\omega)|^2$$

Receiver - global frequency behavior:

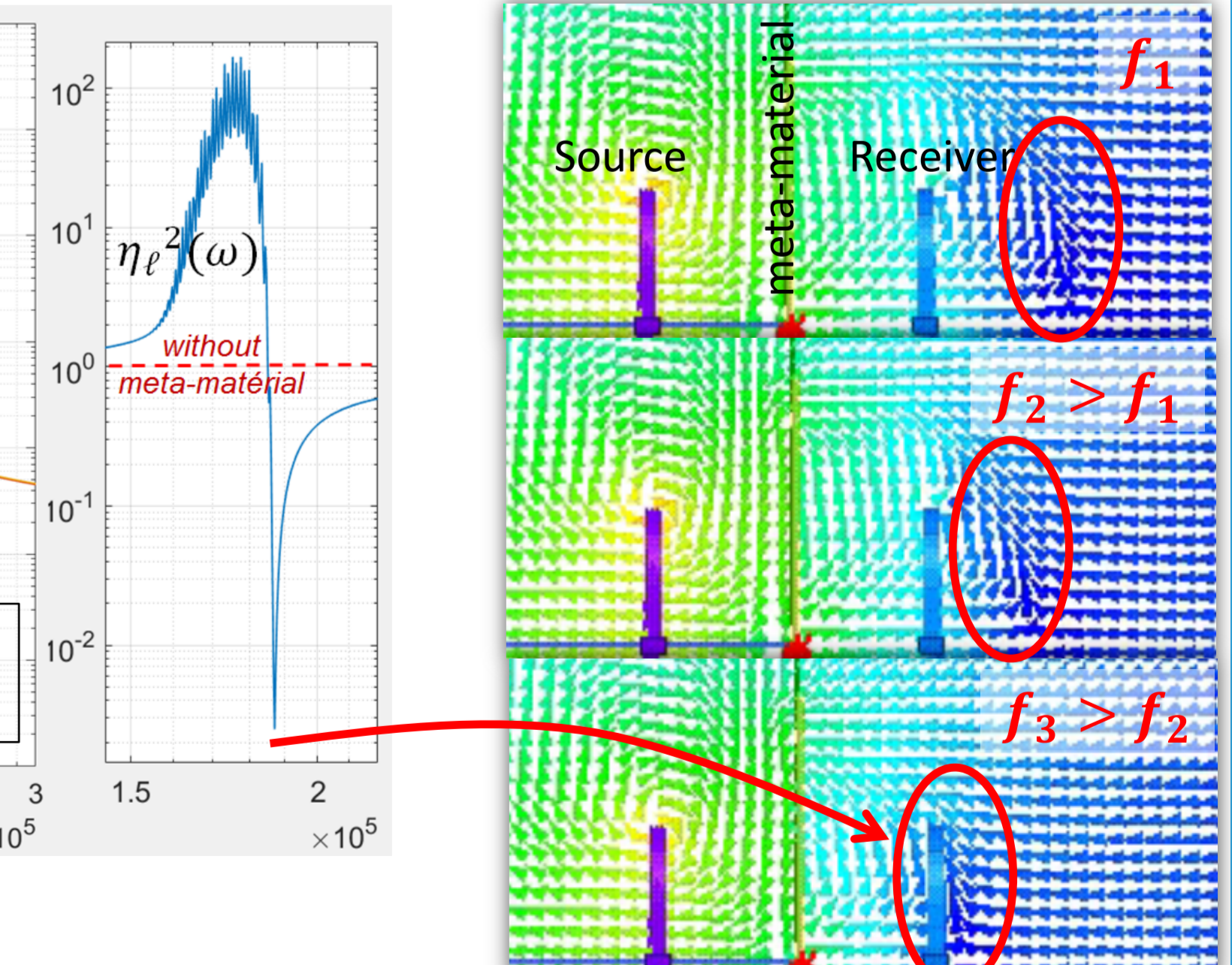
$$\eta(x_r, \omega) = 1 + \sum_{j=1}^N M_{rj} I_j(\omega) / M_{rs} I_s$$

$$\eta^2(\omega) = \frac{1}{\ell} \int_{x_{r0}}^{x_{r0} + \ell} |\eta|^2(x, \omega) dx$$

