



Mathematical Morphology on Tensor Images for Fiber Enhancement

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We introduce morphological methods to analyse images of positive semi-definite matrices (ellipsoids), such as diffusion or structure tensors. Morphological operators cannot be directly defined on these objects because they lack a complete lattice structure. Therefore, we extend recent developments in morphological signal processing on graphs. Graphs are constructed based on all the available information, whereas a scalar signal (anisotropy) is processed on these graphs, successfully enhancing structures such as vessels and fibers.

Basics of Mathematical Morphology

Complete lattices and morphological operators

Complete lattice A complete lattice (L, \leq) is a partially ordered set for which every subset $A \subseteq L$ has a supremum noted $\bigvee A$ and an infimum noted $\bigwedge A$.

In the following, (L, \leq) and (L', \leq') denote two complete lattices.

Dilation, Erosion

- A *dilation* is a mapping $\delta : L \rightarrow L'$ that commutes with the supremum:

for any family $(\mathbf{x}_k)_{k \in K}$ of L , $\delta(\bigvee_{k \in K} \mathbf{x}_k) = \bigvee_{k \in K} \delta(\mathbf{x}_k)$, where K is an index set.

- An *erosion* is a mapping $\varepsilon : L' \rightarrow L$ that commutes with the infimum: $\varepsilon(\bigwedge_{k \in K} \mathbf{x}_k) = \bigwedge_{k \in K} \varepsilon(\mathbf{x}_k)$.

Opening, Closing

- A *closing* is a mapping $\phi : L \rightarrow L$ that is increasing, extensive and idempotent.

- An *opening* is a mapping $\gamma : L \rightarrow L$ that is increasing, anti-extensive and idempotent.

Adjunction A pair of mappings (ε, δ) , $\varepsilon : L' \rightarrow L$, $\delta : L \rightarrow L'$, forms an adjunction if and only if $\forall x \in L, y \in L', \delta(x) \leq' y \iff x \leq \varepsilon(y)$.

