Gridshells without kink angle between beams and cladding panels

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Abstract
In double-curvature gridshells covered with rigid flat panels, panels cannot lay flat on the top surface of beams. In many cases, costly elements need to be inserted in-between to insure a proper beam-panel connection. Precedent research on the geometric rationalization of gridshells has mostly focused on minimizing cladding panel curvature, simplifying node and beam fabrication, and allowing for repeatability of elements. The connection between beams and panels has received very little attention. This paper explores the possible gridshell geometries under the constraint of having a full planar contact between beams and cladding panels. A strategy that consists of folding panels is studied in details. It turns out that a rich variety of panel shapes and folding patterns is possible. Two generation methods for such shapes are proposed: a method for quad meshes of revolution covered with folded panels, and a method for folded hexagonal panels based on the projection algorithm developed by Bouaziz *et al* [1]. The resulting structures are of particular interest for opaque doubly curved facades covered with metal sheets or other foldable material. The good contact between beams and panels also offers the possibility to use the panels as bracing elements. Hence the proposed method proves to be efficient for construction purposes, but also for mechanical behavior.

Keywords: Architectural geometry, structural morphology, gridshell, free-form envelope

1. Introduction
Free-form building envelopes have gained in popularity in the past decade. Many iconic complex doubly-curved facades have been erected. Geometric complexity of envelopes tends to increase drastically the price of a building, because non-standard complex elements have to be fabricated. However, a clever use of geometry can often significantly decrease the complexity of the fabrication process. A recent field of research has focused on the development of geometric methods to ease the fabrication of complex envelope shapes. The scientific community has focused on topics such as panelizing a surface with flat panels ([2], [3], [4]), generating meshes with torsion-free nodes and offsets ([5],[6],[7]…), repeating beams, panels ([8]) or nodes ([9],[10]), or optimizing the geometry to use a certain type of fabrication technic ([11]). Many of these complex envelopes are constituted of cladding panels supported by a beam lattice, such as a gridshell. To the best of our knowledge, no research has been dedicated to the rationalization of the connection between beams and cladding panels.

The main geometric factor that affects the connection between beams and cladding panels is the incidence angle between the panel and the top surface of the beam, which we will refer to as the *kink angle* (see Figure 1). One project for which this angle caused drastic cost increase is the Hungerburg Funicular Station, designed by Zaha Hadid. Because of the high variation of the kink angle, custom panel connections had to be milled for every single beam. Glass covered gridshells are also highly susceptible to the kink angle. Glass panels are usually connected to the gridshell by extruded aluminum profiles. A given profile can accommodate a certain range of kink angles. The higher the kink, the more complicated the profile has to be, and thus the more expansive.

Table 1 shows the kink angle for a range of surface curvature radius and panel width. It also gives the gap between the beam and the panel due to the kink angle for an 80mm wide beam – as shown on Figure 1. Gaps above 2mm appear systematically in areas of high curvature or when panels are relatively large. The kink angle can be significantly higher than in Table 1 if the beam is not oriented such that both adjacent panels have the same kink angle. This typically happens if the planes of symmetry of beams are set vertical in order to facilitate the beam-to-beam connections.

Table 1: Kink angle for various radii of curvature and panel sizes (gap between panel and a 80mm wide beam shown in brackets in mm)

Curvature radius (m)	Panel width (m)				
	0.4	0.6	0.8	1	1.2
3	3.8° (2.7)	5.7° (4)	7.7° (5.4)	9.6° (6.8)	11.5° (8.2)
5	2.3° (1.6)	3.4° (2.4)	4.6° (3.2)	5.7° (4)	6.9° (4.8)
7	1.6° (1.1)	2.5° (1.7)	3.3° (2.3)	4.1° (2.9)	4.9° (3.4)
10	1.1° (0.8)	1.7° (1.2)	2.3° (1.6)	2.9° (2)	3.4° (2.4)
15	0.8° (0.5)	1.1° (0.8)	1.5° (1.1)	1.9° (1.3)	2.3° (1.6)
20	0.6° (0.4)	0.9° (0.6)	1.1° (0.8)	1.4° (1)	1.7° (1.2)
25	0.5° (0.3)	0.7° (0.5)	0.9° (0.6)	1.1° (0.8)	1.4° (1)
30	0.4° (0.3)	0.6° (0.4)	0.8° (0.5)	1° (0.7)	1.1° (0.8)

