



HAL
open science

Mathematical Morphology on Tensor Images for Fiber Enhancement

Samy Blusseau, Isabelle Bloch, Y. Gousseau, S Velasco-Forero, Jesus Angulo

► **To cite this version:**

Samy Blusseau, Isabelle Bloch, Y. Gousseau, S Velasco-Forero, Jesus Angulo. Mathematical Morphology on Tensor Images for Fiber Enhancement. SIAM Conference on IMAGING SCIENCE, Jun 2018, Bologna, Italy. hal-01984204

HAL Id: hal-01984204

<https://hal.science/hal-01984204v1>

Submitted on 16 Jan 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

MATHEMATICAL MORPHOLOGY ON TENSOR IMAGES FOR FIBER ENHANCEMENT



S. Blusseau^{1,2}, I. Bloch², Y. Gousseau², S. Velasco-Forero¹, J. Angulo¹

1: CMM, Mines ParisTech, PSL Research University, France 2: LTCI, Télécom ParisTech, Université Paris-Saclay, France



We introduce morphological methods to analyse images of positive semi-definite matrices (ellipsoids), such as diffusion or structure tensors. Morphological operators cannot be directly defined on these objects because they lack a complete lattice structure. Therefore, we extend recent developments in morphological signal processing on graphs. Graphs are constructed based on all the available information, whereas a scalar signal (anisotropy) is processed on these graphs, successfully enhancing structures such as vessels and fibers.

Basics of Mathematical Morphology

Complete lattices and morphological operators

Complete lattice A complete lattice (L, \leq) is a partially ordered set for which every subset $A \subseteq L$ has a supremum noted $\bigvee A$ and an infimum noted $\bigwedge A$.

In the following, (L, \leq) and (L', \leq') denote two complete lattices.

Dilation, Erosion

- A *dilation* is a mapping $\delta : L \rightarrow L'$ that commutes with the supremum:

for any family $(x_k)_{k \in K}$ of L , $\delta(\bigvee_{k \in K} x_k) = \bigvee_{k \in K} \delta(x_k)$, where K is an index set.

- An *erosion* is a mapping $\varepsilon : L' \rightarrow L$ that commutes with the infimum: $\varepsilon(\bigwedge_{k \in K} x_k) = \bigwedge_{k \in K} \varepsilon(x_k)$.

Opening, Closing

- A *closing* is a mapping $\phi : L \rightarrow L$ that is increasing, extensive and idempotent.

- An *opening* is a mapping $\gamma : L \rightarrow L$ that is increasing, anti-extensive and idempotent.

Adjunction A pair of mappings (ε, δ) , $\varepsilon : L' \rightarrow L$, $\delta : L \rightarrow L'$, forms an adjunction if and only if $\forall x \in L, y \in L', \delta(x) \leq' y \iff x \leq \varepsilon(y)$.