The data

The cone of positive semi-definite matrices S_n^+

No natural lattice ordering

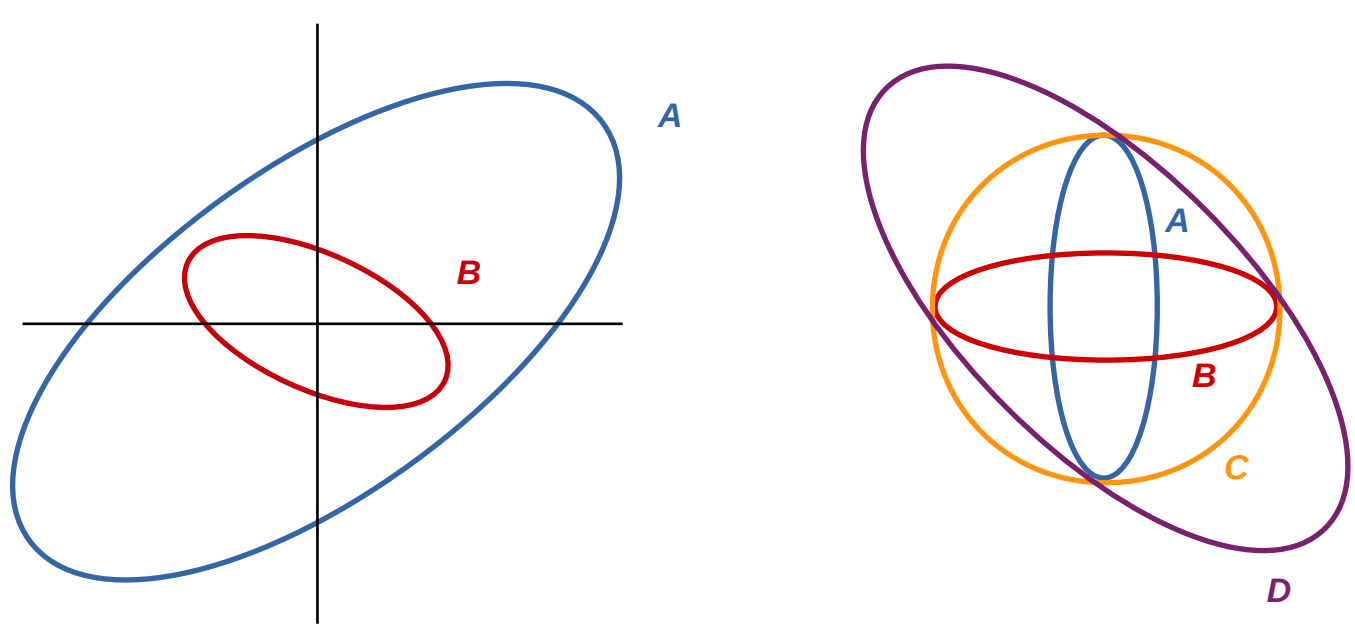


Figure 2: The Loewner ordering is defined by $B \leq A \iff A - B \in S_n^+$ (Left). The example on the right shows that this is not a lattice ordering: if it were, then C would be the supremum of A and B , but D is another upper bound of A and B and yet is not comparable with (and therefore not larger than) C .

Morphological signal processing on graphs

Graph. $G = (V, E)$, weighted and directed graph, containing n vertices.

Adjacency matrix. $W \in Mat_n(\mathbb{R} \cup \{-\infty\})$ such that for any (i, j) , $1 \leq i, j \leq n$, $-\infty \leq w_{ij} \leq 0$, $w_{ii} = 0$, and $-\infty < w_{ij} \iff (i, j) \in E$ (called Conservative morphological weight matrix in [3]).

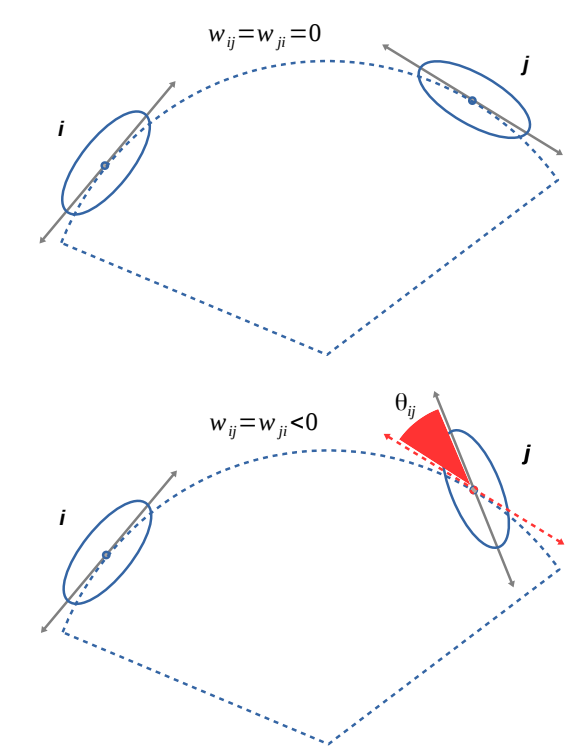
Lattice. We work with vectors $\mathbf{x} \in [0, 1]^n$ supported by the graph G , so the considered complete lattice is $L = ([0, 1]^n, \leq)$ equipped with the usual product partial ordering (Pareto ordering):

$$\mathbf{x} \leq \mathbf{y} \iff x_i \leq y_i \quad \forall i \in \{1, \dots, n\}.$$

Co-circularity graph

In our application the vertices are the pixels or voxels of an image (2D or 3D), thus characterised by their coordinates and a scalar value (the anisotropy) [1].