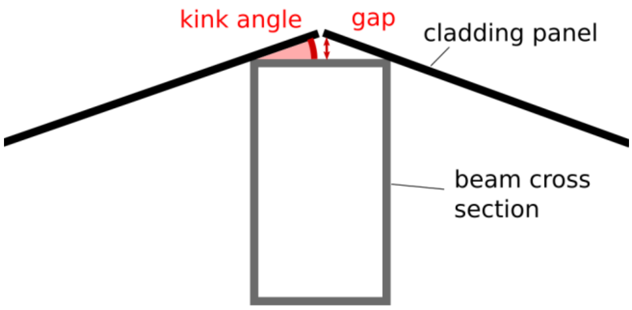

Figure 1: Kink angle between a cladding panel and a beam and corresponding gap

In this paper, we will focus on the following question: what are the possible geometric strategies to design a gridshell where the cladding panels can “lay flat” on the top surface of beams – i.e. with a null kink angle? This property allows the connection between panels and beams to be very simple. It can also allow to make such a connection structural, such that panels can be used for bracing purposes. In this study, we will consider beams for which the top surface is developable, which is the case of rectangular or I cross-section.

Given this constraint, the dihedral angle between adjacent panels is null. Therefore, the curvature of the envelope need to be located inside the panels. For this purpose, panels may be bent or folded. Section 2 will address shortly the first option. Section 3 will explore the different folding methods and the associated mesh topologies. This section is only relevant for cladding materials such as steel, aluminum or polycarbonate. Finally, in section 4 and 5, we will study two particular patterns with folded panels: simply folded quad panels on quadrangular gridshells, and tri-folded hexagonal panels.

2. Developable panels

A first solution to our problem is to bend initially flat panels. Thin panels can be deformed easily into nearly developable surfaces by hot or cold bending. Cold bending of glass panels is a popular method to cover a gridshell without planar faces with glass. This method was for example used for the facades of Strasbourg train station [12] and Vuitton Foundation in Paris.

Since we are dealing with cases where the top surface of the beam is a developable surface, the geometry of this surface can be fully described by one curve and one family of normal vectors, which defines the orientation of the top face at any point of the line. It can be shown ([13]) that there is only one developable surface which is incident to the curve and whose rulings are perpendicular to the normals. Therefore, two panels resting along the same beam belong necessarily to the same developable surface: their rulings are identical. As a result, a structure made of bent panels lying flat on their supporting beams can only describe one single developable surface, as illustrated in Figure 2.

The generation of developable surfaces is a topic that is already well researched. Thus, we will not discuss this option further.

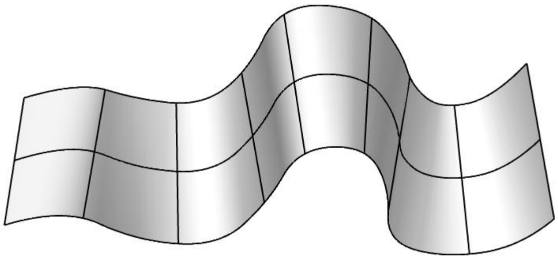

Figure 2 : Developable panels connected without kink angle form one large developable surface

3. Folded panels

A second solution to our problem is to fold the cladding panels. This section will explore folding methods and associated mesh topologies.

3.1. Types of folding

Folding has been the focus of a lot of research, see for example [14] for a review of folds related to structural application. We will here focus on possible ways to fold one panel in the context of our problem.

A first way to fold the panels is to follow the edge of the supporting beams, as shown on Figure 3. This solution can readily be applied to any mesh with planar faces. However, this method requires a lot of folding and cutting. Furthermore, special details are needed at the corner to insure water tightness.

