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Laurent Krähenbühl, Riccardo Scorretti, Arnaud Bréard, Christian Vollaire, Jean-Michel Guichon, et al.. Large surface LC-resonant metamaterials: from circuit model to modal theory and efficient numerical methods. *Compumag* 2019, Jul 2019, Paris, France. hal-01985864

HAL Id: hal-01985864

<https://hal.science/hal-01985864>

Submitted on 5 Apr 2019

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Large Surface LC-resonant Metamaterials: from Circuit Model to Modal Theory and Efficient Numerical Methods

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We study the harmonic magnetodynamic behavior (without wave propagation) of a resonant surface metamaterial, made up of many identical and regularly arranged LC cells. The circuit model gives the exact solution, but is not numerically efficient for the simulation of very large structures (e.g. 1000×1000 cells). For the first time, we highlight the modal character of the solutions, which makes it possible to explain their frequency and spatial related properties. From these results, we show under what assumptions it is possible to homogenize the metamaterial, which opens the way for using this approach together with efficient numerical methods.

Index Terms— LC metamaterials, resonance, circuit analysis, modal analysis, homogenization, matrix reduction, integral equations

I. INTRODUCTION: TYPICAL DEVICE

THE typical device that we consider was first proposed in [1], with 8×8 resonant LC cells arranged on a plane surface (Fig. 1). The goal of the present work is to explain the frequency behavior of these devices and to propose efficient numerical methods to simulate them even for arrangements with a far greater number N of cells (e.g. 1000×1000, $N=10^6$).

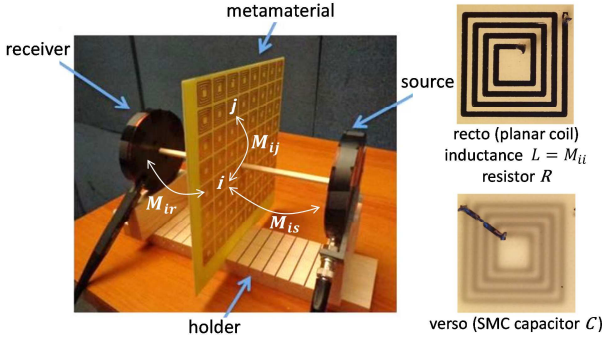


Fig. 1. Experimental setup applied to wireless power coupling studies [1]
Natural frequency of the isolated cell: $f_0 = 1/(2\pi\sqrt{M_{ii}C})$.

II. CIRCUIT ANALYSIS TECHNIQUE (REFERENCE SOLUTION)

For quite small systems, the reference solution is easy to obtain using the circuit analysis technique. We get a complex full $N \times N$ matrix with main terms around the diagonal:

$$\left(R + \frac{1}{j\omega C}\right)L_i + j\omega \sum_{j=1}^N M_{ij}L_j = -j\omega(M_{is}L_s + M_{ir}L_r) \quad (1)$$

We suppose that the excitation current I_s is known and that there is no current in the receiver ($I_r=0$); the method may be applied for other choices by considering the corresponding equations. The mutual inductors (matrix M_{ij} and vectors M_{is} and M_{ir}) may be analytically [2, 3] or numerically computed; it is then possible to obtain the cell currents $I_j(\omega)$ and the flux in the receiver as a function of the angular frequency.

Two global values are particularly significant for the CAD approach; first, the square norm of the cell currents:

$$|I^2|(\omega) = \sum_N |I_j(\omega)|^2 \quad (2)$$

which expresses the intrinsic reaction of the metamaterial; second, the ratio $\eta(x_r, \omega)$ between the flux through the receiver in the x_r position, respectively *with* and *without* presence of the metamaterial, or its norm $\eta_\ell^2(\omega)$ on a length ℓ , which express the frequency effect of the metamaterial for an energy transmission between the source and the receiver:

$$\eta(x_r, \omega) = 1 + \sum_{j=1}^N M_{rj}L_j(\omega)/M_{rs}L_s \quad (3)$$

$$\eta_\ell^2(\omega) = \frac{1}{\ell} \int_{x_{r0}}^{x_{r0}+\ell} |\underline{\eta}|^2(x, \omega) dx \quad (4)$$

For small structures, the circuit analysis gives the exact reference solutions, but it is not practicable for large ones and it does not give the key to understand how the device works.