Measure of co-circularity



Coefficient w_{ij} is computed based on the principal orientations of the ellipsoids in i and j , and their relative positions in the image. Given the angular error θ_{ij} to cocircularity (see illustration), we distinguish two cases:

Binary case with threshold α : $w_{ij} = \begin{cases} 0 & \text{if } \cos(\theta_{ij}) \geq \cos(\alpha) \\ -\infty & \text{otherwise} \end{cases}$

Non-binary case: $w_{ij} = \log(\cos(\theta_{ij}))$

Results and discussion

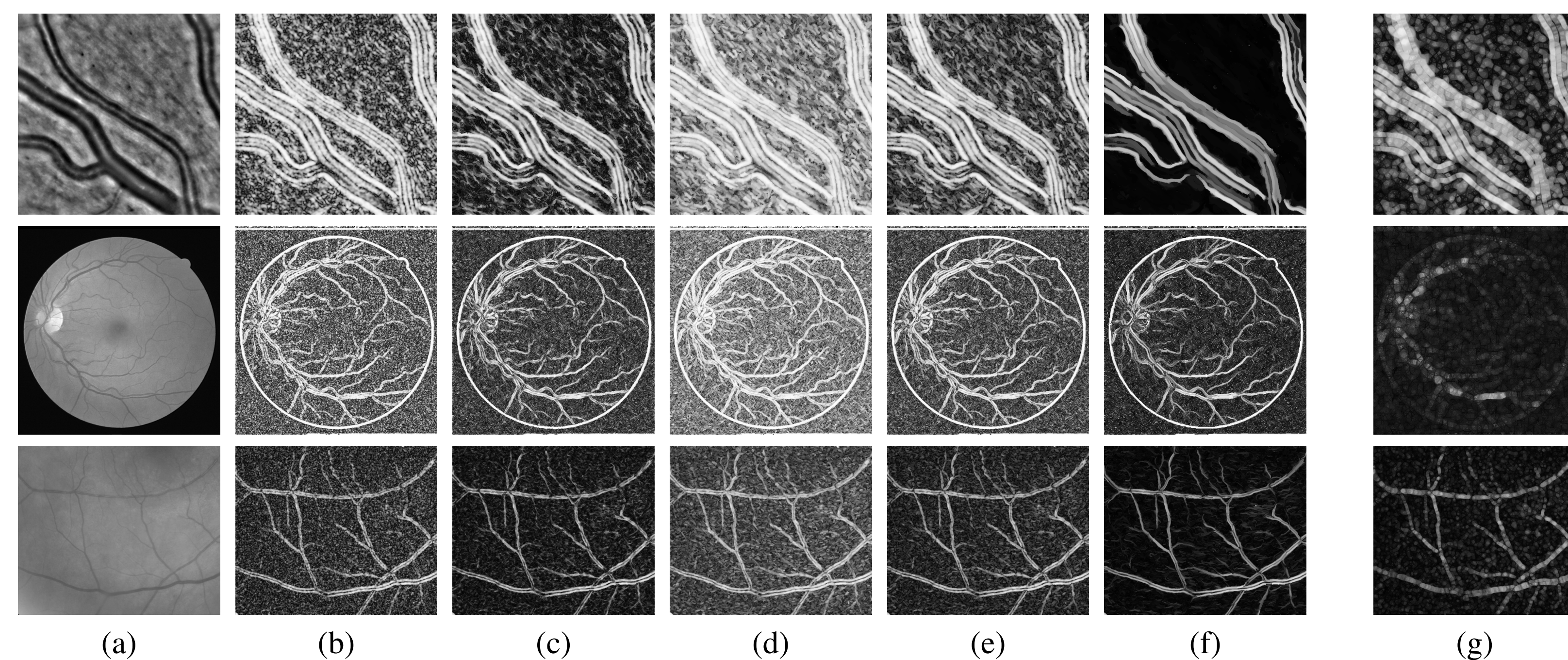


Figure 5: Results of the presented method based on co-circularity graphs (except last column). (a) Input images, (b) Anisotropy images (first and third rows: $\sigma = 3$; second row: $\sigma = 1$), (c) erosions ε_W , (d) dilations δ_W (e) openings γ_W , (f) openings $\gamma_W^{(p)}$ (first row: $p = 10$, second and third rows: $p = 5$). Here the adjacency matrices W are binary, with angular threshold $\alpha = \frac{\pi}{6}$ and spatial neighbourhood parameter k (first row: $k = 7$; second row: $k = 3$, third row: $k = 5$). (g) Results of directional openings by a family of straight lines structuring elements (classical method in MM).