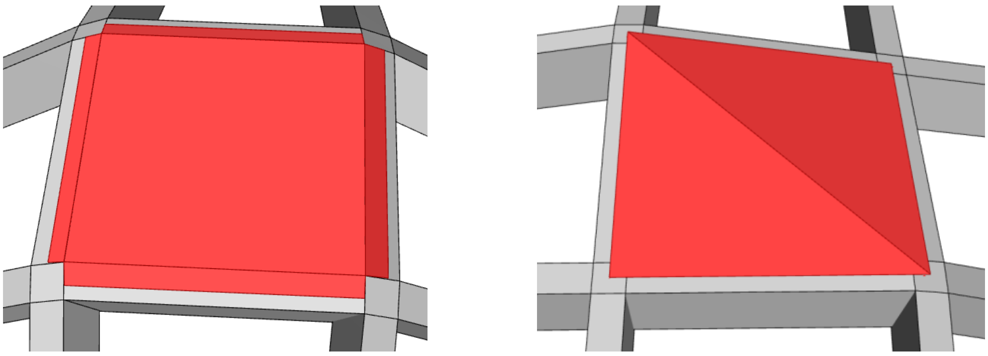

Figure 3: Panels folded along the beam edges (left) and along a diagonal (right)

A second way is to follow the diagonals, as shown on Figure 3. This method has been often used for doubly curved facades with metallic cladding. For examples the facade of Rhike Park in Tbilisi, Georgia and the Bus Umsteigebahnhof Poppenbüttel in Hamburg are covered with simply folded aluminum panels. The Kaiser Hawaiian geodesic dome uses a more elaborate type of folding: quads are folded along their diagonals to form series of pyramids.

For materials that cannot be folded, panels may just be cut along diagonals. This method was for example used in the gridshell of the History Museum of Hamburg ([15]), for which some glass panels had to be cut diagonally to fit the gridshells in highly curved faces.

In origamis folding such as the Miura pattern, multiple fold lines may end at the same vertex. This type of folding is too complicated technologically to be applied to a cladding panel. Thus we will only consider fold lines that start and end on the boundary of a panel.

Fold lines may be curved. In that case the panel is not flat on either side. The fabrication of panels with curved fold lines is highly complex. However, recent projects such as ARUM, the Zaha Hadid exhibit for the 2012 Venice biennale, showed the potential of robotics to make complex curved folds.

Complex panel deformations can be obtained by pressing. Recently, Trautz *et al* [16] proposed a technique to create complex crease patterns on a metal sheet by incrementally pressing it with a moving dice.

Amongst all these folding methods, we shall only consider straight folds along panel diagonals, as this is the technically simplest and the least expensive option.

3.2. Geometry and topology of meshes

Envelopes covered with folded panels can be geometrically described by two meshes, as shown in Figure 4 (in which panels are shown in yellow and grey):

- A *structural mesh*, which represent the top centerline of support beams. This mesh also corresponds to the edges of the panels.
- A *fold mesh* (dashed), which corresponds to all the fold lines. Each node of this mesh has to correspond with one node of the structural mesh. Faces of this mesh are also planar.

As seen on Figure 4, a node of the structural mesh may or may not be a node of the fold mesh. In the second case, the node is necessarily flat: all incident planes are coplanar. Such nodes are very simple to fabricate.

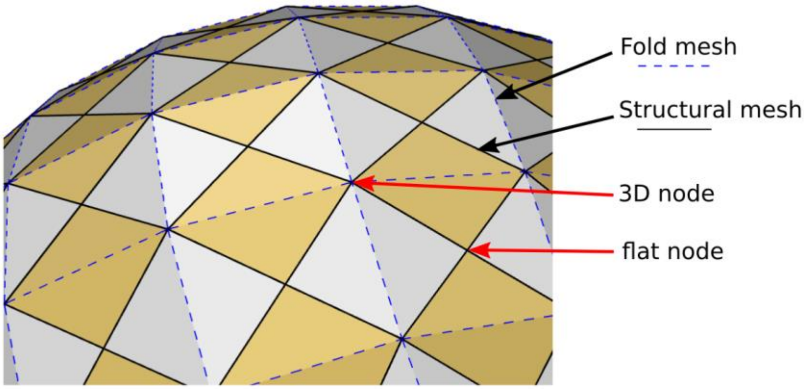

Figure 4: Fold mesh and structural mesh

3.3. Combination of structural and envelope meshes

Figure 4 shows a structure in which both the fold mesh and the structural mesh are quadrangular. However, many other mesh topologies are possible. Figure 5 shows four combinations of structural and fold meshes:

- In pattern (a), a hexagonal beam layout is covered with triply folded hexagons. The fold mesh is triangular. This configuration has nodes with valence 3, which simplifies their fabrication.