The data

The cone of positive semi-definite matrices S_n^+

No natural lattice ordering

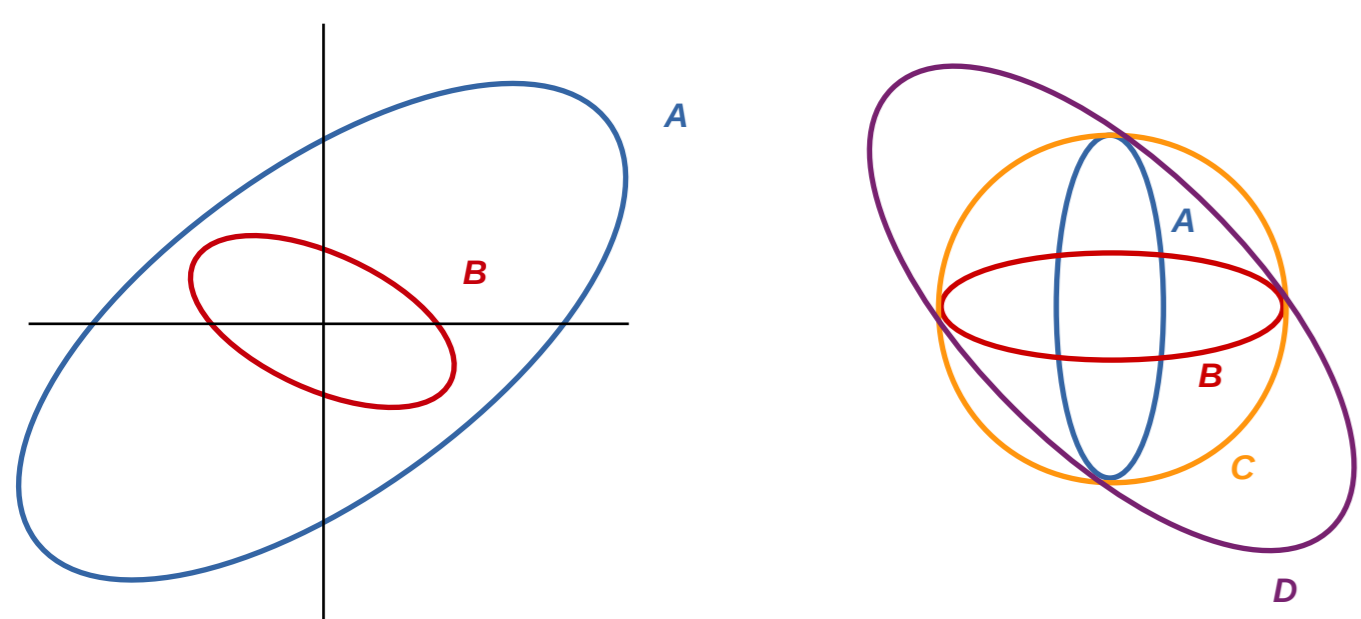


Figure 2: The Loewner ordering is defined by $B \leq A \iff A - B \in S_n^+$ (Left). The example on the right shows that this is not a lattice ordering: if it were, then C would be the supremum of A and B , but D is another upper bound of A and B and yet is not comparable with (and therefore not larger than) C .

Morphological signal processing on graphs

Graph. $G = (V, E)$, weighted and directed graph, containing n vertices.

Adjacency matrix. $W \in Mat_n(\mathbb{R} \cup \{-\infty\})$ such that for any (i, j) , $1 \leq i, j \leq n - \infty \leq w_{ij} \leq 0$, $w_{ii} = 0$, and $-\infty < w_{ij} \iff (i, j) \in E$ (called Conservative morphological weight matrix in [3]).

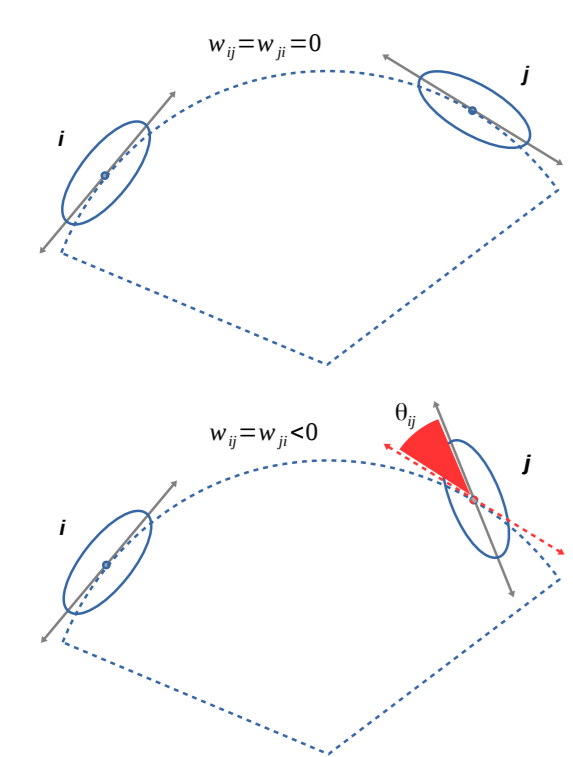
Lattice. We work with vectors $x \in [0, 1]^n$ supported by the graph G , so the considered complete lattice is $L = ([0, 1]^n, \leq)$ equipped with the usual product partial ordering (Pareto ordering):

$$x \leq y \iff x_i \leq y_i \quad \forall i \in \{1, \dots, n\}.$$

Co-circularity graph

In our application the vertices are the pixels or voxels of an image (2D or 3D), thus characterised by their coordinates and a scalar value (the anisotropy) [1].

