Compressions of a polycarbonate honeycomb

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Abstract
The in-plane compressive response of a polycarbonate honeycomb with circular close-packed cells is considered first experimentally then analytically. Under quasi-static uniaxial compression, we observed behaviors strongly depending on the orientation: for one of the two main orientations the compression is homogeneous, while for the other the deformation localizes in a very narrow band of cells. More surprisingly, for not crushing but extreme compression, when the load is released, the deformation is reversed, the localization disappears and the polycarbonate returns to its original shape. In order to explain this strange phenomena, we develop a geometric model of this honeycomb together with an expression of the bending energy. We focus on a basic mechanical element made of an elastica triangle. We also compare our description with previous experimental studies and simulations made with similar material. Finally, to illustrate mathematically this type of behavior, we present a simple model for buckling deformations with two degrees of freedom.

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1 Introduction

Honeycombs are widely used material and it is important to understand the mechanism governing their responses to compressions. Here we consider a polycarbonate honeycomb with circular close-packed cells, (as shown in the following pictures) that we submitted to a in-plane quasi-static uniaxial compression, with different orientations. Since the common tangents to the circles of the plane initial configuration form an hexagonal mesh, there are two different natural directions of compression, namely when the vertical compression axis is either parallel or perpendicular to a tangent. In the first case, the circles are positioned in columns, in the second case they are in a staggered arrangement. We observed that, as expected, in the first case the deformation was homogeneous, compressing all the rows. However, in the second case we observed a localization along few horizontal rows. More surprisingly the second kind of deformations were also reversible, contradicting usual expectations.

To understand these phenomena, we first compared our observations with other published experiments, mainly the excellent work of S. Papka et S. Kyriakides [3], which is accompanied with coherent simulations performed by a detailed computer model. While their aim was to describing the different steps of a complete crushing of a material, the first steps of their described experiments coincide with ours. So our first contribution is the observation of what happens when one releases the load after the localization is achieved, and the surprising fact is that the process is reversible.

Our second contribution is the description of a geometric model for the deformation of the circular close-packed cells and the corresponding hexagonal mesh, controlled by the evolution of the total bending energy of the mechanism. For that purpose, we propose an alternative decomposition of the material in an aggregate of curved triangles instead of the obvious aggregate of circles. We also develop spline approximate models of their geometry. All of this allows us to provide a rational explanation of the considered phenomena.

The last section will be dedicated to the presentation and the mathematical analysis of a simple mechanical model with two degrees of freedom. The target will be to somehow illustrate, with a much simpler model, the observed localization of the deformations which combines a kind of buckling with a rotation; in particular the process is reversible.

We thank the CNRS Fédération Doeblin, for its support during the preparation of this work.

1.1 Experiments

We submitted a circular polycarbonate honeycomb to a in-plane quasi-static uniaxial compression, along two compression directions either parallel or perpendicular to a cell tangent. We observed two completely different behaviors: from the one hand a homogeneous deformation and for the other hand a localization.

Figure 1 shows the first and last step of a compression (before a release of the load); we observe that the middle rows of the cells are strongly bent on the left or on the right and almost crushed. In other words the deformation localizes along the middle rows.

We also notice a quasi invariance by some horizontal translations.

Fig. 1 Compression of a circular honeycomb in a staggered stack

The very surprising fact is that when we stop the compression at this point and leave the material free to relax, it returns to its initial configuration.

In an important paper [3], S. Papka et S. Kyriakides studied the crushing of the same kind of polycarbonate honeycombs that we are considering.

We refer to the figure 2 of page 243 of the article [3] of S. Papka et S. Kyriakides which describes the sequence they observed through 8 successive images of deformed configurations, corresponding to response to a long compression. The picture labeled (0) to (3) are completely conform to our observations, with the same kind of localization phenomena (before we release the load).

S. Papka et S. Kyriakides also mentioned accurate measures of the geometric characteristics of the cells of the honeycomb, pointing out variations of wall thickness and "ovalisation", illustrated by a picture that we sketch in the left panel of Figure 2. These facts were also confirmed in an article [1] by L.L.

Hu, T.X. Yu, Z.Y. Gao and X.Q. Huang, who also provided a convincing photo taken with a microscope.

We emphasize that, on the picture, we can notice that near the stick points, the ovals coincide at an higher order than an usual tangent point. We based our modelization on this important observation.