- In pattern (b), quads are folded along their diagonal and assembled in a tiling which is the dual of the Archimedean tiling referred to as (3^4,6). The fold mesh is a Kagome mesh. This pattern has flat nodes of valence 3 and 6 and 3D nodes of valence 4.
- Pattern (c) is the one shown on Figure 4, with quadrangular fold and structural meshes.
- In pattern (d), folded quad panels are assembled to form a hexagonal fold mesh, while the structural mesh has the topology of the dual of the Archimedean tiling (3,4,6,4).

We will study in more details patterns (c) in section 4, as it is simple and efficient mechanically. We will also see in Section 5 that pattern (a) is much less constrained geometrically for a gridshell with torsion-free nodes thanks to the lower valence of the nodes.

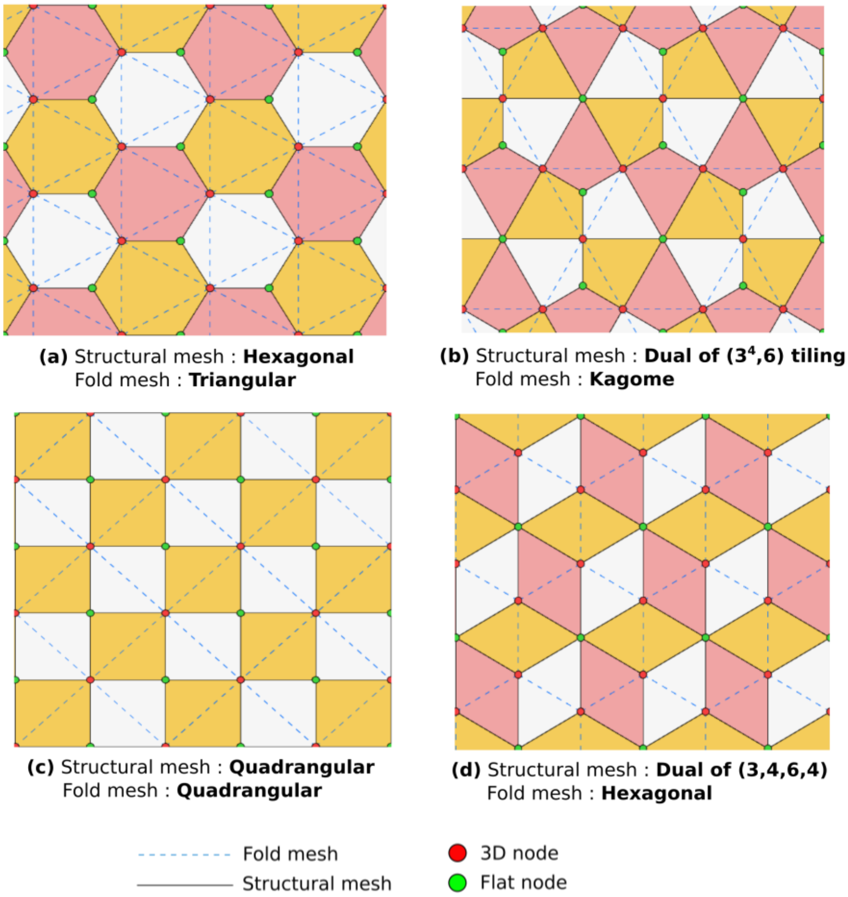

Figure 5: Various combinations of structural and envelope meshes

3.4. Structural considerations

Since panels lay flat on the top beam surface, the connection between beams and panels can be structural – for example bolted or nailed. Panels can then be used as bracing elements for the beams. In that case, diagonal cables or moment connection are not needed anymore, thus lowering the cost of the structure. Since this paper focuses on form exploration, mechanical performance and structural details will be addressed in future work.