Examples

Proposition 1. If (ε, δ) is an adjunction, then $\phi = \varepsilon\delta$ is a closing and $\gamma = \delta\varepsilon$ is an opening.

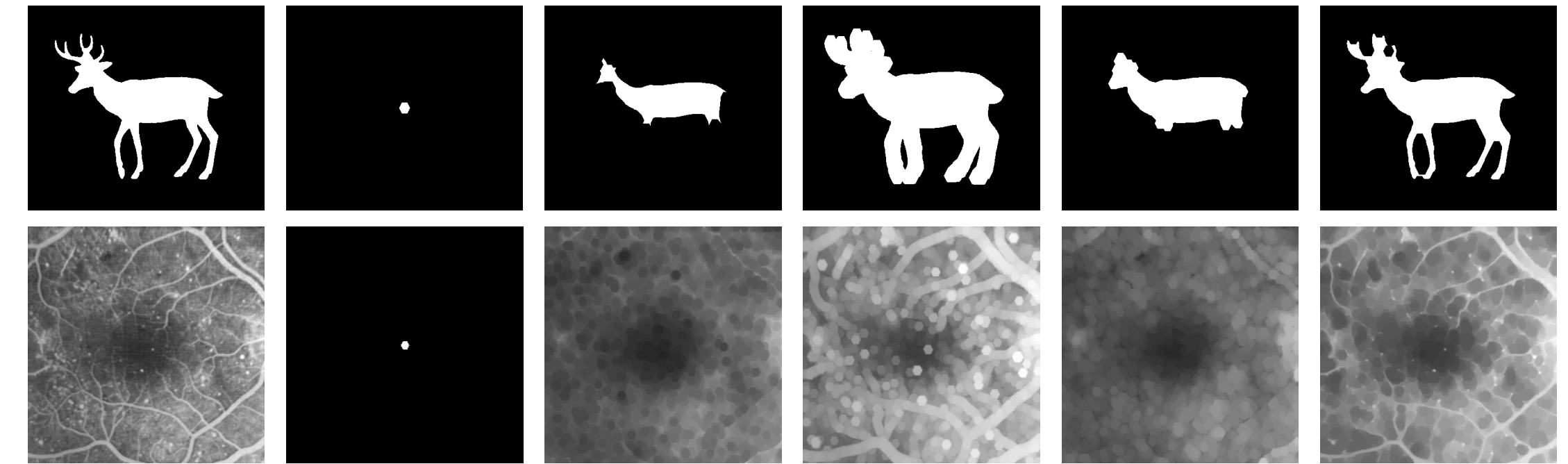


Figure 1: Example of morphological operators depending on a structuring element. (a) Input image: binary image on the first row, greyscale image on the second row. (b) Structuring element (SE) B . (c) Erosion ε_B . (d) Adjoint dilation δ_B . (e) Opening $\delta_B\varepsilon_B$. (f) Closing $\varepsilon_B\delta_B$.

Structure tensors for greyscale 2D images

We consider a greyscale image f , whose pixels coordinates are noted (x, y) .

Scale zero tensor $T_0 : (x, y) \mapsto \nabla f(x, y) \cdot \nabla f(x, y)^T$

Scale $\sigma > 0$ tensor $T_\sigma = G_\sigma * T_0$
where G_σ is the σ -scale Gaussian kernel.

Anisotropy image $A_\sigma = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$,
where $0 \leq \lambda_2(x, y) \leq \lambda_1(x, y)$ are the eigenvalues of $T_\sigma(x, y)$.