Measure of co-circularity



Coefficient w_{ij} is computed based on the principal orientations of the ellipsoids in i and j , and their relative positions in the image. Given the angular error θ_{ij} to cocircularity (see illustration), we distinguish two cases:

Binary case with threshold α : $w_{ij} = \begin{cases} 0 & \text{if } \cos(\theta_{ij}) \geq \cos(\alpha) \\ -\infty & \text{otherwise} \end{cases}$

Non-binary case: $w_{ij} = \log(\cos(\theta_{ij}))$

Results and discussion

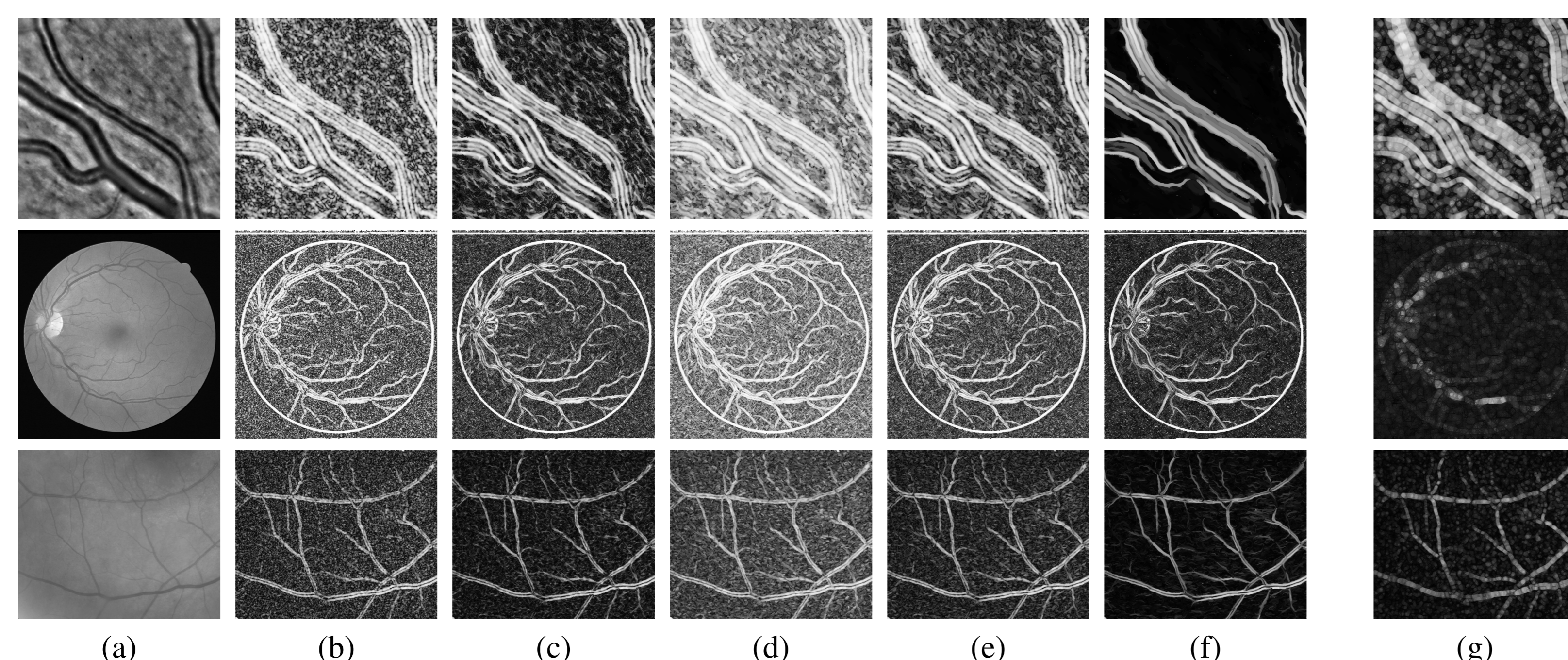


Figure 5: Results of the presented method based on co-circularity graphs (except last column). (a) Input images, (b) Anisotropy images (first and third rows: $\sigma = 3$; second row: $\sigma = 1$), (c) erosions ε_W , (d) dilations δ_W (e) openings γ_W , (f) openings $\gamma_W^{(p)}$ (first row: $p = 10$, second and third rows: $p = 5$). Here the adjacency matrices W are binary, with angular threshold $\alpha = \frac{\pi}{6}$ and spatial neighbourhood parameter k (first row: $k = 7$; second row: $k = 3$, third row: $k = 5$). (g) Results of directional openings by a family of straight lines structuring elements (classical method in MM).