4. Quad meshes with torsion-free nodes

In this section, we analyze the geometry of meshes with the quadrangular pattern shown in Figure 5 (c). This pattern can be applied to any quadrangular mesh with planar faces – referred to as *PQ mesh*. We will focus here on meshes with torsion free nodes, which are much more cost effective and also more challenging geometrically.

For such meshes, the fold mesh is a PQ mesh. Inside each face of this mesh, there is a flat node of the structural mesh. The axis of this node is necessarily perpendicular to the face. For nodes to be torsion-free, this axis needs to be coplanar with the axes of the adjacent nodes of the structural mesh, which are 3D nodes. This property is shown on Figure 6:

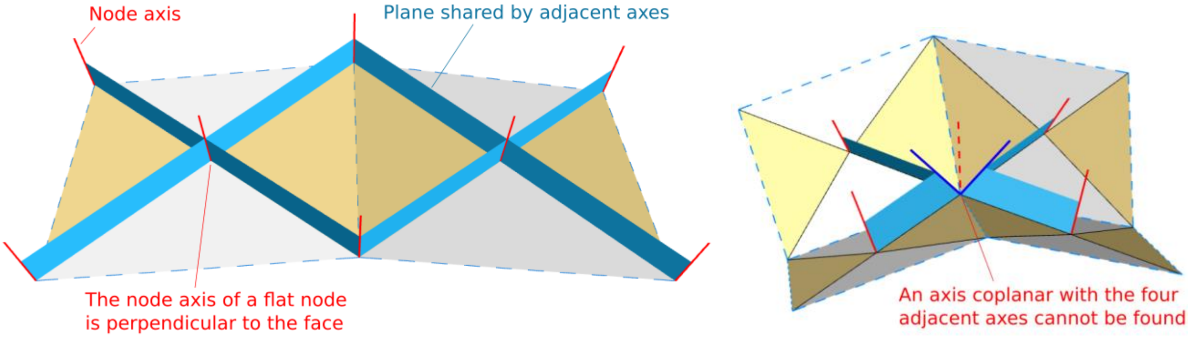

Figure 6: Left: Axes of a torsion-free mesh with folded quads.
Right: Torsion-free nodes cannot be obtained at a 3D node with negative curvature.

Such meshes cannot be negatively curved. One way to understand this is by looking at a fold mesh, as shown on Figure 6. If the node has a negative curvature, we cannot find an axis which is coplanar with the axes of the flat nodes.

If the width of faces of the mesh tends towards zero, such that the mesh tends towards a smooth surface, the fold mesh will tend towards a conjugate networks, as it is a PQ mesh ([17]). On the other hand, the structural mesh (and the diagonals of the PQ mesh) will tend towards principal curvature lines, because of the torsion-free property of the normals. These properties are shown on Figure 7*Figure 7*. They constrain entirely the orientation and aspect ratio of faces of the mesh. This aspect ratio can be directly estimated using the Dupin indicatrix of the surface, which is an ellipse of aspect ratio k_1/k_2 whose main axes are aligned with principal curvature directions (where k_1 is the maximum principal curvature and k_2 the minimum one). The fold mesh being a PQ mesh, nodes of a given face can be approximated by this ellipse. Since the diagonals are aligned with curvature direction, the length ratio of the diagonals is roughly k_1/k_2.

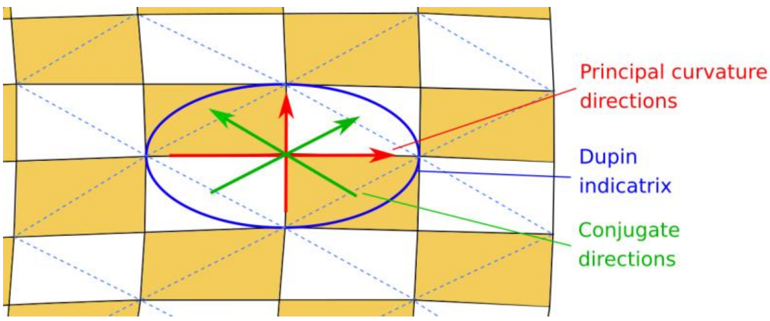

Figure 7: Constraints on the aspect ratio and orientation of a quad of the fold mesh

Because of the high level of constraints, it was found that optimizing a mesh to give it all the above properties gives poor results. However, it is possible to generate meshes of revolutions in a parametric manner that fulfill all these properties exactly. An example is shown on Figure 8. The geometry of this dome was adjusted such that the planes of symmetry of the beams at the planar nodes intersect at 90 degrees. This property simplifies node fabrication.