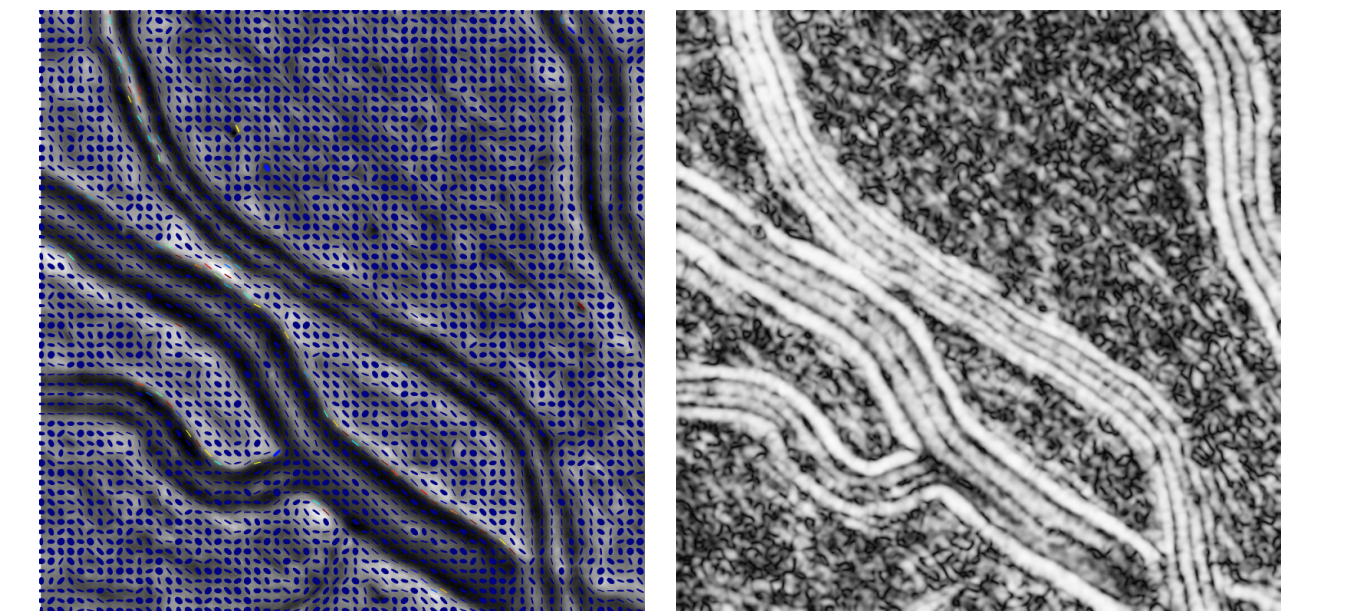


Figure 3: Image of structure tensors T_σ (obtained thanks to [2]) and the corresponding anisotropy image ($\sigma = 3$).

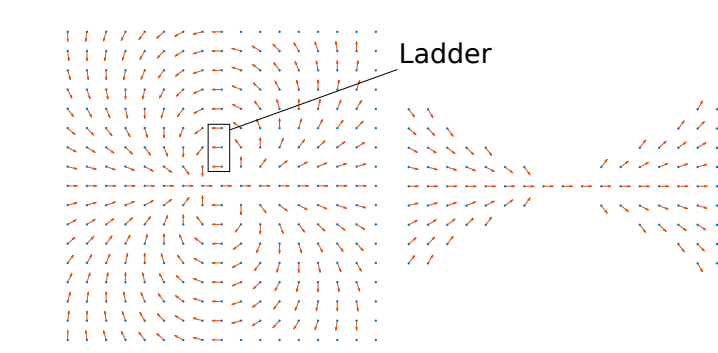
Dilations, erosions, openings. We define the dilation δ_W and its adjoint erosion ε_W on L by

$$\delta_W(x)_i = \max_{1 \leq j \leq n} (x_j + w_{ij}) = \max_{j \in \mathcal{N}_i} (x_j + w_{ij}), \quad \varepsilon_W(x)_i = \min_{1 \leq j \leq n} (x_j - w_{ji}) = \min_{j, i \in \mathcal{N}_j} (x_j - w_{ji}) \quad (1)$$

where $\mathcal{N}_i \doteq \{j \in \{1, \dots, n\}, (i, j) \in E\}$ is the set of neighbours of vertex i in G . We also consider their iterated erosions ε_W^p and dilations δ_W^p , from which we build the openings $\gamma_W^{(p)}$ for $p \in \mathbb{N}$:

$$\varepsilon_W^p = \varepsilon_W \circ \dots \circ \varepsilon_W, \quad \delta_W^p = \delta_W \circ \dots \circ \delta_W, \quad \gamma_W^{(p)} = \delta_W^p \varepsilon_W^p.$$