Examples

Proposition 1. If (ε, δ) is an adjunction, then $\phi = \varepsilon\delta$ is a closing and $\gamma = \delta\varepsilon$ is an opening.

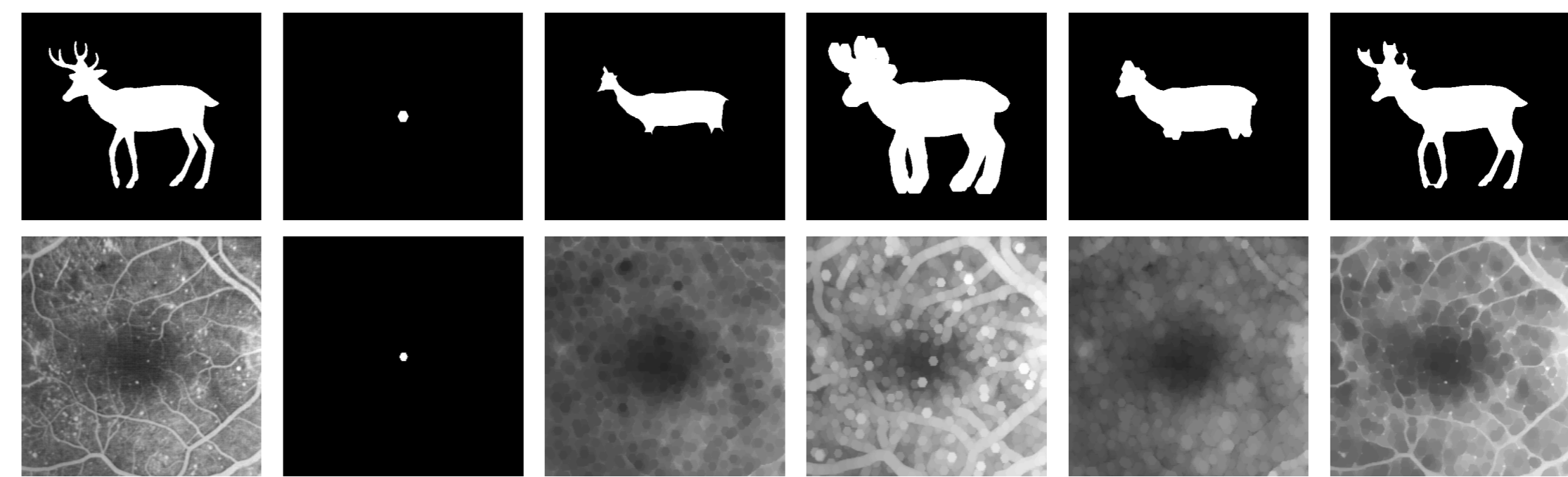


Figure 1: Example of morphological operators depending on a structuring element. (a) Input image: binary image on the first row, greyscale image on the second row. (b) Structuring element (SE) B . (c) Erosion ε_B . (d) Adjoint dilation δ_B . (e) Opening $\delta_B\varepsilon_B$. (f) Closing $\varepsilon_B\delta_B$.

Structure tensors for greyscale 2D images

We consider a greyscale image f , whose pixels coordinates are noted (x, y) .

Scale zero tensor $T_0 : (x, y) \mapsto \nabla f(x, y) \cdot \nabla f(x, y)^T$

Scale $\sigma > 0$ tensor $T_\sigma = G_\sigma * T_0$
where G_σ is the σ -scale Gaussian kernel.

Anisotropy image $A_\sigma = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$,
where $0 \leq \lambda_2(x, y) \leq \lambda_1(x, y)$ are the eigenvalues of $T_\sigma(x, y)$.