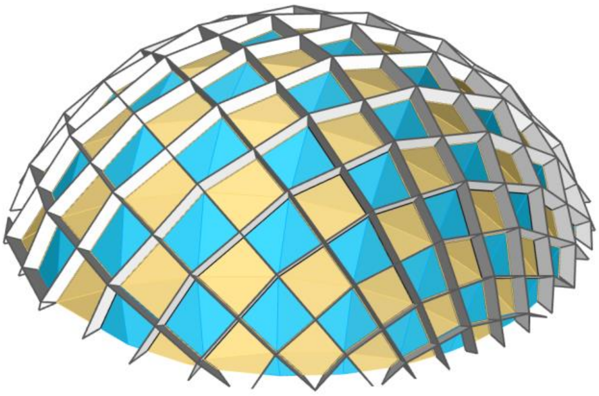

Figure 8: Mesh of revolution with torsion-free nodes

5. Tri-folded hexagonal panels on torsion-free gridshell

One major drawback of quad meshes studied in the previous section is that they do not allow negative curvature. Thanks to the lower valence of the nodes, the pattern shown on Figure 5 (a) does not have this limitation, and its geometry is much less constrained. As a result, meshes can be generated by optimization methods. Such a method was implemented with the plugin Kangaroo2 for Rhino – Grasshopper, which uses the algorithm developed by Bouaziz et al [1]. Figure 9 shows a mesh generated with this method and the set of constraints associated to a face. In areas of negative curvature, hexagons do not need to be in a bow-tie shape, because they do not need to be planar. This gives a significant structural and aesthetic advantage compared to hexagonal gridshells with planar faces.

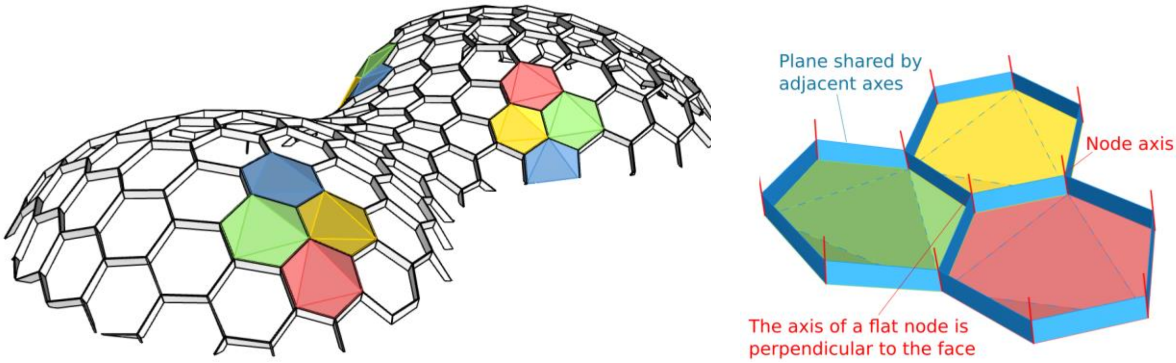

Figure 9: Left: A torsion-free mesh that can be covered with tri-folded hexagons lying flat on beams.
Right: Geometrical constraints associated with the mesh

Conclusion and future work

This paper presented different ways to design a gridshell under the constraint of having panels lying flat on the top surface of beams. The proposed methods are of particular interest for opaque envelopes with high curvature, coarse panelization, or when cladding panels are aimed at bracing the beam network. Future work will address the mechanical behavior of the proposed solutions in this last case. Also, the feasibility of folded hexagonal sandwich panels will be studied. A prototype is being done at scale 1:1 to investigate possible fabrication details. This work is an example of how constraints can paradoxically stimulate creativity to find structural and technical solutions.

Acknowledgements

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