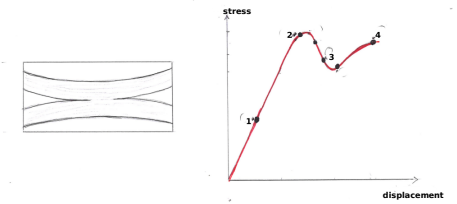


Fig. 2 Left panel: Sketches of the microscopic geometry near a stick point. Right panel: Non linear behavior of the response.

S. Papka et S. Kyriakides recorded the compressive response in order to describe the different steps of the crushing, in the right panel of Figure 2 we sketched a graph summarizing for our own experiment (before the final crushing) the non-linear evolution of the stress against the displacement. The number on this graph correspond to those of Figure 2.0.2.

Finally, we also report that they provided a computer model of the compression process. Their problem was discretized using the software ABAQUS with quadratic beam elements; the simulation of crushing of a honeycomb was conform to the observations summarized in the figure cited above and allowed to describe precisely the process.

2 Geometric Modelization

We consider that the basic polycarbonate material of the curved walls is isotropic and hyper elastic which implies that it admits reversible elementary deformations.

We will assume that the length of each arc of curve between two stick points does not vary during a deformation and that the deformations keep the sections

in the same plane, so all the study will be conducted in 2D. An invariance by horizontal translations is also assumed.

We consider that the glue between the “circular” packed cells play a key role that we formalize through the two following geometric hypothesis: During all the compression and release process we consider that the curvatures at the sticking points are zero. At the beginning of the compression, the 3 semi-tangents at the stick points of a curved triangle meet in a same point. This property remains true for small deformations or when the triangle admits a symmetry axis. Assuming this simplifying property, we can associate to the polycarbonate an abstract hexagonal mesh (made of small tangent segments) which resembles an hexagonal honeycomb and follows the deformation of the polycarbonate.

It makes sense to view the polycarbonate honeycomb not as an aggregate of packed circles, but as an aggregate of curved triangles (represented by Bezier splines) that can be deformed, provided that any two adjacent semi-tangents coincide.

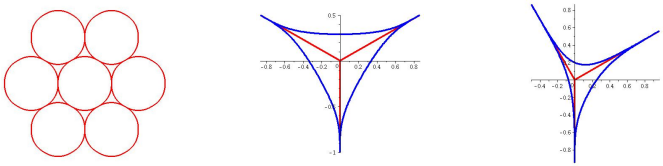


Fig. 3 Model of curved triangle, undeformed and deformed.

We now present an explanation of the unexpected phenomena observed in our experiments.

Let us emphasize, with the illustration of Figure 2, that the initial curved triangles are positioned differently relatively to the load, so that after the first step of the compression the remaining axial symmetries are different.

Let us consider in both cases, (stacks in columns or staggered) the effects of a compression.

Fig. 4 Two distinct directions of conserved symmetry.

2.0.1 Stack “in columns’

- The compression respects the horizontal symmetry of each curved triangle, of each oval, and of each hexagon formed by the tangents.
- The stack of hexagons made by the tangents deform (reversibly) to a flattened stack of hexagons.

2.0.2 ”Staggered“ stack

- After a certain load the ”horizontal“ side of the triangle reaches a maximum allowed length, then either it stops deforming or it buckles and a vertex bends (randomly on the left or on the right), then the triangle rotates in that direction.
- This behavior localizes along few rows where imperfections create a weakest resistance to buckling.
- In this process these curved triangles eventually rotate by an angle of $\frac{\pi}{6}$, and behaves in a way similar to the one described for the curved triangles in a stack “in columns”.

The whole phenomena is illustrated on the sequence of four images shown in Figure 2.0.2, their ordering 1 to 4 correspond to those of Figure 2. To emphasize the described process, we chose three different colors to single out the behavior of each of the curved triangles surrounding an oval in a localizing row. Note that a central symmetry is clearly conserved.