III. MODAL APPROACH

To simplify the explanations, we establish here the modal character of the solutions only for 2D field problems with 1D metamaterials (Fig. 2); the extension to the real 3D situation with 2D metamaterial, will be presented in the extended paper.

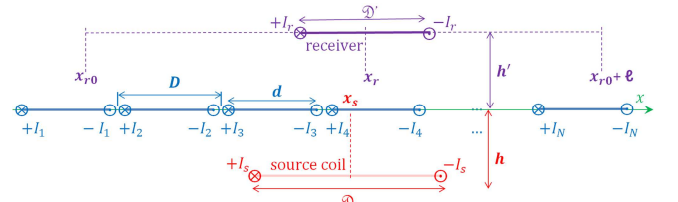


Fig. 2. 2D problem with a linear metamaterial

Let's develop in spatial Fourier series, over a length $L=N'D$ (typically $N' \geq N$), the source flux $\phi_{is} = M_{is}I_s$ and the current solution I_j . For the cell i located at the abscissa x_i we get:

$$\phi_{is} = \sum_{k=0}^{N'/2} \phi_{ks} \cos\left(2\pi k \frac{x_i}{L} - \psi_k\right) \quad (5)$$

$$I_j = \sum_{k=0}^{N'/2} I_k \cos\left(2\pi k \frac{x_i}{L} - \varphi_k\right) \quad (6)$$

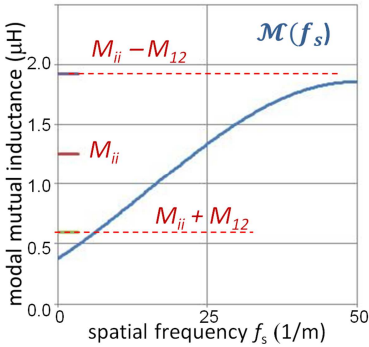
Assuming that the length of the metamaterial is infinite, i.e. the excited area is small compared to this length, we can rewrite the circuit equations as *independent* modal equations:

$$\left(R + \frac{1}{j\omega C} + j\omega \mathcal{M}_k\right) \underline{I}_k = -j\omega \underline{\phi}_{ks} \quad k = 0 \dots N'/2 \quad (7)$$

where the mutual inductance \mathcal{M}_k of the mode k corresponding to the spatial frequency $f_s = k/(N'D)$, is defined by:

$$\mathcal{M}_k = \mathcal{M}(f_s) = \sum_{j=-\infty}^{+\infty} M_{ij} \cos 2\pi k \frac{x_j - x_i}{L} \quad (8)$$

(the result does not depend on i because of the infinite limits). Even if the number of equations $(1+N'/2)$ is of the same order of magnitude as for the circuit method, the modal method is much more efficient because these equations are independent. Moreover, we can evaluate and use only the predominant modes (e.g. 10%), without perceptible loss of accuracy.



Data (see Fig. 2) for the numerical results of Fig. 3 and Fig. 4:

Metamaterial :
 $D = 0.010$
 $d = 0.009$
 wire $\phi = 0.001$
 $N = 361$
 $R = 1m\Omega$ $C = 1.95\mu F$
 Source and receiver :
 $h = h' = 0.10$
 $\mathcal{D} = 0.15$
 $\mathcal{D}' = 0.10$

Fig. 3 Typical behavior of the modal mutual inductance \mathcal{M}_k (8) in function of spatial frequency

The modal equation (8) highlights an electric resonance frequency $1/(2\pi\sqrt{\mathcal{M}_k C})$ for each spatial mode k , fastest spatial variations corresponding to lowest electric resonance frequencies. The two characteristic quantities (2) and (4) typically have the aspects given in Fig. 4:

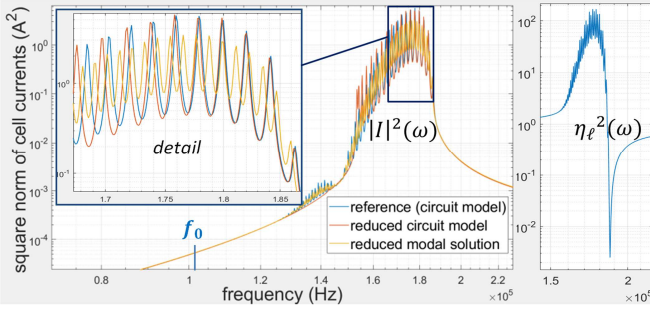


Fig. 4 : Typical intrinsic (left) and transfer (right) behaviors (2) and (4). Detail window (§IV): reference solution in blue; the reduced modal solution (yellow) is roughly correct; the reduced circuit solution (red) is more accurate.

Note that in the real situation of §I, we would obtain a modal mutual \mathcal{M} with two indices, and bidirectional modes (Fig. 5).

IV. HOMOGENIZATION

If the elementary cells are small with respect to the excitation and reception coils dimensions, the amplitude of the higher modes is small and their effect will be negligible on the solution \underline{I} and on the flux in the receiver ($|I|^2$ (2), η (3) or η_ℓ^2 (4)). It is then possible to reduce the number of degrees of freedom of the solution without substantially reducing its accuracy. By way of example, the solution presented in Fig. 4 (left) was calculated in the following 3 ways:

- complete reference circuit solution, using $N=361$ cell currents (361×361 full matrix);
- reduced modal solution, using only the 10% main modes (= low-pass spatial filtering, 37 unconnected equations); the global behavior is correctly described, but the resonances are not precisely obtained (peak values and frequencies), because the metamaterial is considered as infinite (the source field is also repeated at infinity);
- reduced circuit solution, obtained by projection of the unknowns I_i and of the circuit equations on a mesh 10 times rougher than the cell dimension (37×37 full matrix). Like the reduced modal method, this method results in low-pass filtering of the reference solution, but this time it is possible to take into account the finite dimension of the metamaterial. Accuracy is excellent for the frequency band of interest (see “detail” on Fig. 4, left).

This result implies that a way to achieve an homogenization should be found. We will show in the extended paper that for surface metamaterials, the reduced matrix can be efficiently constructed directly using a 2D surface mesh much rougher than the size of the elementary cells. The currents in the cells are represented by a surface density of normal dipoles whose effect, calculated by an integral method, replaces the calculation of mutual inductances. It is thus possible to take into account metamaterial of complex, non-planar shapes.

The homogenization step consists in modifying the classical integral method to introduce (i) the close interactions (known analytically) that the integral method with coarse mesh cannot describe precisely, and (ii) the $(R + 1/j\omega C)$ terms of (1).

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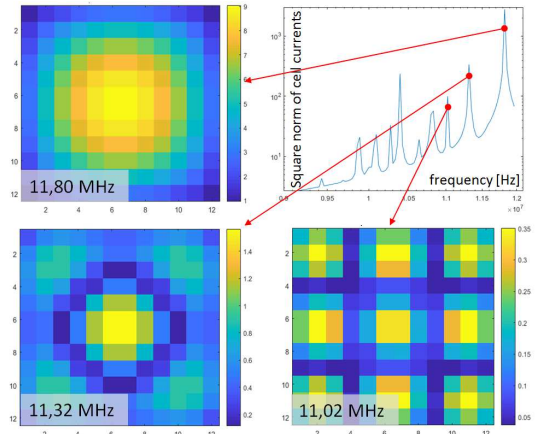


Fig. 5. 2D resonant solutions (12×12 cells, $f_0=10\text{MHz}$) obtained with the complete circuit equations [4]. Colors represents the cell current modulus I_i .