Meta-material



Receiver

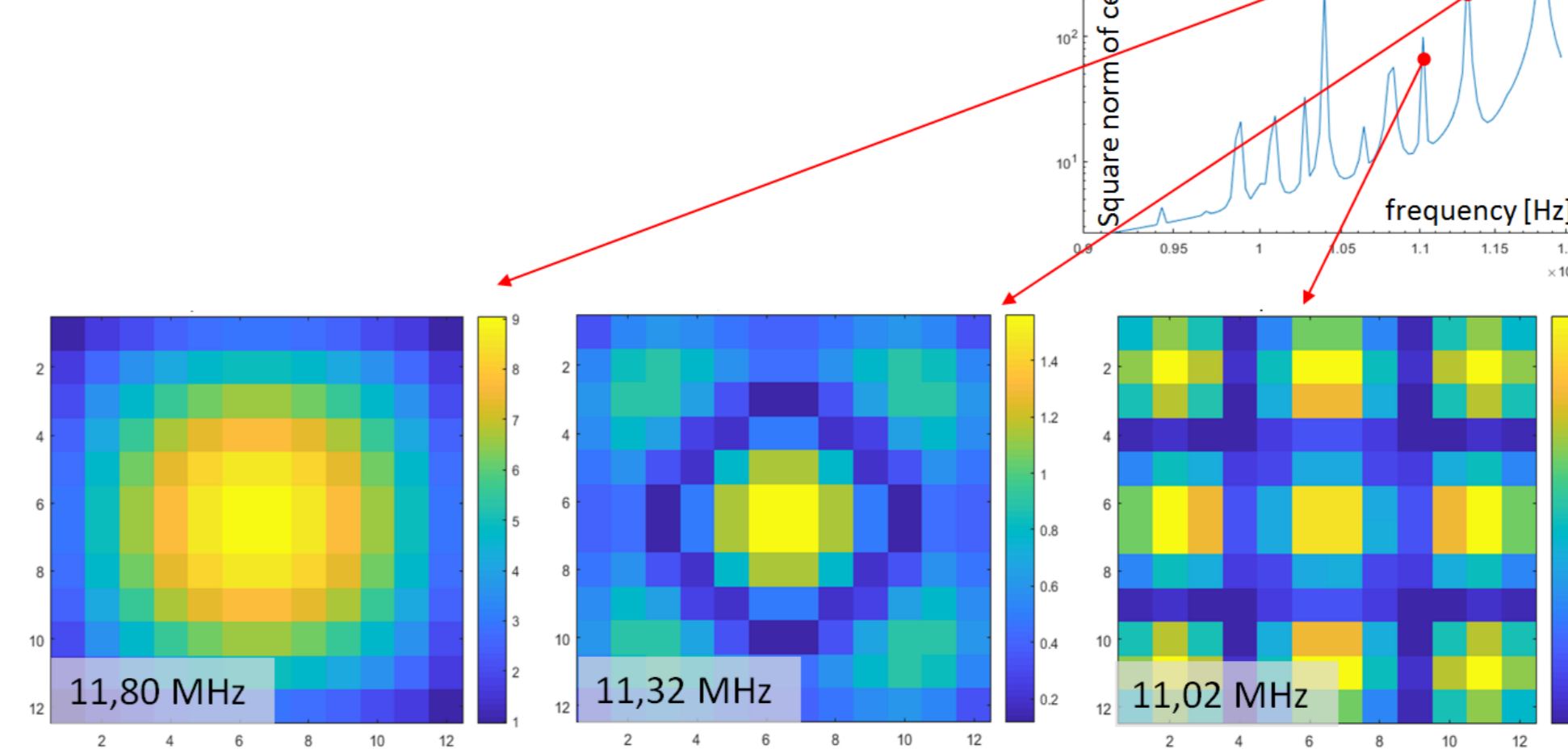


Displacement of the anti-resonance area with frequency changes

Reduction methods

- 1) **Modal solutions** - use only the most energetic modes. Typically $N \rightarrow N/10$
 Equivalent to filter (neglect) the higher space frequencies
- 2) **Circuit equations** - typically $(N \times N) \rightarrow (N/10 \times N/10)$ by using:
 - rough mesh representation of cell currents I_i to reduce the number of unknowns
 - same linear combination of equations to reduce the number of circuit equations.
 Equivalent to filter (neglect) the higher space frequencies
- 3) **Mutual inductance matrix M_{ij}**
 - "Exact" values (analytic values, FEM or PEEC values) only for neighbour cells i, j
 - far cells: just distance-dependent values

PEEC method for 2D structure
 12x12 cell test with Altair Flux PEEC



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