Additional constraints



Spatial constraint: Only pairs of vertices whose coordinates lie in a limited spatial neighbourhood can be neighbours. Denoting u_i and u_j the coordinates of pixels (or voxels) i and j , we impose $w_{ij} > -\infty \iff \|u_i - u_j\|_\infty \leq k$, for a fixed integer k .

Conic constraint: Co-circularity allows ladder configurations. We avoid these by an additional constraint.

Property of the graph

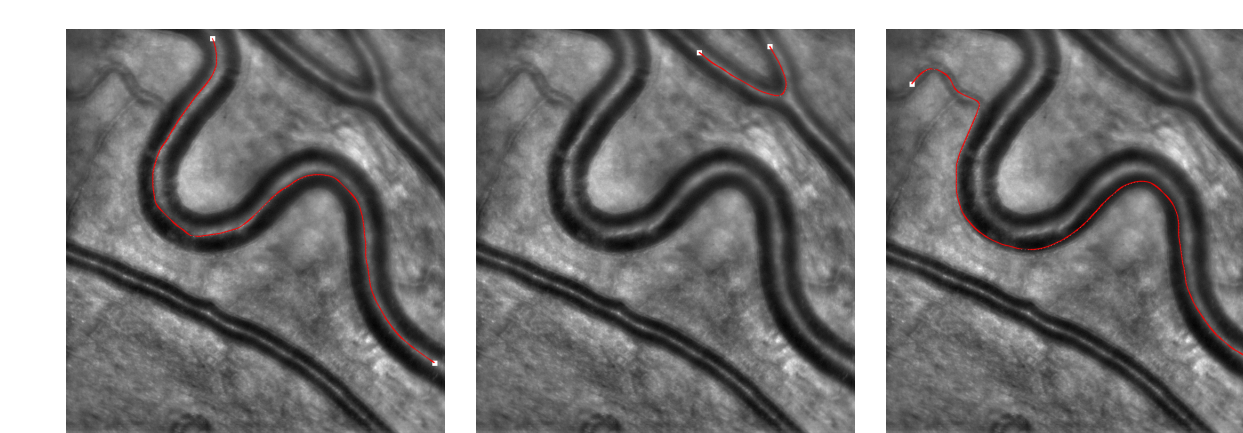


Figure 4: Maximal weight paths between pairs of vertices in a binary co-circularity graph. The weight of a path i_1, i_2, \dots, i_q is the non-positive sum $\sum_{l=1}^{q-1} w_{i_l i_{l+1}}$. The paths with large weights are those with weights close to 0, and which therefore have the strongest contribution in the dilations and erosions defined above.

Contributions We introduced a framework that generalises mathematical morphology for signals on graphs. The operators we defined based on the co-circularity of structure tensors show interesting behaviours for the processing of anisotropic structures.

Future work

- multi-scale approach or an automatic scale determination,
- comparison with other methods, such as path openings and other morphological connected filters,
- exploration of the properties of non-binary co-circularity graphs,
- links between these operators and the max-plus algebra [1].

References

- [1] S. Blusseau, S. Velasco-Forero, J. Angulo, and I. Bloch. Tropical and morphological operators for signal processing on graphs. In *25th IEEE International Conference on Image Processing (ICIP)*, 2018.
- [2] G. Peyré. The numerical tours of signal processing. *Computing in Science & Engineering*, 13(4):94–97, 2011.
- [3] S. Velasco-Forero and J. Angulo. Nonlinear operators on graphs via stacks. In F. Nielsen and F. Barbaresco, editors, *Geometric Science of Information, Proceedings*, pages 654–663. Springer International Publishing, 2015.