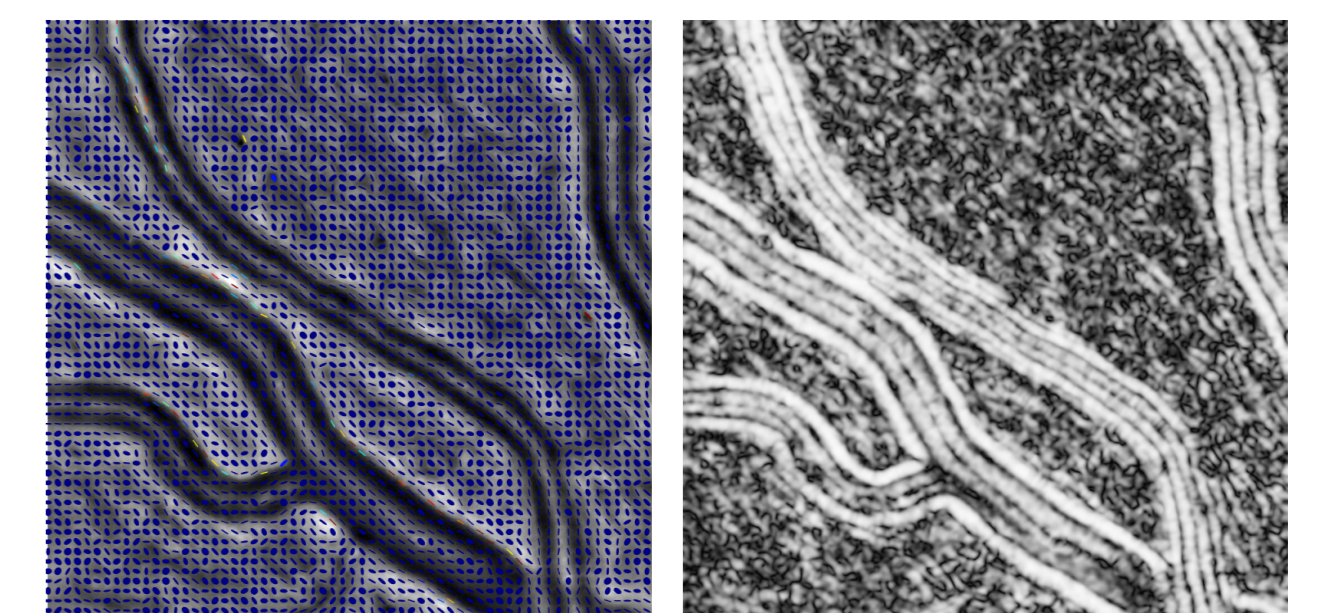


Figure 3: Image of structure tensors T_σ (obtained thanks to [2]) and the corresponding anisotropy image ($\sigma = 3$).

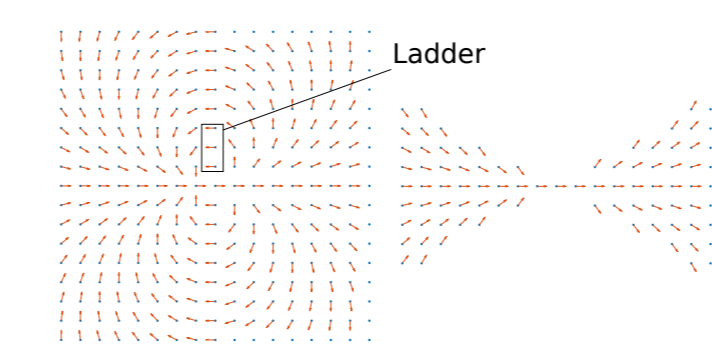
Dilations, erosions, openings. We define the dilation δ_W and its adjoint erosion ε_W on L by

$$\delta_W(x)_i = \max_{1 \leq j \leq n} (x_j + w_{ij}) = \max_{j \in \mathcal{N}_i} (x_j + w_{ij}), \quad \varepsilon_W(x)_i = \min_{1 \leq j \leq n} (x_j - w_{ji}) = \min_{j, i \in \mathcal{N}_j} (x_j - w_{ji}) \quad (1)$$

where $\mathcal{N}_i = \{j \in \{1, \dots, n\}, (i, j) \in E\}$ is the set of neighbours of vertex i in G . We also consider their iterated erosions ε_W^p and dilations δ_W^p , from which we build the openings $\gamma_W^{(p)}$ for $p \in \mathbb{N}$:

$$\varepsilon_W^p = \varepsilon_W \circ \dots \circ \varepsilon_W, \quad \delta_W^p = \delta_W \circ \dots \circ \delta_W, \quad \gamma_W^{(p)} = \delta_W^p \varepsilon_W^p.$$

