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# Hypermedia Synchronization: Modeling and Optimization with Graphs 

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## Introduction

The exploding use of Internet and of hypermedia documents has turned crucial the necessity to dispose of robust online algorithms to manage complexity and interactivity. One of the resulting problems which has emerged recently is the synchronization of hypermedia documents by considering that each object can be compressed or delayed like an elastic spring. The heterogeneity of the objects that compose a hypermedia document turns their presentation in time and space a hard problem. On the other hand, interactivity imposes that real-time updates of the schedule of the document should be possible, increasing the need for faster decision-making algorithms.

As explained in [2], such documents are composed of media objects (audio, video, text, image...), which duration of presentation must be adjusted to satisfy a set of temporal constraints that express the progress of the animation as defined by the author. But for these constraints to be satisfied, the author must accept some flexibility on the duration (that we call ideal) of presentation of each object, pauses being totally forbidden if not explicitly wanted. To estimate the quality of an adjustment, a cost function, usually convex (cf. figure 1), is introduced for each object. To sum up, the problem we attempt to solve here is to find an adjustment of best quality, i.e. which minimizes the sum of the costs of the media objects.


Figure 1: Examples of cost functions. a) Piecewise linear with a single ideal value. b) Non-linear, but convex and derivable. c) Piecewise linear with several ideal values. d) Discrete values.

The set of temporal constraints can be modeled as a directed graph $G=(X ; U)$ where $X$ is a set of nodes, $U$ a set of arcs, $m=|U|$ and $n=|X|$. The nodes represent events (the start or the end of presentation of an object). The arcs express duration constraints between nodes. With each arc $u$ is associated a time domain $S_{u}$, an ideal duration $o_{u}$ and a cost function $c_{u}$ defined on the domain $S_{u}$. An arc $u=(x ; y)$ between two nodes $x$ and $y$ means the event $x$ precedes $y$ and they are separated by a duration $\theta_{u} \in S_{u}$, the ideal value being $o_{u}$.

Let $\pi: X \mapsto \mathbb{R}$ be a potential function which assigns a date to each event node of the graph. Then the duration $\theta_{u}$ of an object associated with an arc $u=(x ; y)$ can be seen as the difference
of potentials $\theta_{u}=\pi_{y}-\pi_{x}$, in other words, $\theta=\left(\theta_{u}\right)_{u \in U}$ is a tension vector on the graph (e.g. [3]). Denoting by $A$ the incidence matrix of the graph, i.e. matrix $A$ of dimension $(m \times n)$ with the elements $a_{x u}$ equal to -1 (if $u$ leaves $x$ ), +1 (if $u$ comes to $x$ ) or 0 (any other case), the problem, called the minimum cost tension problem, is simply formulated as following:

$$
\left\{\begin{array}{l}
\text { minimize } \sum_{u \in U} c_{u}\left(\theta_{u}\right) \\
\text { with } \theta=A^{T} \pi, \theta \in S
\end{array}\right.
$$

## 1 Continuous Domain and Convex Costs Minimization

In a first step, we consider the domain $S_{u}$ of an arc $u$ as a continuous interval $\left[a_{u} ; b_{u}\right]$ and suppose the cost function $c_{u}$ to be convex two-piecewise linear cost as shown by figure 1a, the adaptation to more pieces of the methods described here is straightforward. Hence, we consider the cost function $c_{u}$ as following:

$$
c_{u}\left(\theta_{u}\right)=\left\{\begin{array}{l}
c_{u}^{1}\left(o_{u}-\theta_{u}\right), \text { if } \theta_{u}<o_{u} \\
c_{u}^{2}\left(\theta_{u}-o_{u}\right), \text { if } \theta_{u} \geq o_{u}
\end{array}\right.
$$

With convex piecewise linear costs, it is possible to model the problem with linear programs. It is a solution widely used in practice for the synchronization problem (e.g. [4]). Another way to solve the problem is the out-of-kilter algorithm first introduced for the minimum cost flow problem by D.R. Fulkerson in 1961 and then for the minimum cost tension problem by J.M. Pla in 1971. We proposed an adaptation of that method to piecewise linear costs. That algorithm is pseudopolynomial, $O\left(m^{2}(A+B)\right)$ operations where $A=\max _{u \in U}\left\{a_{u} ; b_{u}\right\}$ and $B=\max _{u \in U}\left\{c_{u}^{1 ;} ; c_{u}^{2}\right\}$. A polynomial method is presented in [3] but is only really efficient in practice for a special class of graphs (Penelope's graphs). More recently, [1] presents an algorithm to solve a more generic problem called the convex cost integer dual network flow problem, the algorithm consists in transforming the minimum cost tension problem into a minimum cost flow problem, solved with the well-known cost-scaling method. This algorithm is polynomial, $O\left(m n^{2} \log n A\right)$ operations, and proves to be very efficient in practice. After presenting briefly all these methods, we discuss a practical comparison of the methods and how they can be used in hypermedia synchronization systems and software.

## 2 Continuous Domain and Alterations Minimization

As explained in [5], authors of hypermedia documents often need to minimize the number of media objects that are not scheduled at their ideal value. It means the cost function $c_{u}$ of an arc $u$ is defined as following:

$$
c_{u}\left(\theta_{u}\right)=\left\{\begin{array}{l}
0, \text { if } \theta_{u}=o_{u} \\
1, \text { if } \theta_{u} \neq o_{u}
\end{array}\right.
$$

The definition domain $S_{u}$ of a duration $\theta_{u}$ is still supposed to be continuous as presented in the previous section. However, due to the nature of the cost functions, the techniques described before can not be used directly, since the problem now belongs to the family of mixed integer programming. We propose a method based on linear relaxation to solve the problem. We show that it is still possible to use the tension formulation within the relaxation, which allows us to
define efficient strategies based on methods presented in section 1 to get very good solutions in a reasonable amount of time.

## 3 Discrete Domain and Convex Costs Minimization

We consider now another variation of the problem described in section 1. Instead of considering a continuous domain for the tension of each arc, we are now dealing with discrete domains, e.g. the tension of each arc $u$ can only have some values belonging to a set $S_{u} \in \mathbb{R}^{k}$. We propose an integer formulation that involves binary/decision variables for each possible value of each set $S_{u}$. This kind of problem has been shown to be much harder to solve within the context of linear programming. Preliminary results are shown.

## References

[1] Ravindra K. Ahuja, Dorit S. Hochbaum, and James B. Orlin. Solving the Convex Cost Integer Dual Network Flow Problem. In 7th International IPCO Conference, 1999.
[2] M. Cecelia Buchanan and Polle T. Zellweger. Specifying Temporal Behavior in Hypermedia Documents. In European Conference on Hypertext '92, pages 262-271, 1992.
[3] Malika Hadjiat. Penelope's Graph: a Hard Minimum Cost Tension Instance. In Theoretical Computer Science, volume 194, pages 207-218. Elsevier Science, 1998.
[4] Michelle Y. Kim and Junehwa Song. Multimedia Documents with Elastic Time. In Multimedia '95, pages 143-154, 1995.
[5] M.T. Medina, C.C. Ribeiro, and L.F.G. Soares. Automatic Scheduling of Hypermedia Documents with Elastic Times. In Parallel Processing Letters, 2002.