When the load is released, each flattened curved triangle reacts elastically, in particular the ones in the upper and lower rows which did not buckle or rotate recover their original form. In the localized rows, the vertices which were forced to meet are anew separated, can relax, the process is then inverted and

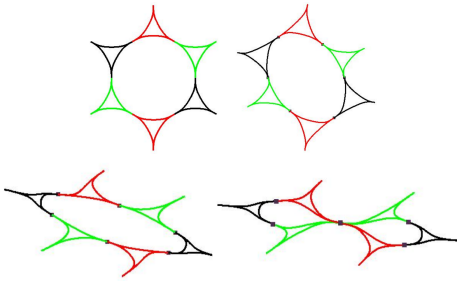


Fig. 5 Deformations and rotations around an oval.

the curved triangles in the localization row rotate in the opposite direction.

3 A simple model for buckling deformations

In this section we present a model with 2 degrees of freedom made of 3 bars linked with 2 hinges; one bar is linked to a fixed hinge whereas the third one is linked to an hinge which can move horizontally and on which a horizontal load is applied. The system may be described with 3 angles of the bars with respect to the unloaded configuration ϕ_1, ϕ_2, ϕ_3; see the figure below. The work of the applied load is Pw; where the horizontal displacements of the hinges are:

$u_0 = 0, u_1 = l_1(1-\cos(\phi_1))$, $u_2 = l_1(1-\cos(\phi_1)) + l_2(1-\cos(\phi_2))$,

$w = l_1(1-\cos(\phi_1)) + l_2(1-cos(\phi_2)) + l_3(1-\cos(\phi_3))$.

The horizontal ones are: $x_0 = 0, x_1 = l_1\sin(\phi_1), x_2 = l_2\sin(\phi_2), x_3 = 0$ and we have: $x_2 - x_1 = l_3\sin(\phi_3)$; $\phi_1 = \arcsin(\frac{x_1}{l_1})$, $\phi_2 = \arcsin(\frac{x_2}{l_2})$ and $\phi_3 = \arcsin(\frac{x_2-x_1}{l_3})$.

We assume that torsion springs are active on hinges A_1 and A_2; we denote with θ_1, θ_2 the relative angle of rotation of each bar with respect to its neighbor; $\theta_1(x_1,x_2) = \phi_1(x_1,x_2) - \phi_3(x_1,x_2)$, $\theta_2(x_1,x_2) = \phi_2(x_1,x_2) + \phi_3(x_1,x_2)$.

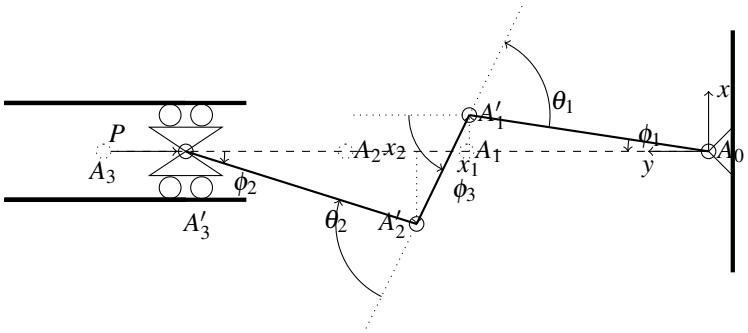
Assuming the torsion springs to be linear elastic, the strain energy is $\frac{1}{2}(K_1\theta_1^2 + K_2\theta_2^2)$. Then the total energy involving the work of the applied load is

$$V(x_1,x_2) = \frac{1}{2}[K_1(\theta_1(x_1,x_2))^2 + K_2(\theta_2(x_1,x_2))^2] - Pw(x_1,x_2). \qquad (1)$$

With this energy, we get the equilibrium equation: $\frac{\partial V}{\partial x_1} = 0$, $\frac{\partial V}{\partial x_2} = 0$.

To simplify the computations, we assume here that $l_1 = l_2 = l_3 = 1$ and that $K1 = K2 = 1$. Then by symmetry, the equilibrium can be reached only if $x_2 = x_1$ or $x_2 = -x_1$.

For $P < 1$, then $x = 0$ is the only stable equilibrium. For $1 < P < 3$, then $x = 0$ is an unstable equilibrium and two stable equilibriums appears for $x_2 = x_1$ and $P = \frac{arcsin(x_1)}{x_1}$ i.e $|x_1|$ almost equal to $\sqrt{(6(P-1)}$. For $P > 3$, two stable equilibriums appears for $x_2 = -x_1$ and $|x_1|$ almost equal to $\sqrt{(2(P-3)/3}$. It corresponds to the configuration shown in the next figure.



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