Additional constraints



Spatial constraint: Only pairs of vertices whose coordinates lie in a limited spatial neighbourhood can be neighbours. Denoting u_i and u_j the coordinates of pixels (or voxels) i and j , we impose $w_{ij} > -\infty \iff \|u_i - u_j\|_\infty \leq k$, for a fixed integer k .

Conic constraint: Co-circularity allows ladder configurations. We avoid these by an additional constraint.

Property of the graph

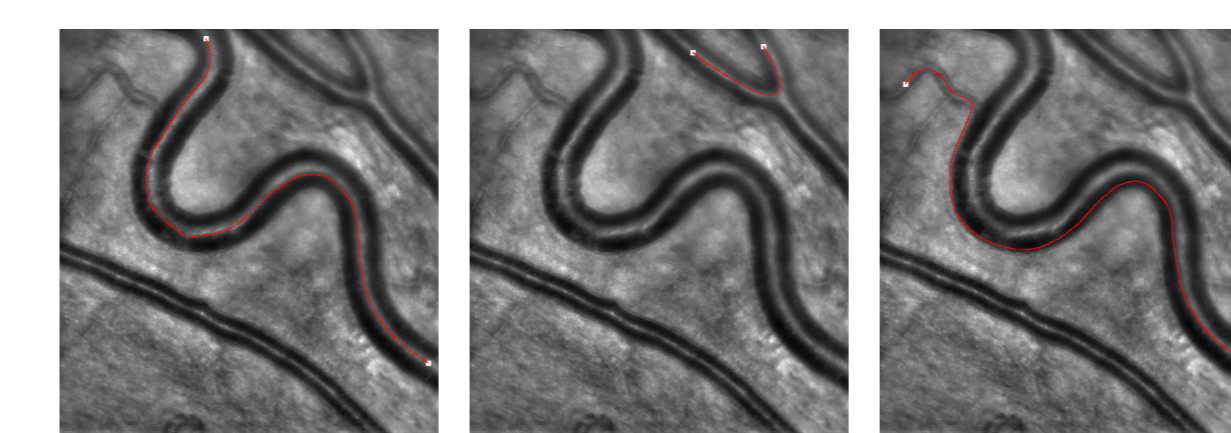


Figure 4: Maximal weight paths between pairs of vertices in a binary co-circularity graph. The weight of a path i_1, i_2, \dots, i_q is the non-positive sum $\sum_{l=1}^{q-1} w_{i_l i_{l+1}}$. The paths with large weights are those with weights close to 0, and which therefore have the strongest contribution in the dilations and erosions defined above.

Contributions We introduced a framework that generalises mathematical morphology for signals on graphs. The operators we defined based on the co-circularity of structure tensors show interesting behaviours for the processing of anisotropic structures.

Future work

- multi-scale approach or an automatic scale determination,
- comparison with other methods, such as path openings and other morphological connected filters,
- exploration of the properties of non-binary co-circularity graphs,
- links between these operators and the max-plus algebra [1].

References

- [1] S. Blusseau, S. Velasco-Forero, J. Angulo, and I. Bloch. Tropical and morphological operators for signal processing on graphs. In *25th IEEE International Conference on Image Processing (ICIP)*, 2018.
- [2] G. Peyré. The numerical tours of signal processing. *Computing in Science & Engineering*, 13(4):94–97, 2011.
- [3] S. Velasco-Forero and J. Angulo. Nonlinear operators on graphs via stacks. In F. Nielsen and F. Barbaresco, editors, *Geometric Science of Information, Proceedings*, pages 654–663. Springer International Publishing, 2015.